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**Combinatorial Exchange Models  
for a User-Driven  
Air Traffic Flow Management in Europe**

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# Sommario

L' Air Traffic Flow Management (ATFM) è il servizio atto a garantire che la capacità del sistema di trasporto aereo venga sempre rispettata ed utilizzata in maniera efficiente. A tale scopo vengono impiegate una serie di misure che spaziano da quelle strategiche a lungo termine fino all'imposizione di ritardi a terra ad un livello tattico. Questi ritardi ATFM sono imposti individualmente, sotto forma di slot, ai singoli voli prima del decollo presso il loro aeroporto di partenza, poichè l'anticipazione a terra di qualsivoglia ritardo previsto nel sistema, implica un costo inferiore ed una maggiore sicurezza.

Tali ritardi vengono assegnati da un'autorità centrale in base ad un principio First-Planned-First-Served, senza prendere in considerazione le preferenze individuali delle compagnie aeree. Tale criterio di assegnazione può implicare un costo aggregato agli utenti maggiore di quello minimo, dal momento che il costo del ritardo è legato da una funzione non-lineare alla durata del ritardo stesso e dipende da molte altre variabili quali il tipo di velivolo, la specifica coppia origine-destinazione, ecc.

Questa tesi affronta il problema della formalizzazione ed analisi di modelli alternativi per l'assegnazione di risorse ATFM, che tengano conto delle preferenze individuali delle compagnie aeree. In particolare vengono analizzati modelli di programmazione matematica che estendono il concetto di slot ATFM correntemente impiegato, a quello di Target Window proposto dal progetto europeo CATS. Tale concetto è in linea con il programma SESAR, recentemente adottato dalla Commissione Europea per sviluppare il sistema di Air Traffic Management di nuova generazione, il quale impone un coinvolgimento diretto degli utenti (le compagnie aeree) ogniqualvolta debbano essere imposte delle limitazioni esterne, che modifichino le richieste originali.

Il primo capitolo fornisce un'introduzione generale al contesto dell'Air Traffic Management e del controllo del traffico aereo. Nel secondo capitolo vengono descritti i principi, i metodi e le performances del sistema di ATFM sia in base all'organizzazione attuale sia in accordo con il concetto SESAR.

In seguito viene descritto matematicamente il problema dell'assegnazione ottima di risorse ATFM e successivamente viene analizzato per rilevare due strutture fondamentali che ne determinano la trattabilità: la prima corrisponde al caso in cui ci sia un'unica risorsa capacitata, mentre la seconda include il caso in cui vi sia un generico numero di risorse capacitate.

Nel capitolo tre vengono dimostrate una serie di proprietà che permettono di studiare l'applicabilità di diversi meccanismi per il calcolo centralizzato della soluzione ottima da parte di un'autorità centrale. Tali meccanismi vengono formulati come particolari tipi di aste, gli scambi, dal momento che richiedono la minimizzazione di un costo a cui sono soggette entità distinte e che permettono contemporaneamente a ciascun partecipante di acquistare e vendere differenti beni indivisibili.

L'ultima parte della tesi inclusa nel capitolo quattro tratta la progettazione di meccanismi di scambio iterativi, la cui applicazione reale presenta diversi vantaggi rispetto all'adozione di modelli centralizzati, dalla distribuzione della complessità computazionale tra i partecipanti, alla preservazione della privacy riguardo alle informazioni degli operatori aerei. In questo caso viene dapprima formulato ed analizzato un modello basato sul rilassamento lagrangiano del problema centrale separabile. Per superare alcuni problemi derivanti da una sua applicazione pratica, viene successivamente formulato uno schema euristico che implementa un meccanismo di mercato. L'algoritmo sviluppato sfrutta alcune proprietà specifiche del problema sottostante per arrivare a soluzioni vicine all'ottimo in tempi accettabili.

I risultati computazionali ottenuti simulandone l'applicazione su dati di traffico reale, mostrano che sono possibili considerevoli riduzioni dei costi rispetto ad una allocazione centrale delle risorse di tipo First-Planned-First-Served.

Il contributo di questa tesi è duplice. Il primo è rappresentato dalla descrizione, modellizzazione ed analisi matematica del problema di scambio

di risorse ATFM, affrontato dalle compagnie aeree nel momento in cui la capacità della rete debba essere razionata tra di loro. Il secondo consiste nell'innovazione metodologica rappresentata dalla formulazione del meccanismo di mercato, che risponde a requisiti pratici e legislativi presenti nel sistema reale e la cui simulazione ha fornito risultati incoraggianti.



# Summary

Air Traffic Flow Management (ATFM) is the service responsible to guarantee that the available capacity of the air transportation system is efficiently used and never exceeded. It guarantees safety of air transportation by adopting a series of measures which range from strategic long-term ones to the imposition of ground delays to flights at a tactical level. These ATFM delays are imposed to individual flights at the departure airport prior to their take-off, since it is safer and less costly to anticipate on the ground any delay predicted somewhere in the system. They are assigned by a central authority according to a First-Planned-First-Served principle, without taking into account individual Airlines' preferences. This criteria of assignment can cause an aggregated cost of delay experienced by users, higher than the minimal one, due to the fact that the cost of delay is a non-linear function of the duration and it depends on many variables such as the type of aircraft, the specific origin-destination pair, ecc. This thesis tackles the issue of formalizing and analyzing alternative models for the assignment of ATFM resources which take into account individual airlines preferences. In particular mathematical programming models are analyzed, that extend the concept of ATFM slot currently adopted to the one of Target Window, as proposed in the CATS European project. Such a concept is in line with the SESAR program, recently adopted in Europe to develop the new generation system of Air Traffic Management, which imposes a direct involvement of Airspace users whenever external constraints need to be enforced that modify their original requests.

The first Chapter provides a general introduction to the context of Air Traffic Management and Air Traffic Control. In the second Chapter the principles, methods and performances of the ATFM system are described

according to the current situation as well as to the SESAR target concept.

The problem of optimally assign ATFM resources is then described mathematically and then analyzed to uncover two fundamental structures that determine its tractability: one corresponds to the case in which there is a unique capacity constrained resource while in the second there is an unrestricted number of constrained resources.

In Chapter three a number of properties are proved that give insight into the applicability of different mechanisms for a central calculation of the optimal solution by the ATFM authority. Since such mechanisms involve cost minimization for several agents they are formulated as exchanges, i.e. particular types of auctions in which each participant may buy and/or sell several indivisible goods.

The last part of the thesis included in Chapter four deals with the design of iterative exchange mechanisms, whose application in real world presents several advantages with respect to centralized models, from the distribution of computational complexity among participants to the preservation of disclosure of private information by Aircraft Operators. In this case an optimal model based on the Lagrangian relaxation of the separable central problem is first formulated and analyzed. To overcome practical issues possibly deriving from its application in real operations, an heuristic iterative Market-based mechanism is finally formalized. This algorithm exploits some of the underlying characteristics specific to the problem to derive near-optimal solutions in an acceptable time. Computational results are obtained by simulating its implementation on real traffic data and they show that considerable cost savings are possible with respect to a First-Planned-First-Served central allocation.

The contribute of this thesis is twofold. The first is to provide a mathematical description, modeling and analysis of the ATFM resource exchange problem faced by Airspace users when network capacity needs to be rationed among them. The second consists in the methodological innovation represented by the formulation of the Market Mechanism which is compliant with several requirements represented by legislative and practical constraints and whose simulation provided encouraging results.

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# List of acronyms

Area Control Centre	ACC
ATS Data Exchange Presentation	ADEXP
ATFCM Daily Plan	ADP
Airspace Flow Programs	AFP
Aeronautical Fixed Telecommunication Network	AFTN
ATFM Information Message	AIM
Aeronautical Information Publication	AIP
Aeronautical Information Regulation And Control	AIRAC
Aeronautical Information Services	AIS
ATFCM Notification Message	ANM
ATFM Notification Message	ANM
Air Navigation Services	ANS
Air Navigation Service Providers	ANSP
Aircraft Operator	AO
Airline Operations Centre	AOC
Air Route Traffic Control Centers	ARTCC
Air Traffic Control	ATC
Air Traffic Controller	ATCO
Air Traffic Control System Command Center	ATCSCC
Air Traffic Control Towers	ATCT
Air Traffic Flow and Capacity Management	ATFCM
Air Traffic Flow Management	ATFM

Air Traffic Management	ATM
Business Trajectory	BT
Computer Assisted Slot Allocation	CASA
Contract-based Air Transportation System	CATS
Continuous Double Auction	CDA
Collaborative Decision Making	CDM
Center Radar Approach Control	CERAP
Central Flow Management Unit	CFMU
Modification message	CHG
Cancellation message	CNL
Communication, Navigation and Surveillance	CNS
Contract of Objectives	CoO
Conditionl Route Availability Message	CRAM
Central Route Charges Office	CRCO
Calculated Time Over	CTO
Calculated Take-Off Time	CTOT
Delay message	DLA
Distance Measuring Equipment	DME
Departure Planning Information	DPI
European AIS Database	EAD
European Commission	EC
European Civil Aviation Conference	ECAC
Estimated Off-Block Time	EOBT
Enhanced Tactical Flow Management System	ETFMS
Enhanced Traffic Management System	ETMS
Estimated Time Over	ETO
Estimated Take Off-Time	ETOT
European Union	EU
Federal Aviation Administration	FAA

Functional Airspace Block	FAB
Flow Constrained Area	FCA
Flight Information Region	FIR
Flight Suspension Message	FLS
Flow Management Position	FMP
First Planned First Served	FPFS
Flight Plan	FPL
Flight Update Message	FUM
Ground Delay Programs	GDP
Ground Holding	GH
Ground Stop	GS
High Density Rules	HDR
Human In the Loop	HIL
International Air Transport Association	IATA
International Civil Aviation Organisation	ICAO
Integrated Initial Flight Plan Processing System	IFPS
Instrument Flight Rules	IFR
Integer Programming	IP
Key Performance Area	KPA
Key Performance Area	KPA
Key Performance Indicator	KPI
Linear Programming	LP
Miles In Trail	MIT
United States National Airspace System	NAS
Network Management Function	NMF
Notice To Airmen	NOTAM
Performance Review Report	PRR
Route Availability Document	RAD
Ration By Schedule	RBS

Reference Business Trajectory	RBT
Ready For Improvement	RFI
Replacement Flight Plan Procedure	RFP
Reduced Vertical Separation Minima	RVSM
Slot Allocation List	SAL
Slot Allocation Message	SAM
Slot Credit Substitutions	SCS
Single European Sky	SES
Single European Sky ATM Research	SESAR
Slot Issue Time	SIT
Societe Internationale de Telecommunications Aeronautiques	SITA
Slot Missed Message	SMM
Set Packing Problem	SPP
Slot Revision Message	SRM
System Wide Information Management	SWIM
Terminal Manoeuvring Area	TMA
Traffic Management Units	TMU
Terminal Radar Approach Control	TRACON
Target Take Off Time	TTOT
Target Window	TW
User Driven Prioritization Process	UDPP
Upper Information Region	UIR
United States	US
Vickrey-Clarke-Groves	VCG
VHF Omnidirectional Range	VOR

# Chapter 1

## Introduction: Air Traffic Management Principles

Commercial aviation has experienced a spectacular development in the XXth century; the outstanding technological progresses have allowed more and more people to access to air travel for both leisure and business, national markets have become more and more connected to each other, thus stimulating competition, global trade and tourism. With the terrific increase in the number of air movements and thus in density of air traffic, the captain was no more able to carry out all the manouvres in a safe manner and started to delegate more and more control to ground based stations. This was the start of Air Traffic Control systems worldwide in the early 1930 with the first radio-equipped control towers, followed by the routine use of radar for approach and departure control after World War II. The International Civil Aviation Organisation (ICAO) was established at the Chicago convention in 1944 to coordinate and regulate international air travel. In this circumstance it was established a first fundamental set of rules of airspace, of safety standards and rights, which were subscribed by most nations. Nowadays ICAO still promulgates rules and procedures for the safety of air traffic, through annexes and documents. A key example is the Doc 4444: Rules of the Air and Air Traffic Services, which constitutes a reference for the correct implementation of Air Traffic Control procedures, whose enforcement is nevertheless

delegated to contracting States. ICAO also establishes the responsibilities for Air Traffic Management (ATM) which encompasses the following areas:

- Airspace Management, which includes all those tasks related with the planning of the Airspace infrastructure, its organization at a strategic level (e.g. the design of the route network) and at a tactical one (e.g. the dynamic use of Airspace, the civil/military coordination)
- Air Traffic Services, which includes Air Traffic Control (ATC) to flights, flight information services and the alerting service.
- Air Traffic Flow Management, whose main role is to regulate traffic in order to ensure that the available capacity of the system resources (airports and airspace) is always respected and used efficiently.

More in general one refers to Air Navigation Services (ANS) as to a wider set of tasks which comprises ATM, as well as the Communication, Navigation and Surveillance (CNS) system, the meteorological services system and other services which are auxiliary to aviation. According to ICAO rules each State is responsible for providing Air Traffic Services as a public service and has complete sovereignty over the Airspace within national boundaries. Those services are delivered to users by Air Navigation Service Providers (ANSP), which in Europe are authorities independent from the civil aviation authority whose role is the supervision and enforcement of standards and regulations for civil aviation. In the United States the Federal Aviation Administration (FAA) undertakes both these functions. In Europe the ANSPs are usually organised at a national level either as government departments, operated by civil servants, or as autonomous bodies belonging to the State or as privatised companies (fully or partly). An example of this last type of status is the UK ANSP (NATS), which is a public-private partnership jointly owned by an Airline group consisting of 7 airlines, the Government, Airports Authority and NATS employees. ANSPs in Europe are mostly financed through the collection of air navigation service charges, which are levied for each flight performed under Instrument Flight Rules (IFR) in the Flight Information Regions (FIRs). The EUROCONTROL Central Route Charges Office



Figure 1.0.1: Map of the European Civil Aviation Conference (ECAC)

(CRCO) collects all those charges from Airspace Users and redistribute the right amount to ANSPs on the base of the distance flown over each Member State crossed and, less than proportionately, of the aircraft weight. This is one of the roles of EUROCONTROL, the European Organisation for the Safety of Air Navigation which includes today 38 Member States. It was established in 1960 with the mission of coordinating, complementing and integrating the different air navigation services in Europe in order to improve the overall performance, safety and sustainability of the whole European system. The EUROCONTROL Central Flow Management Unit (CFMU) for instance, which is based in Brussels, centrally manages the Air Traffic Flow Management service for the 44 states participating in the European Civil Aviation Conference (ECAC), represented in Figure 1.0.1.

The principal stakeholders that interact on a daily basis to operate the Air Traffic Management system can be grouped into three main categories:

- Air Navigation Service Providers, who enable the safe and expeditious flow of air traffic by delivering Air Traffic Services within national boundaries;

- Airport operators, who provide the interface between ground and air operations and constitute access points to the network;
- Aircraft Operators (AOs), they are the airspace users carrying on flights to move passengers and freights around the globe, which is the the main goal of civil aviation. Commercial Airlines operating scheduled flights constitute the principal member in this category, while all other civil aviation operations for remuneration or hire are grouped under the classification of General Aviation.

## 1.1 The Air Traffic Control system

The two factors underlying the need for ATC are safety and efficiency. Airspace users must have in fact enough space to avoid the risk of near misses or collisions but at the same time the individual use of airspace shall be minimal, within the constraint of safety, in order to maintain operational efficiency. All the ATC system components described hereafter are conceived to conciliate these two objectives.

### 1.1.1 Flight categories

Meteorological conditions constitute a strong constraint for air traffic, one of which is represented by the flight rules under which a flight might be operated. Instrument flight rules are regulations and procedures for flying aircraft by referring only to the aircraft instrument panel for navigation. A civil flight which is operated under IFR rules can rely on both on-board instruments and on-ground instructions provided by Air Traffic Controllers (ATCOs) via VHF radio channels on the basis of the radar information they observe. Only IFR-rated pilots can operate under IFR procedures, allowing them to fly while looking only at the instrument panel, even in the case that visibility conditions are poor such as when the aircraft crosses clouds and it follows ATC instructions to maintain separation from other aircraft.

This ensures that the pilot is always aware of the current situation around him even in conditions of scarce visibility. In contrast, a flight operated

under Visual Flight Rules (VFR) is independent from ATC and the pilot is responsible for maintaining separation from other vehicles and to determine the correct route with the help of geographical landmarks. Hence a VFR flight is subject to minimum visibility criteria constraints for safety reasons.

Most scheduled flights operate under IFR, while VFR ones are mostly non-commercial, private recreational aircraft flights and can be executed whenever meteorological conditions meet the minimum requirements, referred as visual meteorological conditions. Whenever these conditions are not met only IFR flights can be operated under instrument meteorological conditions, according to which the pilot controls the aircraft relying on flight instruments while ATC provides separation. This means that the aircraft is kept away from obstacles and other aircraft using the clearance issued by ATCO, which contains instructions on heading or route to follow, altitude and limits of validity, after which a new clearance from ATC is needed. This clearance is based on the radar information that ATCOs observe, or through aircraft position reports in areas where radar coverage is not available, sent as voice radio transmissions. Aircraft position reports are not necessary if ATC communicates that the aircraft is in radar contact.

ICAO specifies that the minimum vertical separation for IFR flight is 1000 feet in the airspace between ground and the altitude of 29000 feet, i.e Flight Level (FL) 290, and 2000 feet above FL 290. However since 1997 most Countries in the world have adopted Reduced Vertical Separation Minima (RVSM) procedures, which fix the safe vertical separation to 1000 feet also in the airspace above FL 290, thus increasing the maximum amount of aircraft that an airspace can safely host, i.e the effective Airspace capacity. Horizontal safe separation is usually fixed at 5 NM en route airspace and 3 NM in terminal airspace.

### 1.1.2 The Geographical Organisation of Airspace

ANSPs provide ATC services within the FIRs of their competence, which mostly coincide with national boundaries, with some minor exceptions due to operational requirements such as multilateral agreements, through which an

ANSP may delegate the responsibility for ATC provision in limited portions of airspace to a neighbor State, in order to facilitate cross-border procedures. The Airspace may be further classified as Upper Information Region (UIR) in the area above a certain flight level. However a standard for the lower limit of a UIR has not been established so far, thus existing UIRs extending from FL 195 (i.e. 19500 feet altitude) to unlimited, others from FL 285 to unlimited, ecc. The Area Control Centre (ACC) is the ground-based facility responsible for controlling IFR aircraft at higher altitudes while en route, within a particular FIR or UIR. During the execution of a flight and according to the number of FIRs crossed by the aircraft, the responsibility may pass from one ACC to a neighboring one through the so called handover procedure. Besides grouping together all the instruments (prevalently radar screens) used by ATCOs to monitor the progress of commercial flights and to guarantee the safe separation among them, usually each ACC also houses a military ATCOs dedicated solely to the provision of ATC services to military traffic while in pursuance of operative missions. A Flow Management Position (FMP) is also present in each European ACC, to establish ATFM measures in collaboration with EUROCONTROL CFMU through a dedicated terminal.

ATCOs working within an ACC communicate via radio with pilots of IFR flights passing through the Center's airspace. A Center's communication frequencies (typically in the very high frequency modulation aviation bands, 118 MHz to 137 MHz, for overland control) are published in aeronautical charts and manuals, and are also announced to a pilot by the previous controller during a handover. Airspace controlled by an ACC is further split into smaller ATC sectors of Airspace which may vary in number depending on many factors (from 2 to 25 across the ECAC area), to allow ATCOs to better manage the traffic. Each sector is assigned to a control team, typically composed of 2 members: the executive controller (who is responsible for separation and sequencing flights and issues instructions/clearances to pilots to provide separation) and the planning controller (who coordinates and approves the flow of traffic in its sector determining as far as possible conflict free trajectories). The Airspace extending in the vicinity of one or more major airports is classified as Terminal Manoeuvring Area (TMA) and

is designed to handle traffic arriving or departing the airport(s) included in it. Normally only IFR flights are allowed in it, since the density of traffic in this area and its complexity (aircraft climbing or descending are more difficult to manage for both ATCOs and pilots) impose a precise instrumental control.

### 1.1.3 The Route Structure

International air traffic is channeled along specified ATC routes and each of them is part of a network of routes which is generally fixed within a FIR. This permits ATCOs to manage traffic more easily since aircraft follow predetermined paths. An ATS route is defined by route specifications which include a route designator, reporting requirements and the track to or from significant waypoints. Those waypoints may correspond to the location of ground-based nav aids, such as VHF Omnidirectional Range (VOR) and Distance Measuring Equipment (DME) stations used by pilots since early 1950s to determine the relative aircraft position, in which case they have a three-letter identifier. Otherwise the waypoint has a five-letters identifier if it is just a geographical coordinate (e.g. ODINA, KOMUR,...). Most of five-letters waypoints define routes in upper airspaces, but they can also be present in lower airspace.

In the en-route airspace the main routes currently consist of airways with the usual width of 5NM and upper air routes with no defined width, which always follow straight lines between consecutive pairs of waypoints. Although the route structure is generally considered fixed, it is in fact subject to continuous refinements in order to optimize the traffic paths and minimize the actual distances flown between origin/destination pairs. EUROCONTROL since 1994 has been entrusted with the responsibility to organise and carry out the necessary co-ordination of planning and implementation activities for improving and upgrading the ATS route network in the ECAC area.

All operational changes in the route structure can be introduced according to the Aeronautical Information Regulation And Control (AIRAC) cycle, which lasts 28 days during which all the information relative to the specific

AIRAC are valid. To guarantee the correct integration of information into different systems, users must receive a paper copy at least 28 days in advance of the effective date. Whenever major changes are planned and where additional notice is desirable and practicable, a publication date of at least 56 days in advance of the effective date should be used, in accordance with ICAO Annex 15 document about Aeronautical Information Services (AIS).

## **1.2 The Airline Management of Flight Operations**

Now that an outline of the air transport infrastructure has been sketched, it is interesting to focus on the methodologies and principles that determine the execution of a flight, from the airline scheduling construction as a response to passengers' demand to the interface between Aircraft operators and traffic regulating authorities required to ensure safety of operations.

### **1.2.1 Airline schedule development**

Whereas the ATC is distributed geographically, the Airspace users, who are prevalently Airlines operating fleets of aircraft, have centralized much of their planning and control activities. The prevalent model for a major Airline operations is in fact today represented by hub-and-spoke networks. According to this model flights connect a unique central hub airport to a number of smaller peripheral airports, thus forming a star network (Economides, 1996). This allows a single Airline to offer its customers a number of possible (indirect) city connections which grows more than linearly with the number of spokes departing from hub. In fact by adding one new round-trip flight from a hub where an Airline already connects to  $N$  cities, will create  $2 \times (N + 1)$  additional origin-destination (O/D) pairs. This organization of the network makes it possible for passengers departing from outlying airports to connect to a variety of destinations, thus generating for the Airline increasing returns to scale.

On the contrary, low cost and new entrant carriers typically prefer to adopt a Point-to-Point network, where direct links are established between any pair of airports for which there is enough demand volume to justify the costs of establishing the link itself. Under this type of network configuration, all the nodes (i.e. the local airports) are at the same hierarchical level thus giving airlines more flexibility in managing demand fluctuations and eliminating the constraint of connections between successive flights that causes the typical traffic peaks at hub airports.

In general hub-and-spoke airlines want in fact to maximize the number of possible connecting markets for passengers, but they also want to minimize the passenger travel time, so they usually manage departures and arrivals from/to the hub into clusters occurring at periodically spaced times, according to which a great number of arrivals from spoke origins during time period  $t$ , is followed by departures to spoke destinations at successive periods  $T > t$ . This can obviously cause congestion at hub airport during the traffic waves and a greater network sensitivity to delay than in the case of non-hub networks (see Mayer and Sinai, 2003).

To efficiently manage these complex interactions in a rapid and informed manner, this model of operations has led most of the biggest Airlines worldwide to install their Airline Operations Center (AOC) at the hub Airport. Whenever a disruption occurs affecting the operational schedule (e.g. a crew member did not report for duty or an aircraft experiences malfunction), specialized teams of people work under the supervision of an operations control manager, to solve the specific problem through a process known as Disruption Management or Operations Recovery (see e.g. Kohl et al., 2007). From the AOC the Airline can monitor the state of all its flights, checking if they are following the schedule that was previously defined by other areas of the company. The execution of a flight is in fact a result of a long and complex process that an Airline may start years before the day of operations.

This process usually starts with the schedule design, which defines markets to serve and the service frequency as well as the departure time for each flight leg. This first step is strategic since it determines the competitive position of an Airline on the market and is determinant for its profitability.

The schedule design is typically performed manually by Airline experts or by optimization models which determine incremental changes to existing flight schedules (see Barnhart et al., 2003). This is due to the extreme complexity of modeling the real problem, which needs to capture critical interactions among internal Airline resources and external resources such as Airport slots (cf. Section 1.2.2).

The second step is the fleet assignment, i.e. the decision of what size of aircraft to assign to each flight, based on the Airline fleet. This problem can be formulated as a multicommodity network flow problem (see Hane et al., 1995), with nodes representing times and locations of leg departures and arrivals and arcs representing either flight legs or idle aircraft on the ground.

After these decisions are made, the next step is to decide the assignment of individual aircraft of the type previously determined, to flight legs, in order to determine aircraft rotations, which are mainly subject to maintenance constraints.

Successively each flight is assigned to a specific crew, thus determining a crew schedule where the objective is to minimize the cost of selected pairings (i.e. the multiple days working schedules that specify number of hours worked per day, rest hours between working periods, the time the crew is away from home,...) and then these pairings are assigned to crew members with monthly rosters. The crew pairing and rostering problems are typically modelled as set partitioning problems, with binary decision variables corresponding to each possible pairing and to each crew assignment respectively. Due to the billions of variables involved researchers have proposed branch and price techniques to generate solutions without explicitly considering all of them (Barnhart et al., 1998).

### **1.2.2 The Strategic Assignment of Airport Slots**

The allocation of airport capacity constitutes an administrative approach to the strategic management of demand, since all the Airlines that intend to schedule a flight movement to and from a coordinated airport need to be assigned an airport slot for this purpose. In the European Union the

system for airport slot allocation is based on the IATA Worldwide Scheduling Guidelines (Council Regulation No 95/93 amended by Regulation 793/2004), as well as in most Countries worldwide. The Airport slot is defined, according to International Air Transport Association (2009), as “the scheduled time of arrival or departure at the terminal. An allocated slot will take account of all the coordination parameters at the airport, e.g. runway(s), taxiways, aircraft parking stands, gates, terminal capacity (e.g. check-in and baggage delivery), environmental constraints e.g. night restrictions, etc.”.

According to the level of congestion, three types of Airport are defined by IATA: fully coordinated (or level 3, requiring the assignment of a slot by the airport coordinator), schedule-facilitated (or level 2 for which there is potential for congestion at certain periods of time, voluntary cooperation with a schedule facilitator is required), non-coordinated (or level 1, where there is low congestion and airport capacity is adequate to meet users' demand). As of 2009, 156 airports worldwide were designated fully-coordinated (International Air Transport Association, 2009), including practically every European major airport, and used the IATA schedule coordination approach. To this purpose the IATA Schedules Conferences take place twice a year, in advance of the summer and winter schedules, to allocate airport slots to Airlines. Each fully coordinated airport must specify a declared capacity (in number of aircraft movements per unit of time) taking into account all the constraints affecting availability of resources. Users interested in scheduling operations at these airports must send a formal request for each desired slot. The available capacity is then allocated following two fundamental criteria: historical precedence (the so-called grandfather rights) and time adjustments of historical slots. This first allocation is conditional to the fact that such slots were effectively used in the previous equivalent season for at least 80% of the times (the use-it-or-lose it rule). After this first assignment to incumbent airlines, a slot pool is created with the remaining slots, 50% of which is allocated free of charge by the slot coordinator to new entrant airlines, i.e. airlines holding less than five slots at that particular airport if the request is accepted. The size of the pool may only permit new entry from very small carriers offering low frequency services, which are unlikely to pose significant

competitive challenges to high frequency services offered by established carriers (DotEcon Ltd., 2001). The remaining slots in the pool are allocated giving priority to year-round commercial air services.

The slots at a coordinated airport, allocated after this first stage, can be exchanged or transferred between airlines on a one for one basis by any number of airlines, conditionally to the approval of the coordinator. Exchanges can involve a compensation if this is not prohibited by the laws of the relevant country, while slot transfers between Airlines can only occur for slots not allocated to new entrants.

Under the EC Slot Regulation, exchange and transfer are allowed under certain conditions:

- slots may be transferred by an air carrier from one route or type of service to another route or type of service operated by the same air carrier;
- slots may be transferred unilaterally within the same commercial family;
- slots may be exchanged one by one, subject to confirmation by the slot coordinator.

The text of the current EC Regulation is silent on the question of exchanges with monetary transfers associated, i.e. the so called secondary trading. However in a formal EC Communication the European Commission stated that “the exchanges of slots for monetary and other consideration, more commonly referred to as secondary trading, are taking place at a number of congested Community airports. This has had certain advantages, notably in allowing the creation of additional services on specific routes. The text of the current Regulation is silent on the question of exchanges with monetary and other consideration to reflect differences in value between slots at different times of day and other factors. Given that there is no clear and explicit prohibition of such exchanges, the Commission does not intend to pursue infringement proceedings against Member States where such

exchanges take place in a transparent manner, respecting all the other administrative requirements for the allocation of slots set out in the applicable legislation.” (COM (2008) 227, 31-4-2008)

Such communication effectively allows for secondary airport slot trading, in particular in response to the events actually observed in real situation. One of the practices that raised questions about its legitimacy is the artificial exchange one, i.e. the practice of exchanging a valuable slot for a so-called “junk slot” at a commercially less attractive time, which is returned to the coordinator after the exchange concludes. Artificial exchanges are likely to be accompanied by monetary compensations (de Wit and Burghouwt, 2008). As an example, in 1998 Air UK ceased to British Airways slots previously used to connect London Heathrow with Guernsey, in exchange for less valuable slots. The Guernsey government decided to summon the Heathrow slot coordinator but the English High Court held that the artificial exchanges, whose meaning was not qualified by the provisions of the Regulation, were exchanges in the ordinary meaning of the language and not unilateral slot transfers. Also side payments in exchange for airport slots have been employed in practice, as it was the case in December 2007 when Alitalia ceased three pairs of airport slots at Heathrow in three separate deals for a total of €92 million (Done, 2007). Another example of side payments is the acquisition by Virgin Atlantic and Qantas Airlines of three pairs of Heathrow slots each from the small British regional airline Flybe for a total amount of £ 40 million (Kilian, 2008).



