Study of the Neutral Decays of the $K_L$ for the CP Violation Measurement at KLOE
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Introduction

The main purpose of the KLOE experiment is to measure the $CP$ violation parameters in the neutral kaon system to a precision of $\mathcal{O}(10^{-4})$. The KLOE detector is running at the DAΦNE collider, a $\phi$-factory built at Laboratori Nazionali di Frascati, near Rome. KLOE started taking data in April 1999. Since the beginning, some problems in the accelerator limited seriously the luminosity, causing a delay in the physics program of the KLOE experiment. Then, a large number of them were understood and partly solved, allowing almost 200 pb$^{-1}$ to be collected in 2001.

In this thesis the measurement of the ratio of $\Gamma(K_L \rightarrow 2\pi^0)$ to $\Gamma(K_L \rightarrow 3\pi^0)$ is presented. Such a measurement is important, because it is affected by the same systematics as the neutral part of the double ratio, used in the determination of $\Re(e'/e)$. In the analysis $\sim$23 pb$^{-1}$, collected in 2000 and available on disk, were used. The result is dominated by the limited statistics, nevertheless it can give a first insight into the present level of the systematic uncertainty on $\Re(e'/e)$ due to the $K_L \rightarrow \pi^0\pi^0$ decay channel. In the following, the contents of this work are outlined.

In the first chapter, an introduction to the topic of $CP$ violation is given. The peculiar properties of the neutral kaon system due to the weak interactions are described and the phenomenology of $CP$ violation in the neutral kaon system, as well as the relevant physical parameters for indirect and direct $CP$ violation, are introduced. Then, the origin of $CP$ violation is explained in the framework of the Standard Model and, finally, the status of theoretical estimates of the direct $CP$ violation parameter $\Re(e'/e)$ and the most recent experimental measurements are briefly reported.

In the second chapter, the DAΦNE collider and the KLOE experimental apparatus are outlined. In the first section, DAΦNE is briefly described. The second part is devoted to the KLOE detector: the structure of the electromagnetic calorimeter, of the drift
chamber, and of the quadrupole tile calorimeter are explained. For each subdetector, its performance, as measured by using real data sets, is given.

In the third chapter, three fundamental and related aspects of the analysis are briefly described. In the first section, the measurement of the boost and the vertex position of the $\phi$ meson is treated. In the second section, the method used to identify the $\phi \rightarrow K_SK_L$ events is explained. In the last section, the kinematical reconstruction of the $K_L$ vertex is described for the case when the $K_L$ decays into pionic fully neutral states.

In the fourth chapter, the measurement of the branching ratio of the $K_L$ meson into three neutral pions is presented. In the first part the method used to perform the measurement is sketched out, then the selection of the sample and the calculation of the efficiencies are described. In the last section, the result is given.

In the fifth chapter, the measurement of the $K_L$ branching ratio into the $CP$ violating final state with two neutral pions is treated. In the first part, after a brief description of the method, the event selection is described. In particular, the kinematical cuts to reduce the background and the fit procedure to estimate the signal are explained. Then, the determination of the geometrical acceptances and the measurement of the efficiencies are described. Finally, the result is given.

In the sixth chapter, the analyses of the previous two chapters are combined to give the ratio of the decay amplitude $\Gamma(K_L \rightarrow 2\pi^0)$ to $\Gamma(K_L \rightarrow 3\pi^0)$. In this case, some of the acceptances and efficiencies calculated for the measurements of the absolute branching ratios simplify either completely or partially, reducing the total systematic uncertainty.
Chapter 1

CP violation in the $K^0$-$\bar{K}^0$ system

In this chapter an introduction to the topic of CP violation is given. The peculiar properties of the neutral kaon system due to the weak interactions are described and the phenomenology of CP violation in the neutral kaon system, as well as the relevant physical parameters for indirect and direct CP violation, are introduced. Then, the origin of CP violation is explained in the framework of the Standard Model and, finally, the status of theoretical estimates of the direct CP violation parameter $\Re(e'/e)$ and the most recent experimental measurements are briefly reported.

1.1 The discrete symmetry transformations

The fundamental constituents of matter, quarks and leptons, and the strong and electroweak forces, interplaying between them, are described by the Standard Model of Elementary Particles [1, 2]. The elementary particles and their interactions are treated in the theoretical framework of the Relativistic Quantum Field Theory [3], where the quarks ($u, d, s, c, b, t$) and the leptons ($e, \mu, \tau, \nu_e, \nu_\mu, \nu_\tau$) are interpreted as quanta of the matter fields and the forces are mediated by the exchange of virtual particles ($g, \gamma, Z, W$), called force carriers.

A basic aspect of our description of elementary interactions is represented by the invariance of the Lagrangian, and consequently of the physical system it represents, under some kind of transformations of the space-time coordinates (space and time translations, spatial rotations, Lorentz transformations, space and time inversions) and of the particles'
internal degrees of freedom (such as flavor, color, and lepton and baryon numbers). Such transformations are referred to as symmetry transformations and are strictly connected to the existence of conserved quantum numbers in the physical systems under consideration. The conservation laws play a twofold role: on the one hand the conserved quantum numbers allow us to label uniquely the particle states and to establish selection rules for the reactions, on the other hand the symmetry principles represent a very convenient guideline for the theoretical formulation of the interaction Lagrangian.

Among the symmetry transformations, traditionally the discrete transformations, and the corresponding invariance of the laws of Nature, have ever attracted the interest of Physicists. This is even more so in the most recent years for the potential insight into new Physics, that they seem to allow. These symmetries are parity or space inversion \((P)\), time inversion \((T)\) and charge conjugation or particle-antiparticle conjugation \((C)\):

\[
P: \quad (t, \vec{x}) \rightarrow (t, -\vec{x}),
\]

\[
T: \quad (t, \vec{x}) \rightarrow (-t, \vec{x}),
\]

\[
C: \quad \text{particle} \rightarrow \text{antiparticle}.
\]

\((1.1)\)

\(P, T, \) and \(C\) are evidently discrete symmetries. The Hermitian operators, which implement \(P\) and \(C\) transformations in the Hilbert space, have discrete eigenvalue spectra (whereas \(T\) has no observable eigenvalues).

Originally, \(P, T, \) and \(C\) were believed to be symmetries of the natural laws. Indeed, as far as electromagnetic and strong interactions are considered, \(P, T, \) and \(C\) are experimentally seen to separately hold. In the Fifties, nevertheless, an unexpected revolution took place. It began with the so-called \(\theta-\tau\) puzzle \([4]\): two particles had been found, \(\theta\) and \(\tau\), which had the same mass and quantum numbers, but decayed to final states with opposite parity, namely, two and three pions in s-wave. The puzzle was solved in 1956 by T.D. Lee and C.N. Yang \([5]\) by hypothesizing that \(\theta\) and \(\tau\) were indeed the same particle (the kaon) and that parity is violated by weak interactions. The confirmation came the following year from an experiment by C.S. Wu et al. \([6]\), studying the \(\beta\)-decay of polarized \(^{60}\)Co nuclei, which demonstrated that \(P\) and \(C\) are violated maximally. The \(CP\) transformation, which consists in a parity flip followed by charge conjugation, was still believed a symmetry of Nature. But in 1964, by observing \(K_L \rightarrow \pi\pi\), J.M. Christenson, J.W. Cronin, V.L. Fitch and R. Turlay \([7]\), announced the surprising result that \(CP\) symmetry was violated in hadronic decays of the neutral kaons, opening a question about the nature of such a violation which has been firmly established only quite recently.

Moreover \(P, T, \) and \(C\) are connected by the \(CPT\) theorem, which is one of the most
important and fundamental principles of Quantum Field Theory. This theorem states that every Quantum Field Theory, relativistically covariant, that admits a minimum energy state and obeys the principle of microcausality (this principle requires that independent measurements can always be done on two space-time points which are outside each other’s light cone), is invariant under the combination of the three operations \( P, T, \) and \( C \) taken in any order. As a consequence of the \( CPT \) theorem, particles and antiparticles have the same mass and lifetime, and magnetic moments equal in magnitude but opposite in sign. So far, all the experimental tests have confirmed the validity of \( CPT \) invariance, obtaining extremely stringent limits on the \( CPT \) parameters, e.g.:

\[
\frac{M_{K^0} - M_{\bar{K}^0}}{M} \lesssim 10^{-18} .
\]  

Thus \( CPT \) invariance will be assumed in the sequel.