## Chapter 2

# Air Traffic Flow and Capacity Management

The Air Traffic Flow Management (ATFM) is the branch of ATM dedicated to regulate flights in order to ensure that the available capacity of the system is efficiently used and never exceeded, in order to enable a safe, order and expeditious flow of traffic. In Europe this concept has recently evolved to the wider Air Traffic Flow and Capacity Management (ATFCM), to underline its role in managing the balance between demand and capacity by coordinating all actors involved.

In Europe the centralized ATFM service is provided by EUROCONTROL CFMU, as prescribed by ICAO doc. 4444 that sets down three main phases around which ATFM is organized:

1. Strategic Flow Management takes place from several months up to seven days prior to the day of operation and includes all the long term activities relative to the elaboration of the route allocation plans. These mainly consist in the identification of major congestion problem and the proposal of measures to alleviate them, the planning for extra traffic due to exceptional events (e.g. the 2006 soccer world cup in Germany), the assessment of the impact of actions proposed and the partial elaboration of the contingency routing scheme to be adopted in case of serious disruptions of ATS.

2. Pre-Tactical Flow Management takes place during six days prior to the day of operation and consists of planning and coordination activities aimed at the identification of potential overloads and the preparation of preventive measures. These measures can vary from the activation of mandatory routes, to the negotiation with local FMPs to increase capacity by proper ATC sectors configurations, to slot allocation regulations. The output is the ATFCM Daily Plan (ADP) published via ATFCM Notification Message (ANM) and Network News.
3. Tactical Flow Management takes place on the day of the operation, until the departure of the flight. This phase updates the daily plan according to the actual traffic and capacity. The management of the traffic is made through slot allocation and/or ad-hoc re-routings.

## 2.1 The Airspace Capacity

The main objective ATFM is to guarantee flights' safety throughout the network and this is achieved by monitoring air traffic and balancing with available capacity through appropriate measures such as ATFM slots. The capacity of an airspace depends on the individual capacity of the sectors composing it. In Europe the capacity of a sector is defined as the maximum number of aircraft that can enter the sector during a specified period, while still permitting an acceptable level of controller workload (Majumdar, 2007). The workload experienced by ATCOs is thus the factor determining capacity, which is usually assessed based on task time obtained from observation of ATCOs actions, for example through real time Human-In-the-Loop (HIL) simulations. Task times are successively used to determine effective workload induced by traffic, usually through model-based simulations. Recent studies indicate that the workload experienced by controllers is affected by a complex interaction of a number of factors (Majumdar and Ochieng, 2002, Mogford et al., 1995) related with (i) the situation in the airspace, since several features of air traffic and of sector geography interact to produce air traffic ATC complexity; (ii) the state of the equipment, determined by

the design, reliability accuracy of equipment both on-ground and on-board; (iii) the state of the controller, e.g. the controllers age, experience, decision making strategies.

These parameters can be thought of as the drivers of controller workload, and consequently of airspace capacity, i.e. airspace capacity drivers. To adjust capacity a number of measures can be taken, to modify these drivers. At a strategic level for example, airspace management actions aim at designing routes, individual sectors' geometry and sector configurations that minimize controller workload. Complexity factors are also taken into account when measuring the performances of the ATM system. The EUROCONTROL Performance Review Commission relies its assessment upon the indicator of interactions, i.e. the simultaneous presence of two aircraft in a volume of Airspace of  $197.5 \text{ NM}^3$  (see ATM Cost-effectiveness (ACE) Working Group, 2006).

At a pre-tactical level increases of capacity can be obtained by implementing or extending the opening time of different configurations of sectors, as a trade-off between the number of controllers required by a specific configuration and the mismatch between planned demand and available capacity.

## 2.2 Information Exchanges between Airlines and ATFM

The final process that an Airline must execute for each scheduled flight is to file a Flight Plan (FPL).

According to the current system all civil IFR flights that are intended to operate within the Integrated Initial Flight Plan Processing System (IFPS) zone, which is the area covered by the ATS facilities of the ECAC States, must have a valid FPL filed. The FPL has to be sent to the IFPS between 3 and 120 hours before the Estimated Off-Block Time (EOBT) by the Aircraft Operator. Repetitive FPLs for the flights scheduled with regular frequencies can be filed to the RPL system at CFMU only once at the beginning of the season and are then they are regularly processed by the IFPS system 20

hours before EOBT.

This system receives all FPL data and sent them to two CFMU units located in Haren (Brussels) and Bretigny-sur-Orge (Paris) in order to process them by sharing the workload between 2 facilities and to make the system redundant. Two networks are used for this purpose, the AFTN (Aeronautical Fixed Telecommunication Network) and the SITA (Societe Internationale de Telecommunications Aeronautiques), which form together a high speed Wide Area Network. The former has direct access points located at almost all significant ground points in the world, such as airports, ACCs, ecc. while the latter is accessed directly by airlines, computer reservation systems, airports and governments around the world. The FPLs are first automatically checked for validity with respect to the correct format (which can be either ICAO or ADEXP) and with respect to all the aeronautical information contained in the European AIS Database (EAD). This unique source of information merge into one single database the Aeronautical Information Publication (AIP) detailing all regulations, procedures and other information issued by the national civil aviation administrations, the Notices to Airmen (NOTAMs) concerning all the temporary changes in conditions to aeronautical facilities, services or procedures, as well as other documents relevant to the safe operation of aircraft. Another CFMU publication that must be considered by AOs when compiling flight plans is the Route Availability Document (RAD), which is updated at each AIRAC and integrates both structural and ATFCM requirements as a listing of all restrictions on routes such as city-pair level cappings or route limitations state by state.

Once the FPL has been acknowledged as valid by the IFPS, an ACK message is sent back to the sender and no further action is required by the AO until the day of operations. The invalid FPLs can either be corrected manually by IFPS staff, in which case a MAN message is returned to the sender that has no action to perform, or rejected by a REJ message, in which case a new valid FPL has to be resubmitted to the system.

All the FPLs accepted are assigned a unique identifier and then sent to all the ATS Units concerned with the management of the flight and to the Enhanced Tactical Flow Management System (ETFMS).

Also the 4D profile (i.e. location + time) for each flight is estimated, based on the aircraft performances and the information contained in the FPL such as route, level, speed, and time estimates (EUROCONTROL Central Flow Management Unit, 2009b).

Repetitive Flight Plans can be sent to the RPL Unit by e-mail or through SITA network once for all seasonal scheduled flights, and they represent almost 50% of all flight plans submitted to IFPS.

There are a variety of software tools available to flight planners to help in compiling FPLs, which integrate all the up-to-date aeronautical information available (i.e NOTAM, AIS, ecc.) but also optimization engines to calculate optimal flight plans according to weather conditions and users' preferences such as the cost index, a parameter expressing the cost of time relative to the cost of fuel for each flight according to the company business objectives. This flight planning tools can be used at pre-tactical or tactical level for generating FPL (rather than RPL) that are optimal according to latest meteo conditions, to the Conditionl Route Availability Message (CRAM), establishing on the day before operations which conditional routes are open and which are closed for military exercises, the ATFM Notification Message (ANM), listing planned ATFM measures the day before operations and the ATFM Information Message (AIM), listing information about ATFM measures on the day of operations.

## 2.3 The Tactical Assignment of ATFM Slots

The ATFM slot allocation measure is based on the universally accepted principle that delays on the ground are safer and less costly than those in the air. Any forecast delay somewhere in the system is thus anticipated at the departure airport prior to the take-off and the traffic is controlled in a safe and simple manner. In the rest of this section the process of ATFM slot allocation is described, in accordance with the mechanisms employed under both the European and the United States system.

### 2.3.1 The European Enhanced Tactical Flow Management System

During the 48 hours prior to the execution of a flight, the Enhanced Tactical Flow Management System is feeded with flight plan data coming from IFPS. This information is then compared by the ETFMS with the flow restrictions, known as ATFM regulations (or simply regulations), declared to users through ANM and AIM messages. Each regulation specifies the area affected, the maximal rate of flights that the area can accept and the period of activation. The maximal rate establishes the limit on the number of flights that can enter a certain element of the airspace per period of time. ATFM regulations are established during the pre-tactical phase based on the traffic forecasts, available from FPLs and other past data, and on the assessment of the impact of regulations on traffic flows, which strongly relies on the experience of operators both at CFMU and at the national FMP positions. Whether a flight is permitted to depart at its planned time depends on the effect of flow restrictions placed on the airports and airspaces through which the flight is planned.

The Computer Assisted Slot Allocation (CASA) system, a module within the ETFMS, constitutes the main tool to implement the slot allocation procedure. The CASA system is largely automatic and centralised and works in a passive mode from a user perspective. In fact the sole act of filing a flight plan effectively constitutes a request for a slot, since CASA uses FPLs to calculate for each flight the Estimated Take Off Time (ETOT), by summing the taxi time to the Estimated Off Block Time (EOBT) published in the FPL. Successively an Estimated Time Over (ETO) is calculated for each point of entry into each sector crossed. This allows the system to assign to each flight a first provisional slot based on the ETO on the restricted location which constitutes a capacity constrained resource. For each constrained resource, the number of slots available is defined by its maximal hourly rate of acceptance multiplied by the number of hours of activation. This set of slots constitutes the Slot Allocation List (SAL) for the given capacity-constrained resource.

Each flight is given a provisional slot based on its ETO on the restricted location. Each slot has capacity of 1 flight, hence if the slot corresponding to the original ETO of the flight has not been already assigned to another flight, it is available and the flight will not be delayed. Otherwise the slot is assigned to the flight with the lowest ETO, according to a First-Planned-First-Served (FPFS) principle, and the flight with the greatest ETO receives a later slot, corresponding to a new Calculated Time Over (CTO) which will be greater than the original ETO. In the case a flight is subject to several ATFM regulations, the highest delay, caused by the slot in the most penalizing regulation crossed, is forced also in the other ones. At a certain time before the flight EOBT, known as Slot Issue Time (SIT) occurring 2 hours before EOBT at the earliest, the operator of the regulated flight and the ATC concerned, receive a Slot Allocation Message (SAM), which defines the Calculated Take-Off Time (CTOT) for the flight. This implies that aircraft must take off during the time range between  $CTOT - 5$  minutes and  $CTOT + 10$  minutes. This is the only effective ATFM slot that the flight must respect, being calculated backward from the CTO on the most penalizing regulation. The AO and ATC are jointly responsible for CTOT compliance at the departure aerodrome. This implies that AOs need to plan the departure of a flight, taking into account the taxi-time, in order to ensure that the aircraft will be ready for start up in sufficient time to comply with its CTOT. On the other hand ATC at the airport must include CTOT in the ATC clearance procedure taking into account all applicable ATFM slots when clearance is issued. If an AO cannot respect its assigned slot, it must be communicate the revised EOBT through a Delay message (DLA) or a Modification message (CHG) and will receive a new CTOT. In the case the AO cannot estimate a new EOBT, it must send a Slot Missed Message (SMM) and the ETFMS returns a Flight Suspension Message (FLS) (EUROCONTROL Central Flow Management Unit, 2009b).

A flight that has been allocated a CTOT may still reduce the delay in the case some other regulated flight is canceled and its slot becomes available. In this case a flight which is in the default Ready For Improvement (RFI) status, will receive a Slot Revision Message (SRM) in case of improvement.

Two flights operated by the same AOs may swap their slots if they have both a CTOT issued and they are both subject to the most penalizing regulation. The AO must submit the swap request (maximum one swap per flight) to CFMU either directly to the Central Flow Helpdesk or via an FMP. CFMU confirms the feasibility of the slot swap only in the case it has no negative network effect on the system (EUROCONTROL Central Flow Management Unit, 2009b).

Another measure that AOs can adopt for trying to reduce ATFM delays is to re-route a flight through longer but less congested area(s), in order to avoid the regulated one(s). This can be done by sending either a CHG message or a Cancellation message (CNL) and then re-filing the flight plan using the Replacement Flight Plan Procedure (RFP), according to the information about current ATFM measures contained in the AIM.

### **2.3.2 U.S. ATFM Environment and Systems**

While both European and United States air navigation systems are operated with similar technology and operational concepts they present some fundamental differences. The US sky is managed as a unique National Airspace System (NAS) through standardized automations and procedures.

There is a unique service provider, the Federal Aviation Administration (FAA) which coordinates from the Air Traffic Control System Command Center (ATCSCC) in Virginia all the Traffic Management Units (TMUs) located at the 21 Air Route Traffic Control Centers (ARTCCs) around the Country. Each TMU is responsible for the management of traffic problems that are within the scope of the ARTCC. The final level of the hierarchy consists of TMUs at the Terminal Radar Approach Control (TRACON) facilities, the Center Radar Approach Control (CERAP) facilities, and Air Traffic Control Towers (ATCT). The TRACON and CERAP TMUs manage problems specific to the terminal areas under their control.

The Enhanced Traffic Management System (ETMS) is used by the FAA to perform all ATFM functions, according to various sources of information such as the Official Airline Guide for Airline schedules, the airline flight data

messages during daily operations, messages from ARTCCs and TRACONs facilities, weather and geographical information as well as information from other international ATC systems such as European departure messages.

ETMS processes these data in order to maintain a set of databases capable of representing current and projected traffic demand data for a 24 hours look ahead period. Flight data messages sent by airlines are used to correct predictions that are continuously refined until real-time data from airborne flights are processed to update the current and projected NAS picture.

Besides re-routing actions, the main other tools used as ATFM actions are:

- Ground Delay Programs (GDP): when the demand for arrivals into an airport is predicted to exceed significantly the available capacity, aircraft are delayed on the ground previous to their departure according to Ground Holding (GH) procedures.
- Airspace Flow Programs (AFP): Similar to GDPs, when excessive congestion is predicted for an area of airspace, defined as a Flow Constrained Area (FCA), GH is adopted as an ATFM measure.
- Ground Stops (GS): an extreme form of GH, according to which all departures of aircraft bound for a particular destination airport are temporarily postponed.
- Miles-In-Trail (MIT) restriction: when a regional ATS provider impose such a restriction, the adjacent upstream regional ATS provider has responsibility for maintaining a traffic flow at or below the restricted level. This can be achieved by airborne holdings, reroutings or by issuing similar flow restrictions on flights further upstream. In this way, it is possible for a flow restriction to propagate through much of the airspace system, possibly eventually leading to ground holds at airports of origin.

Thus any problems arising on the day of operations due to en route capacity constraints or congestions at different airports, are typically addressed

separately through different GDPs and AFPs. In particular if a flight is already subject to a GDP, it is exempted from any other possible AFP derived by a FCA it may cross (see Volpe National Transportation Systems, 2007). Only under particular conditions, such as in situations of adverse weather affecting wide regions of airspace, these ATFM measures become dependent among each other.

Another important difference is the strategic management of airport resources: In Europe in fact traffic at major airports (i.e. coordinated airports) is controlled in terms of volume and concentration through the assignment of airport slots in the strategic phase, several months in advance to the day of operations (cf. Section 1.2.2). In the U.S. instead only four airports are subject to High Density Rules (HDR) limiting the maximum number of IFR takeoffs/landings per hour during certain hours of the day (New-York JFK, New-York LaGuardia, Chicago OHare and Washington Ronald Reagan). Differently to an EU airport slot, a slot allocated at an U.S. Airport subject to HDR, only refers to runway use and separate negotiations are necessary to acquire other airport resources (e.g. gates, check-in desks, baggage handling systems, etc.). At all other U.S. airports airline scheduling is unrestricted and demand levels are controlled by airlines and adapted depending on the expected cost of delays and the expected value of operating additional flights. This results in a higher variability of departure delays whenever there is a mismatch between scheduled demand and available airport capacity.

### **2.3.2.1 Collaborative Decision Making in the U.S.**

These specific features of the U.S. ATFM system have enabled the adoption of a vast Collaborative Decision Making (CDM) program. CDM is a joint government-industry initiative aimed at improving the ATFM by increasing the exchange of information and improving decision-support tools.

The CDM effort (Wambsganss, 2001) was initiated in the '90s in response for dissatisfaction expressed by both Airspace users and FAA on the way GDPs were planned and controlled at that time. In particular the FAA had realized the need for informing users with the most up-to-dated information

whenever ground delays or ground stops had to be imposed on the flights they operated. There was the general feeling among AOs that the allocation procedures used by the ATCSCC were always fair and efficient. In particular each airline wished to gain more control over the specific allocation of ATFM delays to its own flights. On the other hand the ATCSCC had realized the importance of being informed with the decision of users regarding flight cancellation and delays due to any reason non imputable to GDP, having them an impact on the efficiency of resources' allocation during GDPs and hence on global delays.

The CDM philosophy can be interpreted as the application application of the principles of information sharing and distributed decision making to ATFM.

The first procedure used under U.S. CDM to allocate resources during GDPs is the Ration-by-schedule (RBS) algorithm, that produces an initial assignment of slots to flights. Each airline can then decide to modify this slot-to-flight assignment for its own flights, through cancellation and substitution procedures. In practice these processes transform the slot-to-flight first assignment obtained by RBS allocation into a slot-to-airline assignment, since AO are free to swap slots between pairs of flights or to cancel a less profitable flight and assign its slot to another flight, thus reducing its delay. After this intra-airline exchange, the compression algorithm, which is carried out by the FAA, is run to maximize slot utilization by performing inter-airline slot exchanges, in order to ensure that no slot goes unused.

The final step, Compression, maximizes slot utilization by performing an inter-airline slot exchange. The reason for this is that cancellations and substitutions may create "holes" in the current schedule; that is, there will be arrival slots that have no flights assigned to them. The purpose of the compression step is to move flights up in the schedule to fill these slots, according to the priority rule that first candidates to fill a hole are those flights operated by the same Airline; in the case that no flight of the same Airline is available to receive the slot (e.g. their scheduled time is later than the slot), it is assigned to the earliest scheduled flight operated by any other Airlines and so on. Then the Airline that released the slot is awarded with

the earliest slot that becomes vacant through the process and can be used by one of its flights.

Hence the compression procedure in this case can be interpreted as an exchange among airlines of the slots distributed through the initial RBS allocation. Despite of the competition that may exist among different airlines, each of them has an incentive to release slots even if a competitor airline will profit directly, since it will also enjoy benefits through a chain of delay reductions with respect to the first RBS allocation. Two groups of flights are exempted from the basic RBS allocation: flights that are currently airborne, since they can not be assigned a ground delay, and flights selected according to the distance of their departure airports from the GDP (arrival) airport (Ball and Lulli, 2004). The motivation for the latter exemption is the great level of uncertainty associated with capacity constraints 4 or 5 hours in advance of the arrival, which can make unnecessary the ground delay suffered by the flight.

The Slot Credit Substitutions (SCS) constitutes a more dynamic form of slot exchange, which has been recently introduced by the CDM working group (Vossen and Ball, 2006b). Through SCS an Airline can submit a cancellation request for a flight, associated with slot release, conditional to the assignment of a later slot to be used by another flight in the fleet, which would thus reduce its delay. The FAA monitors such requests on a continuous basis, and, if possible, immediately implements the associated exchange(s) of slots. SCS can be viewed as a real-time version of the Compression procedure, which is a batch process run periodically and provides increased trading opportunities over compression.

## **2.4 Performances of the Current ATFM System**

Air Traffic Flow Management, coupled with Capacity management into the wider scope of ATFCM, is the primary means of ensuring flight punctuality and efficiency, whilst managing at best the available capacity on the

air and on the ground. The ATFM system is mainly based, under the current mode of operation, on regulating mechanisms that cause traffic flow regulation under certain conditions. The most largely used control action at a tactical phase is the imposition to aircraft of a calculated take-off time, which is the final outcome of a longer and complex process which has been outlined in the previous section. This section describes and reports a high level analysis about the main indicators of the performances of the ATFM system.

### 2.4.1 Delays in Air Traffic Management

There are different perceptions of aircraft delays by different types of system actors. Airlines and passengers have interest in reducing to the minimum arrival delays and are less concerned about departure delays (although they are correlated). Airport operators and handlers are interested in maintaining on-time operations both on aircraft arriving and departing. Some margins of tolerance with respect to timings are always present in air transportation in order to be able to cope with small disturbances that may happen during operations. There are tolerances associated with almost each type of timing during aircraft operations; for example the ATFM slot is defined as the time interval  $[CTOT - 5minutes; CTOT + 10minutes]$ , that mostly reflects the grade of predictability associated with the taxi time duration. Flight operations occurring with a deviation of less than 15 minutes with respect to their scheduled time are usually referred as on-time. These tolerances introduce a certain flexibility that makes the entire system more robust to small variations in the duration of operations.

#### 2.4.1.1 The Notion of Delay

According to Institut du Transport Aérien (2000), delays affect flight operations at several phases during its preparation, from the strategical one to the pre-tactical to the execution. At the strategical phase in fact, when an airline establishes a flight program, it will lengthen the scheduled duration of flight times or turnaround times at a given airport, according to previous

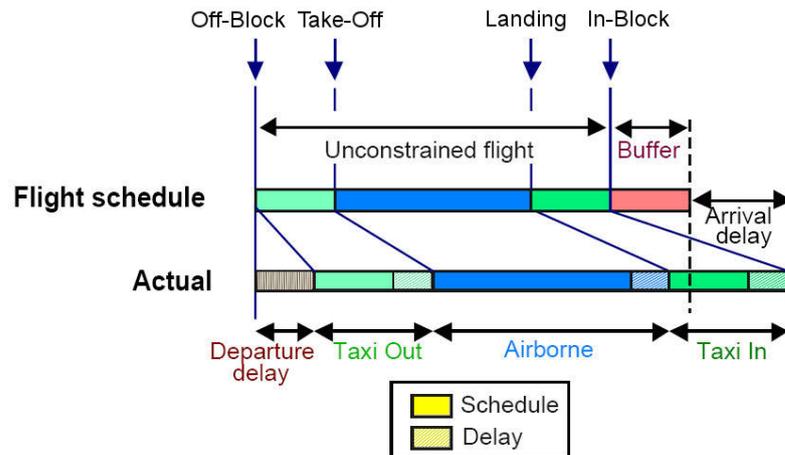


Figure 2.4.1: Delay composition in flight operations (Institut du Transport Aérien, 2000)

experience of delay encountered under the same circumstances. This allows to absorb more easily delays that are likely to occur.

During the day of operations then each flight can be decomposed into four main parts: turnaround process on the ground, out-bound taxiing at the departing airport, en-route phase and in-bound taxiing at the arrival airport. The arrival delay is just the linear composition of the delays occurring at these different phases minus the buffer (cf. Fig. 2.4.1).

Delays imputable directly to a specific operation related to the execution of a flight are called primary delays. These delays can successively propagate throughout the day on subsequent flight legs, thus becoming reactionary delays. These knock-on delays might be experienced either by the same aircraft causing the primary delay (i.e. rotational reactionary delay) or by others (i.e. non-rotational reactionary delay) (Cook, 2007). Airlines typically insert larger buffers into the schedules of earlier flights in the morning, since these have a greater operational impact due to the largest number of reactionary delays they may cause during all subsequent operations.

However the number of buffer minutes padded into a schedule is the result of a compromise and all airlines have their models to decide the correct amount, if any at all. In principle the right duration of the strategic buffer

can be considered as the one that equals the expected cost of the tactical delay it is intended to absorb, plus an extra margin for uncertainty.

#### 2.4.1.2 The Causes of Delay

Flight delays are caused by a variety of reasons related either directly or not with the execution of the flight affected. Among the primary direct causes of delay there are the turnaround procedures, which is the set of operations that take place between the time aircraft is in-block at the gate and its push-back for executing the next leg. These include passenger and baggage boarding and un-boarding, aircraft cleaning, refueling, ecc. All these operations are subject to stochasticity in duration and can cause a delay (Fricke and Schultz, 2009). Once the turnaround process has terminated the aircraft is usually ready for take-off, hence it can push-back from the gate conditionally to ATC clearance and possibly to ATFM slot. From the time it leaves the gate it is subject to taxi time variations (first out-bound from departing airport and secondly in-bound the destination one) which mainly depend on the location of the stand with respect to the runway and on airport congestion; after the take off it is subject to in-flight time variation (mainly due to winds encountered en-route and tactical manoeuvres imposed by ATC), until it finally lands at the destination airport. In order to standardize the reporting of delay occurrences IATA Delay Codes were created to provide airlines with a unique codification to specify the responsible for the delay, the phase and the actors penalized. They can be roughly divided into causes imputable to Airline processes, Airport limitations, En-route restrictions or weather conditions. Very often airlines and ground handling companies have bonus-malus based contracts which establish monetary compensations due to delays in providing services, calculated based on the delay reports compiled by the captain using such codes.

Figure 2.4.2 reports the contribution to primary departure delays of different causes during year 2008, showing that delays caused by ATFM measures constitute nearly 30% of all primary sources of delay in Europe.

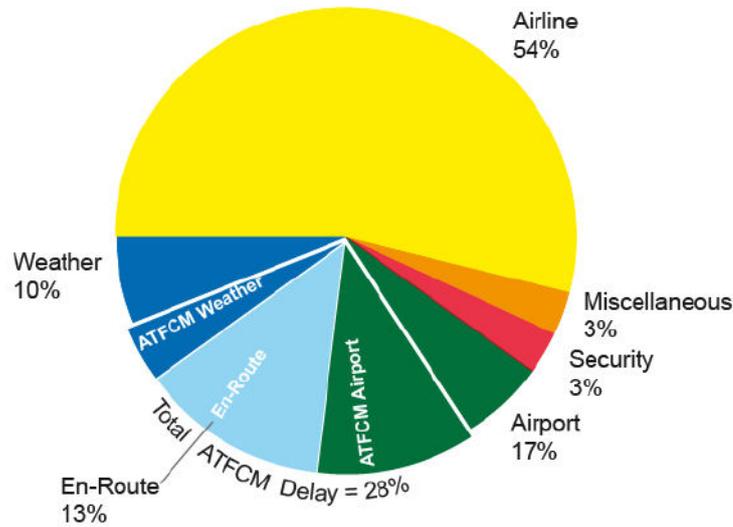


Figure 2.4.2: Primary departure delay causes during year 2008 (source: EUROCONTROL Central Office for Delay Analysis, 2008)

### 2.4.1.3 The Cost of Delays

It is interesting to consider the impact of flight delays in terms of costs born by the Airspace users. There are two main types of costs induced by delays: operational costs and strategic costs.

Operational costs are those caused either directly by a primary delay, or indirectly due to a reactionary delay and are composed by the costs experienced by passengers, who are the final users bearing the opportunity cost represented by operational delays, since the time lost is not employed to work or to derive utility from leisure. Also the Aircraft Operators have a cost directly linked with operational delays, since they have to employ human and other resources to recover for delays and to maintain the schedule, and also to compensate passengers according to regulations such as European Regulation 261/2004 or to corporate policies. In addition to this “hard costs”, Airlines typically experience in the long term other “soft costs”, such as loss of market share due to passenger dissatisfaction (Cook et al., 2009).

The second type of delay costs are the indirect ones, caused by the antic-

ipation of delays into schedules at the strategic phase through buffers. These strategic delays increase the schedule robustness and network stability on one hand, but on the other hand may cause an aircraft sub-optimal utilization and unnecessary consumption of ground resources such as gates (Cook, 2007). Although these costs are usually not taken into consideration explicitly by airlines' accounts, so they are in a certain sense "hidden", they are nonetheless real costs representing the opportunity of being able to use such resources in another way, or to save money by not having them.

It is generally agreed that that predictability, or rather lack of it, is an underpinning cause of the financial losses suffered as a consequence of delay. If all delays could be predicted with confidence to be exactly 10 minutes, then schedules could be re-adjusted accordingly, and there would subsequently be no tactical delay costs as such, apart from the opportunity costs of not using the 10 minutes.

A first preliminary study commissioned by EUROCONTROL to Institut du Transport Aérien (2000) estimates that the cost to airlines due to operational deviation from scheduled times is between €1.6 Billions and €2.3 Billions for year 1999 and that the cost to passengers is of the same order of magnitude. A successive study performed by the Transport Studies Group of the University of Westminster and published by EUROCONTROL (Cook et al., 2004) constitutes the most comprehensive report on the cost of delays in the air traffic management system up to date. It gives delay cost figures quoted by type of aircraft, phases of flight, type of delay and delay duration according to the data provided by several Airlines, handling agents, aircraft operating lessors, airports, EUROCONTROL and IATA. Costs of delay are calculated at the tactical and strategic levels for 12 different types of aircraft and according to 3 different cost scenarios.

The cost figures provided by authors show that airborne delays are typically more expensive than delays consumed at-gate, thus validating one of the fundamental principles of ATFM delays. They also point out that the tactical cost of delay is a non-linear function of the length of delay, since costs per minute are considerably higher for longer duration delays. They consider therefore two specific delay durations to be representative of short

(i.e. 15 minutes) and long (i.e. 65 minutes) delays. Authors specify that these values are not strict but rather have to be considered indicative for delays of the same order of magnitude. In particular the 65 minute value represents a typical amount of delay that causes a series of costs (e.g. a missed connection) which would not be incurred for shorter delay durations, such as 15 minutes. Additionally for each delay duration they provide a low, base and high cost scenario to better represent different situations and for the purposes of comparison, according to the aircraft load factor, the number of connecting passengers, the range of the flight leg, the fuel price range. For example for a delay around 15 minutes it is relatively unlikely that crew runs out of hours and that passengers miss connections, thus these costs are only assigned to the high cost scenario for 15 minute delays. This is a far more realistic approach than treating all 15 minute delays as alike.

The network impact of primary delays is also quantified through base multipliers (1.20 and 1.80) for the two delay durations considered. Each minute of primary delay hence causes, on average, 30 to 40 seconds of reactionary delay. Successively these time figures are transformed into cost figures. The resulting cost curve between 15 and 65 minutes delay is likely to be complex and irregular, so results are mainly provided in tables of values for each type of delay, categorized according to two delay duration classes and further split into the 3 scenarios for each of the 12 aircraft types.