1.2 The neutral kaon system

This section is devoted to the neutral kaon system. In the first part the \( K^0-\bar{K}^0 \) mixing, induced by the weak interactions and responsible for the matter-antimatter oscillations, is described. In the second part the topic of \( CP \) violation is discussed. “Indirect” and “direct” \( CP \) violation mechanisms are introduced and the status of the direct \( CP \) violation description within the Standard Model is briefly reviewed.

1.2.1 \( K^0-\bar{K}^0 \) mixing

The strong and electromagnetic interactions conserve strangeness and have well defined strangeness eigenstates, \( K^0 (\bar{s}d) \) with \( S = +1 \) and \( \bar{K}^0 (s\bar{d}) \) with \( S = -1 \). On the other hand, the weak interactions do not conserve strangeness. Consequently kaons, which are the lightest strange particles, can decay to non-strange final states (\( |\Delta S| = 1 \) processes) via weak interactions. Moreover, the neutral kaons can mix together and oscillate between \( K^0 \leftrightarrow \bar{K}^0 \), for example through a common intermediate decay state (\( |\Delta S| = 2 \) processes), such as \( K^0 \rightarrow (2\pi, 3\pi) \rightarrow \bar{K}^0 \) and vice versa.

Under the discrete transformations \( P, C, \) and \( T \), the strong Hamiltonian eigenstates \( |K^0\rangle \) and \( |\bar{K}^0\rangle \) transform in the rest frame \((\vec{p} = 0)\) in the following way [8]:

\[
P |K^0\rangle = -|K^0\rangle , \quad P |\bar{K}^0\rangle = -|\bar{K}^0\rangle , \\
C |K^0\rangle = e^{i\alpha} |\bar{K}^0\rangle , \quad C |\bar{K}^0\rangle = e^{-i\alpha} |K^0\rangle , \\
T |K^0\rangle = e^{i(\theta - \alpha)} |K^0\rangle , \quad T |\bar{K}^0\rangle = e^{i(\theta + \alpha)} |\bar{K}^0\rangle ,
\]  

(1.3)
where $\alpha$ and $\theta$ are arbitrary phases. Since strangeness is conserved in strong and electromagnetic interactions, it is possible to redefine the $|K^0\rangle$ and $|\bar{K}^0\rangle$ phases:

$$
|K^0\rangle \rightarrow e^{-i\xi \hat{S}} |K^0\rangle = e^{-i\xi} |K^0\rangle , \\
|\bar{K}^0\rangle \rightarrow e^{i\xi \hat{S}} |\bar{K}^0\rangle = e^{i\xi} |\bar{K}^0\rangle ,
$$

where $\hat{S}$ is the strangeness operator: $\hat{S} |K^0\rangle = |K^0\rangle$ and $\hat{S} |\bar{K}^0\rangle = -|\bar{K}^0\rangle$. The choice of $\xi$ is completely arbitrary, but the physically observable quantities must be independent of any phase convention.

In particular, a convenient convention is the choice $\xi = (\pi - \alpha)/2$, such that the transformations of $|K^0\rangle$ and $|\bar{K}^0\rangle$ under CP are given by:

$$
CP |K^0\rangle = |\bar{K}^0\rangle , \\
CP |\bar{K}^0\rangle = |K^0\rangle ,
$$

whereas under CPT:

$$
CPT |K^0\rangle = -e^{i\theta} |\bar{K}^0\rangle , \\
CPT |\bar{K}^0\rangle = -e^{-i\theta} |K^0\rangle .
$$

From Eq. (1.5) it is evident that $|K^0\rangle$ is the CP conjugate of $|\bar{K}^0\rangle$ and vice versa. If CP were an exact symmetry also of weak interactions, so that it could be diagonalized simultaneously with the total Hamiltonian, the physical states would be the CP eigenstates:

$$
|K_1\rangle = \frac{1}{\sqrt{2}} (|K^0\rangle + |\bar{K}^0\rangle) , \\
|K_2\rangle = \frac{1}{\sqrt{2}} (|K^0\rangle - |\bar{K}^0\rangle) .
$$

Indeed, $|K_1\rangle$ and $|K_2\rangle$ exhibit definite CP parity values, $CP = +1$ and $CP = -1$ respectively.

The neutral kaons decay principally into the final states $\pi\pi$, $\pi\pi\pi$ or $\pi\ell\nu$. Regarding the individual hadronic final states, $\pi\pi$ has $CP = +1$, $\pi\pi\pi$ has $CP = (-1)^{\ell+1}$. Therefore, as far as the CP eigenstates are concerned, with CP conservation, $|K_1\rangle$ is expected to decay predominantly into two-pion final states (the decay mode into $\pi\pi\pi$ is kinematically suppressed by the angular momentum barrier due to $\ell \geq 1$), whereas $|K_2\rangle$ can only decay into $\pi\pi\pi$ with a much longer lifetime, but not to $\pi\pi$. The (large) difference in lifetimes essentially reflects the difference in phase space.

It is convenient to treat the $K^0-\bar{K}^0$ system using a two-component formalism similar to the nucleon isospin. A neutral kaon state $|\psi(t)\rangle$ at a time $t$ can in general be considered
as a superposition of $|K^0\rangle$ and $|\bar{K}^0\rangle$ states: $|\psi(t)\rangle = \psi_1(t)|K^0\rangle + \psi_2(t)|\bar{K}^0\rangle$. The time evolution of $|\psi(t)\rangle$ is given by the two-component Schrödinger equation:

$$i \frac{\partial}{\partial t} \begin{pmatrix} \psi_1(t) \\ \psi_2(t) \end{pmatrix} = H \begin{pmatrix} \psi_1(t) \\ \psi_2(t) \end{pmatrix}. \quad (1.8)$$

We assume $t$ to be the proper time, and $H$ is a $2 \times 2$ matrix, whose entries are defined from the Hamiltonian as $\langle i|H|j\rangle$ with $i, j = K^0, \bar{K}^0$, and, in particular, $K^0-\bar{K}^0$ mixing is represented by the off-diagonal elements. In general, $H$ is complex and can be expressed in terms of two Hermitian matrices, $M$ (the mass matrix) and $\Gamma$ (the decay matrix):

$$H = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix} = \begin{pmatrix} M_{11} - \frac{i}{2} \Gamma_{11} & M_{12} - \frac{i}{2} \Gamma_{12} \\ M_{21} - \frac{i}{2} \Gamma_{21} & M_{22} - \frac{i}{2} \Gamma_{22} \end{pmatrix} = M - \frac{i}{2} \Gamma. \quad (1.9)$$

The time evolution matrix $H$ is invariant under $CPT$ if $H_{11} = H_{22}$, i.e.

$$M_{11} = M_{22} \quad \text{and} \quad \Gamma_{11} = \Gamma_{22}. \quad (1.10)$$

Eq. (1.10) is a necessary, but not sufficient, condition to have invariance under $CP$. To insure $CP$ invariance the off-diagonal elements must satisfy the condition $H_{12} = H_{21}$, i.e.

$$M_{12} = M_{12}^* \quad \text{and} \quad \Gamma_{12} = \Gamma_{21}^*, \quad (1.11)$$

taking into account that both $M$ and $\Gamma$ are Hermitian. This condition depends on the phase choice; in fact, redefining the phases of the strangeness eigenstates as in Eq. (1.4), the off-diagonal entries transform in the following way: $M_{12} \rightarrow e^{-2i\xi} M_{12}$ and $\Gamma_{12} \rightarrow e^{-2i\xi} \Gamma_{12}$. The condition that insures $CP$ invariance, independently from the phase choice, is:

$$\arg \left( \frac{M_{12}}{\Gamma_{12}} \right) = 0, \quad (1.12)$$

that means that the off-diagonal matrix elements must have the same phase.