### **2.4.2 Analysis of the Performances**

The EUROCONTROL Performance Review Commission publishes each year the Performance Review Report (PRR), which presents a series of statistics and indicators related to the previous year regarding the main Key Performance Areas (KPA) of Safety, Punctuality and Predictability, Capacity/delays, Flight-Efficiency, Cost-Effectiveness and Environmental impact. This constitutes since 1998 one of the most complete and reliable sources of public-domain information regarding the performances of the ATS system in Europe. According to the PRR 2008 (EUROCONTROL Performance Review Commission, 2009), traffic in the European Statistics Reference Area

reached 10.1 million general traffic movements of IFR flights in 2008 (60% were traditional scheduled while 20% were low-cost flights). Despite the slow-down of traffic increase which reduced from the average 5% annual value observed in the period 2003-2007 to a 0.4% for the period 2007-2008, mainly due to the economic downturn experienced in this last period, the record of 34105 daily movements was registered on June, 27<sup>th</sup> 2008.

According to CFMU figures there were approximately 28000 flights per day on average during year 2008 and on average 5600 flights were regulated daily through ATFM slots, thus producing an average daily delay of approximately 65000 minutes, with peaks during the summer season of up to 111000 minutes (EUROCONTROL Central Flow Management Unit, 2009a)

EUROCONTROL Performance Review Commission (2009) states that although the majority of European ACCs met or even exceeded their capacity plans, the failure to deliver capacity as planned or inadequate plans in a few ACCs negatively impacted the whole European network performance. 8% of flights were subject to en-route ATFM delays in 2008 thus producing an average delay figure of 1.9 minutes per flight (for the summer period), caused prevalently by inadequate capacity and shortage of ATC staff (76% of the causes for en-route ATFM delays). This marked an increase with respect to 2007, when the average delay per flight was 1.6 minutes, despite the modest traffic increase. This was mainly due to global reduction in the effective capacity experienced during 2008, which decreased for the first time after 10 years. According to EUROCONTROL Performance Review Commission (2009) this was caused prevalently by the lack of adequate ATC staffing, preventing ACCs to open their maximum sector configuration at peak hours. Consequently the gap between effective capacity and air traffic demand continued to widen (cf. Fig.2.4.3). Similarly airport ATFM measures were an important cause of delay (11% of the primary causes of delay in 2008, 32% of ATFM delays according to EUROCONTROL Central Flow Management Unit, 2009a) and they were mostly caused (65%) by the 20 busiest European airports. EUROCONTROL Performance Review Commission (2009) estimates that en-route and airport ATFM delays combined caused an aggregated cost to Airspace users of €1.5 Billions in 2008.

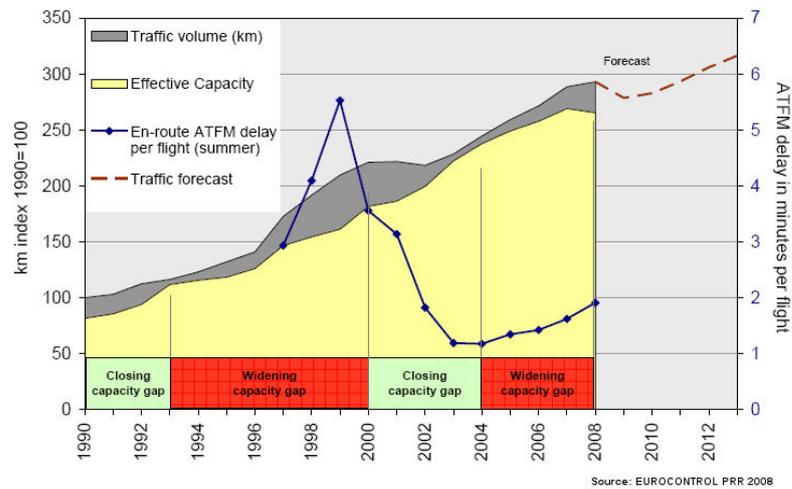


Figure 2.4.3: Demand-capacity gap for en-route Airspace resources (source: EUROCONTROL Performance Review Commission, 2009)

Besides average figures it is important to consider also standard deviation relative to delay figures, since this gives a valuable indicator regarding the predictability of ATS system, with a direct impact on the strategic costs of delay (cf. Section 2.4.1.3). The standard deviation for the time variability regarding general operations is 20 minutes according to EUROCONTROL Performance Review Commission (2009); most of this variance was concentrated at the airport and it is mainly caused by delayed operations but also by operations occurring earlier than their scheduled time. This is especially verified for arrivals, since 20% of the occurrences were registered more than 7 minutes in advance. While from an airline perspective an early flight just results in an opportunity cost due to under utilization of resources, from an airport and ATC point of view, flights ahead of schedule can represent a management issue as much as delayed flights due to gate availability, variability of traffic flows, etc.

Punctuality average levels for arrivals are similar in the U.S. and Europe, albeit with higher delay variability in the U.S.. This is mostly caused by stronger weather disruptions but also by congestion at airports, whose scheduled number of movements is closer to visual meteorological conditions, causing higher delays when low visibility is experienced. The percentage of

flights arriving on time coincides under the two systems, while remarkable differences are registered for arrival punctuality figures. In Europe in fact arrival and departure punctualities almost coincide, with very low variability registered between scheduled and observed en-route duration. In the U.S. on the contrary departure punctuality is significantly higher than punctuality at arrival and this difference is attributable to en-route ATFM measures such as MIT procedures (cf. Section 2.3.2).

EUROCONTROL Performance Review Commission (2009) concludes that improvements in on-time performance and a reduction in variability require a simultaneous tightening of airline, airport and ATM processes, and imply a better coupling between them in the aviation network. Airspace is becoming more and more congested, and traffic forecast will grow steadily over the next 15 years.

## **2.5 Reforming the Current ATM System**

In the previous section it has been highlighted the strong impact on punctuality and predictability of the current organization of Air Traffic Management system. Delays are mainly due to structural limitations which cause a lack of capacity hardly removable, for example runways at airports are major constraints on airport capacity and their expansion could result complicate if not impossible. However these structural limitations seem to be coupled with inefficiencies in the use of the available capacity, under the current ATM system architecture.

### **2.5.1 The Single European Sky Initiative**

The Single European Sky (SES) represents an ambitious program undertaken by the European Commission (EC) to solve these deficiencies with a long-term strategy. It represents a legislative approach aimed to meet future capacity and safety needs at a global European level rather than at local one, with the objectives of restructuring the European airspace as a function of air traffic flows, creating additional capacity and increasing the overall

efficiency.

### **2.5.1.1 The SES I Package**

The SES initiative was launched in 2000 by the EC following the severe flight delays that were experienced in Europe in 1999. A High Level Group was established, composed by representatives of the majority of stakeholders, and the EC drafted a legislative package at the end of 2001, building on the recommendations of its report. This first package, known as SES I, entered into force in April 2004 and comprises four basic regulations, which provide the framework for the creation of additional capacity and for improved efficiency and interoperability of the ATM system in Europe:

- The Framework regulation (EC No 549/2004) - laying down the framework for the creation of the Single European Sky;
- The Service provision regulation (EC No 550/2004) - on the provision of air navigation services in the Single European Sky;
- The Airspace regulation (EC No 551/2004) - on the organisation and use of airspace in the Single European Sky;
- The Interoperability regulation (EC No 552/2004) - on the interoperability of the European Air Traffic Management network.

The four basic regulations are complemented by more detailed implementing rules adopted by the EC after discussion within the Single Sky Committee. Industry is invited to advise the EC on actions to be taken on the basis of the regulations through an Industry Consultation Body.

Member States are responsible for the correct implementation of the EC rules. National Supervisory Authorities (which in practice coincide with the national civil aviation authorities) have to make sure that services are delivered to the highest standards in accordance with the legal requirements.

EUROCONTROL has the role of preparing various implementing rules on the basis of mandates issued by the European Commission and in close co-ordination with all relevant stakeholders. The final result of the mandate

work is a report including a draft implementing rule. The European Commission submits then the draft implementing rule under its responsibility to the Single Sky Committee and adopts it following the favourable opinion of the Committee.

### **2.5.1.2 The SES II Package and Functional Airspace Blocks**

In October 2009, in response to strong demand from industry, Member States and other stakeholders to simplify and increase the effectiveness of the regulatory framework for aviation in Europe, the SES II package (Regulations EC No 1070/2009, EC No 1108/2009) was adopted by the EC towards more sustainable and better performing aviation. This amends the SES I package, with the aim of improving the performance of air navigation services and network functions in the Single European Sky, on the basis of a report submitted in July 2007 by the High Level Group containing a set of recommendations on how to improve the performance and governance of the European aviation system. One of the main enhancements introduced with SES II is the establishment of performance targets in the fields of safety, capacity, flight and cost efficiency and the environment subject to a periodic process of review, monitoring and benchmarking based on key performance indicators for air navigation services.

One of the key tools to achieve the performance targets, which has been strengthened by SES II, is the process of integration of European national airspaces in Functional Airspace Blocks (FABs), that will be configured as macro volumes of airspace based on operational requirements and established regardless of State boundaries, where the provision of air navigation services and related ancillary functions are integrated.

The motivation behind the requirement of FABs implementation is the fragmentation of the European system, resulting from the fact that air traffic control has been historically associated with sovereignty and hence confined within national borders. This fragmentation results today in a patchwork of national systems, where 47 out of the total 69 ACCs operate with 10 sectors or fewer at maximum configuration thus being below their optimal

economic size. The duplication of systems results in high costs of purchase, maintenance and certification of equipments that are not inter-operable and require increased coordination at the interfaces between them, resulting in additional ATCOs workload for handover. Also the costs of research, training and administration do not benefit from economies of scale. The overall order of magnitude of the costs of fragmentation in the European ATM system was estimated at some €880 Millions - €1,4 Billions (Helios Economics and Policy Services, 2006). Thus FABs are considered key enablers for enhancing cooperation between air navigation service providers in order to improve performance and create synergies. This implies the institution of ATC systems established by mutual agreement between all the Member States, irrespective of national borders and designed to increase flight efficiency on the base of traffic flows rather than State boundaries. The FAB concept was developed in the SES I package in terms of configuration of the upper airspace as a continuum managed by a single ANSP or by a joint venture of ANSPs through common systems and procedures. The institution of FABs has encountered numerous hurdles after the adoption of SES I package, due to the complex issues regarding national sovereignty and systems integration.

This has led EC to strengthen the requirement on the creation of FABs through SES II, by requiring Member States to establish FABs by the end of 2012 at the latest and to designate one or more service providers that might provide services also in a Member State other than that in which they have their principal place of operation. Thus the focal point of FAB has been moved from a simple configuration of upper airspaces into the integration of service provisions. At the request of all Member States concerned with a FAB project, the EC can designate a natural person as FAB coordinator in order to facilitate negotiation process, to overcome difficulties and to speed up the establishment of FABs. FAB implementation must be justified by the overall added value, including optimal use of resources, on the basis of cost-benefit analyses and supported by a safety case.

The process of FABs set up shall be facilitated by the European Network Management Function (NMF) to ensure the convergence of national ATM systems through the preparation of implementing rules and monitoring.

Traffic Flow Management, in particular regarding slot coordination and allocation, will be subject to regulation according to SES II. The implementing rules shall include consistency between flight plans and airport slots and the necessary coordination with adjacent regions, in accordance with the recommendations contained in a previous EC communication (EC No 819-2006), stating that an appropriate balance between market-led solutions (market mechanisms for slot allocation) and regulatory measures (Single European Sky and airport safety oversight) must be sought to deal with the expected capacity crunch;

The network management function more in general comprises a range of tasks, exercised by different actors, whose modalities of execution will be developed in implementing rules guaranteeing public interest impartiality and ensuring appropriate industry involvement. These tasks will include:

- route network design;
- management of scarce resources through a centralised inventory;
- traffic flow management, in particular slot coordination and allocation as a function of the required time of arrival.

The last bullet in particular is intended to enhance predictability and ultimately the management of infrastructure elements in accordance with the Single European Sky ATM Research Program (SESAR) proposals.

### **2.5.2 The SESAR Program**

SESAR represents the technological and industrial component of the Single European Sky, adopted in March 2004 to develop the new generation ATM system capable of providing the technological leap required to keep pace with the increasing air transport demand foreseen over the next 30 years, while ensuring the safety and fluidity of operations.

The SESAR project is organised in three phases. The definition phase (2004-2008) has delivered the SESAR Master Plan (SESAR Consortium,

2008) establishing the different technological stages, priorities and timetables. The second phase (2008-2013), managed by the SESAR Joint Undertaking, concentrates efforts on the research and development activities that will enable the development of the technologies which will underpin the new generation of systems. The following deployment phase (2014-2020 and beyond) will embrace the production and implementation of the new ATM infrastructure.

The SESAR Consortium, which carried out the definition phase study, brought together for the first time in history the major stakeholders in European aviation including airspace users, ANSPs, airport operators and the supply industry (European and non-European), plus a number of associated partners, including safety regulators, military organisations, staff associations (pilots, controllers and engineers) and research centres which worked together with the significant expertise of EUROCONTROL.

The SESAR Concept of Operation is the result of a top-down approach that led to the identification of an operational evolution road map starting from five high level performance targets:

- Enable a 3-fold increase in capacity (w.r.t 2005 figures) which will also reduce delays, both on the ground and in the air;
- Improve the safety performance by a factor of 10;
- Enable a 10% reduction in the effects flights have on the environment;
- Provide ATM services at a cost to the airspace users which is at least 50% less.

From these strategic performance targets, a number of Key Performance Indicators (KPI) have been agreed in 11 Key Performance Areas (KPA), as developed by ICAO, clustered into 3 groups:

- Societal KPAs: safety, security, environmental sustainability;
- Operational KPAs: capacity, cost effectiveness, efficiency, flexibility, predictability;

- Enablers KPAs: access and equity, participation, interoperability

The 11 KPAs constitute the base for the SESAR Performance Framework which has been developed to define the axes along which benefits have to be introduced by the program. The performance targets have been the fundamental input for the development of a SESAR concept of operations for 2020 as disclosed in SESAR Consortium (2007). This concept focuses on global interoperability of the number of subsystems that compose the ATM system, with user-driven management of flight trajectories in 4D (i.e. space + time), as well as on the exchange of relevant flight information between all airborne and ground partners, in order to allow users to directly take informed and non-imposed decisions, based on the most accurate and complete information, whenever non-nominal situations occur. This concept has been designed for compatibility with other worldwide initiatives, such as NextGen in USA, in order to ensure global interoperability.

### 2.5.2.1 The Business Trajectory

The key concept to achieve SESAR performance targets is the new management of all operations based on the trajectory of the flight, in order to execute each flight as close as possible to the intentions of its owner, i.e. the airspace user. For commercial flights these intentions are specified in a Business Trajectory (BT), with a required precision associated in all 4 dimensions (i.e. space+time). Whenever ATM constraints need to be applied, including those arising from infrastructural and environmental restrictions and regulations, it will be the individual user who will find the best alternative BT within these constraints, according to his internal business objectives. The life cycle of the Business Trajectory starts in the long term phase, possibly years before flight execution under the form of Business Development Trajectory, which is not shared outside the Airspace User organisation. Once it has reached a certain level of stability, it is made available to the ATM System for planning purposes thus becoming Shared Business Trajectory, through which potential discrepancies with network constraints might already be detected and the Airspace Users will be notified with the request to adjust

their BT. This process will be iterative and will rely on CDM mechanisms in order to solve mismatches between demand and capacity and will ultimately lead from the Shared Business Trajectory to the Reference Business Trajectory (RBT), which is the trajectory that the Airspace User agrees to fly and the ANSPs and Airports agree to facilitate; it represents the objective to be achieved and will be progressively authorised during flight execution. Most of the time indications contained in the RBT are estimates, however some may be target times to facilitate planning and some of them may be constraints to assist in particular in queue management when appropriate.

### **2.5.2.2 System Wide Information Management**

The trajectory management concept entails the systematic exchange of aircraft trajectory data between various participants in the ATM process, which will be supported by a System Wide Information Management System (SWIM) enabling all participants to have access to a unique and accurate source of data to plan and execute operations. SWIM will integrate Air-Ground and Ground-Ground information sharing by the exchange of real-time information between different actors belonging to the ATM system, during all phases of flight, thus forming a connected ATM network in which the nodes (including aircraft, AOCs, Airports, ACCs, etc.) can provide and consume information. Such a system requires to shift from the current point-to-point message exchange to the sharing of information within a common virtual information pool, organized around a number of central themes, called data domains, to separate different types of available information (e.g. flight data, surveillance data, aeronautical data, meteorological data, ATFCM data, etc.) (Trausmuth and Hirschberger, 2008).

### **2.5.2.3 Collaborative Decision Making**

The SESAR Target Concept claims that, on the basis of common situational awareness enabled by SWIM, in the future ATM system Airspace users will be fully involved in the process of demand and capacity balancing, through the implementation of ad-hoc collaborative processes. This will

happen either during the agreements on adjustments to traffic demand or individual trajectories when ANSPs and Airports cannot provide sufficient capacity (i.e. the aforementioned modifications to Shared Business Trajectory) or in the User Driven Prioritization Process (UDPP) which will be designed to prioritise traffic queues caused by unexpected capacity shortfalls. (SESAR Consortium, 2007) In particular SESAR states that the Airspace users among themselves can recommend to the Network Management a priority order for flights affected by delays caused by an unexpected reductions of capacity. The airspace users will respond in a collaborative manner to the Network Management with a demand that best matches the available capacity.

This implies the application of the Collaborative Decision Making principle (cf. Section 2.3.2) to the European Air Traffic Flow Management, which has been limited so far just to Airport environment. Since 2001 in fact a number of European Airports in collaboration with EUROCONTROL have implemented methodologies and tools to increase the operational efficiency of airport procedures, in particular the aircraft turn-round process, through the sharing of accurate and timely information about available resources and operational status of different partners such as Aircraft Operators, Air Traffic Control, handling agents and the airport management. The main result of the Airport CDM process is a very accurate calculation of the aircraft Target Take Off Time (TTOT), which allows to build robust ground planning. However these benefits could be extended to the entire network if the information available at a CDM Airport would be available also outside the scope of the airport, in order to allow also en-route ATC and arrival airport to be updated in advance about flight punctuality (Iagaru et al., 2009). Munich Airport is the first aerodrome in Europe which shares, since June 2007, its local information with CFMU through the exchange of Departure Planning Information (DPI), that provides CFMU with a reliable estimate about aircraft departure time, and Flight Update Messages (FUM), that provides informs the airport about modifications in the estimated time of aircraft arrival.

This collaborative management of flight updates forms a part of the sec-

ond level of Airport CDM implementation, together with the calculation of variable taxi times (EUROCONTROL, 2003). The full exploitation of Airport CDM potential lies however in the third level, which includes the collaborative pre departure sequence, as a fundamental process for the efficient use of ground resources and the direct involvement of Aircraft Operators into the departure sequencing moving away from the first-come-first-served principle.

Airport CDM has increased the involvement of aircraft operators and of airport operations in the process of Air Traffic Management (ATM), improving the predictability of events and enhancing efficiency in resources utilisation, thus introducing benefits for all actors without requiring huge investments (a benefit-to-cost ratio of 9 has been estimated in EUROCONTROL, 2008). To realise however all the potential benefits of the CDM philosophy, a European-wide approach is necessary in the implementation of the CDM elements as recognized by SESAR.

### 2.5.3 Contract-based Air Transportation System

One of the possible implementations of the Business Trajectory concept proposed in SESAR, is the Contract of Objectives (CoO) investigated by the Contract-based Air Transportation System (CATS) project. This is an European project selected during the fourth call of the 6<sup>th</sup> EC Framework Program for research and launched in November 2007 with a three year duration. The main objective of the CATS Project is to experimentally assess the operational validity the CoO.

The CATS Project hence provides a solution to the need of determining “how to deal with business trajectories in the strategic, tactical and operational phases of flight”, as highlighted in SESAR Consortium (2006).

The CoO constitutes a formal and collaborative commitment between the main actors involved in the flight operations (i.e. Airports, ANSPs and Airline), that establishes a set of 4D windows, called Target Windows (TWs), inside which each one engages in delivering its services to flight execution, from gate to gate. This allows to define for each aircraft a number of TWs located at the transfer of responsibility areas along flight trajectory (e.g.

between 2 ACCs) with specific volume and temporal duration, determined according to resource availability (e.g. capacity) and downstream constraints (e.g. punctuality at the destination).

TWs are negotiated among actors starting from the long term planning phase, when the first TWs, related to departure and arrival airports, are defined on the base of airspace user demands and Airport resource plans. At this phase no major changes are introduced with respect to the current system, since the concept is consistent with the airport slot coordination process (cf. Section 1.2.2). If there is no mismatch between demands and airport resources, the initial plans are accepted, otherwise a negotiation process begins between airports and users until satisfactory solutions are found. If an agreement cannot be found between airspace users and airports, the network management function can play the role of moderator, facilitator or decision maker.

Once the initial plan is designed, the first phase concludes according to SESAR and the BT changes its status from business development trajectory to shared business trajectory. Accordingly the TWs related to departure and arrival airports become public information and it is the time for ANSPs to intervene in order to adapt the airspace structure and the working methods to the airspace user demands. The purpose is to adjust the availability of ANSP resources in order to avoid congestions and to fit users' demand as far as possible, in accordance with the SESAR Target Concept advocating a collaborative refinement of the business trajectories (SESAR Consortium, 2007). ANSPs can thus build a resource plan on the basis of the business trajectories, while a phase of negotiations between all the actors enables the ultimate production of Reference Business Trajectory, which consists, in the CATS operational view, in a collection of Target Windows located at each transfer of responsibility area between actors.

Subsequently during the short term planning phase, the CoO is adapted through the negotiation mechanism to the real situation occurring on the day of operations, according to weather conditions which can be observed or accurately predicted and to other disruptions until it is conclusively signed just before aircraft push-back.

Any divergence from this definitive CoO causing non-compliance with at least one TW, due to either operational issues or to divergence from what previously scheduled, still remains possible but triggers a specific decision-making process at a system level, called renegotiation. This process is performed by the involved actors, relying on SWIM network facilities, minimizing data exchanged to avoid the saturation of the SWIM network. A renegotiation proposing a revision of one or more TWs, may be proposed by ANSP, airport, airline or aircrew. The revision of the TW shall be based a CDM process involving all the concerned actors, whenever the time horizon allows, and in particular letting the user decide according to its business preferences. In certain cases, e.g. if the impact of renegotiation is observable only by two neighboring ACCs, the process is limited to the impacted actors and the outcome of the TW renegotiation process is then made available through the SWIM network. Whenever the situation is urgent, the ATCOs may decide to immediately and locally revise the trajectory for safety and separation purposes, without applying a CDM process.

It is important to highlight that the CoO is not free from conflicts, hence aircraft still need separation through ATC clearances which are supposed to let aircraft cross its agreed TWs, unless unforeseen disruptions occur (e.g. due to conflict resolution manoeuvres in a managed airspace). For this reason TWs are defined as temporal and spatial intervals rather than points, since this allow the tactical operations to unroll with a certain flexibility to ensure resilience in case of disruption and conflict management. Stochasticity will always be a component of the system hence the CATS concept proposes to keep it under certain margins, by managing disruptions via the size of the TWs and to limit the side effects of any divergence.

Through TWs in fact all actors can gain mutual awareness of each other objectives and to monitor in real time the evolution of the pre-established plans. At a conceptual level, the CoO and TWs can be regarded as an operational way of achieving the establishment of the ATM performance partnership recommended by SESAR Consortium (2006). TWs in fact ensure a common translation and representation of the performance targets to be achieved by the overall ATM chain.

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At a second more operational level, TWs unequivocally identify the transfer of responsibility areas between partners, and at the same time they constitute a way of managing uncertainty and monitoring disruptions. Measurement of compliance with TWs established during the negotiation process could represent a new and reliable metric for assessing the quality of a provided service.



## Chapter 3

# The Central Resource Allocation Problem in the Context of ATFM

Previous chapters have described the set of resources required by Aircraft Operators to perform a flight, which can be distinguished into internal (i.e. aircraft, crew,...) and external, constituted by Airport facilities and Airspace capacity. The availability, or rather the scarcity, of external resources have an economical impact on the flight profitability, which is properly taken into account by the AO during the planning phase, e.g. through the obtainment of the proper airport slots. Decisions dictated by the availability of such resources are made directly by the AO depending on its position on the market and its business model. Conversely for Airspace resources the assignment is currently decided by a central regulator on the base of a FPFS principle and then imposed under the form of ATFM slots to flights, without considering their economic impact on individual users affected. While this mechanism is easy to implement and guarantees equity of treatment and fairness of the assignment, it also creates a potential for improvement as recognized by SESAR, which instead proposes to give individual users the power to make decisions when ATM constraints need to be enforced.

This is also the fundamental concept at the base of CDM mechanisms em-

ployed in the U.S., where the different nature of ATFM problem allowed for introduction of slot-exchange mechanisms. Airspace users can take independent actions (such as conditional cancellations under SCS, cf. Section 2.3.2.1) to distribute delay among their flights as they prefer. The different nature of European ATFM problem, where multiple regulations can be imposed on a flight, does not permit a straightforward application of U.S. CDM applications such as Compression and SCS but demand for more complex mechanisms for slot exchanges.

In this chapter a number of auction-based models are developed that could be employed at a tactical or pre-tactical level for establishing priorities among flights subject to ATFM regulations, assuming that an adequate communication network such as SWIM (cf. Section 2.5.2.2) enables the exchange of information among stakeholders and that some airlines may be interested in paying for delay reductions or receiving compensations for delay increases, as already suggested by Vossen and Ball (2006b) and Ball et al. (2005).

### 3.1 Equity and Fairness in ATFM

When regulating air traffic, the ATFM authority has the obligation of assigning resources in the most equitable and impartial manner to users, that must have equal rights to access to these resources independently on their identities. In the context of resource allocation equity can be defined as a state in which each user's welfare is increased to the maximum extent possible, given the limited resources, after taking proper account of disparate claims and individual circumstances (Hoffman and Davidson, 2003). The most difficult part of this definition to implement is perhaps the concept of taking in proper account different preferences, since preferences are subjectively defined by the claimants.

The time estimates contained in the flight plan are currently taken as representative of the individual user preference and employed to sequence aircraft under the current FPPS principle.

However SESAR states that improved priority management alternatives shall be provided to airspace users to satisfy their needs, going beyond the

FPFS principle. These new priority rules are considered performance enablers and included within the Equity KPA, and have to be applied in a transparent and correct manner (EUROCONTROL, 2009).

Since money constitute a universal metric to define preferences we argue that it could be employed also in the assignment of priorities in ATFM. Moreover, since ATFM delays constitute a cost for aircraft operators that depend on several factors (cf. Section 2.4.1.3), payments involved in the implementation of a fair assignment can be directly comparable with the cost of delay suffered. Hence a key assumption made in the rest of this thesis, is that both utility derived by the assignment and the cost for achieving it are measured in monetary units. This assumption implies that there are actually two types of goods in the resource allocation settings we consider: the first is the limited capacity and the second is money.

## 3.2 Literature Review

Different types of measures can be undertaken to solve congestion problems and they vary according to the time horizon necessary for their application. Odoni (1987) provides the first systematic description of the different categories of possible ATFM initiatives.

Long term solutions have a lookahead time from 5 to 10 years and include the building of new infrastructure to accommodate traffic (e.g. runways, airports) or the introduction of new standards for ATC (e.g. methodologies, technologies). Their implementation usually requires high costs and could result complicated or even impossible due to environmental constraints or safety requirements.

Medium term measures have a planning horizon of 6 months to 2 years and try to alleviate congestion by modifying spatial or temporal traffic patterns. They can vary from strategic flow management actions discussed in chapter 2, which are decided several months in advance based on traffic forecasts, to the adoption of demand management policies for the assignment of Airport resources. These latter can be either purely administrative as it is the case for the strategic assignment of Airport slots (cf. Section 1.2.2) or purely

economic, such as congestion pricing, or configured as a combination of them (see Fan, 2004 for a thorough analysis).

Short term solutions are those adopted at a tactical level to guarantee safety by avoiding congestion, essentially through ground holdings and re-routing under the current European system. Ground holding practice, i.e. the action of delaying a flight's take-off beyond its scheduled departure time, is motivated by the principle that delays on the ground are safer and less costly than those in the air. Hence any forecast delay somewhere in the system is anticipated at the departure airport prior to the take-off (Ball and Lulli, 2004). Since re-routing cannot be imposed by CFMU and must be accepted by Aircraft Operator, it requires some coordination activity and is therefore less used than ground holding. The dynamic nature of the ATFM slot allocation process requires it to be fast, reliable and equitable as it must react to continuous changes in the environment to which it is applied.

### **3.2.1 Medium Term Measures to Manage Demand**

Then it is not surprising that research efforts on demand management have concentrated primarily on medium-term initiatives such as auctions and congestion pricing, considered as alternative to administrative procedures for the assignment of resources.

Rassenti et al. (1982) propose a sealed-bid type of combinatorial auction (see Section 3.2.3.1 for a definition of different types of auctions) for the long-term strategic allocation of airport slots in the U.S.. Such a system requires a complete upfront revelation of airlines information in order to calculate the allocation, while the revenues from the auction are collected by airports. Authors recognize that this point can be matter for debate since airports with limited capacity would rise their revenues by imputing rents on scarce commodities. Donohue et al. (2003) propose the use of auctions to allocate airport arrival slot to balance demand with capacity at slot-constrained airports, while Le et al. (2004) formalize an auction model for the strategic assignment of airport slot which combines economic and administrative measures and provide a case study. Ball et al. (2005) observe that U.S.

airport slots are scarce commodities with both a private and a common values. Then, to discover the slot market price, they suggests to allocate slots through combinatorial auctions. They provide a thorough analysis of the objectives and issues associated with slot auctions at different planning levels, from the strategic to the tactical one. DotEcon Ltd. (2001) and more recently NERA consulting (2004) provide an analysis of the effects of current practices for the assignment of strategic airport slots in Europe and an assessment of several potential market-based schemes.

Raffarin (2002) proposes a Vickerey-Clarke-Groves (VCG) sealed-bid type of auction for the assignment of ATFM slots under the European system, 80% to be allocated in the strategic phase and the other 20% on the day of operations. Author proves that such a mechanism achieve an efficient allocation and allows the discovering of Airlines' private values for resources. Results are based on theoretical analysis without the support of simulations due to lack of data.

One interesting point of VCG mechanisms is that they charge each participant a price equal to the external cost caused on society. Congestion pricing theory is based on this same fundamental principle, which is achieved through a different mechanism, since there are no predefined slots and each flight is free to schedule operations as long as it pays an appropriate congestion fee. This toll forces users to internalize the external cost they cause. A number of studies deals with the problems of determining appropriate landing fees to manage congestion at airports (Levine, 1969, Daniel, 1995, Hansen, 2002) or in the en-route airspace (Andreatta and Odoni, 2003).

However Schank (2005) argues that institutional barriers may prevent effective implementation of congestion pricing at airports and provides several case studies as a support. In particular 2 cases presented show that a particular group of aircraft was discriminated by the toll charging policy and this led to a substantial opposition which caused its dissolution. This demonstrates that political and social equity problems may eventually prevent the adoption of congestion pricing schemes, if adequate alternatives are not provided to the displaced passengers.

### 3.2.2 Short Term Measures to Manage Demand

Following the seminal work of Odoni (1987), a number of researchers have focused their activity on the development of optimization models and algorithms for the assignment of ground delays as a short-term measure to regulate traffic flows. The problem of assigning ground delays to a set of flights in order to minimize an aggregated cost function, given airport capacity constraints, is known as Ground Holding Problem (GHP).

In its basic version the GHP assumes that only one airport in the system is subject to capacity constraints which are imposed only on arrival flights. This problem is referred to as the Single Airport Ground Holding problem (SAGHP) and has been formulated for different cases.

Andreatta and Romanin-Jacur (1987) analyze and solve the problem with a dynamic programming algorithm for a single time period and one airport subject to a capacity limit which is a random variable whose distribution is known. Terrab and Odoni (1993) formulate the problem as a minimum cost network flow for the multi-period deterministic case in which the future airport capacity profile is determined in advance. They also extend the algorithm proposed in Andreatta and Romanin-Jacur (1987) to the multi-period case with stochastic capacities and propose several heuristics to achieve approximate solutions due to the complexity of the exact dynamic programming approach.

Richetta and Odoni (1993) extend these results by presenting a stochastic linear programming solution to the static version of SAGHP. Hoffman and Ball (2000) propose a variant of SAGHP by adding banking constraints that force sets of flights to be temporally grouped in order to allow the transfer of passengers, baggages and crews for aircraft operating under a hub-and-spoke system (cf. Section 1.2.1).

Dell'Olmo and Lulli (2003) formulate a dynamic programming algorithm that optimally allocates airport capacity to both arrival and departure flights, using the concept of capacity envelope according to which arrival and departure capacities are interdependent and connected by a convex piecewise-linear functional relationship.

In the case a network of airports is considered and propagation of delay can occur between successive flights, the problem of determining individual flight delays to minimize the total delay cost is referred to as the Multi Airport Ground Holding problem (MAGHP). Under this general case the problem is typically modeled through an integer program that increases computational burden with respect to the SAGHP. This latter in fact can be encoded through a pure linear program giving integer solutions due to the total-unimodular constraint matrix associated.

Vranas et al. (1994a) first proposed several exact formulations as well as an heuristic for the static deterministic version of the MAGHP. An extension to the dynamic stochastic case is provided in Vranas et al. (1994b).

A further extension of the MAGHP is the one that also includes constraints on the capacities of en-route sectors of airspace and determines optimal speed adjustments of aircraft, besides their release time into the network. This is known as Air Traffic Flow Management problem (TFMP). Bertsimas and Stock Patterson (1998) formulate an integer program with binary variables for the deterministic version of the TFMP, which also includes the rerouting option since each flight has a pre-determined set of routes available. Authors prove that their formulation is NP-hard, however in many practical cases the linear programming relaxation of the integer problems yields integer optimal solutions and is solvable in polynomial time, hence it is computationally efficient. This is due to the fact that several of the constraints in the model provide facets of the convex hull of solutions.

Bertsimas and Stock Patterson (2000) consider an extension of the TFMP problem in which aircraft can be dynamically rerouted through the system. Their model is formulated as a dynamic, multi-commodity, integer network flow model with side constraints, whose solutions indicate aggregate flows obtained by solving a lagrangian relaxation of the linear program in which capacity constraints are dualized. Successively a rounding heuristic is employed to decompose aggregate flows into a set of individual flight paths and finally an integer packing problem is solved to obtain individual flight routes.

Bertsimas et al. (2008) extend the model in Bertsimas and Stock Patterson (1998) by adding routing decision capabilities to it. This is achieved

through a compact formulation based on a description of the overall network as a directed graph, where nodes represent capacity constrained elements and arcs define their sequence relations.

Alonso et al. (2000) formulate the TFMP through two versions of a stochastic model based on a multistage scenario approach, which extend the deterministic model of Bertsimas and Stock Patterson (1998).

Lulli and Odoni (2007) formulate a deterministic model for the optimal assignment of delays to flights in order to guarantee the respect of airports' and sectors' capacities under the European system. They show that under particular circumstances, due to the complex traffic flows interactions generated by en-route airspace capacities, the assignment of airborne delay in addition to ground delay can be beneficial.

The common characteristic of all these models is the presence of a unique central decision maker, the ATFM authority, which is in charge of assigning individual delays to flights in order to minimize a global objective function, obtained by aggregating the direct operating costs caused to all regulated flights by ATFM restrictions. This approach is consistent with the current mode of operations of the European ATFM system, where the CFMU centrally calculates and imposes ground holdings to flights according to a FPFS heuristic. This criteria for delay assignment does not consider the cost caused to users, which can be correctly estimated only by individual airlines depending on their internal priorities and business models.

This principle is at the base of the Collaborative Decision Making program undertaken by the FAA to partially decentralize decision making regarding the assignment of ground delays to flights (cf. Section 2.3.2.1). The CDM implementation in the U.S. system allows airlines to exert active control on their aircraft, by providing incentives to share up-to-date information and to cooperate in determining resource allocation. Vossen and Ball (2006a) demonstrate that the inter-airline slot exchange procedure implemented through the Compression algorithm (cf. Section 2.3.2.1) can be interpreted as a mediated bartering, in which the FAA acts as a "broker", matching offers proposed by the airlines.

The idea behind the Compression algorithm is to reward airlines for slots

they release, thus encouraging airlines to report cancellations.

### 3.2.3 The Use of Auctions in Resource Allocation

Auctions constitute protocols that use the preference expressed by participants to determine an allocation of resources and a set of respective payments. Auctions provide a tool to allocate resources more efficiently than other mechanisms and have been used in a variety of settings where self-interested agents compete over the allocation of goods. The most important auctions applications include spectrum allocation in several countries all over the world (see Banks et al., 2003; Grimm et al., 2003), electricity supply (see de Castro et al., 2008; Elmaghraby, 2005), procurement of goods in industrial settings (Bichler et al., 2005; Hohner et al., 2003), allocation of bus routes (Bichler et al., 2005) and online advertisement positioning (Edelman et al., 2007). Auctions have also been proposed by researchers for bandwidth allocation (Maille and Tuffin, 2004), scheduling problems (Wellman et al., 2001; Kutanoglu and Wu, 1999) and also for the specific problem of resources in ATM as illustrated in Section 3.2.1.

#### 3.2.3.1 Types of Auctions

The literature on auctions identifies a wide variety of auction types that I will briefly outline here. The interested reader can refer to Krishna (2009) and the literature cited therein, for a complete description of auction theory. The single-good type constitutes the most familiar class of auctions, where there is one seller, multiple potential buyers and one unique good for sale. These auctions are called single-sided, since there are multiple agents only on one side of the market. The open outcry English auction is the oldest and perhaps most prevalent auction format, according to which an auctioneer announces a starting price for the item and increases it incrementally until it remains only one bidder interested. As soon as the last bidder remains alone in the bidding process, it is assigned the object and pays the price announced, at which the second last bidder refuses to stay. Hence the price the winner has to pay equals the second highest bid from which it inherits

the classification as second-price auction.

The Vickrey auction is a second-price auction in which each bidder submits its bid to the auctioneer privately, thus preventing other bidders from knowing the amount. The bidder submitting the highest bid is assigned the item and pays a price equal to the next highest bid (i.e. the highest rejected bid). This is known as a second-price, sealed-bid type of auction.

A sealed bid type of auction where the highest bid is assigned the item and pays a price equal to its bid is known as first-price auction.

A first-price open outcry auction is the Dutch auction, in which the auctioneer first calls out a price and decreases this price incrementally until the first bidder is willing to accept it, thus being assigned the item and having to pay this bid.

Under the assumption that all agents are risk neutral and have private statistically independent values drawn from the same distribution, Vickrey (1961) proves that the revenues under all these auction formats are equivalent. Myerson (1981) generalizes this result of revenue equivalence to all auction formats that are allocative efficient, i.e. those implementing a solution that maximizes the sum of bidders' utilities over all allocations.

A quite different setting is the one in which there are multiple items that have to be sold by an auctioneer. In the case that there is still only one kind of good available and the different items are identical copies of that good the auction is known as multiunit. A number of different cases is grouped under this category, depending on the characteristics of individual demands (either for single units or multiple units) and supply (limited or unlimited). Usually in multiunit auction theory it is assumed that the marginal value of an additional unit for buyer decreases with the number of units they receive.

Differently when the items auctioned are not identical but constitute a variety of different goods available in the same market, combinatorial auctions are used to assign them. They are employed for example when auctioning paths in networks as bidders can express complex bids on packages of items which are valuable only if acquired together, since a path might be useless if incomplete.

In two-sided auctions instead there are many buyers and sellers who trade

in a market and both bids (i.e. offers to purchases) and asks (i.e. offers to sells) are allowed. A typical example is the stock market, where many agents interact to buy and sell different stocks. In the particular case that all agents trade many units of the same identical good (e.g. the shares of a given company), the auction is called double auction. The main subclasses of double auctions are the Continuous Double Auction (CDA) and the periodic double auction, which differ in the timing of trades occurrence. In CDA bids and offerings are matched immediately as they are revealed, while in the periodic double auction trades occur at some predetermined time, when the maximal amount of trade is computed on the base of the bids previously received.

Combinatorial exchanges are particular double auctions in which multiple buyers and sellers trade different heterogeneous goods. This type of mechanisms constitute a perfect candidate for application in Air Traffic Flow Management problems, where there are several competing airlines and the reallocation of different resources demands for complex trading combinations to fully achieve economic efficiency.

### 3.3 Definition of the Problem

As already highlighted in section 3.1, we assume that both the utility derived by the assignment of ATFM resources and the cost for achieving it can be measured in monetary units and this utility is transferable among Aircraft Operators under the form of payments. While any type of currency can be adopted we employ Euro (€) for simplicity of treatment. Hence there are two types of goods in the resource allocation settings we consider: the first is the limited capacity and the second is money. We rely on TW concept defined in Section 2.5.3 to support a clear and practical extension of the actual assignment of ATFM capacity. By restricting TWs uniquely on their temporal dimension, they represent time slots that allow to explicitly allocate network resources to users whenever there is mismatch between demand and capacity. With a slight abuse of terminology we use the terms allocation and assignment as synonyms.

Our purpose is to find a mechanism for the assignment TWs that, besides guaranteeing safety by adopting the same definition of capacity employed today, it satisfies some of the following desirable properties commonly used in mechanism design (see Krishna, 2009 for a formal treatment):

1. **Allocative efficiency:** if utility is transferable among all agents, a mechanism that maximizes the sum of individual utilities (i.e. the sum of utilities calculated from the values communicated to the auctioneer) is called allocative efficient. Thus an exchange mechanism which is allocative efficient maximize the total increase in value over all agents. Pareto efficiency property instead, implies that the mechanism attain an allocation, for which no other allocation exists, that makes at least one agent better off without making at least one agent worse off. Allocative efficiency can be defined in an ex-post and ex-ante sense. Ex-ante efficiency takes preferences over expected allocations in consideration, whilst ex-post analyzes preferences over realized allocations.
2. **Incentive compatibility:** Incentive compatibility refers to the validity of the information communicated by agents to the auctioneer (i.e. the values attached by a flight  $f$  to a certain allocation  $j$ ). A mechanism is incentive compatible if the equilibrium strategy for agents is to report their preferences truthfully. Agents may have an incentive to untruthfully report their preferences in order to increase their individual utility.
3. **Individual rationality:** this property requires that the utility after participating in the mechanism is higher than before for all participating agents. Otherwise the agent would decide not to take part in the mechanism.
4. **Budget balance:** a mechanism is said to be strictly budget balanced if the amount of prices sums up to 0 over all agents. In this case the mechanism neither requires payments from outside (i.e. there is no subsidization requirement) nor generate a surplus, but just redistributes the payments among the agents. Budget balance property implies that the resource allocation can be performed at no costs. A mechanism is

weakly budget balanced if it can potentially produce a surplus, i.e. the global amount of prices sums up to a positive quantity.

An ideal mechanism would verify all the above properties, it would achieve a minimal cost assignment computed on the base of truthful information communicated by agents and it would guarantee that every participants is better-off without requiring external subsidization. Unfortunately the impossibility theorem by Myerson and Satterwhite (1983) states that no mechanism can satisfy at the same time incentive compatibility, individually rationality, budget balance and efficiency at the equilibrium. In the following we thus relax the incentive compatibility constraint in order to guarantee a mechanism that is individual rational and budget balance and that implements the efficient allocation according to the reported information. We thus make the assumption that participants are price-takers, i.e. they do not declare untruthful valuations to modify prices at their advantage.

### 3.4 Mathematical Description of the Problem

Let  $\mathcal{F} = \{1, \dots, F\}$  be the set of flights subject to ATFM regulations during a certain period  $\mathcal{T} = [T_{min}; T_{max}]$  and  $\mathcal{S} = \{1, \dots, S\}$  a set of sectors and airports which are regulated (capacity constrained) during  $\mathcal{T}$ . Each flight  $f \in \mathcal{F}$  is expected to cross a sequence of elements  $S_f \subseteq \mathcal{S}$  according to its flight plan, hence it will need to be assigned a time slot for each  $s \in S_f$ .

This is an extension of the system employed today for the European ATFM, under which just one departure ATFM slot is assigned to flight  $f$ , according to its most penalizing regulation. This extension is in line with the Target Window concept proposed by CATS (cf. Section 2.5.3), if we restrict the 4 dimensions of TW (i.e. space+time) just to the time. Hence in the following we use the term TW to indicate a time slot assigned to a flight on each regulated resource (e.g. airport, en-route sector, ecc.) it planned to cross, to balance global demand with the available capacity.

According to the CFMU definitions (EUROCONTROL Central Flow Management Unit, 2009b), a regulated resource  $s \in \mathcal{S}$ , with capacity limited to  $K$  entries per hour from  $st\_time$  to  $end\_time$ , has an associated Slot Allocation List  $L_s = [1, \dots, N_s]$ . Each time slot  $j = [I_j, U_j] \in L_s$  represents a TW having capacity of one flight where:

$$\begin{aligned} N_s &= \left\lfloor \frac{end\_time - st\_time}{\frac{60}{K}} \right\rfloor \\ I_j &= \left\lfloor st\_time + (j - 1) \cdot \frac{60}{K} \right\rfloor \quad \text{with } j \in \{2, \dots, N_s\} \\ U_j &= I_{j+1} - 1 \quad \text{with } j \in \{1, \dots, N_s - 1\}, \end{aligned}$$

and  $I_1 = st\_time$ ,  $U_N = end\_time$ .

Each Flight Plan indicates an estimated time of entry into each element  $s \in S_f$  traversed by flight  $f$ , i.e.  $E_f^s$ . Then  $f$  is allocated a list of TWs  $q_f = [TW_1, \dots, TW_{|S_f|}]$ , where  $TW_i$  is the TW assigned on the  $i^{th}$  element of  $S_f$  and can not be earlier than  $E_f^i$  since flights cannot be anticipated, i.e.  $E_f^i \leq U_{TW_i}$  for all  $TW_i \in q_f$ .

We assume that for each flight  $f \in \mathcal{F}$ , if  $|S_f| > 1$  then the flying time between pairs of consecutive resources,  $(i, j)$  with  $i, j \in S_f$  and  $j = i + 1$ , is fixed. This implies that  $|I_{TW_j} - I_{TW_i}| \leq E_f^j - E_f^i \leq |U_{TW_j} - U_{TW_i}|$ .

An assignment  $q$  will cause a positive delay to flight  $f$  if and only if  $E_f^i < I_{TW_i}$  for some  $TW_i \in q$  and the amount of delay will be:

$$d_f^q = \begin{cases} \max_{i \in S_f} (I_{TW_i} - E_f^i) & \text{if } E_f^i \leq I_{TW_i} \quad \forall i \in S_f \\ 0 & \text{otherwise} \end{cases}$$

Hence each TW assignment  $q$  to a flight  $f$  implies a nonnegative cost of delay  $C(f, q) \geq 0$ , which depends on the amount of delay itself and on the flight receiving the delay. We consider the cost  $C(f, q)$  as a non-linear increasing function of delay  $d_f^q$ , bounded from below by  $C(f, q) = 0$  when  $d_f^q = 0$ .

An assignment  $q$  is feasible for flight  $f$  if and only if (i) it contains one

TW for each  $i \in S_f$  and each pair  $(i, j)$  of consecutive TW is connected by the fixed flying time  $E_f^j - E_f^i$ , (ii) it assigns a nonnegative delay to  $f$  and (iii) the delay it assigns is bounded, i.e.  $d_f^q < MaxDel_f$  where  $MaxDel_f$  is a fixed parameter for each flight beyond which the flight prefers to be canceled.

We indicate with  $Q_f$  the set of all TW assignments that are feasible for flight  $f$ .

A pair of consecutive flights  $(f, f') \in \mathcal{F}$  such that  $f'$  follows  $f$ , can be merged to form a unique flight  $f''$  such that  $S_{f''} = S_f \cup S_{f'}$  and the airport turnaround time is defined by  $E_{f'}^1 - E_f^{|S_f|}$ .

### 3.5 The Single Capacity Constrained Resource Case

We consider first the particular case in which there is a unique capacity constrained resource  $s$ , and all flights  $f \in \mathcal{F}$  compete for TWs on this resource. The minimal cost assignment of TWs to flights is then obtained by solving the following problem:

$$\min \sum_{f \in \mathcal{F}} \sum_{k \in Q_f} C(f, k) x(f, k) \quad (1a)$$

subject to

$$\sum_{f \in \mathcal{F} | k \in Q_f} x(f, k) \leq 1 \quad \forall k \in L_s \quad (1b)$$

$$\sum_{k \in Q_f} x(f, k) = 1 \quad \forall f \in \mathcal{F} \quad (1c)$$

$$x(f, k) \geq 0 \quad \forall f \in \mathcal{F}, k \in Q_f. \quad (1d)$$

The objective function (1a) minimizes the aggregated cost of delay caused by the regulation, subject to the constraints that each target window  $k \in L_s$  can be assigned at most to one flight (1b), and all flights affected by the regulation must have a TW allocated (1c). The variable  $x(f, k)$  is equal to 1 if and only if flight  $f$  is allocated to TW  $k$ , and 0 otherwise. This

is guaranteed by constraints (1d), since the integrality of the the decision variables  $x(f, k)$  is ensured by the particular structure of problem (1) as an assignment problem, also referred to as the bipartite weighted matching (see Papadimitriou and Steiglitz, 1998). This is due to the total unimodularity property of the constraints' matrix associated with the problem, implying that all the basic feasible solutions are integer and equal either 0 or 1.

In the case that all flights have the same unitary cost of delay, an allocation  $x^F$  obtained by applying the FPFS principle is optimal for problem (1).

**Theorem 1.** *If  $C(f, k) = d_f^k$  for all  $f \in \mathcal{F}, k \in L_s$ , a TW allocation  $x^F$  obtained with FPFS rule, which sorts flights  $f \in \mathcal{F}$  by ascending value of  $E_f$  and allocates each flight to the first available TW, is optimal for problem (1).*

*Proof.* By closing parallel with the proof of Theorem 3.2 in Vossen and Ball, 2006a where all TW are composed by only one unit of time, i.e.,  $I_j = U_j \forall j \in L_s$ . Suppose without loss of generality that  $E_{f_i} < E_{f_j}$  for all  $f_i, f_j \in \mathcal{F}$ . Suppose  $A_1$  is an assignment obtained by solving problem (1) and  $A_2$  is an assignment obtained by FPFS, then the flight assigned to the first TW in  $L_s$  ( $TW_1$ ) must be the same. Otherwise it means that  $A_2$  assigns  $TW_1$  to a flight  $f_i$  while  $A_1$  assigns  $TW_1$  to a different flight  $f_j \neq f_i$  and assigns another TW  $TW_k$  to  $f_i$  with  $i, k > 1$ . Then the  $A_2$  assignment implies that  $E_{f_i} < E_{f_j}$  but this contradicts the optimality  $A_1$  which could be improved by exchanging  $f_i$  with  $f_j$ . The same argument can be applied to the remaining  $TW \in L_s$ .  $\square$

Thus in the special case of a single capacity constrained resource, the FPFS policy produces an allocation of TWs to flights that minimizes the global delay. However if we weigh delays by the specific unitary costs of delay suffered by flights, theorem (1) does not hold anymore. In this case to achieve the cost-minimizing assignment, problem (1) should be solved with weights  $C(f, k)$  which can be estimated by aircraft operators and then communicated to a central authority in charge of solving the problem.

Under this scenario however there would obviously be an incentive for users to declare weights  $\widehat{C}(f, k) > C(f, k)$ , higher than true value in order

to receive a less penalizing assignment. To avoid this issue aircraft operators could be charged, on the basis of the values reported, an appropriate price  $p(k^*)$  for the TW  $k^*$  assigned by solving problem (1) with weights  $\widehat{C}(f, k)$  declared by users. Prices  $p(k^*)$  could be calculated by solving the following problem dual to (1):

$$\max \sum_{f \in \mathcal{F}} u(f) - \sum_{k \in L_s} p(k) \quad (2a)$$

subject to

$$u(f) - p(k) \leq C(f, k) \quad \forall f \in \mathcal{F}, k \in Q_f \quad (2b)$$

$$p(k) \geq 0 \quad \forall k \in L_s \quad (2c)$$

We assume in the following that the total cost of a TW assignment  $k$  for each flight is represented by a quasilinear function composed by the sum of the cost of delay caused by the TW  $k$  plus the price paid for being assigned the specific TW. Hence the optimal dual variables  $u^*(f)$  and  $p^*(k^*)$  represent respectively for each flight  $f \in \mathcal{F}$  the total cost and price for the optimal assignment  $k^*$ , since  $u^* = p^*(k^*) + C(f, k^*)$  for the complementary slackness condition.

Optimal prices  $p^*$  have the property of being market clearing, i.e. no flight prefers to be assigned another TW  $j \neq k^*$  and being charged the correspondent price  $p^*(j)$ , since this would imply a higher cost. This can be easily proven by considering that at the optimum, due to complementary slackness conditions it is  $p^*(k^*) + C(f, k^*) \leq p^*(j) + C(f, j)$  for all  $f \in \mathcal{F}, j \neq k^* \in Q_f$ . At the same time the incentive for declaring  $\widehat{C}(f, k) > C(f, k)$  would be removed since a flight could end up paying a higher price  $\widehat{p}^*(\widehat{k}^*) > p^*(k^*)$ . Moreover Leonard (1983) proves that by charging prices  $p^*$  (the minimal set in case of multiple dual solutions) not only there is no incentive to misrepresent, but also that the surplus from telling the truth is always at least as great as that from lying, thus these prices are incentive compatible.

However it should be noted that by charging flights a price for TW as-

signed could be perceived as unfair by users. Compared with the FPFS allocation currently implemented, some flights could gain a better position and being charged accordingly, some others might increase their delay and additionally face a payment. In fact let us indicate  $A = \{a_1, \dots, a_F\}$  the assignment of TWs to flights obtained through FPFS, for each  $f \in \mathcal{F}$  the cost of delay caused by this assignment is  $C(f, a_f)$ . On the other hand by imposing the auction mechanism composed by the optimal assignment for problem (1) with payments optimal for its dual problem (2) the cost experienced by each flight  $f \in \mathcal{F}$  becomes  $p^*(k^*) + C(f, k^*)$  and there is not guarantee that  $p^*(k^*) + C(f, k^*) < C(f, a_f)$

To ensure that all participants are better off after the implementation of the optimal assignment, we propose to start from the current FPFS assignment and to consider it as the initial endowment guaranteed to all flights. This implies that a first TW assignment  $A = \{a_1, \dots, a_F\}$  is implemented according to the current FPFS mechanism, and these allocation is considered as a baseline asset at flights' disposal. Subsequently the optimal exchange of TWs among flights is calculated by solving the following problem:

$$\max \sum_{f \in \mathcal{F}} \sum_{k \in Q_f} [C(f, a_f) - C(f, k)] \cdot x(f, k) \quad (3a)$$

subject to

$$\sum_{f \in \mathcal{F} | k \in Q_f} x(f, k) \leq 1 \quad \forall k \in L_s \quad (3b)$$

$$\sum_{k \in Q_f} x(f, k) = 1 \quad \forall f \in \mathcal{F} \quad (3c)$$

$$x(f, k) \geq 0 \quad \forall f \in \mathcal{F}, k \in Q_f. \quad (3d)$$

Instead of minimizing the cost of the assignment, we look for the exchange of TWs among flights of maximal value, where  $V(f, k) = [C(f, a_f) - C(f, k)]$  is the value obtained by flight  $f$  by exchanging its FPFS assigned TW  $a_f$  with the optimal TW  $k$ . It will be  $V(f, k) > 0$  if the delay caused by TW  $q$  is lower than delay caused by  $a_f$ ,  $V(f, k) < 0$  otherwise.

The assignment obtained by solving problem (3) is the same minimal-cost assignment obtained by solving problem (1), since they share all constraints, the term  $\sum_{f \in \mathcal{F}} \sum_{k \in Q_f} C(f, a_f) \cdot x(f, k)$  in the objective function (3a) is a constant due to constraints (3c) and maximizing the remaining part  $\sum_{f \in \mathcal{F}} \sum_{k \in Q_f} [-C(f, k)] \cdot x(f, k)$  is equivalent to the objective function (1a). Hence the minimal cost assignment corresponds to the maximum value exchange. The dual problem becomes:

$$\min \sum_{f \in \mathcal{F}} u(f) + \sum_{k \in L_s} p(k) \quad (4a)$$

subject to

$$u(f) + p(k) \geq [C(f, a_f) - C(f, k)] \quad \forall f \in \mathcal{F}, k \in Q_f \quad (4b)$$

$$p(k) \geq 0 \quad \forall k \in L_s \quad (4c)$$

If each flight  $f \in \mathcal{F}$  is charged a price  $p^*(k^*)$  for the optimal assignment  $x(f, k^*)$  and is compensated by  $p^*(a_f)$  for the relinquished TW previously assigned by FPFS, its cost after the exchange becomes  $C(f, k^*) + p^*(k^*) - p^*(a_f)$ , while the initial cost of delay after FPFS is  $C(f, a_f)$ . We prove that each flight  $f \in \mathcal{F}$  can only reduce its cost of assignment by exchanging TW  $a_f$  with optimal TW  $k^*$  at prices  $p^*$ :

**Property 1.** *The cost reduction  $r(f) = C(f, a_f) - C(f, k^*) - p^*(k^*) + p^*(a_f)$  obtained by each flight  $f \in \mathcal{F}$  by purchasing TW  $k^*$  optimal for problem (3) at the dual price  $p^*(k^*)$  and selling its FPFS assigned TW  $a_f$  at the dual price  $p^*(a_f)$  is always non negative.*

*Proof.* Due to complementary slackness conditions at optimum it is  $u^*(f) + p^*(k^*) = [C(f, a_f) - C(f, k^*)]$ , while  $u^*(f) + p^*(j) \geq [C(f, a_f) - C(f, j)] \quad \forall f \in \mathcal{F}, j \neq k^* \in Q_f$ . Hence  $r(f) = C(f, a_f) - C(f, k^*) - p^*(k^*) + p^*(a_f) \geq 0$  for an  $a_f \in Q_f$  with  $a_f \neq k^*$ .  $\square$

The optimal exchange for problem (3) is thus Individual Rational, since every participant will have a non negative cost reduction by implementing it, i.e. a profit with respect to not participating in the exchange.

If problem (3) admits at least one feasible solution, i.e. there are enough TW  $k \in L_s$  to implement a feasible allocation to all flights  $f \in \mathcal{F}$ , there is at least one optimal exchange that is strictly Budget Balanced, i.e. all the payments made and received by participants sum up to zero. The following two lemmas are necessary to prove that the exchange is also budget balanced.

**Lemma 1.** *For every TW allocation  $K^*$  optimal for problem (3) and TW allocation  $A$  obtained by applying FPFS algorithm on the same set of flights  $\mathcal{F}$ , a TW  $k \in L_s$  which is not assigned in  $A$  is neither assigned in  $K^*$ .*

*Proof.* By contradiction. Let us assume that  $\hat{j}$  is the first empty TW not assigned in  $A$ , but allocated in  $K^*$ . Let  $F^A$ , respectively  $F^{K^*}$ , be the set of the flights allocated to the TW  $1, \dots, \hat{j}$  in  $A$ , respectively in  $K^*$ . The minimality of  $\hat{j}$  guarantees that  $F^{K^*} \setminus F^A \neq \emptyset$ . Then consider a flight  $\hat{f} \in F^{K^*} \setminus F^A$ . In  $A$ , flight  $\hat{f}$  is allocated to a TW  $\hat{l} > \hat{j}$ . Define the allocation  $\hat{x}^A = \{\hat{x}_{1i}^A, \dots, \hat{x}_{F\hat{j}}^A\}$  such that

$$\hat{x}_{fj}^A = \begin{cases} 0 & \text{if } j = \hat{l} \text{ and } f = \hat{f} \\ 1 & \text{if } j = \hat{j} \text{ and } f = \hat{f} \\ x_{fj}^{K^*} & \text{otherwise} \end{cases} .$$

That is,  $\hat{x}^A$  induces the same TW allocations of  $A$  except for flight  $\hat{f}$  which is allocated to TW  $\hat{j}$  instead of  $\hat{l}$ . The feasibility of  $A$  and the fact that  $\hat{f}$  is allocated to TW  $\hat{j}$  in  $K^*$  imply the feasibility of  $\hat{x}^A$ . Also the cost caused by  $x^{K^*}$  is strictly greater than the corresponding cost of  $\hat{x}^A$  as  $E_{\hat{f}} \leq U_{\hat{j}} < I_{\hat{l}}$ . This is in contradiction with the optimality of  $K^*$ , hence we cannot assume that  $\hat{j}$  is not assigned in  $x^A$ , but it is  $x^{K^*}$ . An analogous argument proves that we cannot assume that  $\hat{j}$  is empty in  $x^{K^*}$ , but is allocated in  $x^A$ . Then, we must conclude that  $\hat{j}$  cannot exist.  $\square$

The above lemma implies that any TW allocated under the FPFS policy is also allocated, possibly to a different flight, under the optimal exchange.