Non-conservation of $CP$ by the weak Hamiltonian implies that $CP$ and $H$ cannot be diagonalized simultaneously, therefore that $|K_1\rangle$ and $|K_2\rangle$ do not coincide with the physical mass eigenstates. Assuming $CPT$ invariance, the latter are given by the linear combinations:

$$|K_S\rangle = \frac{1}{\sqrt{1 + |\epsilon|^2}} (|K_1\rangle + \epsilon |K_2\rangle), \quad (1.13)$$

$$|K_L\rangle = \frac{1}{\sqrt{1 + |\epsilon|^2}} (|K_2\rangle + \epsilon |K_1\rangle),$$
Chapter 1: CP violation in the $K^0 - \bar{K}^0$ system

<table>
<thead>
<tr>
<th>$M_{K^0}$</th>
<th>$497.672 \pm 0.031$ MeV/$c^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_S = 1/\Gamma_S$</td>
<td>$(8.935 \pm 0.008) \times 10^{-11}$ s</td>
</tr>
<tr>
<td>$\tau_L = 1/\Gamma_L$</td>
<td>$(5.17 \pm 0.04) \times 10^{-8}$ s</td>
</tr>
<tr>
<td>$M_L - M_S$</td>
<td>$(3.489 \pm 0.008) \times 10^{-12}$ MeV/$c^2$</td>
</tr>
</tbody>
</table>

Table 1.1: Measured values of $K_S$ and $K_L$ parameters [9].

or, using Eq. (1.7):

$$|K_S\rangle = \frac{1}{\sqrt{2(1 + |\varepsilon|^2)}} \left( (1 + \varepsilon) |K^0\rangle + (1 - \varepsilon) |\bar{K}^0\rangle \right) ,$$

$$|K_L\rangle = \frac{1}{\sqrt{2(1 + |\varepsilon|^2)}} \left( (1 + \varepsilon) |K^0\rangle - (1 - \varepsilon) |\bar{K}^0\rangle \right) .$$

The subscripts $S$ and $L$ stand for short and long after the huge difference in $K_S$ and $K_L$ lifetimes ($\tau_L \approx 580 \tau_s$). $\varepsilon$ is a complex mixing parameter, which depends on the phase convention chosen for $|K^0\rangle$ and $|\bar{K}^0\rangle$. It is given by:

$$\frac{1 - \varepsilon}{1 + \varepsilon} = \sqrt{\frac{M_{12}^* - \frac{i}{2} \Gamma_{12}^*}{M_{12} - \frac{i}{2} \Gamma_{12}}} .$$

The relative misalignment between the phases of $M_{12}$ and $\Gamma_{12}$ is responsible for CP violation in the mixing. If Eq. (1.12) held, $\varepsilon$ would turn out to be a pure phase, that could be cancelled out by a phase redefinition in Eq. (1.4), and in this case no CP violating mixing would appear.

From the diagonalization of Eq. (1.9) it follows:

$$M_{11} = M_{22} = (M_L + M_S)/2$$

$$M_{21} = -\Re M_{12} \sim (M_L - M_S)/2 ,$$

$$\Gamma_{11} + \Re \Gamma_{12} \sim \Gamma_S ,$$

$$\Gamma_{11} - \Re \Gamma_{12} \sim \Gamma_L ,$$

where $M_{S,L}$ and $\Gamma_{S,L}$ are the masses and the decay widths of the physical states $|K_{S,L}\rangle$ (see Tab. 1.1).

Having $|K_{S,L}\rangle$ definite mass and lifetime, they evolve according to the exponential laws $|K_{S,L}(t)\rangle = |K_{S,L}(0)\rangle e^{-iM_{S,L}t} e^{-\frac{1}{2} \Gamma_{S,L} t}$, see Eq. (1.8). $|K^0\rangle$ and its CP conjugate $|\bar{K}^0\rangle$ have now to be considered as an admixture of the physical shortlived $|K_S\rangle$ component and the physical longlived $|K_L\rangle$ component. This implies that, given a pure beam of $K^0$ ($\bar{K}^0$) at
1.2 The neutral kaon system

t = 0, there exist a probability of finding a $\bar{K}^0$ ($K^0$) at time $t$:

$$P_{K^0 \to \bar{K}^0}(t) = \left| \langle \bar{K}^0(t) | K^0 \rangle \right|^2$$