An important consequence of Lemma 1 is that the generic asymmetric assignment problem (3) can be decomposed into a set of smaller symmetric assignment problems. To prove this last statement, we need to introduce the

following notation. Let  $i$  and  $l$  be two empty TW in  $x^F$  such that  $U_i < I_l$ . We define as  $B_{il}^A \subseteq L_s$  (respectively  $B_{il}^{K^*} \subseteq L_s$ ) the set of TW allocated in  $A$  (respectively in  $K^*$ ) between the empty TW  $i$  and  $l$ . Let  $F_{il}^A \subseteq \mathcal{F}$  (respectively  $F_{il}^{K^*} \subseteq \mathcal{F}$ ) be the corresponding set of flights assigned to the  $B_{il}^A$  (respectively  $B_{il}^{K^*}$ ) TW.

**Lemma 2.** *For every TW allocation  $K^*$  optimal for problem (3) and TW allocation  $A$  obtained by applying FPFS algorithm on the same set of flights  $\mathcal{F}$ , if  $i$  and  $l$  are two empty TW in  $A$  such that  $U_i < I_l$  and no other empty TW exists between  $i$  and  $l$ , then a flight is assigned a TW between  $i$  and  $l$  in  $A$  if and only if it is assigned a TW between the same empty ones  $i$  and  $l$  also in  $K^*$ , that is  $B_{il}^{K^*} = B_{il}^A$  and  $F_{il}^{K^*} = F_{il}^A$ .*

*Proof.* From Lemma 1, it follows that  $B_{il}^{K^*} = B_{il}^A$ . Let  $f \in F_{il}^A$  be a flight allocated to TW  $h$  in  $B_{il}^A$ , i.e.  $I_h > U_i$  and  $U_h < I_l$ . It follows that  $E_f > U_i$ , otherwise TW  $i$  would be assigned to flight  $f$ . Hence also in  $K^*$  flight  $f$  cannot be assigned a TW earlier than  $i$ . Similarly no flight assigned to a TW later than  $l$  can receive a TW  $h$  earlier than  $l$ , hence flight  $f$  cannot be postponed in  $K^*$  to any free TW later than  $l$ . Thus, if in  $K^*$  TW  $h$  is assigned to a flight preceding TW  $i$  in  $A$ , it follows that there must be an empty TW in  $K^*$  that was not empty in  $A$ , and this contradicts Lemma 1. Hence, we must have that  $F_{il}^{K^*} = F_{il}^A$ . A symmetric argument holds for the only if part of the statement.  $\square$

The above Lemma 2 implies that the optimal TW exchange obtained by solving problem (3) on  $\mathcal{F}$  and  $L_s$  can be obtained by solving problem (3) on subset of TW consecutive allocated by FPFS on the respective group of receiving flights. Or equivalently the solution of the asymmetric assignment problem (3) is found by solving a sequence of smaller symmetric assignment problems, one for each set of TW  $B_{il}^A$  and flights  $F_{il}^A$ .

A further consequence of Lemma 2 is that the exchange is budget balanced.

**Property 2.** *The exchange mechanism that assigns to every  $f \in \mathcal{F}$  the TW  $k_f^* \in L_s$  optimal for problem (3), charges the dual price  $p^*(k_f^*)$  for it and*

refund the dual price  $p^*(a_f)$  for the *FPFS* released *TW* is strongly budget balanced.

*Proof.* The statement is an immediate consequence of the fact that  $F_{il}^{K^*} = F_{il}^A$  for any pair of *TW*  $i$  and  $l$  considered in Lemma 2. This implies that no flight  $f \in F_{il}^A$  cedes its *FPFS* assigned *TW* to the auctioneer, neither any flight  $f \in F_{il}^A$  acquires a *TW* that was unassigned by *FPFS* from the auctioneer. The flights just sell and buy *TW* to and from each others.  $\square$

Hence the exchange is coordinated by a benevolent auctioneer, which does not have any profit by participating in the mechanism, neither has to subsidize it.

Another consequence of Lemma 2 is that, from a game theoretic perspective, we can describe the exchange mechanism as a permutation game (Tijss et al., 1984). This is a particular type of coalitional game with transferable utility (i.e. a TU game) described by the pair  $(\mathcal{F}; \theta)$  where  $\mathcal{F}$  is our set of flights and  $\theta : 2^{\mathcal{F}} \rightarrow R$  is the characteristic function:

$$\theta(D) = \sum_{f \in D} C(f, a_f) - \min_{\pi \in \Pi_D} \sum_{f \in D} C(f, \pi(f))$$

for all subsets  $D \subset \mathcal{F}$ , with  $D \neq \emptyset$  and  $\theta(\emptyset) = 0$ , where  $C(f, a_f)$  is the cost suffered by flight  $f$  for being assigned *TW*  $a_f$ ,  $\Pi_D$  is the class of all  $D$ -permutations and  $C(f, \pi(f))$  the cost suffered by flight  $f$  for being assigned *TW* originally assigned to  $\pi(f)$ . The characteristic function  $\theta(D)$  represents the maximal cost saving that the coalition  $D$  can obtain by being assigned the optimal *TW* compared to the situation in which every player receives the *FPFS* *TW*  $a_f$ . In the TU game  $(\mathcal{F}; \theta)$  a payoff allocation  $p = (r(1), r(2), \dots, r(F))$  is the vector of the amounts of profits allocated to each player. A payoff allocation is individually rational if  $r(f) \geq \theta(\{f\})$  for all  $f \in \mathcal{F}$ , it is efficient if  $\sum_{f \in \mathcal{F}} r(f) = \theta(\mathcal{F})$ . In this context, the imputation set  $\mathcal{I}(\mathcal{F}, \theta)$  of the game is the set of the payoff allocations that are efficient and individually rational, while the core  $\mathcal{C}(\mathcal{F}, \theta) \subseteq \mathcal{I}(\mathcal{F}, \theta)$  of the game is the set of the payoff allocations that are efficient and coalition rational, that is  $\sum_{f \in D} r(f) \geq \theta(D)$ , for all  $D \subseteq \mathcal{F}$ . Hence when the core is not empty there is

at least a payoff allocation which belongs to it, meaning that no sub-coalition  $D \subseteq \mathcal{F}$  has an incentive to break the grand coalition  $\mathcal{F}$  and share the payoff it is able to obtain independently, because this can only decrease.

Curiel and Tijs (1986) state that the core of the permutation game is not empty and that there is a bijective relation between the elements of the core and the optimal solutions of the problem (4). In particular, the profits  $r(f)$  as defined in Property 1 define a core payoff allocation. Given the core characteristics, the next property follows.

**Property 3.** *The exchange mechanism that assigns to every  $f \in \mathcal{F}$  the TW  $k_f^* \in L_s$  optimal for problem (3), charges the dual price  $p^*(k_f^*)$  for it and refund the dual price  $p^*(a_f)$  for the FPFS released TW, is coalition rational and defines Pareto optimal profits whose overall value is equal to  $\sum_{f \in \mathcal{F}} r(f) = \sum_{f \in \mathcal{F}} [C(f, a_f) - C(f, k_f^*)]$ .*

It is important to note that the linear problem (3) always produce an optimal solution which corresponds to a feasible exchange, i.e. an integer solution that maximizes the value over all possible exchanges. This is guaranteed by the existence of at least one feasible solution represented by the FPFS allocation and the particular description of the single capacity constraint problem as a bipartite weighted matching. Hence also its dual has at least one solution providing prices  $p^*$  which are market clearing, i.e. according to which each flight executes the profit maximizing exchange. However due to the multiple degeneracy of the primal problem (3) there would typically be multiple solutions to the dual, each one representing a different market clearing vector of TW prices  $p^*$  which implies a different vector of flight payoffs  $r$ , whose global value is nevertheless constant due to property 3. In these cases a particular pair of primal dual solutions could be selected to force further properties such as pairwise-monotonicity (Miquel, 2009) to try to have all flights obtaining non-null profits.

There are some situations where a flight may find convenient to misrepresent its true cost of delay to increase its payoff, at the expense of other flights, as shown in Appendix B. This is not surprising due to the impossibility theorem of Myerson and Satterwhite (1983), since we require individual

rationality and budget balance constraints to hold. However the number of situations in which this fact can be exploited seems to be limited and it would be necessary to have a perfect knowledge of other players' values in order to be sure that the payoff from the exchange does not decrease.

### 3.6 The General Case: Multiple Capacity Constrained Resources

We now turn to the general case in which there are multiple capacity constrained resources (i.e. sectors and airports),  $\mathcal{S} = \{1, \dots, S\}$  with  $|\mathcal{S}| > 1$  and each flight  $f \in \mathcal{F}$  can plan to use a combination of resources  $S_f \subseteq \mathcal{S}$  according to its flight plan. Hence  $f$  needs to be assigned one  $TW_f^s \in L_s$  for each  $s \in S_f$ . This implies that although all single TWs individually represent a valuable resource, a TW bundle  $B = \{TW_f^1, \dots, TW_f^{|S_f|}\}$  has a non-additive value over the single items  $TW_f^i$  composing it, since only complete packages are valuable. This value  $v(f, B) = -C(f, B)$  can be expressed as the opposite of the cost of delay it induces on flight  $f$ . We can identify two important kinds of non-additivity:

- **Substitutability:** a valuation function  $v$  exhibits substitutability if there exist two sets of goods  $B_1, B_2$ , such that  $B_1 \cap B_2 = \emptyset$  and  $v(B_1 \cup B_2) < v(B_1) + v(B_2)$ . When this condition holds, we say that the valuation function  $v$  is sub-additive. When two items are strict substitutes their combined value is the same as the value for either one of the goods.
- **Complementarity:** a valuation function  $v$  exhibits complementarity if there exist two sets of goods  $B_1, B_2$ , such that  $B_1 \cap B_2 = \emptyset$  and  $v(B_1 \cup B_2) > v(B_1) + v(B_2)$ . When this condition holds, we say that the valuation function  $v$  is super-additive. When two distinct items complement each other their combined value is higher than the sum of their individual values.

Let us assume again, as it was for the previous case of single capacity constrained resource, that valuation functions are quasi-linear in the money,

i.e. the total cost for  $f$  of a bundle  $q \in Q_f$  with price  $p(q) \geq 0$  is  $TC(f, q) = C(f, q) + p(q)$ . Moreover we assume that there are no externalities, since each participant's valuation depends only on the subset of items assigned and does not depend on the allocations and payments of the other agents.

When these types of valuation functions are in place in an auction, the bidders face the exposure problem: a bidder might aggressively bid for a set of goods with the purpose of winning the entire bundle, but succeed in winning only a subset of the goods, thus paying too much. In particular this problem is likely to arise in situations where bidders' valuations exhibit complementarities, because in these cases bidders might be willing to pay substantially more for bundles of goods than they would pay if the goods were sold separately.

To overcome this problem combinatorial auctions allow bidders to bid directly on bundles of goods and auctioneer to sell all goods in a single auction. This eliminates the exposure problem because bidders are guaranteed that their bids are satisfied all-or-nothing.

We suppose again that a FPFS assignment  $A = \{a_1, \dots, a_F\}$ , where  $a_f = \{TW_f^1, \dots, TW_f^{|S_f|}\}$  is a bundle of TW, constitutes an initial endowment for each flight. According to the notation defined in section 3.4, we can formulate the optimal TW exchange problem for the general case as the following binary IP program:

$$Z_{IP-E} = \max \sum_{f \in \mathcal{F}} \sum_{q \in Q_f} V(f, q) x(f, q) \quad (5a)$$

subject to

$$\sum_{f \in \mathcal{F}} \sum_{q \in Q_f: q \ni k} x(f, q) \leq 1 \quad \forall s \in \mathcal{S}, k \in L_s \quad (5b)$$

$$\sum_{q \in Q_f} x(f, q) = 1 \quad \forall f \in \mathcal{F} \quad (5c)$$

$$x(f, q) \in \{0, 1\} \quad \forall f \in \mathcal{F}, q \in Q_f \quad (5d)$$

Where  $V(f, q) = [C(f, a_f) - C(f, q)]$  is the value obtained by flight  $f$  by exchanging bundle  $a_f$  with bundle  $q$  and will be positive if the delay caused by bundle  $q$  is lower than delay caused by  $a_f$ , negative otherwise. Constraints (5b) impose that for all resources  $s \in \mathcal{S}$ , each TW  $k \in L_s$  is assigned at most to one flight, while each flight must receive exactly one bundle of TW  $q$  in its set of feasible requests  $Q_f$  according to constraints (5c). Thus the objective is to find the exchange of TWs among flights that maximizes the value over all feasible exchanges.

A feasible solution will exist if and only if there are enough TWs and requests, such that the assigned bundles are pairwise disjoint, i.e. they do not share any TW. In order to guarantee the existence of a feasible solution we assume that each regulated resource  $s \in \mathcal{S}$  has an infinite capacity after the termination of its regulation (as assumed in Terrab and Odoni, 1993) and that each flight has a request  $q_w \in Q_f$  that includes only TWs after the termination of each regulation traversed. The cost  $C(f, q_w)$  associated to this bundle will be equal to either the cost of delay caused by such a bundle or to the cost of cancellation in the case this delay exceeds  $MaxDel_f$ .

**Property 4.** *The problem of determining the maximum value TW exchange is NP-hard.*

*Proof.* By reduction of problem (5) to the following equivalent Weighted Set Packing Problem as defined in Rothkopf et al. (1998):

$$Z_{SPP} = \max \sum_{f \in \mathcal{F}} \sum_{q \in Q'_f} V(f, q)x(f, q) \quad (6a)$$

subject to

$$\sum_{f \in \mathcal{F}} \sum_{q \in Q'_f: q \ni k} x(f, q) \leq 1 \quad \forall s \in \mathcal{S}', k \in L_s \quad (6b)$$

$$x(f, q) \in \{0, 1\} \quad \forall f \in \mathcal{F}, q \in Q'_f \quad (6c)$$

where the request sets  $Q'_f$  are obtained by appending one dummy item  $TW_d^f$  on an dummy resource  $d$  to each bundle  $q$ , such that each flight  $f \in \mathcal{F}$  has one different dummy TW associated and this same TW is appended to all its requests  $q \in Q'_f$ . Then the new set of resources is  $\mathcal{S}' = \mathcal{S} \cup d$  and this

prevents multiple assignments to the same flight, due to the set of constraints (6b). The introduction of dummy items in order to transform requests into mutually exclusive is commonly referred to as the  $OR^*$  language and is due to Fujishima et al. (1999).  $\square$

Let us define  $Z_{LDP-E}$  as the objective value of the linear relaxation of problem (5), whose dual problem is

$$Z_{DLP-E} = \min \sum_{f \in \mathcal{F}} u(f) + \sum_{s \in \mathcal{S}} \sum_{k \in L_s} p(k) \quad (7a)$$

subject to

$$u(f) + \sum_{s \in \mathcal{S}} \sum_{k \in (L_s \cap q)} p(k) \geq V(f, q) \quad \forall f \in \mathcal{F}, q \in Q_f \quad (7b)$$

$$p(k) \geq 0 \quad \forall s \in \mathcal{S}', sl \in L_s \quad (7c)$$

Let us assume for the moment a linear structure of prices, i.e. for of each bundle of slots  $q \in Q_f$  its price will be  $p(q) = \sum_{k \in q} p(k)$ .

The complementary slackness conditions for the linear programming relaxation of problem (5) and its dual (7) are:

$$x^*(f, q) > 0 \Rightarrow u^*(f) + \sum_{s \in \mathcal{S}} \sum_{k \in (L_s \cap q)} p^*(k) = V(f, q) \quad \forall f \in \mathcal{F}, q \in Q_f \quad (8a)$$

$$u^*(f) + \sum_{s \in \mathcal{S}} \sum_{k \in (L_s \cap q)} p(k) > V(f, q) \Rightarrow x^*(f, q) = 0 \quad \forall f \in \mathcal{F}, q \in Q_f \quad (8b)$$

$$\sum_{f \in \mathcal{F}} \sum_{q \in Q_f: q \ni k} x(f, q) < 1 \Rightarrow p(k) = 0 \quad \forall s \in \mathcal{S}, k \in L_s \quad (9a)$$

$$p(k) > 0 \Rightarrow \sum_{f \in \mathcal{F}} \sum_{q \in Q_f: q \ni k} x(f, q) = 1 \quad \forall s \in \mathcal{S}, k \in L_s \quad (9b)$$

### 3.6.1 Walrasian Equilibrium with Linear Prices

Let us define the **demand** of a flight  $f$  as a bundle  $q_f^*$  that minimizes its total cost (i.e. delay + payment) over all  $q \in Q_f$  given its cost function  $C(f, \cdot)$  and individual TW prices  $p$ . Thus it will be:

$$C(f, q_f^*) + \sum_{k \in q_f^*} p(k) < C(f, q) + \sum_{k \in q} p(k) \quad \forall f \in \mathcal{F}, q \neq q_f^* \in Q_f$$

There may be more than one such a bundle, in which case each of them is called a demand. Because of complementary slackness condition (8a), for every flight with  $x^*(f, q_f^*) > 0$ , constraint (7b) is binding while for all other bundles  $q \neq q_f^* \in Q_f$  a strict inequality holds. Hence the optimal solution to problem (5), whenever it coincides with the solution to its linear programming relaxation, constitutes a demand for each flight.

A vector of nonnegative prices  $p^*$  and a TW allocation  $T^* = \{q_1^*, \dots, q_F^*\}$  form a **Walrasian equilibrium** if for every flight  $f \in \mathcal{F}$ ,  $q_f^*$  is a demand of flight  $f$  at prices  $p^*$  and for any TW  $j$  that is not allocated (i.e.  $j \notin \bigcup_{f \in \mathcal{F}} q_f^*$ ) we have  $p^*(j) = 0$ . The set of prices  $p^* = \bigcup_{s \in \mathcal{S}, k \in L_s} p^*(k)$  is the set of market clearing prices.

Then it follows the Theorem 2:

**Theorem 2.** *If an integral optimal solution exists for the linear programming relaxation of problem (5), then a Walrasian equilibrium also exists formed by the optimal primal solution  $T^*$  and by the optimal dual solution  $p^*$  (Bikhchandani and Mamer, 1997).*

*Proof.* If  $T^* = \{q_1^*, \dots, q_F^*\}$  is a feasible allocation (i.e. an integer solution) obtained by solving the linear programming relaxation of problem (5) and  $p^*$  and  $u^*$  are optimal solutions to the its dual problem (7), then because of complementary slackness condition (8a),  $q_f^*$  is a demand for each flight  $f$ , while for condition (9a) any TW  $j$  that is not allocated has a price equal to zero, i.e. for all  $k \in q$  and  $q \in Q_f$  such that  $x(f, q) < 1$  then  $p^*(j) = 0$ .  $\square$

Hence this implies an even stronger condition that individual rationality: not only every flight diminishes its total cost by implementing the exchange,

but also no flight may prefer another exchange different from  $T^*$  because this is the exchange with maximal value, given prices  $p^*$ . Then the exchange is Pareto efficient.

The resulting mechanism that assigns to each flight a bundle  $q_f^*$ , optimal for problem (5) as well as for its linear programming relaxation, charges the dual prices  $\sum_{k \in q_f^*} p^*(k)$  and refund the dual prices  $\sum_{k \in a_f} p^*(k)$  for the FPFS released bundle is weakly budget balanced, hence it can produce a monetary surplus for the auctioneer.

In fact all TWs unassigned under FPFS which are assigned in the optimal allocation have a price  $p^*(k) > 0$  according to (9b). This price has to be payed by the receiving flight but is not due to anyone since that slot was unallocated under FPFS. On the contrary, according to (9a) any TW  $i$  that is not assigned in the optimal allocation has a price  $p^*(i) = 0$ , meaning that the flight giving up a slot received under FPFS which is not allocated by the market mechanism does not receive any compensation for it. This is in contrast with the case of a single capacity-constrained resource, where all the TWs unassigned by FPFS (and only them) remain unassigned in the market allocation, and the mechanism is strongly budget balanced (cf. Property 2).

Consider in fact the case of 2 capacity constrained resources  $(A, B)$  and 3 flights:  $f_1$  crossing only resource  $A$  in  $E_1^A$ ,  $f_2$  crossing  $A$  and  $B$  respectively in  $E_2^A > E_1^A$  and  $E_2^B$  and  $f_3$  crossing only resource  $B$  in  $E_3^B > E_2^B$ . Suppose that  $E_1^A$  and  $E_2^A$  correspond to the same  $TW_1^A$ ,  $E_2^B$  correspond to  $TW_1^B$ , while  $E_3^B$  correspond to another  $TW_2^B$  successive to  $TW_1^B$ . The FPFS principle then will assign a delay to  $f_2$  which will move to  $TW_2^A$  and  $TW_2^B$  due to the precedence of  $f_1$  in  $A$  and as a consequence  $f_3$  will be assigned  $TW_3^B$ . Hence  $TW_1^B$  remains unassigned under FPFS but it can be assigned by the market mechanism if prices  $p^*$  are such that:

$$\begin{cases} C(f_1, TW_2^A) + p^*(TW_2^A) < C(f_1, TW_1^A) + p^*(TW_1^A) \\ C(f_2, (TW_1^A; TW_1^B)) + p^*(TW_1^A) + p^*(TW_1^B) < \\ C(f_2, (TW_2^A; TW_2^B)) + p^*(TW_2^A) + p^*(TW_2^B) \\ C(f_3, TW_2^B) + p^*(TW_2^B) < C(f_3, TW_3^B) + p^*(TW_3^B) \end{cases}$$

Bikhchandani and Ostroy (2002) prove that a Walrasian equilibrium for the general TW exchange problem exists if and only if the integrality gap between problem (5) and its LP relaxation is null.

### 3.6.1.1 Structures for the Existence of Walrasian Equilibria

Under certain special restrictions on the agents' valuation functions, on the structure of requests or on the structure of prices however the existence of an equilibrium can be guaranteed. This is the case for example of the single capacity-constrained resource exchange as we proved in section 3.5, where valuations do not exhibit complementarity since each agent is interested only in one TW. Such unitary demand restriction turns the valuation functions to belong to a broader class referred to as gross-substitute valuations (Kelso and Crawford, 1982; Gul and Stacchetti, 1999). Under the gross-substitute assumption it is verified that for every pair of price vectors  $p' \geq p$  (component-wise comparison), the optimal TW package demanded by a flight  $f$  at prices  $p'$  contains all the items in the optimal package demanded by  $f$  at prices  $p$ , whose price remained constant. Whenever there are complementarities in valuations functions as in the case of multiple constrained resources, gross-substitutes property does not hold anymore.

Yet another restriction that guarantees the existence of a Walrasian equilibrium is on the structure of the requests. Consider a tree  $T$ , i.e. a connected graph without cycles, with a distance function  $d$  associated and for each vertex  $v \in T$  let  $N(v, r)$  denote the set of all vertices in  $T$  that are within distance  $r$  from  $v$ . Consider that vertices in  $T$  represent individual TWs connected by a path with no cycles. Then if all the requests are constrained to be of the type  $N(v, r)$ , the constraint matrix associated with the LP relaxation of problem (5) is balanced, i.e. it contains only 0-1 elements and has no square submatrix of odd order with exactly two 1's in each row and column (Schrijver, 1986). In this case the integrality gap between problem (5) and its LP relaxation is null.

Rothkopf et al., 1998 and de Vries and Vohra, 2003 identify other sufficient conditions that guarantee the integrality of the solutions to linear programs,

most of which correspond with the Set Packing formulation of problem (6), also referred to as the OR version of the problem. However the unique assignment constraints (5c) in our formulation, which is usually referred to as the Exclusive OR (XOR) version, destroy most of the properties of the matrix associated to the correspondent OR formulation that guarantee integrality.

### 3.6.2 Non-linear Prices

Bikhchandani and Ostroy (2002) extend the concept of Walrasian equilibrium to a general concept of competitive equilibrium, by assuming non-linear prices for packages of TWs. They re-formulate the maximum value assignment through the following linear program:

$$\max \sum_{f \in \mathcal{F}} \sum_{q \in Q_f} V(f, q) x(f, q) \quad (10a)$$

subject to

$$\sum_{q \in Q_f} x(f, q) = 1 \quad \forall f \in \mathcal{F} \quad (10b)$$

$$\sum_{f \in \mathcal{F}, q \in Q_f: q \ni b} x(f, q) \leq \sum_{m \ni b} y(m) \quad \forall b \in B \quad (10c)$$

$$\sum_{m \in M} y(m) \leq 1 \quad (10d)$$

$$x(f, q), y(m) \geq 0 \quad \forall f \in \mathcal{F}, q \in Q_f, m \in M \quad (10e)$$

Where the notation is the same as in model (5), with the addition of the set  $B$ , composed by all the feasible TW bundles which belong to the set of requests for at least one flight, i.e.  $B = \{\cup b : b \in Q_f, \forall f \in \mathcal{F}\}$ . This formulation extends the LP relaxation of problem (5) by adding  $M$ , which denotes the set of feasible partitions of TW into packages.

For example if we have 2 sectors ( $P, Z$ ) with their allocation lists  $S_P = \{1..7\}$  and  $S_Z = \{1..7\}$  and 2 flights ( $f_1, f_2$ ) which cross both  $P$  and  $Z$ , then each flight will need a package composed by two TWs, one for each crossed sector. In this case we indicate  $[(1, 4), (2, 5)]$  as a feasible partition of TWs

into packages which assigns TW 1 in sector  $P$  and TW 4 in sector  $Z$  to one flight and TW 2 in sector  $P$  and TW 5 in sector  $Z$  to the other flight.

The TWs not assigned can be considered to form a dummy package which completes the partition, while variable  $y(m) = 1$  indicates that allocations of TWs to flights must be restricted to the bundles in partition  $m \in M$ . For example if partition  $[(1, 4), (2, 5)]$  is selected, then the only valid allocations are those that assign  $(1, 4)$  to some flight and  $(2, 5)$  to the other one. Constraints (10c) and (10d) replace constraint (1b) and impose that each TW is not allocated more than once. The dual is:

$$\min \sum_{f \in \mathcal{F}} u(f) + \pi \quad (11a)$$

subject to

$$u(f) + p(q) \geq V(f, q) \quad \forall f \in \mathcal{F}, q \in Q_f \quad (11b)$$

$$\pi - \sum_{b \in m} p(b) \geq 0 \quad \forall m \in M \quad (11c)$$

$$p(b), \pi \geq 0 \quad \forall f \in \mathcal{F}, b \in B \quad (11d)$$

Constraints (11b) and (11c) correspond to primal variables  $x(f, q)$  and  $y(m)$  respectively, while variables  $u_i, p(b)$  and  $\pi$  correspond to constraints (10b), (10c) and (10d) respectively.

Variables  $p(b) \quad \forall b \in B$ , can be interpreted as bundle prices, which are now nonlinear since  $p(b) \neq p(b_1) + p(b_2)$ , for some  $b = b_1 \cup b_2$  and  $b_1 \cap b_2 = \emptyset$ . Variable  $u(f) = \max_{q \in Q_f} \{V(f, q) - p(q)\}$  can be interpreted as the maximal utility to flight  $f$  at prices  $p(q)$  and  $\pi = \max_{m \in M} \sum_{b \in m} p(b)$  as the maximal price volume which can be generated by the exchange.

Constraint (11b) is equivalent to the original constraint (7b) which imposed individual rationality, then it continues to hold that the exchange between the FPFs assigned bundles and the bundles optimal for model (10) at prices  $p^*(b)$  is individual rational.

Prices however are no more linear, so Bikhchandani and Ostroy (2002) extend the definition of Walrasian equilibrium (i.e., a pricing equilibrium with

linear, anonymous prices) to the general one of **competitive equilibrium**, which is defined as an allocation  $T^*$  and a set of prices  $p^*$ , according to which each flight receives the utility maximizing bundle at prices  $p^*$  and the allocation  $T^*$  is the one with maximal prices among all feasible partitions. Optimal dual prices support a competitive equilibrium whenever the solution to the primal is integral. However it is not guaranteed that problem (10) always gives integer solutions.

Let's assume for example that there are 3 flights  $(f_1, f_2, f_3)$ , each one crossing 2 consecutive sectors in the same order, and their requests (and costs associated) are the following:

$$\begin{aligned} Q_{f_1} &= \{(1, 4), (0); (1, 5), (57); (2, 5), (76); (2, 6), (171); (3, 6), (190)\} \\ Q_{f_2} &= \{(1, 3), (0); (1, 4), (24); (2, 4), (36); (2, 5), (96); (3, 5), (108)\} \\ Q_{f_3} &= \{(1, 4), (0); (2, 4), (7); (2, 6), (42); (3, 6), (49); (3, 7), (84)\} \end{aligned}$$

The set  $M$  will be composed by 10 different feasible partitions:

$$\begin{aligned} M &= \{[(1, 4), (2, 5), (3, 6)]; [(1, 4), (2, 5), (3, 7)]; [(1, 4), (3, 5), (2, 6)]; \\ &[(1, 5), (2, 4), (3, 6)]; [(1, 5), (2, 4), (3, 7)]; [(2, 5), (1, 3), (3, 6)]; \\ &[(2, 5), (1, 3), (3, 7)]; [(2, 6), (1, 3), (3, 7)]; [(2, 6), (1, 4), (3, 7)]; \\ &[(3, 6), (1, 3), (2, 4)]\} \end{aligned}$$

Suppose that FPFS assigns bundle  $(1, 5)$  to  $f_1$ ,  $(2, 4)$  to  $f_2$  and  $(3, 6)$  to  $f_3$ . In this case the optimal solution to problem (10) is fractional ( $x(1, 1) = x(1, 3) = x(2, 1) = x(2, 3) = y(1) = y(10) = \frac{1}{2}, x(3, 4) = 1$ ) and the objective value is  $Z_{LP}^* = 37$ . The optimal feasible integer allocation ( $x(1, 3) = x(2, 1) = x(3, 4) = y(6) = 1$ ), obtained by substituting  $x(f, q), y(m) \in \{0, 1\}$  in constraint (10e) implies an optimal exchange with value  $Z_{IP}^* = 17$ . In this case there are not dual variables corresponding to supporting prices, since there is a gap between the integer program and its linear relaxation.

Parkes (2001) proves that the following conditions are sufficient for the solution of problem (10) to be integral:

- The **safe bids** condition: each pair of bundles in the set of requests is non-disjoint, i.e., they share at least one item:  $\forall q_1, q_2 \in Q_f \quad q_1 \cap q_2 \neq \emptyset$ ;
- The supermodular valuations condition:  $\forall q_1, q_2 \subseteq M \quad V(f, q_1) + V(f, q_2) \leq V(f, q_1 \cup q_2) + V(f, q_1 \cap q_2)$ .

The latter condition implies that all TW bundles are complements for flights, which is not the case since each flight just requires 1 TW on each resource and the cost of delay represented by two non intersecting bundles is just the min cost of delay between them.

The safe bids condition is neither automatically verified since non intersecting bundles can belong to the request set of a flight. It could be artificially imposed by adding one flight-specific dummy TW to each bundle in the flight's request set. However this pre-processing transforms the problem in another specific version of the third-order formulation (12) proposed by Bikhchandani and Ostroy (2002):

$$\max \sum_{f \in \mathcal{F}} \sum_{q \in Q_f} V(f, q) x(f, q) \quad (12a)$$

subject to

$$\sum_{q \in Q_f} x(f, q) = 1 \quad \forall f \in \mathcal{F} \quad (12b)$$

$$x(f, q) \leq \sum_{m \ni [f, q]} y(m) \quad \forall f \in \mathcal{F}, q \in Q_f \quad (12c)$$

$$\sum_{m \in M'} y(m) \leq 1 \quad (12d)$$

$$x(f, q), y(m) \geq 0 \quad \forall f \in \mathcal{F}, q \in Q_f, m \in M' \quad (12e)$$

This formulation extends the linear programming relaxation of model (10) by modifying  $M$  into  $M'$ , which is the set of all feasible partitions of TW into packages to be specifically assigned to different flights, where  $[f, q] \in m$  indicates that a flight-partition  $m \in M'$  contains bundle  $q$  designated for the specific flight  $f$ . Variable  $y(m)$  corresponds to the selection of a flight-partition  $m \in M'$ .

For example with 2 flights  $(f_1, f_2)$  and 2 TWs  $(1, 2)$  belonging to both request sets  $Q_{f_1}$  and  $Q_{f_2}$ , the set of agent-partitions is:

$$M' = \{[(f_1, 1), (f_2, 2)], [(f_1, 2), (f_2, 1)]\}$$

then if partition  $[(f_1, 1), (f_2, 2)]$  is selected the only valid allocations are those that assign TW 1 to  $f_1$  and TW 2 to  $f_2$ .

The difference with model (10) is in constraint (10c), which becomes (12c), all other constraints remaining the same. For the 3 flights example reported before, the set  $M'$  of agent-partitions enlarges to 84 elements, since in this formulation the model distinguishes for example between allocation  $[(1, 4), (2, 5), (3, 6)]$  and another feasible allocation  $[(1, 4), (3, 6), (2, 5)]$ .

The dual is:

$$\min \sum_{f \in \mathcal{F}} u(f) + \pi \quad (13a)$$

subject to

$$u(f) + p(f, q) \geq V(f, q) \quad \forall f \in \mathcal{F}, q \in Q_f \quad (13b)$$

$$\pi - \sum_{[f, q] \in m} p(f, q) \geq 0 \quad \forall m \in M \quad (13c)$$

$$u(f), p(f, q), \pi \geq 0 \quad \forall f \in \mathcal{F}, q \in Q_f \quad (13d)$$

The third-order equilibrium obtained by solving problem (12) on our former example implies the allocation  $(x(1, 3) = x(2, 1) = x(3, 4) = y(6) = 1)$  which is integral and supported by non-anonymous bundle prices  $p(f, q)$ , which are dual variables corresponding to primal constraints (12d). For example at equilibrium, the same bundle of slot  $(1, 4)$  is priced differently depending on the requesting flight:  $p(1, (1, 4)) = 118$ ,  $p(2, (1, 4)) = 83$ ,  $p(3, 1, 4) = 49$ , even if it is not part of the optimal allocation.

Although it is always possible to add inequalities to a linear program to make the optimal solution integral (see e.g. Schrijver, 1986), the formulation by Bikhchandani and Ostroy (2002) is very powerful because it has a natural

economical interpretation of dual variables as non-linear and non-anonymous auction prices.

Bikhchandani and Ostroy (2002) prove the following theorem:

**Theorem 3.** *The optimal solution to linear problem (12) is always integral and therefore a competitive equilibrium with non-linear and non-anonymous prices always exist.*

Therefore by employing prices on packages rather than on individual TWs and by discriminating among different flights, one can ensure the existence of a competitive equilibrium, which constitutes an extension of the classical Walrasian equilibrium with linear prices. However this guarantee comes at the expenses of the computational complexity, due to the exponential increase of the solution space in the number of feasible allocations and flights, since in practice problem (12) stores one  $y$  variable for every possible solution.

Additionally the bundle prices (both the anonymous obtained by model (11) when they exist and non-anonymous obtained by model (13)) do not guarantee the exchange to be budget balanced, since the sum of the prices of FPFS packages can be larger than the total price of market packages. This implies that the central authority should pay more to the flights as compensation for the packages they release than it receives from them for the optimal packages.

### 3.6.3 The AkBA Model

Wurman and Wellman (2000) formulate the ‘Ascending k-Bundle Auction mechanism’ (AkBA), which can always determine equilibrium prices supporting the combinatorial exchange problem (5), under certain assumptions. In particular this mechanism requires prices to be non-linear, the valuation functions of the agents to be monotone (i.e.  $V(f, q_1) \leq V(f, q_2)$  for all  $q_1 \subseteq q_2$ ), and each participant to require at most one bundle. This last condition can be modified into an equality constraint for our problem (constraint (5c)) all the rest remaining unchanged.

After solving problem (5) to determine the optimal allocation  $T^* = \{q_1^*, \dots, q_F^*\}$  and the correspondent welfare function value  $Z_{IP-E}$ , the mech-

anism employs a dual program due to Leonard (1983), to compute minimal prices for assignment problems. We can adapt this model in order to fit with our problem, in the following way:

$$\min \sum_{q \in T^*} p(q) \quad (14a)$$

subject to

$$u(f) + p(q) \geq V(f, q) \quad \forall f \in \mathcal{F}, q \in T^* \quad (14b)$$

$$p(q) \geq 0 \quad \forall f \in \mathcal{F}, q \in T^* \quad (14c)$$

$$\sum_{f \in \mathcal{F}} u(f) + \sum_{q \in T^*} p(q) = Z_{IP-E} \quad (14d)$$

where  $T^*$  is the set composed by all bundles allocated by the primal problem (5)<sup>1</sup> and  $Z_{IP-E}$  is the welfare attained by the optimal exchange. The combinatorial exchange problem (5) restricted to bundles  $q \in T^*$  is defined as the assignment subproblem because it omits from the formulation all the bundles unassigned in the optimal allocation. The price vector  $p^*$  obtained by solving problem (14) is a price equilibrium for the assignment subproblem, because constraints (14b) impose that for each flight  $f$  no other bundle than the one received can increase its utility. In particular this solution is the minimum price equilibrium. In their original formulation Wurman and Wellman (2000) formulate a model to obtain maximum price equilibria by simply changing the objective function (14a) into  $\min \sum_{f \in \mathcal{F}} u(f)$ , all other constraints remaining the same. In our setting however this problem is unbounded since  $u(f) \geq 0$ .

Once all the bundles  $q_f^* \in T^* = \{q_1^*, \dots, q_F^*\}$  have been priced and the correspondent utilities  $u^*(f)$  determined, the price on unassigned bundles  $b \in B_0$  is calculated according to the following:

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<sup>1</sup>In their formulation Wurman and Wellman (2000) include in  $T^*$  one dummy item  $\phi_i$  for each unallocated agent  $i$  and they impose a value  $V(i, \phi_i) = 0$  for it. This is required by the inequality constraint they have in the primal problem instead of constraint (5c) as in our case

$$p(b) = \max_{f \in \mathcal{F}} [V(f, b) - u^*(f)] \quad \forall b \in B_0$$

An exchange mechanism that adopts those non-linear prices is therefore individually rational due to constraints (14b), but not budget balanced, since prices  $p(b)$  on bundles that were allocated by FPFS which remain non allocated in the optimal allocation  $T^*$ , have to be payed by the auctioneer in order to compensate flights releasing them.

### 3.6.4 Vickrey-Clark-Groves (VCG) prices

The Vickrey-Clarke-Groves (VCG) class of auction (Vickrey, 1961; Clarke, 1971; Groves, 1973) is central in auction theory and mechanism design. VCG auction is a sealed-bid, one-shot type of combinatorial auction which is incentive compatible and produces an allocation that is economically efficient. It provides a dominant-strategy solution to the combinatorial exchange problem, which is centrally solved by the auctioneer who calculates optimal allocation and prices based on the information announced by participants. The particular form of payments make truth-revelation the dominant strategy for each of them, independently from what is the information communicated by others.

The efficient exchange  $T^*$  with optimal value value  $Z_{IP-E}(\mathcal{F})$  is first calculated centrally according to the problem (5) including all  $f \in \mathcal{F}$ . Problem (5) is successively re-computed with each flight  $g \in \mathcal{F}$ , together with its FPFS allocated bundle  $a_f$ , taken out of the exchange in turn:

$$Z_{IP-E}(\mathcal{F} \setminus g) = \max \sum_{f \in (\mathcal{F} \setminus g)} \sum_{(q \in Q_f: q \cap a_f = \emptyset)} V(f, q) x(f, q) \quad (15)$$

subject to constraints (5b), (5c), (5d)

Then each flight  $g$  payment is calculated as follows:

$$p_{vcg}(g) = V(g, q_f^*) - [Z_{IP-E}(\mathcal{F}) - Z_{IP-E}(\mathcal{F} \setminus g)]$$

Each flight thus pays a price equal to its value for the exchange, dis-

counted by the incremental benefit arising from its inclusion into the mechanism. This last term is called the **marginal product** of flight  $g$ . It can be proven that it is always non-negative since the correspondent coalitional game is superadditive, i.e. the characteristic welfare function is superadditive in the number of participants. This is equivalent to prove that, since the exclusion of a flight from the mechanism also implies the exclusion of its FPFS assigned slots, then the maximum value attainable by the exchange without one flight  $g$  will be lower than the maximum value including this flight, since the opportunities for exchanges will increase for both the group  $\mathcal{F}\setminus g$  and the flight  $g$ , i.e.  $Z_{IP-E}(\mathcal{F}) - Z_{IP-E}(\mathcal{F}\setminus g) \geq 0$ .

A negative payment means that the flight receives a compensation, while a positive payment means that it must pay the exchange. In both cases the term  $[Z_{IP-E}(\mathcal{F}) - Z_{IP-E}(\mathcal{F}\setminus g)]$  represents a ‘bonus’, since negative payments are larger (in absolute value) and positive payments are smaller.

The profit deriving from the allocation  $q_g^*$  to flight  $g \in \mathcal{F}$  is:

$$r_{vcg}(g) = V(f, q_g^*) - p_{VCG}(g) = Z_{IP-E}(\mathcal{F}) - Z_{IP-E}(\mathcal{F}\setminus g)$$

The VCG exchange is thus individual rational since each flight  $f$  will derive a profit from the exchange which is always non negative.

The VCG exchange mechanism is **strategyproof** since a flight  $g$  has no interest in misrepresenting its true value for an exchange. In fact if it declares  $\hat{V}(g, \cdot) \neq V(g, \cdot)$  then the central authority chooses the allocation  $\hat{q}^*$  based on this information, i.e. by maximizing  $\sum_{f \in \mathcal{F}\setminus g} \sum_{q \in Q_f} V(f, q)x(f, q) + \sum_{i \in Q_g} \hat{V}(g, i)x(g, i)$  and its profit becomes:

$$\begin{aligned} \hat{r}_{vcg}(g) &= V(g, q_g^*) - \hat{V}(g, q_g^*) + Z_{IP-E}(\mathcal{F}) - Z_{IP-E}(\mathcal{F}\setminus g) \\ &= V(g, q_g^*) + \sum_{f \neq g} V(f, q_f^*) - Z_{IP-E}(\mathcal{F}\setminus g) \end{aligned}$$

The only term which depends on  $g$  is the true valuation  $V(g, q_g^*)$ , thus utility of the exchange does not depend directly on the declared value  $\hat{V}(g, \cdot)$ . The optimal trade instead is calculated upon the declared values in order to

maximize the global welfare, thus a flight should announce  $\hat{V}(g, \cdot) = V(g, \cdot)$  in order to align the optimal solution to the exchange with its own interests.

The implementation of the VCG exchange requires agents to submit the complete set of their valuation functions and successively the resolution of  $F + 1$  optimization problems.

Unfortunately when this VCG pricing rule is adopted in our TW exchange model, the resulting mechanism is individual rational, strategy-proof and allocative efficient but not budget balanced and this comes at no surprise given the impossibility result in Myerson and Satterwhite (1983), which proves that no economical efficient exchange mechanism can guarantee at the same time Individual Rationality, Budget Balance and Incentive Compatibility.

In order for the mechanism to be budget-balanced it should verify that  $\sum_{i \in \mathcal{F}} p_{VCG}(f) \geq 0$ , i.e. the total amount paid by flights must be greater than or equal to the total amount they receive as compensation from the exchange. This is not verified in our setting, in fact if we consider the previous example with 3 flights, we have  $Z_{IP-E}(\mathcal{F}) = 17$ ,  $Z_{IP-E}(\mathcal{F} \setminus i) = 0 \quad \forall i = 1, \dots, 3$  since no exchange takes place without one flight participating and  $p_{vcg}(1) = -36$ ,  $p_{vcg}(2) = 19$ ,  $p_{vcg}(3) = -17$ , hence the central authority must pay 34 at the end of the exchange. Even if we impose  $p_{vcg}(3) = 0$  since flight 3 maintains its FPFs assigned bundle, the central authority must pay 17.

### 3.6.5 VCG-based Prices

To overcome this issue and guarantee the budget balance property, Parkes et al. (2001) propose a payment rule for an exchange that is individual rational and budget balanced and that minimize the distance to VCG payments, thus being quasi strategyproof.

They formalize the problem as the following linear program:

$$\min_{\Delta} L(\Delta, \Delta_{VCG}) \tag{16a}$$

subject to

$$\sum_{f \in \mathcal{F}} \Delta(f) \leq Z_{IP-E}(\mathcal{F}) \quad (16b)$$

$$\Delta(f) \leq \Delta_{VCG}(f) \quad \forall f \in \mathcal{F} \quad (16c)$$

$$\Delta(f) \geq 0 \quad \forall f \in \mathcal{F} \quad (16d)$$

where  $\Delta_{VCG} = (\Delta_{VCG}(1), \dots, \Delta_{VCG}(F))$  is the vector of flights' marginal products as calculated under the classic VCG mechanism (i.e.  $\Delta_{VCG}(g) = Z_{IP-E}(\mathcal{F}) - Z_{IP-E}(\mathcal{F} \setminus g)$ ), which are also referred to as VCG discounts.

The objective of problem (16) is to find discounts  $\Delta = (\Delta(1), \dots, \Delta(F))$  which minimize the distance to VCG discounts according to a suitable distance function  $L(\Delta, \Delta_{VCG})$ . Constraints (16c) ensure that for each flight  $f \in \mathcal{F}$  its VCG discount is an upper bound on the implemented discount, while constraint (16b) ensures budget balance and constraints (16d) ensure individual rationality.

The payment rule is thus formulated to minimize the distance to VCG payments, under different metrics. The simplest distance metric considered by Parkes et al. (2001) is  $L_2(\Delta, \Delta_{VCG}) = \sum_{f \in \mathcal{F}} (\Delta_{VCG}(f) - \Delta(f))^2$ . Then rather than solving problem (16a) directly, they compute an analytic expression for the family of solutions that corresponds to each distance function. Each family of solution constitutes a parametrized payment rule.

For example the Threshold payment rule corresponds to the distance metric  $L_2$ . According to this rule  $\Delta^*(f, C_t) = \max(0, \Delta_{VCG}(f) - C_t)$  depends on the selection of the parameter  $C_t$ , which is selected at its optimal value if it is the smallest  $C_t$  for which the objective function (16b) holds. The payment rule is then implemented by assigning the discount  $\Delta^*(f, C_t^*) = \Delta_{VCG}(f) - C_t^*$  to all flights  $f$  with  $\Delta_{VCG}(f) > C_t^*$ , while all other flights receive a discount  $\Delta^*(f, C_t^*) = 0$ .

Experimental and theoretical analysis performed by Parkes et al. (2001) on several distance functions  $L$ , suggests that this Threshold rule has useful incentive properties and provides allocative efficiency higher than other rules, by removing easy opportunities for manipulation. In particular this rule minimizes the maximal amount that the agent can increase its utility with

some bid by misrepresenting its valuation, all other agents' valuations held constant.

Although residual potential benefits for non-truthful bidding remain, as in all mechanisms which are Individual Rational and Budget Balanced, the strategic behavior for agents with incomplete information about the preferences and strategies of other agents is made difficult.

The computational burden of the central authority is however non negligible under both VCG and VCG-based mechanisms, since  $F + 1$  NP-hard problems must be solved in both cases.

Table 3.1 resumes the characteristics of an exchange mechanism according to the different pricing rules and valuation functions of the bidders:

Prices	Condition for existence	Outcome of the Exchange		
		Ind. Rat.	B. Bal.	Inc. Comp.
Linear	unit-demand	Yes	Yes	No
	gross-substitutes val	Yes	Yes	No
Non-Linear	safe bids	Yes	No	No
	supermodular val	Yes	No	No
Non-Linear AND Non-Anonymous	general val	Yes	No	No
Non-Linear (AkBA)	monotone val	Yes	No	No
VCG	general	Yes	No	Yes
VCG-based	general	Yes	Yes	No

Table 3.1: Properties of different payment functions

For the Non-Linear pricing rule, another assumption that is always required is that each agent must value all bundles. Since the number of possible bundles is exponential with the number of TW available, an automatic rule is usually adopted to evaluate the price of a package not explicitly communicated by the participant. According to this rule, the same value of a given package  $b_0$  is attached to all packages that contain this bundle. However the number of variables still grows exponentially, even if the communication of all the correspondent values is not necessary (Parkes et al., 2001).

# Chapter 4

## Iterative Market Mechanisms

The chapter presented models for determining the welfare maximizing TW exchange that required the participants to directly send their valuation to a central authority. To centrally solve the problem such models need in principle to elicit from all flights  $f \in \mathcal{F}$  their feasible requests  $q \in Q_f$ , as well as the non-linear cost of delay attached  $C(f, q)$ . The optimal exchange is then calculated centrally according to this information, thus implementing a single-round, sealed-bid type of exchange (de Vries and Vohra, 2003). This type of mechanism suffers from the following issues:

- The high computational effort for Airlines of calculating the complete sets of requests and the value associated;
- The high computational cost for the central authority which must determine the welfare maximizing exchange, which is (in the general case) NP-hard (cf. 4).
- The high communication cost of sending the complete set of values over a network;
- The complete disclosure for Airlines of confidential information, which might be considered as private values in a highly competitive environment as commercial aviation.

- The lack of dynamism, since all bids from participants must be communicated before a deadline.

In this section we consider iterative exchange models that allow agents to indirectly send information about their valuations. Under these class of exchange protocols, the mechanism repeatedly interacts with the different agents, aiming to adaptively elicit enough information about their preferences as to be able to find a good (optimal or close to optimal) allocation. The idea is that the adaptivity of the interaction with the bidders may allow pinpointing the information that is relevant to determine the exchange without requiring full disclosure of agents valuations.

This may not only reduce the communication complexity of transferring all the required information to the central authority, but also preserve some privacy about agents' valuations, only requiring the disclosure of information that is really necessary to compute a solution. In addition in the real-life setting, aircraft operators may need a non-negligible effort even for determining their own valuation (e.g. data collection and manpower) and iterative mechanisms may assist them with estimating their valuations by focusing their attention only to relevant data and possibly without requiring precise point estimates but rather valid bounds.

Since the computational burden to determine the exchange is no more entirely on the central authority but is shared by participants, iterative mechanisms are sometimes qualified as distributed or de-centralized. This emphasizes the fact that even if coordination and enforcement tasks are still performed by a central authority, final outcomes are iteratively elaborated through distributed computation of individual optimal solutions given public prices.

Iterative auctions are modeled in general by considering the bidders as “black-boxes”, represented by oracles, where the central auctioneer repeatedly queries these oracles. Several types of queries are possible; value queries are those in which the central authority asks participants to report their values for particular exchanges. Demand queries imply that the central authority communicates prices and the participants report the bundle demanded at these prices. Order queries elicit the relative preference between pairs of

solutions, while bound queries ask if a certain solution is worth at least a given value. These and other types of queries are analyzed in Sandholm and Boutilier (2005).

Parkes et al. (2008) provide a model for the general iterative combinatorial exchange problem that allows full expressiveness of the possible trades. It employs linear price feedback and elicits information from bidders by querying upper and lower values on exchanges. We rather exploit the particular structure of our TW exchange problem, in which the central authority possesses some information regarding flights requests and delay cost structures. This allows to simplify the elicitation by simply employing demand queries which are easier to answer and more transparent from a user perspective.

## 4.1 An Iterative Algorithm for Gross-substitutes Valuations

The most natural type of iterative mechanism for the determination of the optimal exchange is the one in which individual TW prices increase gradually until no TW that is provisionally assigned to one flight is demanded by another. Intuitively, at this point demand equals supply and we are close to a Walrasian equilibrium discussed in Section 3.6.1. This type of algorithm will converge to a solution only in those settings in which a Walrasian equilibrium is guaranteed to exist, namely when the gross-substitutability assumption holds. This is the case of the unique capacity constrained resource, when all TW packages requested by flights reduce to singletons.

The method of Bertsekas (1990) represents one of the earliest attempts to solve the assignment problem in a distributed fashion and it can be adapted to our problem as it follows.

The algorithm starts with the FPFS assignment  $a_f$  allocated to each flight  $f \in \mathcal{F}$  and prices  $p(k) = 0$  for all  $k \in L_s$  where  $s$  is the capacity constrained resource. Then iteratively the central authority proposes to all flights in turn the current TW prices  $p(k)$ ; in response they announce the TW  $j_f$  that maximizes the profit of the exchange with its FPFS assigned TW  $a_f$ , i.e.

$$j_f = \operatorname{argmax}_{k \in Q_f} \{C(f, a_f) - C(f, k) + p(a_f) - p(k)\},$$

and propose for this TW a higher bid price  $\hat{p}(j_f) = p(j_f) + \gamma_i$  where  $\gamma_i \geq 0$  is the cost caused by executing the second best TW exchange after than  $j_f$ , i.e.,  $\gamma_i = \max_{k \in Q_f: k \neq j_f} \{C(f, a_f) - C(f, k) + p(a_f) - p(k)\} - \max_{k \in Q_f} \{C(f, a_f) - C(f, k) + p(a_f) - p(k)\}$ .

The price of each TW is set equal to the highest bid price and it is allocated to the correspondent bidder. If another flight was already assigned this TW earlier in the iteration, it becomes unassigned. This procedure iteratively continues until all flights  $f \in \mathcal{F}$  have a TW allocated. When this condition holds, it has been proven by Bertsekas (1990) that this allocation is also optimal for problem (3) and that the associated prices  $p^*(k)$  are equal to the optimal solutions to problem (4) for all  $k \in L_s$ .

To avoid cycling caused by degeneracy, occurring when a flight  $f$  is indifferent in exchanging or not its *FPFS* TW as the exchange gives no profit, i.e.,  $r(f) = 0$ , in these cases the algorithm sets the profit  $r(f)$  equal to  $-\epsilon$  with  $0 < \epsilon < \frac{1}{|B_{jk}^F|}$ , where  $|B_{jk}^F|$  is the number of TW between  $j$  and  $k$  consecutively assigned under *FPFS* as defined in section 3.5.

This iterative mechanism implements a primal-dual algorithm: it starts with a feasible solution to problem (4) in which  $p(k) = 0$  for all  $k \in L_s$ , and as long as the complementary-slackness conditions are unsatisfied proceeds by improving the solution of the dual program (i.e., increasing some prices).

## 4.2 An iterative algorithm for the general problem

The model presented in previous section 4.1 constitutes an ascending price auction algorithm adapted to our TW exchange problem. There is a single monotonic price trajectory for each TW and the algorithm converges to a Walrasian equilibrium in the cases in which participants present gross-substitute valuation functions.

We want to implement an iterative mechanism that converges under the most general conditions, i.e. when valuation functions are non-linear and exhibit complementarity over different TWs. For this purpose we analyze in the following a distributed algorithm that exploits the decomposition properties of the original problem (5) into independent sub-problems which are locally solvable by Aircraft Operators, without requiring other information than the one relative to each individual flight. Under such mechanism the central authority simply acts as a coordinator, by verifying the acceptability of the flight requests and signaling back to them the unbalance between demand and capacity, through appropriate prices.

In fact, by dualizing constraints (1b) the corresponding Lagrangian relaxation of problem (5) is:

$$\begin{aligned} ZLR_{LP-E}(\lambda) = \max & \sum_{f \in \mathcal{F}} \sum_{q \in Q_f} V(f, q)x(f, q) + \\ & + \sum_{s \in \mathcal{S}, k \in L_s} \lambda_k (1 - \sum_{f \in \mathcal{F}, q \in Q_f: q \ni k} x(f, q)) \end{aligned} \quad (17a)$$

subject to

$$\sum_{q \in Q_f} x(f, q) = 1 \quad \forall f \in \mathcal{F} \quad (17b)$$

$$x(f, q) \geq 0 \quad \forall f \in \mathcal{F}, q \in Q_f \quad (17c)$$

We have thus removed the explicit capacity constraints (1b) from the problem formulation and made them part of the objective function (17a) with associated Lagrange multipliers  $\lambda$ . In this way the solution to problem (17) needs not to be feasible for the original problem (5), however we obtain a problem with an interesting structure. In fact problem (17) is separable into  $F$  problems, one for each flight which can be solved locally by Aircraft Operators, according to problem (18):

$$\begin{aligned}
ZLR_{LP-E}(f, \lambda) = \max \sum_{q \in Q_f} V(f, q)x(f, q) + \\
+ \sum_{s \in S, k \in L_s} \lambda_k (1 - \sum_{q \in Q_f: q \ni k} x(f, q))
\end{aligned} \tag{18a}$$

subject to

$$\sum_{q \in Q_f} x(f, q) = 1 \tag{18b}$$

$$x(f, q) \geq 0 \quad \forall q \in Q_f \tag{18c}$$

Problem (18) is a linear problem and can be locally solved in polynomial time by Aircraft Operators. For each TW  $k$ , its price  $\lambda_k$  is calculated centrally according to the excess of demand for it and then communicated to Aircraft Operators, which will in turn modify their demand according to such prices. The following algorithm (19) can be employed to calculate prices:

$$\lambda_k^{t+1} = \max(0, \lambda_k^t - \Phi^t \cdot SG_k^t) \tag{19a}$$

$$SG_k^t = 1 - \sum_{f \in \mathcal{F}} \sum_{q \in Q_f: q \ni k} x(f, q) \tag{19b}$$

Where  $\Phi^t$  is a positive stepsize chosen at iteration  $t$  and  $SG_k^t$  is a subgradient of  $ZLR_{LP-E}(\lambda)$  at any  $\lambda$  for which  $x$  solves problem (17). Thus ideally the central authority seeks the prices  $\lambda$  that solve the following problem (20), dual to problem (17):

$$ZLR_{DLP-E} = \min_{\lambda} ZLR_{LP-E}(\lambda) \tag{20a}$$

subject to

$$\lambda \geq 0 \tag{20b}$$

Since  $ZLR_{LP-E}(\lambda)$  is a convex, piecewise linear, non-differentiable function, this problem is typically solved through the subgradient algorithm

(19). The resulting procedure iteratively alternates a central price-calculation phase (problem 19) with a local optimization one which finds the maximal value TW-exchange at current prices (problem (18)  $\forall f \in \mathcal{F}$ ). Held et al. (1974) prove that by appropriately choosing the stepsize  $\Phi^t$  such that  $\Phi^t \rightarrow 0$  and  $\sum_t \Phi^t \rightarrow \infty$  for  $t \rightarrow \infty$ , the procedure converges to a solution which minimizes  $ZLR_{LP-E}(\lambda)$ . By weak duality and linear programming theory it will be verified in general that  $Z_{IP-E} \leq Z_{LP-E} \leq ZLR_{LP-E}(\lambda)$ , while it will be  $Z_{IP-E} = Z_{LP-E}$  in the case of null gap between the integer program and its linear relaxation and  $Z_{LP-E} = ZLR_{LP-E} = \min_{\lambda} ZLR_{LP-E}(\lambda)$  when the subgradient algorithm converges to an optimal solution for problem (20). However this solution will be an optimal exchange for the original problem (5) if and only if the gap between  $Z_{IP-E}$  and its linear relaxation  $Z_{LP-E}$  is null, a condition which can only be guaranteed in the case of gross-substitute valuations, such as when all participants compete for TW on a unique resource. In this case when  $Z_{LP-E} = ZLR_{LP-E}(\lambda^*)$ , variables  $\lambda^*$  correspond to optimal equilibrium prices  $p^*$ , which can be obtained by centrally solving problem (7).