$$= \frac{1}{2} \left[ e^{-\Gamma_{st}t} + e^{-\Gamma_{Lt}t} + 2 e^{-\frac{1}{2} \left( \Gamma_{st} + \Gamma_{Lt} \right)t} \cos(\Delta m t) \right],$$

$$P_{\bar{K}^0 \to K^0}(t) = \left| \langle K^0(t) | \bar{K}^0 \rangle \right|^2$$

$$= \frac{1}{2} \left[ e^{-\Gamma_{st}t} + e^{-\Gamma_{Lt}t} - 2 e^{-\frac{1}{2} \left( \Gamma_{st} + \Gamma_{Lt} \right)t} \cos(\Delta m t) \right].$$

Equation (1.17)

These matter-antimatter oscillations are $|\Delta S| = 2$ processes. From the frequency of the oscillatory term, the quantity $\Delta m = M_L - M_S$ has been measured (Tab. 1.1). Eq. (1.17) refers to the $CP$ conserving case. In the $CP$ violating case, interference effects arise, that essentially modulate the coefficient and the argument of the oscillatory term, as will be shown in the sequel.

Another peculiar phenomenon related to the $K^0$-$\bar{K}^0$ mixing is the “regeneration”. The relevant eigenstates for the strong interactions of neutral kaons are the $K^0$ ($S = +1$) and $\bar{K}^0$ ($S = -1$) states. The $K^0$ and $\bar{K}^0$ have different quark content ($d\bar{s}$ and $\bar{d}s$, respectively) and so they interact differently with ordinary matter, which contains $u$ and $d$ but not $\bar{u}$ and $\bar{d}$. If a pure $K_L$ beam passes through a slab of matter, the $\bar{K}^0$ component has more interaction channels open than the $K^0$ component, so that it is more strongly absorbed. Thus, the beam will emerge as a fairly pure $K^0$ beam, which is again a coherent superposition of $K_L$ and $K_S$.

1.2.2 Mechanisms of $CP$ violation

Until 1964, $CP$ had been considered as an exact symmetry of Nature and the states $|K_1\rangle$ and $|K_2\rangle$ were thought to be mass eigenstates. The observation, in that year, that also $|K_2\rangle$ (actually, $|K_L\rangle$) decays into a two-pion final state with a branching ratio of order $O(10^{-3})$, demonstrated that $CP$ does not commute with the weak Hamiltonian and that the physical states have to be considered as an admixture of a $CP$-even component and a $CP$-odd component, as anticipated in the previous section in Eq. (1.13).

Indeed, the $CP$ violating decay $K_L \to \pi\pi$ can proceed, as exemplified in Fig. 1.1, either via the mismatch between the $CP$ eigenstates $|K_{1,2}\rangle$ and the mass eigenstates $|K_{S,L}\rangle$, introduced by the $CP$-violating impurity in the $K^0$-$\bar{K}^0$ mixing (“indirect” $CP$ violation), or via the “direct” $|\Delta S| = 1$ $CP$-violating transition $K_2 \to \pi\pi$. As said above, the indirect $CP$ violation effect is induced by the mixing parameter $\bar{\varepsilon}$ of Eq. (1.13), which is of the order $O(10^{-3})$ according to the branching ratio measurement.

Historically, to account for indirect $CP$ violation, a “super-weak” model was developed [10]: a $|\Delta S| = 2$ Hamiltonian was postulated, which causes $K^0 \leftrightarrow \bar{K}^0$ and hence
Chapter 1: CP violation in the $K^0$-$\bar{K}^0$ system

![Diagram](image)

**Figure 1.1:** Direct and indirect CP violation

$K_S \rightleftharpoons K_L$ transitions. In this case, CP violation arises entirely from the mass matrix. Models where there is a "direct" CP violation are called "milliweak". The combination of the two mechanisms of CP violation in the $K^0$-$\bar{K}^0$ system induces a new parameter, $\varepsilon'$, relative to direct CP violation, in addition to the mixing parameter $\varepsilon$.

In the specific case of the decays of the neutral kaons into $\pi\pi$, direct and indirect CP violation are usually parameterized in terms of the ratios of the transition matrix elements:

$$\eta_{00} = |\eta_{00}| e^{i\phi_{00}} = \frac{\langle \pi^0\pi^0 | \mathcal{H}_W | K_L \rangle}{\langle \pi^0\pi^0 | \mathcal{H}_W | K_S \rangle}, \quad (1.18)$$

and

$$\eta_{+-} = |\eta_{+-}| e^{i\phi_{+-}} = \frac{\langle \pi^+\pi^- | \mathcal{H}_W | K_L \rangle}{\langle \pi^+\pi^- | \mathcal{H}_W | K_S \rangle}, \quad (1.19)$$

where $\mathcal{H}_W$ represents the $|\Delta S| = 1$ weak Hamiltonian. The magnitudes and phases of $\eta_{00}$ and $\eta_{+-}$ are physical observables and have been measured precisely with time interference methods:

$$|\langle \pi\pi | \mathcal{H}_W | K^0 \rangle|^2 = \frac{1}{2} (1 - 2 \Re \varepsilon) |\langle \pi\pi | \mathcal{H}_W | K_S \rangle|^2 \times$$

$$[|\eta_{\pi\pi}|^2 e^{-\Gamma_L t} + e^{-\Gamma_S t} + 2 |\eta_{\pi\pi}| e^{-(\Gamma_S + \Gamma_L)t/2} \cos(\Delta m t - \phi_{\pi\pi})]$$

$$|\langle \pi\pi | \mathcal{H}_W | \bar{K}^0 \rangle|^2 = \frac{1}{2} (1 + 2 \Re \varepsilon) |\langle \pi\pi | \mathcal{H}_W | K_S \rangle|^2 \times$$

$$[|\eta_{\pi\pi}|^2 e^{-\Gamma_L t} + e^{-\Gamma_S t} - 2 |\eta_{\pi\pi}| e^{-(\Gamma_S + \Gamma_L)t/2} \cos(\Delta m t - \phi_{\pi\pi})].$$

The measurements of $\eta_{00}$ and $\eta_{+-}$ give [9]:

$$|\eta_{00}| = (2.262 \pm 0.017) \times 10^{-3}, \quad \phi_{00} = 43.2^\circ \pm 1.0^\circ, \quad (1.21)$$

$$|\eta_{+-}| = (2.276 \pm 0.017) \times 10^{-3}, \quad \phi_{+-} = 43.3^\circ \pm 0.5^\circ.$$  

From Bose symmetry, a two-pion state in s-wave must have isospin $I = 0$ or 2. The amplitudes for $|K^0\rangle$ and $|\bar{K}^0\rangle$ to decay into two pions with definite isospin $I$ can be written...
1.2 The neutral kaon system

as [8]:

\[
\langle (\pi\pi)|\mathcal{H}_W|K^0 \rangle = A_I e^{i\delta_I} ,
\]
\[
\langle (\pi\pi)|\mathcal{H}_W|\bar{K}^0 \rangle = A^*_I e^{i\delta_I} ,
\]

where \( A_I \) are the weak transition amplitudes and the \( \delta_I \) are the phase shifts which arise from the isospin conserving final state interactions among the pions.

By decomposing the amplitudes \( \langle \pi\pi|\mathcal{H}_W|K_{S,L} \rangle \) into the amplitudes to \((\pi\pi)_I\) states with definite isospin \( I = 0, 2 \) through the Clebsch-Gordan coefficients, Eqs. (1.18) and (1.19) can be written as

\[
\eta_{00} = \varepsilon - \frac{2\varepsilon'}{1 - \omega \sqrt{2}} \approx \varepsilon - 2\varepsilon' ,
\]
\[
\eta_{+-} = \varepsilon + \frac{\varepsilon'}{1 + \omega / \sqrt{2}} \approx \varepsilon + \varepsilon' .
\]