Unfortunately even in the case of gross-substitutability there is no guarantee of convergence in a finite number of steps. By stopping the procedure when a feasible exchange for the original problem (5) is demanded at current prices  $\lambda$ , the optimality of the solution will be verified if and only if the complementary slackness condition  $\sum_{s \in \mathcal{S}, k \in L_s} \lambda_k (1 - \sum_{f \in \mathcal{F}, q \in Q_f: q \ni k} x(f, q)) = 0$  holds and  $ZLR_{LP-E}(\lambda) = Z_{IP-E}$  (see Proposition 3 in Larsson et al. (1999) for a formal proof). This is only verified in the case that all unassigned TWs  $k$  such that  $x(f, q) = 0$  for all  $f \in \mathcal{F}$  and  $q \in Q_f$  such that  $k \in q$ , have a price  $\lambda_k = 0$ . Hence when the procedure converges to a solution which is capacity compliant, in general it will be  $ZLR_{LP-E}(\lambda) \geq Z_{IP-E}$ , some lagrangian multipliers  $\lambda_k \geq 0$  will be higher than minimal prices and the solution will not constitute a Walrasian equilibrium. However the correspondent exchange will still be an equilibrium solution and will guarantee individual rationality and weak budget balance. We propose in the following section an heuristic approach for determining an individual rational and budget balanced exchange through a distributed iterative mechanism.

## 4.3 An Heuristic Approach

In order to implement a distributed market mechanism that achieves, in a reasonable amount of time, an exchange which is individual rational and budget balanced we propose in this section a practical heuristic algorithm.

### 4.3.1 The Cost Structure

Given the trajectory of a flight which is published through the Flight Plan and given the set of capacity-constrained resources  $\mathcal{S}$ , it is possible for the central authority to generate the ordered list of resources  $S_f$  crossed by flight  $f$ , as well as the time relation between each pair of consecutive resources, due to the estimated entry time  $E_f^i$  in each element  $i \in S_f$ .

The bundle  $R_f(1) = \{TW_1^f(1), \dots, TW_{|S_f|}^f(1)\}$  constitutes the most preferred bundle and represents the case in which no restrictions were applied to the flight, i.e.  $d_f^{R_f(1)} = 0$  and  $R_f(1)$  has a null cost of delay associated  $C(f, R_f(1)) = 0$ .

From these values it is easy to build a set of acceptable bundles for each flight  $f$ , i.e. the set of requests  $Q_f = \cup_{i=1}^{MaxRqf} R_f(i)$ , where  $R_f(MxRq)$  is the least acceptable request for flight  $f$ , causing the maximum acceptable delay  $MaxDel_f$ .

We will say that for flight  $f$  a request  $A$  is preferred to request  $B$  and we will indicate  $A \succ B$  if and only if  $d_f^B > d_f^A$ .

Since the most preferred bundle  $R_f(1)$  is the one original requested by flight  $f$  and since we cannot anticipate but only delay flights, it will be  $R_f(1) \succ B$  for all  $B \in Q_f \setminus R_f(1)$ . Two different requests  $(A, B) \in Q_f$  can never be indifferent for  $f$ , otherwise they coincide. Further we can assume that for each flight  $f$  the cost  $C(f, \cdot)$  is an increasing function of delay, thus without loss of generality we can impose that  $C(f, B) > C(f, A)$  for all the pairs  $(A, B) \in Q_f$  such that  $A \succ B$ . It follows that the set of requests  $Q_f$  is a totally ordered set and this structure will be exploited in the heuristic for minimizing the elicitation of information about different exchange values.

Hence we can denote a feasible assignment of bundles to flights as a tuple  $(b_1, \dots, b_F) \in Q^F$ , such that no intersecting bundles are assigned (i.e.

$\cap_{f=1}^F b_f = \emptyset$ ) and where  $Q^F = Q_1 \times Q_2 \times \dots \times Q_F$  is the Cartesian product of the sets of flight requests. Since  $Q_f$  can be put in bijective correspondence with its rank set, each feasible assignment can be univocally defined through the tuple of the corresponding ranks  $(r_1, \dots, r_F)$ , where  $R_f(r_f) = b_f$  for all  $f \in \mathcal{F}$ . Then the optimal exchange will be a rank set  $r^* = (r_1^*, \dots, r_F^*)$  that maximizes the value of the exchange with a given FPFs assignment  $(a_1, \dots, a_F)$ , i.e. such that for all other feasible rank sets  $r = (r_1, \dots, r_F)$  it will be  $\sum_{f \in \mathcal{F}} V(f, R_f(r_f)) \leq \sum_{f \in \mathcal{F}} V(f, R_f(r_f^*))$ .

This implies that the optimal rank set  $r^*$  will be non dominated by any other feasible one, i.e. there will be no other feasible rank set  $r$  such that  $r_f \leq r_f^*$  for all  $f \in \mathcal{F}$  and  $r_g < r_g^*$  for at least one  $g \in \mathcal{F}$ , otherwise  $\sum_{f \in \mathcal{F}} V(f, R_f(r_f)) > \sum_{f \in \mathcal{F}} V(f, R_f(r_f^*))$ . This allows to restrict the solutions space to the set of rank sets which are not dominated by any other one, or equivalently to the set Pareto-efficient assignments  $P$ :

$$P = \{(R_1(r_1), \dots, R_F(r_F)) \in Q^F : \bigcap_{f \in \mathcal{F}} R_f(r_f) = \emptyset \text{ AND } r_f < r'_f \forall f \in \mathcal{F}, r'_i \notin P\}$$

The optimal solution will be the allocation with maximal global value among all the Pareto-efficient solutions. Unfortunately the cardinality of  $P$  grows superlinearly with  $MxRq$ , i.e. with the number of admissible requests in each flight's request set and can easily reach thousands of elements even with a dozen flights and  $MxRq = 20$  for each flight. Hence it results impractical to explicitly ask each participant the value of the exchanges in the Pareto-set. The ordered structure of requests however can help in reducing the number of demand queries performed in a sub-gradient procedure.

### 4.3.2 Modified Subgradient

Let us recall the Lagrangian problem (17) formulated in section 4.2 and the subgradient algorithm (19) for solving it. A formula for  $\Phi^t$ , the stepsize

in the subgradient algorithm, which has been proven effective in practice is:

$$\Phi^t = \frac{\mu^t(ZLR_{LP-E}(\lambda^t) - Z_{IP-E}^*)}{\sum_{s \in \mathcal{S}, k \in L_s} (1 - \sum_{f \in \mathcal{F}, q \in Q_f: q \ni k} x^t(f, q))^2}$$

where  $0 < \mu^t \leq 2$ ,  $x^t$  is the vector of solutions to problem (18) at iteration  $t$  according to slot prices  $\lambda^t$ . Usually the scalar  $\mu_t$  is taken at its higher values during first iterations and halved whenever  $ZLR_{LP-E}(\lambda^t)$  has failed to decrease in a specified number of iterations (see Fisher, 1985). In our case the central authority does not know the exchange values  $V(f, q)$  and thus cannot calculate neither  $ZLR_{LP-E}(\lambda^t)$  nor  $Z_{IP-E}^*$ . We then modify formula (4.3.2) in the following formula (21):

$$\hat{\Phi}^t = \frac{\mu^t(UB_{Z^*} - ZLB_{IP-E})}{\sum_{s \in \mathcal{S}, k \in L_s} (1 - \sum_{f \in \mathcal{F}, q \in Q_f: q \ni k} x^t(f, q))^2} \quad (21)$$

where  $UB_{Z^*}$  is an upper bound on the optimal value of the exchange which is held constant through iterations  $t$ . It can be established through simulations and determined for standard cases depending on the cardinality of  $\mathcal{F}$  and of  $\mathcal{S}$ . Instead  $ZLB_{IP-E}$  is a lower bound on the optimal value of the exchange for each instance, which is dynamically adjusted through the course of the distributed mechanism. At iteration  $t$  the central authority can in fact calculate for each bundle  $q$  demanded by flight  $f$  at current prices  $\lambda^t$ , a lower bound on the exact value  $V(f, q)$  for the exchange:

$$LB(f, q) = \sum_{s \in \mathcal{S}, k \in L_s: q \ni k} \lambda_k^t - \sum_{s \in \mathcal{S}, k \in L_s: a_f \ni k} \lambda_k^t \quad (22)$$

For all  $f \in \mathcal{F}$  and  $q \in Q_f$ , the lower bound can be initialized to  $LB(f, q) = 0$  if  $d_f^q \leq d_f^{a_f}$  and to  $LB(f, q) = -\infty$  if  $d_f^q > d_f^{a_f}$ , since we assume non-negative costs of delay. This implies that each flight would exchange its FPFS assigned bundle  $a_f$  with  $q$  for a cost greater or equal to zero whenever  $q$  causes a shorter delay than  $a_f$  or for a negative cost (a taking) whenever  $q$  represents a longer delay than  $a_f$ .

At iteration  $t$  the central authority will calculate the  $LB(f, q)$  value according to formula (22) and it will store it in memory if this value is higher than the previously calculated one.

Also by exploiting the structure of  $Q_f$  as a totally ordered set, it is possible to update with this same value the  $LB(f, b) = LB(f, q)$  all the bundles  $b \in Q_f$  such that  $d_f^b < d_f^q$ , since the value of an exchange is a decreasing function of delay and  $V(f, b) > V(f, j)$ .

The central authority can then solve the following problem:

$$ZLB_{IP-E} = \max \sum_{f \in \mathcal{F}} \sum_{q \in Q_f} LB(f, q)x(f, q) \quad (23a)$$

subject to

$$\sum_{f \in \mathcal{F}} \sum_{q \in Q_f: q \ni k} x(f, q) \leq 1 \quad \forall s \in \mathcal{S}, k \in L_s \quad (23b)$$

$$\sum_{q \in Q_f} x(f, q) = 1 \quad \forall f \in \mathcal{F} \quad (23c)$$

$$x(f, q) \in \{0, 1\} \quad \forall f \in \mathcal{F}, q \in Q_f \quad (23d)$$

Problem (23) is equivalent to problem (5) with  $V(f, q) = LB(f, q)$  for all  $f \in \mathcal{F}$  and  $q \in Q_f$ , then it will be  $ZLB_{IP-E} \leq Z_{IP-E}$ , the strict inequality holding whenever  $LB(f, q) < V(f, q)$  for at least one request  $q \in Q_f$  for some flight  $f \in \mathcal{F}$ . Hence all the integer (feasible) exchanges calculated by solving the linear relaxation of problem (23) with  $V(f, q) = LB(f, q)$  and implemented at prices equal to the dual variables corresponding to constraints (23b), will guarantee individual rationality and weak budget balance, whenever  $ZLB_{IP-E} > 0$ .

The solution obtained (exchanges and prices associated) is not however a Walrasian equilibrium because at the given prices there could be some  $f$  better-off with another exchange than the one implemented and this implies that the solution must be somewhat forced by the central authority. Then after a pre-determined number of iterations or a given elapsed time, if an equilibrium cannot be found by simply alternating local optimization and

price update (i.e. problems (20) and (19)), then the central authority can impose the best solution calculated so far by solving the linear relaxation of problem (23) at the dual prices, i.e. the integer solution which gives the highest positive-value according to  $LB$ , which is the last feasible solution obtained since  $LB$  are always updated by increasing them.

### 4.3.3 Markets and Sub-markets Selection

The complete procedure for TW exchanges can proceed as it follows. The first step is to create a partition of the grand coalition  $\mathcal{F}$  into independent subsets  $M_i \subseteq \mathcal{F}$ , such that for every pair of different flights  $f \in M_i$  and  $g \in M_j$  with  $i \neq j$  it will be  $Q_f \cap Q_g = \emptyset$ . Then each subset  $M_i$  constitutes an independent market, since all the tradable resources will be within the market itself.

From each of these markets  $M_i$ , depending on their dimensions smaller sub-markets (sub-coalitions) of predetermined size  $|SM|$  can be formed and then processed, in order to increase the probability of obtaining integer solutions to the linear relaxation of problem (23), that guarantee the existence of linear prices supporting the exchange (cf. 3.6.1).

In fact by reducing the size of the sub-markets, at the same time the value of the optimal exchange reduces while the percentage of instances for which  $Z_{IP-E} = Z_{LP-E}$  increases. This is due to the fact that by reducing the number of flights, the constraint matrix associated with the LP relaxation of problem (23) has higher probability of being balanced, i.e. not to contain any square submatrix of odd order with exactly two 1's in each row and column. This is a sufficient condition for  $Z_{IP-E} = Z_{LP-E}$  (cf. Section 3.6.1.1).

By forming these sub-markets according to a criteria which first includes flights with higher exchange potential, in few repetitions the procedure can determine the exchanges with highest value and reach a solution that approaches the global optimum as illustrated in charts (4.3.1).

The data correspond to the average on 100 instances obtained by attaching different vectors of costs drawn from the same distribution, to a fixed set of real flight plans data, further described in Section 4.3.5. The Exchange

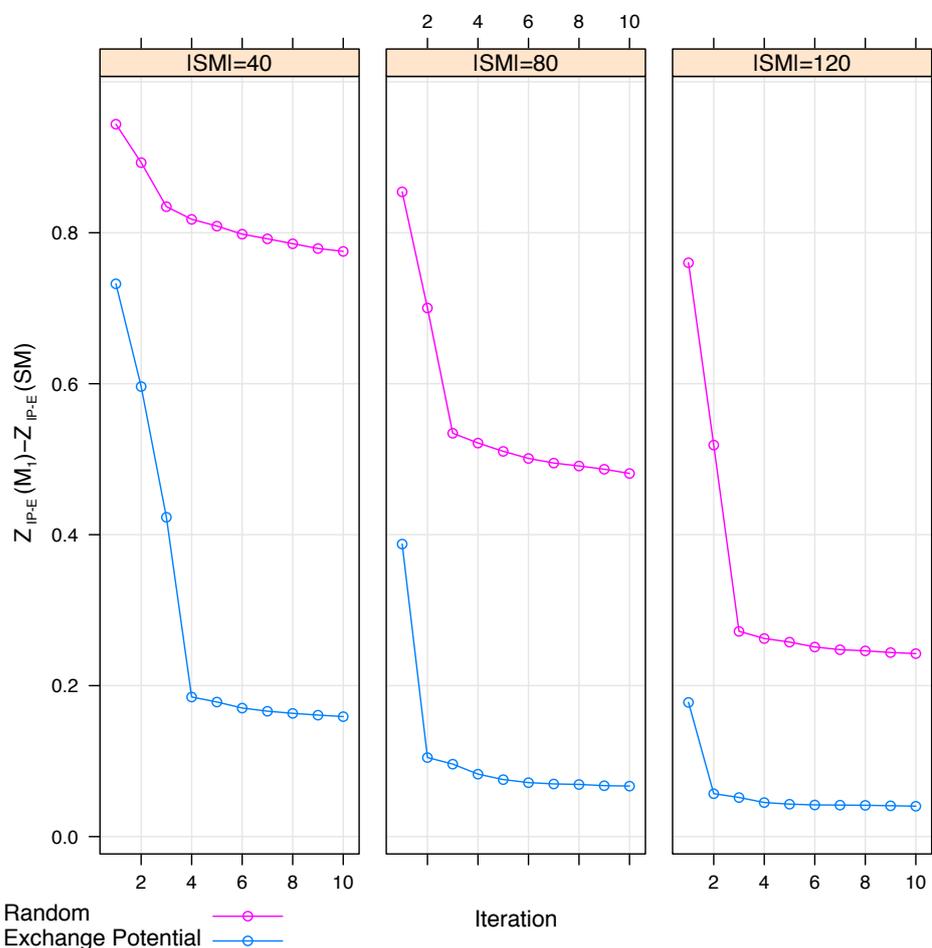


Figure 4.3.1: Gap between the optimal value of the exchange in the Main Market  $M_1$  and in Sub-markets  $SM$ .

Potential series illustrates the case in which 10 sub-markets of fixed size  $|SM|$  are successively created from the main Market  $M_1$  in decreasing order of their potential of exchange, while the random series corresponds to the situation in which sub-markets are formed by including flights randomly from  $M_1$ . The optimal exchange is calculated centrally by solving the LP relaxation of problem (5) on the sub-market  $SM$ , including only flights  $f \in SM$  and the TWs currently assigned to them by either the FPFS allocation or by a previous exchange, and then implemented. For each of the 100 instances this procedure is repeated 10 times by forming sub-markets according to a spe-

cific criteria (i.e. Exchange Potential series) and other 10 times by forming sub-markets randomly (i.e. Random series).

To create sub-markets according to their potential of exchange, flights in the Main Market  $M_1$  are first ordered from the one with the lowest to the one with the highest assigned FPFS request, according to the rank set notation described in Section 4.3.1. Then starting from the head of this ordered list one flight  $f$  is selected as well as its first potential seller  $g$  starting from the tail. A flight  $g$  is a potential seller for flight  $f$  if (i) they share at least one resource  $s$  ( $S_f \cap S_g \neq \emptyset$ ) (ii)  $f$  prefers the TW  $k$  assigned to  $g$  on  $s$  than its currently assigned one  $j$  ( $I_k < I_j$ ) (iii) TW  $k$  is feasible for  $f$  ( $E_f^s \leq U_k$ ). If no potential seller exists the flight next to  $f$  is selected together with its first potential seller.

Only flights which have not been previously included in other sub-markets can be selected. This limits to  $|M_1|/|SM|$  the maximum number of times that the procedure can be repeated. In the case that all flights  $f \in M_1$  have been selected and sub-markets are still to be created, flights at random are included. The complete procedure is described in algorithm (2) in Appendix A.

This procedure has been applied for iterations 1 to 3 in the cases  $|SM| = 80$  and  $|SM| = 120$ , while it has been applied for iterations 1 to 5 in the  $|SM| = 40$  case.

By forming sub-markets with this procedure the exchanges attainable always give a higher global value than in the case trades occur within random coalitions. The higher value exchanges are established during the first iterations, i.e. when trades occur among flights with higher exchange potential, while after 5 iterations only a small number of residual exchanges occur.

The number of instances which give non integer solution to the linear relaxation of problem (5) increases with the size of the sub-market. When  $|SM| = 40$  on average 0.15% of cases are non integer, 3.65% when  $|SM| = 80$  and 4.35% when  $|SM| = 120$ . In these cases the dual variables are not supporting prices and one possible solution could be represented by the exchange optimal for the integer problem (5) and the prices calculated according to a VCG-based payment rule. Even if a competitive equilibrium with linear

prices does not exist for such instances, our heuristic can still converge to a solution which guarantees IR and weak BB. Once the sub-markets have been formed according to criteria described before, the iterative Market Mechanism can be applied to them.

#### 4.3.4 Overall Description of the Iterative Market Mechanism

The diagram in Figure (4.3.2) represents the steps performed by the heuristic procedure. It starts from a given FPFs allocation, that we implemented through algorithm (1) in Appendix A. Independent Markets  $M_i$  are determined and sub-markets  $SM$  are possibly selected through procedure described in algorithm (2) in Appendix A. Each sub-market is then processed by the heuristic which is configured as an iterative exchange with demand-queries. In fact the central authority utilizes linear prices to elicit the preferences of participants.

Individual TW prices are set equal to zero at the beginning, i.e.  $\lambda_k^0 = 0$  for all  $s \in \mathcal{S}, k \in L_s$ . At a subsequent general iteration  $t$  prices  $\lambda_k^t$  are modified according to the subgradient algorithm (19) as a function of the difference between demand and capacity for each TW  $k$ , with stepsize calculated according to formula (21). A flight demand then corresponds to the optimal bundle  $q^*$  such that  $x^t(f, q^*) = 1$ , obtained locally by the Aircraft Operator solving problem (18) with prices  $\lambda^t$ .

The demand of each flight  $f \in SM$  is then used by the central authority to update the lower bounds on actual exchange values through formula (22) for all bundles  $b \in Q_f$  such that  $d_f^b < d_f^{q^*}$ . This allows the central authority to solve the LP relaxation of problem (23) and to determine an Individual Rational and weakly Budget Balanced exchange whenever the solution is integral and has  $ZLB_{LP-E} > 0$ .

In fact when  $ZLB_{LP-E} > 0$  it follows from the definition of lower bound exchange values given in formula (22) that

$$\sum_{f \in SM} \sum_{q \in Q_f: x^t(f, q) = 1} \sum_{s \in \mathcal{S}, k \in L_s: q \ni k} \lambda_k^t > \sum_{f \in SM} \sum_{s \in \mathcal{S}, k \in L_s: a_f \ni k} \lambda_k^t$$

and this implies weak Budget Balance.

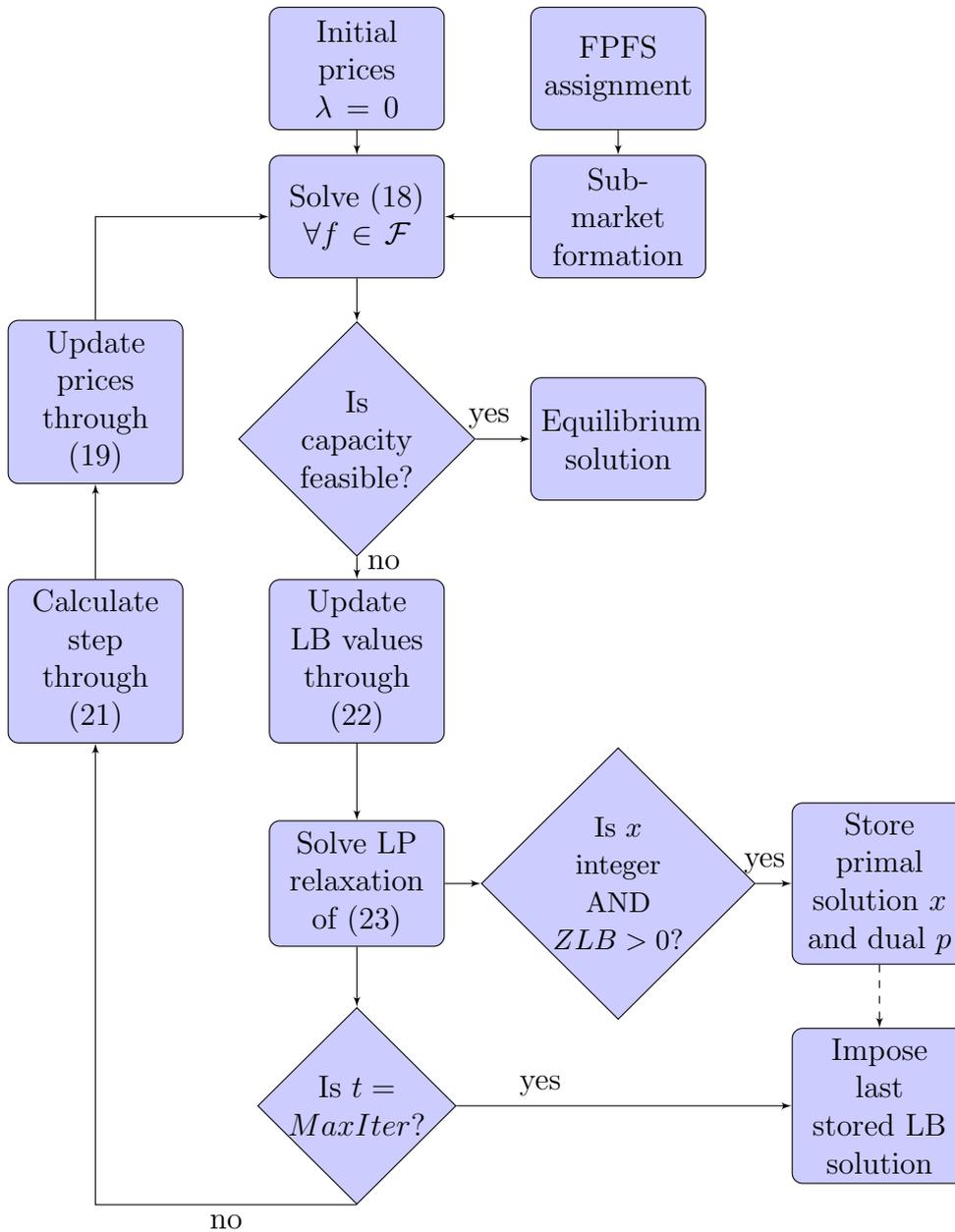


Figure 4.3.2: Schematics of the iterative Market Mechanism

In the case that at a certain iteration  $t < MaxIter$  the TWs demanded by flights constitute a capacity compliant solution, the algorithm stops and the correspondent exchange is executed at prices  $\lambda^t$ . In the case that all unassigned TWs  $k$  have a price  $\lambda_k^t = 0$  the solution corresponds to a Walrasian equilibrium, hence it is equivalent to the exchange which can be centrally calculated by solving problem (5).

When  $t = MaxIter$  the procedure stops and the last feasible solution centrally calculated by solving the LP relaxation of problem (23) is proposed to users. This solution corresponds to a feasible exchange which is weakly Budget balanced and Individual Rational, then all participants (including the central authority) will increase their utility by implementing it. Prices are the solutions of the problem dual to the LP relaxation of problem (23).

### 4.3.5 Computational Results

We simulated the iterative Market Mechanism described in previous section on a sample of traffic retrieved from real CFMU data, relative to the two hours period from 09:00 AM to 11:00 AM on Friday August 15<sup>th</sup>, 2008. There were a total of 60 capacity constrained resources and 482 regulated flights, that were clustered into 3 independent Main markets ( $M1, \dots, M3$ ), with  $|M1| = 425, |M2| = 34, |M3| = 23$ . Market  $M1$  included most of the flights interacting directly or indirectly in the exchange of TW on 58 resources, flights in  $M2$  and only them were affected by a regulation on an upper en-route sector (LECMDOM) in the north of Spain that was limiting traffic flow rate to 43 entries/hour from 09:00 AM to 12:00 AM due to ATC capacity reasons. Flights in  $M3$  and only them competed for the assignment of TW on an upper en route sector (LFMMW2) located in the South of France that was closed from 10.15 AM to 11.30 AM due to ATC routing. In this such cases, i.e. when capacity of a certain resource  $z \in \mathcal{S}$  is null for a given time period  $[st\_time; end\_time]$ , we included just one TW in  $L_z$  with  $I_1 = end\_time$  and infinite capacity in order to make problem feasible without rerouting (the same assumption is used in Terrab and Odoni, 1993).

For each flight  $f \in \mathcal{F}$  we attached a vector cost of delay  $CD_f \in \mathbb{N}^3$ , where

each component represents the per-minute cost of delay according to the magnitude of the delay itself, which has been discretized into the three classes  $[1; 15)$  min,  $[15; 45)$  min,  $[45; MaxDel_f]$  min. Components  $cd_f \in CD_f$  have been randomly drawn from the uniform distribution on the three discrete intervals:

$$cd_f \sim \begin{cases} U(1; 5) & \text{€}/\text{min} & \text{for } d_f \in [1; 15) \\ U(15; 25) & \text{€}/\text{min} & \text{for } d_f \in [15; 45) \\ U(30; 105) & \text{€}/\text{min} & \text{for } d_f \in [45; MaxDel_f] \end{cases}$$

Graphs in Figure (4.3.3) show the results obtained by applying the iterative Market Mechanism described in Section 4.3.4, where  $MaxDel_f$  was fixed for each flight such that  $MaxRq = 200$  for each flight. The value of  $\mu$  in step size formula (21) was fixed at 1.5 at the beginning and modified whenever the value  $ZLB$  was positive and did not change for 3 consecutive iterations:  $\mu \leftarrow \mu/2$  if  $\mu \geq 0.1$ , otherwise it was reinitialized to  $\mu = 1.5$ .

The highest value is achieved by letting all flights  $f \in M_1$  participate in a unique repetition of the mechanism (i.e.  $SM = M_1$ ), however the coordination tasks for the central authority may become slower. In fact at each iteration  $t$  all the optimal demands  $x^*$  at current prices  $\lambda^t$  have to be collected and they are likely to arrive asynchronously, depending on the computational capabilities of the Aircraft Operator. Additionally only a small percentage of solutions obtained by iteratively solve LP relaxation of problem (23) will give feasible integer solutions (26% according to our simulations). This reduces the probability of finding linear prices that support the exchange and may demand for an alternative calculation of prices, such as the VCG-based (cf. Section 3.6.5) thus increasing computational complexity and reducing the transparency of the mechanism, since the prices charged would be different from feedback prices used along mechanism's iterations.

In all the other three cases with  $SM \subset M_1$  the most valuable exchanges occur within the first five repetitions of the procedure on successive submarkets. For the case with  $|SM| = 40$ , 90% of the final value is attained after 4 trades series, while with  $|SM| = 80$  and  $|SM| = 120$  just 2 repetition of the procedure are sufficient to achieve 95% of the final value. This is due

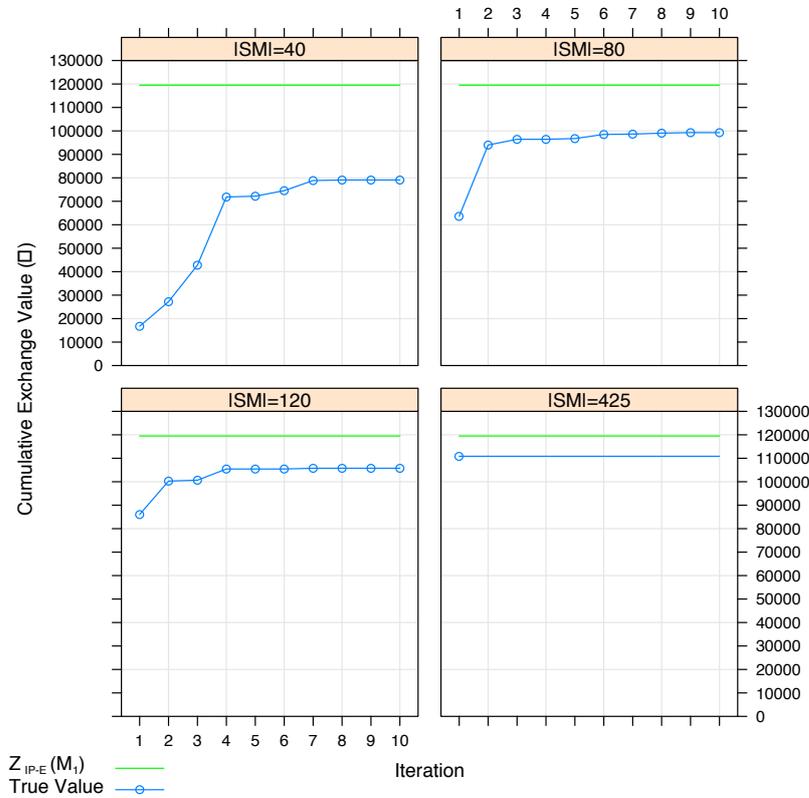


Figure 4.3.3: Value of the exchanges obtained through the iterative mechanism on several Sub-Markets

to the highest number of exchanges which becomes possible by increasing the sizes of coalitions and it is a prove that remarkable cost savings are achievable also by employing a decentralized heuristic market mechanism.

We employed Xpress Mosel v.3.0.0 to code the heuristic procedure and Xpress Optimizer version 20.00.05 to solve all the linear problems. The result figures obtained regarding the cost savings achievable for Aircraft Operators by implementing exchanges are fairly high, especially if we consider that our traffic sample is relative to a 2 hours period. However they represent less than 20% of the total cost of delay originally caused by the FPFS allocation, which equals 740187 €, for the 12989 min of ground delay assigned to flights according to the our algorithm. This figure cannot match exactly the real one computed by the CFMU CASA procedure, due to the differences in both

the behaviors of the algorithms and the environment of application, which is highly dynamical and influenced by many external factors (e.g. cancellations, reroutings, ecc.) in the real world. However this result is perfectly in line with the figures estimated by EUROCONTROL Performance Review Commission (2009), which attribute to direct and reactionary ATFM delays an aggregated cost of  $1.5 \cdot 10^9$  € for year 2008, which correspond to an average cost of about 400000 €/hour if we consider a daily regulated time horizon of 10 hours. EUROCONTROL Performance Review Commission (2009) takes an average cost of delay of 63 €/min equal for all flights, while we simulated all flights with associated their individual costs of delay, in line with figures provided in Cook et al. (2004). The average cost of delay produced by our simulation is 57 €/min.

Graphs in the following figures illustrates the course of the exchanges for three Sub-market sizes, in which we have fixed the cost vector  $CD_f$  for all flights to a single random generated instance. The blue line indicates the true cumulative value of the exchange obtained by the heuristic at the correspondent iteration, while the pink line indicates the correspondent lower bound value centrally calculated by solving problem (23). The subgradient procedure never converged to a capacity feasible solution, however it constituted an efficient algorithm for the pricing problem.

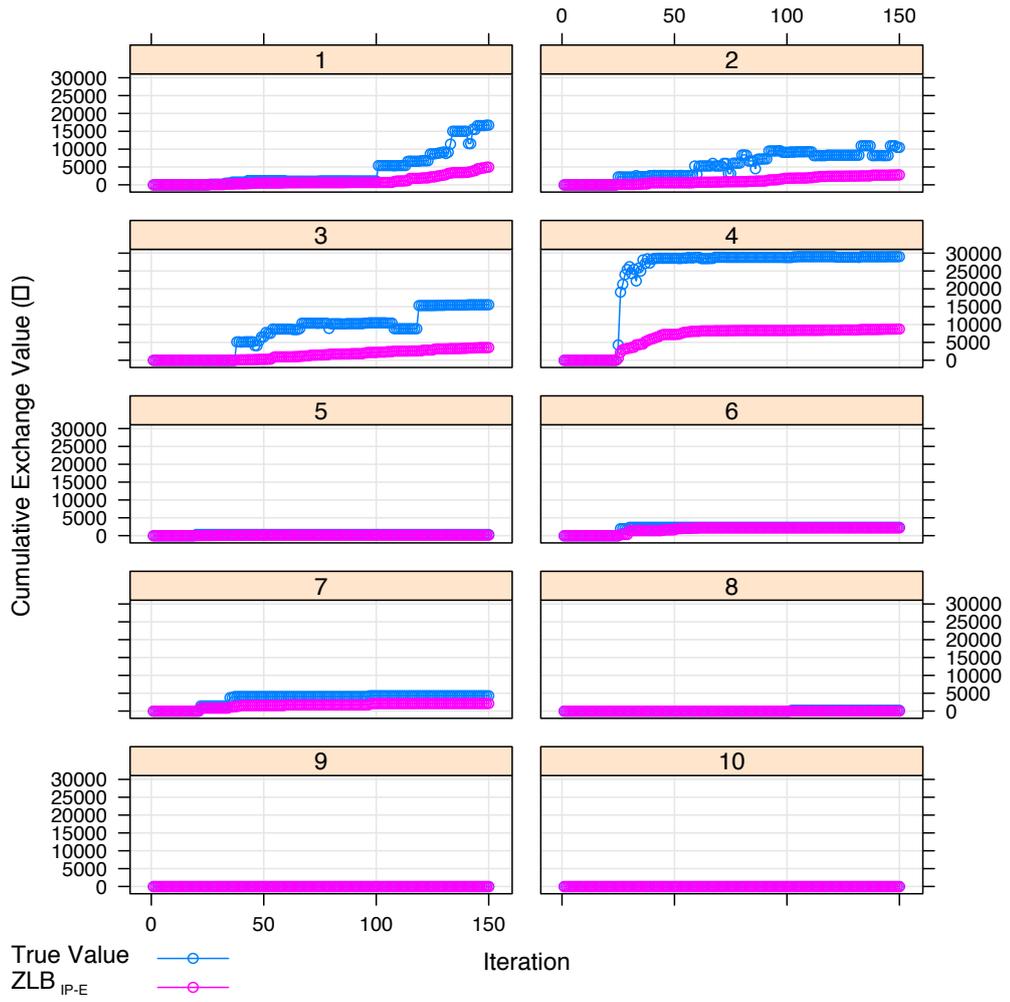


Figure 4.3.4: Course of the iterative mechanism on Sub-Markets with  $|SM| = 40$

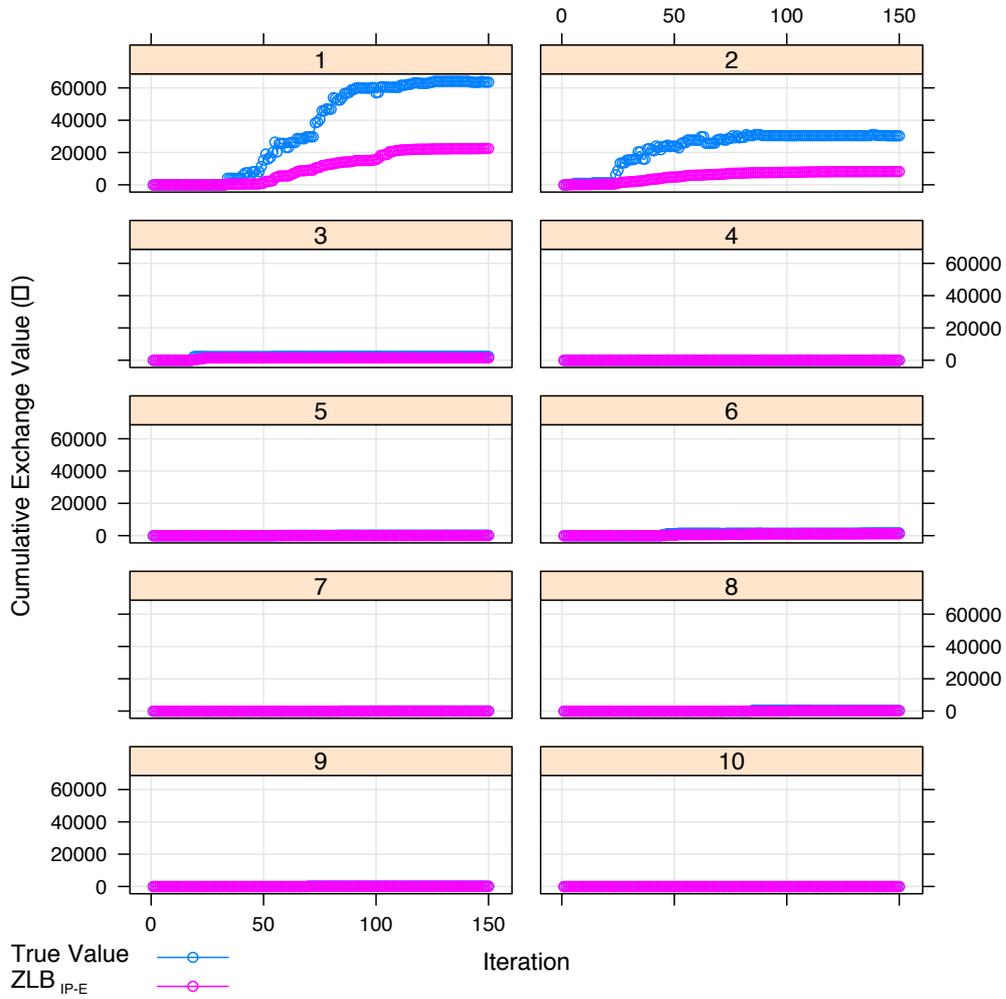


Figure 4.3.5: Course of the iterative mechanism on Sub-Markets with  $|SM| = 80$

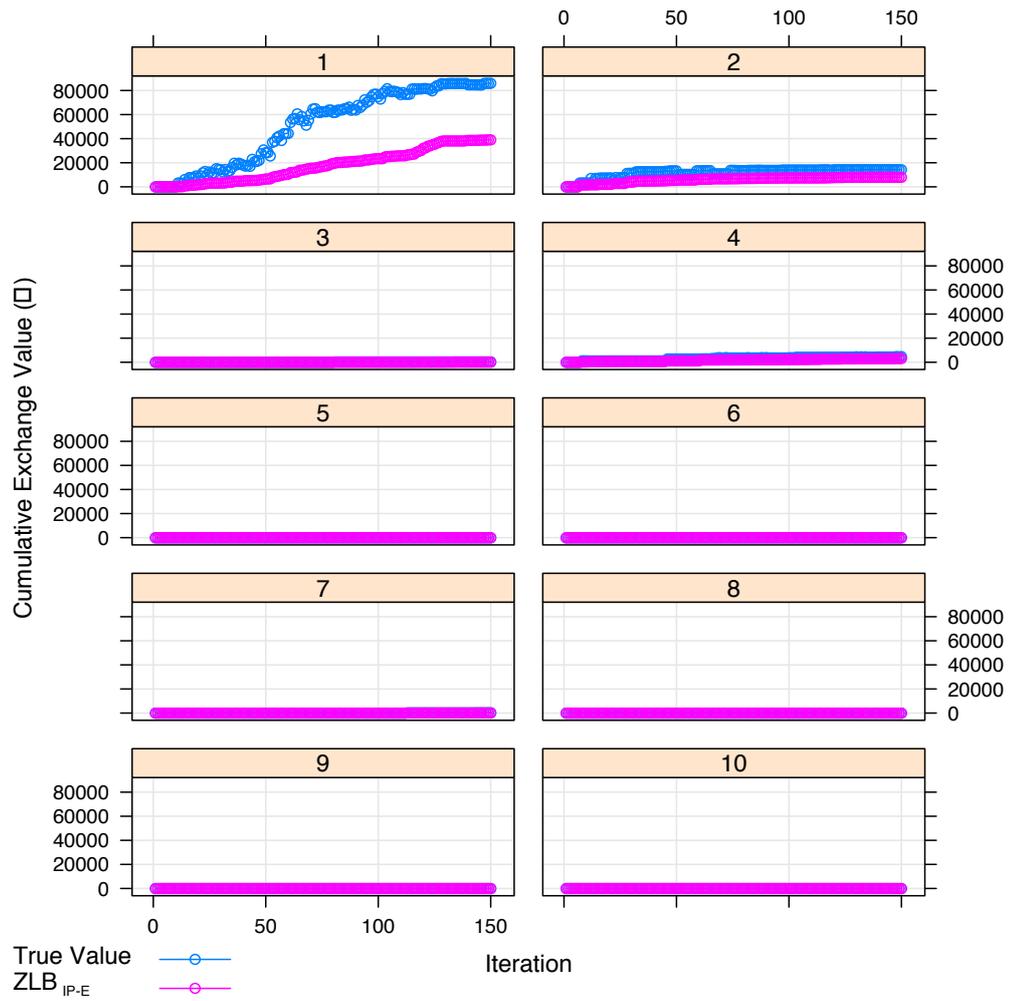


Figure 4.3.6: Course of the iterative mechanism on Sub-Markets with  $|SM| = 120$

Since lower bounds on individual exchanges' values are always adjusted upwards, the global value  $ZLB_{IP-E}$  describes a non-decreasing trajectory until the final value. The true value instead is generally growing but can locally register a decrease during some iteration with respect to the previous iteration, giving a negative relative difference which is not registered by the correspondent LB value. This process of finding the high values exchanges and relative prices could be further fastened if the participants were allowed to update lower bounds not just through their demands but also by explicitly set tighter values according to their internal estimates. The case  $|SM| = 40$  shows the slowest rate of ascent, in fact by stopping the procedure after 100 iterations we get 72% of the value achievable after 150 iterations. In the other cases even by stopping prematurely the procedure we get most of the value: with  $|SM| = 80$  and  $|SM| = 120$  the mechanism reaches 90% of the final value within the first 100 iterations, while in the case  $|SM| = 425$  100% of the final value is reached after the first 94 iterations.

It is hard to provide an estimate of the time required by a real implementation of the mechanism, however it is arguable that by employing a fast communication network as the one provided by SWIM (cf. Section 2.5.2.2) and by automating the Airline interfaces through a proxy for the computation of demands, each iteration will require a time in the order of 5 to 10 seconds, then in less than 30 minutes the most valuable exchanges could be calculated and implemented.



# Chapter 5

## Conclusions

The Air Traffic Flow Management is the complex task of regulating air traffic in order to ensure that the available capacity of the system resources is always respected and used efficiently. In order to achieve this objective, ground delays are systematically imposed to flights which are foreseen to cross congested resources, in order to avoid airborne delays.

The impact of ATFM delays on the costs experienced by airspace users (prevalently commercial Airlines) under the current system is non-negligible, in particular due to the First-Planned-First-Served allocation policy employed today to assign delays to flights, which does not take into account individual users' preferences. This system is likely to be modified under the new paradigm-shift proposed by SESAR for the management of air traffic in Europe in the forthcoming years. According to the SESAR Target Concept, users will be fully involved in the process of demand-capacity balancing through the implementation of ad-hoc collaborative mechanisms that will allow them to cooperatively elaborate solutions that best match their internal business objectives.

This thesis formalizes and analyzes a number of auction-based models and mechanisms that could be employed to assign ATFM resources to competing flights. A key assumption is that both the utility derived by the resource assignment and the cost for achieving it are measured in monetary units and users can transfer this utility among them by adopting proper exchanges.

This assumption implies that there are two types of goods in the resource allocation setting considered: the first is the limited capacity represented by particular resources called Target Windows (TW) and the second is money.

Theoretical results prove that a market equilibrium, representing the optimal solution to the underlying problem, can be guaranteed only under particular conditions that make strong assumptions on the type of users' preferences or on the number of constrained resources or on the structure of prices adopted. Hence an heuristic approach has been undertaken to develop a specific Market Mechanism for the exchange of resources that responds to a number of practical requirements.

This mechanism has been successfully tested on a real sample of traffic data with related costs of delay drawn from a Uniform distribution, in line with reference Cook et al. (2004). All the mathematical models routinely employed by the Mechanism remain linear even if costs of delay experienced by individual flights are non-linear. This is due to the particular formulation of the problem as a combinatorial exchange, that has been shown to be computationally tractable on real instances and to enable the elaboration of good solutions in acceptable times.

The resulting Market Mechanism leverages some of the underlying properties of the specific problem, such as its separability into smaller problems which are locally solvable by airspace users and the characterization of the costs as a non-decreasing function of delays, to determine exchanges of resources among users which are Individual Rational and weakly Budget Balance. These properties imply that all participants as well as the central authority are guaranteed to experience a non negative profit by participating into the mechanism than by accepting the baseline FPFS solution. These features, together with its de-centralized nature and privacy preservation of users' confidential information, make it a good candidate for the adoption as a tactical tool for TW exchanges on a pre-operational phase.

It appears from the results that remarkable cost savings are possible for Aircraft Operators if they were offered the possibility to actively participate in the sequencing of flights imposed by capacity restrictions. Equity is guaranteed by taking as a baseline solution the one obtained by applying a FPFS

priority rule. This sequencing principle is the same adopted under the current ATFM system but it is extended to provide an explicit assignment of multiple TWs on all capacity constrained resources crossed by a flight. This allows Airspace Users to engage in an effective exchange mechanism that permits to considerably reduce their delay-related costs, while at the same time ensuring the respect of capacity constraints.

The introduction of the TW concept presented in the CATS project (cf. Section 2.5.3), constitutes a fundamental tool to achieve CDM capabilities comparable to those already implemented under the U.S. ATFM system. In fact the higher complexity of the European setting deriving from a systematic occurrence of combined en-route and airport regulations, does not allow the direct adoption of CDM mechanisms such as Compression and Slot Credit Substitution (cf. Section 2.3.2.1), which give users high control capabilities on the management of ATFM delays.

At the same time the iterative Market Mechanism prevents the disclosure of users' private information by being decentralized and letting users make independent decisions based on their internal business objectives and according to the value attached by other users to individual TWs on capacity constrained resources. This is in-line with the SESAR target concept (Section 2.5.2) which demand for new methods and tools to manage air traffic flows, that have to respond to a number of requirements in several Key Performance Areas.

While several pricing rules are possible for the combinatorial exchange problem, it seems that a linear one that assigns a price to each TW is the most effective and permits to reach a solution in a vast majority of instances. By explicitly trading individual TWs, possibly bundled in packages, the central authority can assess the value implicitly attached by participants, thus effectively eliciting their preferences. This can provide a baseline for assessing incentive schemes or penalties mechanisms when it comes to tactically enforce the respect of TWs previously traded.

Additionally the dynamic nature of the iterative Market Mechanism proposed makes it suitable for being employed continuously on a rolling horizon basis, since a flight could join it few hours before the take-off, in correspon-

dence to the current Slot Issue Time (cf. Section 2.3.1), and left either when a satisfying exchange occurs or at a fixed time before its original FPFS assigned TW. However this constitute an extension of the mechanism described in Section 4.3.4, that is worth of being further analyzed and simulated.

Another interesting extension of the mechanism could be constituted by including TWs of variable duration, whose price can modify depending on their time extension. This concept should be first validated from the safety perspective, since it implies a new definition of capacity. Rather all the models exposed in this thesis are based on the same definition of capacity adopted under the current ATFM system.

# Appendix A

## Algorithms

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**Algorithm 1** Implement First-Planned-First-Served allocation of flight requests

---

```

1: Input: Set of flights  $\mathcal{F}$ , set of resources crossed  $S_f \ \forall f \in \mathcal{F}$ , set of
   requests  $Q_f \ \forall f \in \mathcal{F}$ , set of resources  $\mathcal{S}$ , slot allocation list  $L_s \ \forall s \in \mathcal{S}$ ,
   estimated time over  $E_f^s \ \forall f \in \mathcal{F}, s \in S_f$ 
2: for all  $f \in \mathcal{F}$  do
3:    $provalloc(f) \leftarrow 0$ 
4: end for
5: while  $AllProcessed = FALSE$  do
6:   for all  $s \in \mathcal{S}$  do
7:      $Ep \leftarrow$  sort flights  $f$  crossing  $s$  by increasing  $E_f^s$ 
8:     for all  $f \in Ep$  do
9:       if  $provalloc(f) = 0$  then
10:         $provalloc(f) \leftarrow AssignFirstFeasible(f, s, 1)$ 
11:         $processed(f, s) \leftarrow TRUE$ 
12:       else if  $processed(f, s) = FALSE$  then
13:         if  $IsFeasible(provalloc(f), f, s) = TRUE$  then
14:            $processed(f, s) \leftarrow TRUE$ 
15:         else
16:           for all  $z \in S_f : z \neq s$  do
17:              $processed(f, z) \leftarrow FALSE$ 
18:           end for
19:            $provalloc(f) \leftarrow AssignFirstFeasible(f, s, provalloc(f))$ 
20:            $processed(f, s) \leftarrow TRUE$ 
21:         end if
22:       end if
23:     end for
24:   end for
25: end while
26:  $noimprovement \leftarrow FALSE$ 
27: while  $noimprovement = FALSE$  do
28:    $noimprovement \leftarrow TRUE$ 
29:   for all  $f \in \mathcal{F}$  do
30:     for all  $q \in Q_f, s \in S_f : q < provalloc(f)$  do
31:       if  $IsFeasible(q, f, s) = TRUE$  then
32:          $provalloc(f) \leftarrow q$ 
33:          $noimprovement \leftarrow FALSE$ 
34:         break
35:       end if
36:     end for
37:   end for
38: end while
39: for all  $f \in \mathcal{F}$  do
40:    $a_f \leftarrow provalloc(f)$ 
41: end for

```

---

The function *AllProcessed* returns *TRUE* if  $processed(f, s) = 1$  for all  $f \in \mathcal{F}, s \in S_f$  and *FALSE* otherwise.

The function *AssignFirstFeasible*( $f, s, k$ ) returns the first requests  $q$  in  $k...MxRq_f$  such that  $TW_i \in q$  has not been assigned to another flight or that was assigned to another flight  $g$  with  $E_g^s > E_f^s$ . In this latter case flight  $g$  becomes status is modified to  $processed(g, s) \leftarrow FALSE$ . In the worst case, i.e. no TW is available for flight  $f$ , it is assigned its last request  $q_w \in Q_f$ . The function *IsFeasible*( $q, f, s$ ) returns *TRUE* if  $TW_s \in q$  for flight  $f$  has not been assigned to any other flight and *FALSE* otherwise.

---

**Algorithm 2** Select sub-market  $SM$  with fixed size  $|SM| = K$

---

Input: Main market  $M_1$ , sub-market size  $K$ , rank set corresponding to the FPFS assignment  $R = (r_1, \dots, r_F)$  such that  $Q_f(r_f) = a_f \quad \forall f \in \mathcal{F}$

- 2: sort flights  $f \in M_1$  by decreasing  $r_f$   
 $SM \leftarrow \emptyset$
- 4:  $i \leftarrow 0$   
**while**  $|SM| < K$  **do**
- 6:    $g \leftarrow M_1(i)$   
        $sg \leftarrow \text{findseller}(g)$
- 8:   **if**  $\text{exists}(g) = \text{TRUE}$  AND  $\text{exists}(sg) = \text{TRUE}$  **then**  
       **if**  $\text{hastraded}(g) = \text{FALSE}$  **then**
- 10:        $SM \leftarrow SM \cup \{g\}$   
       **if**  $|SM| < K$  **then**
- 12:        $SM \leftarrow SM \cup \{sg\}$   
       **end if**
- 14:   **end if**  
        $i \leftarrow i + 1$
- 16: **else if**  $\text{exists}(g) = \text{TRUE}$  AND  $\text{exists}(sg) = \text{FALSE}$  **then**  
       **if**  $i < |M_1|$  **then**
- 18:        $i \leftarrow i + 1$   
       **else**
- 20:       **while**  $|SM| < K$  **do**  
            $SM \leftarrow \text{random}(M_1)$
- 22:       **end while**  
       **end if**
- 24: **else if**  $\text{exists}(g) = \text{FALSE}$  **then**  
       **while**  $|SM| < K$  **do**
- 26:        $SM \leftarrow \text{random}(M_1)$   
       **end while**
- 28: **end if**  
       **for all**  $f \in SM$  **do**
- 30:        $\text{hastraded}(f) \leftarrow \text{TRUE}$
- 32:       **end for**
- 32: **end while**

---

The function  $findseller(f)$  returns the first potential seller for flight  $f$ , if there is one, otherwise it returns a random flight. A flight  $g$  is a potential seller for flight  $f$  if (i) they share at least one resource  $s$  ( $S_f \cap S_g \neq \emptyset$ ) (ii)  $f$  prefers the slot  $k$  assigned to  $g$  on  $s$  than its currently assigned one  $j$  ( $I_k < I_j$ ) (iii) slot  $k$  is feasible for  $f$  ( $E_f^s \leq U_k$ ).



# Appendix B

## Discussion on Incentive Compatibility

This appendix provides a simple example where a flight may find convenient to misrepresent its true cost of delay to increase its payoff, at the expense of other flights, in the simple case of a unique capacity constrained resource  $s$ .

Consider two flights,  $f_1$  and  $f_2$ , and two TWs,  $a$  and  $b$ . Both flights request TW  $a$ . Then we set  $C(f_1, a) = C(f_2, b) = 0$ . We assume that  $0 < C(f_1, b) < C(f_2, b)$  and that  $E_{f_1}^s < E_{f_2}^s$ . The *FPFS* policy allocates flight  $f_1$  at TW  $a$  and flight  $f_2$  at TW  $b$ . Let  $r(f_1)$  and  $r(f_2)$  be the profits of flight  $f_1$  and  $f_2$ , respectively, obtained by selling its FPFS TW and purchasing the other one. We finally assume that flights do not know the costs of delay of each other and that the optimal exchange is centrally calculated by solving problem (3). We want to investigate the opportunity for flight  $f_1$  to cheat about its true cost of delay to get a higher profit. Let  $\hat{C}(f_1, b)$  be the false value of the delay cost displayed by flight  $f_1$  for TW  $b$ , and  $p'(a), p'(b)$  and  $r'(f_1)$  be the corresponding modified TW prices and profit for flight  $f_1$ , respectively.

Flight  $f_1$  (resp.  $f_2$ ) has a nonnegative profit in selling its TW  $a$  (resp.  $b$ ) and purchasing the other TW  $b$  (resp.  $a$ ) at prices  $p(a)$  (resp.  $p(b)$ ) and  $p(b)$

(resp.  $p(a)$ ). In particular,

$$\begin{aligned} r(f_1) &= C(f_1, a) - C(f_1, b) + p(a) - p(b) \\ r(f_2) &= C(f_2, b) - C(f_2, a) + p(b) - p(a) \end{aligned}$$

where  $p(a)$  and  $p(b)$  are the optimal solutions of the following problem (4):

$$\begin{aligned} \min Z &= u(f_1) + u(f_2) + p(a) + p(b) \\ u(f_1) + p(a) &\geq 0 \\ u(f_1) + p(b) &\geq C(f_1, a) - C(f_1, b) \\ u(f_2) + p(a) &\geq 0 \\ u(f_2) + p(b) &\geq C(f_2, b) - C(f_2, a) \\ p(a), p(b) &\geq 0 \end{aligned}$$

In the  $(p(a), p(b))$  space the optimal region is  $C(f_1, b) - C(f_1, a) \leq p(a) - p(b) \leq C(f_2, a) - C(f_2, b)$ . This optimal region has only two finite vertices, i.e.,  $(p(a) = C(f_1, b) - C(f_1, a), p(b) = 0)$  and  $(p(a) = C(f_2, a) - C(f_2, b), p(b) = 0)$ . Using a standard algorithm, as the simplex or the dual simplex algorithm to solve the problem, the optimal solution is always point  $(p(a) = C(f_1, b) - C(f_1, a), p(b) = 0)$ . Hence the profit of flight  $f_1$  in selling its TW  $a$  at price  $p(a) = C(f_1, b) - C(f_1, a)$  and purchasing the TW  $b$  at price  $p(b) = 0$  is  $r(f_1) = 0 - C(f_1, b) + C(f_1, b) - 0 = 0$ .

As long as  $\hat{C}(f_1, b) \leq C(f_2, b)$  the TW allocation remains the same and the optimal solution obtained by the simplex algorithm is  $(\hat{p}(a) = \hat{C}(f_1, b) - \hat{C}(f_1, a), \hat{p}(b) = 0)$ . Hence the modified profit is  $\hat{r}(f_1) = C(f_1, a) - C(f_1, b) + \hat{C}(f_1, b) - \hat{C}(f_1, a)$ . Then  $\hat{r}(f_1) - r(f_1) = \hat{C}(f_1, b) - C(f_1, b)$ . Then if  $0 \leq \hat{C}(f_1, b) < C(f_1, b)$  it follows that  $\hat{r}(f_1) - r(f_1) < 0$ , and if  $C(f_1, b) < \hat{C}(f_1, b) < C(f_2, b)$  we have  $\hat{r}(f_1) - r(f_1) > 0$ . When  $\hat{C}(f_1, b) > C(f_2, b)$ , the TW assignment changes and becomes identical to the FPFS allocation. Hence in this case  $\hat{r}(f_1) = 0$ .

As the value  $C(f_2, b)$  is not known to flight  $f_1$ , we conclude that the flight taking the first TW under the FPFS allocation knows that it does not have to

display a false delay cost lower than the true one because this choice may lead to a profit  $\hat{r}(f_1)$  lower than the true profit  $r(f_1)$ . On the other side, if this flight  $f_1$  communicates to the central authority a false delay cost  $\hat{C}(f_1, b)$  higher than the true one, the profit  $\hat{r}(f_1)$  it gets is higher than or equal to the true profit  $r(f_1)$ . Then the mechanism is not incentive compatible because there is no disadvantage for the first flight in the FPFS allocation to appropriately misrepresent its delay costs.

However these findings assume that we know in advance which is the vertex of the optimal region of problem (4) chosen by the solving algorithm. On the contrary, when we consider as TW prices a generic pair of optimal values  $(p(A), p(B))$  the corresponding profit  $r(f_1)$  can be strictly positive, as the mechanism is by construction individual rational. In this situation, it can be risky for flight  $f_1$  to cheat about its cost of delay  $C(f_1, b)$ . In fact, if it sets its false value  $\hat{C}(f_1, b)$  strictly larger than  $C(f_2, b)$  its profit  $\hat{r}(f_1)$  is equal to 0. Since  $C(f_2, b)$  is unknown to flight  $f_1$ , a misrepresentation of its cost of delay may produce a profit  $\hat{r}(f_1)$  lower than the true  $r(f_1)$ .

Hence when one does not know in advance which are the optimal TW prices, it is impossible to identify up to which limit a false value of the delay cost does not lead to a profit  $\hat{r}(f_1) < r(f_1)$ .



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