Here, the parameter \( \omega \) measures the ratio:

\[
\omega = \frac{\Re A_2}{\Re A_0} \approx \frac{1}{22.2} ,
\]

a small number that characterizes the well-known \( \Delta I = 1/2 \) rule for non-leptonic weak interactions. The parameters \( \varepsilon \) and \( \varepsilon' \) are defined, in terms of the \( |K_{S,L}\rangle \) decay amplitudes, as:

\[
\varepsilon = \frac{\langle (\pi\pi)_{I=0}\rangle|\mathcal{H}_W|K_L \rangle}{\langle (\pi\pi)_{I=0}\rangle|\mathcal{H}_W|K_S \rangle} ,
\]

and

\[
\frac{\varepsilon'}{\varepsilon} = \frac{1}{\sqrt{2}} \left\{ \frac{\langle (\pi\pi)_{I=2}\rangle|\mathcal{H}_W|K_L \rangle - \langle (\pi\pi)_{I=0}\rangle|\mathcal{H}_W|K_S \rangle}{\langle (\pi\pi)_{I=2}\rangle|\mathcal{H}_W|K_L \rangle - \langle (\pi\pi)_{I=0}\rangle|\mathcal{H}_W|K_S \rangle} \right\} .
\]

They parameterize indirect (via \( K_1-K_2 \) mixing) and direct (in the \( |K_1\rangle \) and \( |K_2\rangle \) decays) \( CP \) violation, respectively. Clearly, the parameter \( \varepsilon' \) differentiates the \( CP \) violating decays \( K_L \rightarrow \pi^0\pi^0 \) and \( K_L \rightarrow \pi^+\pi^- \). Notice that the values of Eq. (1.21), by themselves, are not able to give a determination of \( \varepsilon' \neq 0 \), but only indicate that \( \varepsilon' \) must be very small, of order \( \leq 10^{-2} \) with respect to \( \varepsilon \). Thus, a dedicated experiment is necessary.

Using Eqs. (1.14) and (1.22), \( \varepsilon \) is expressed as follows:

\[
\varepsilon = \bar{\varepsilon} + i \frac{\Im m A_0}{\Re A_0} ,
\]

where \( \bar{\varepsilon} \) is the mixing parameter introduced in Eq. (1.15). The phase convention dependence of the term \( i \Im m A_0 / \Re A_0 \) cancels the convention dependence of \( \bar{\varepsilon} \), so that \( \varepsilon \) is a
physical observable. In the convention where the phase is chosen such that the amplitude $A_0$ is real, the so-called Wu and Yang convention, the parameter $\varepsilon$ obviously coincides with $\bar{\varepsilon}$. The experimental measurements of $|\varepsilon|$ give the value [9]

$$|\varepsilon| = (2.271 \pm 0.017) \times 10^{-3}, \quad (1.29)$$

see also Eq. (1.21). By applying CPT invariance and unitarity to the evolution matrix of Eq. (1.9), and approximations that apply specifically to neutral kaon decays, the phase of $\varepsilon$ is given approximately by:

$$\phi_\varepsilon \approx \tan^{-1} \left[ \frac{2(M_L - M_S)}{\Gamma_S - \Gamma_L} \right] = 43.49^\circ \pm 0.08^\circ. \quad (1.30)$$

The parameter $\varepsilon'$ can be written as a function of the weak transition amplitudes in the following way:

$$\varepsilon' = e^{i(\pi/2 + \delta_2 - \delta_0)} \frac{\omega}{\sqrt{2}} \left( \frac{\Im A_2}{\Re A_2} - \frac{\Im A_0}{\Re A_0} \right). \quad (1.31)$$

In order to have $\varepsilon' \neq 0$ two conditions must hold simultaneously:

- $\text{arg}(A_0) \neq \text{arg}(A_2)$,
- $\delta_0 \neq \delta_2$,

in agreement with the general statements that requires at least two different electroweak phases and two different strong phases to observe direct CP violation. The phase shifts $\delta_0$ and $\delta_2$ are measured from $\pi-\pi$ scattering, giving:

$$\phi_{\varepsilon'} = \frac{\pi}{2} + \delta_2 - \delta_0 \simeq 48^\circ \pm 4^\circ, \quad (1.32)$$

so that, to a good approximation, $\varepsilon'$ and $\varepsilon$ can be considered as relatively real.

Eq. (1.31) shows that the phenomenon of direct CP violation resides entirely in the interference between the weak amplitudes $A_0$ and $A_2$, that carry different complex phases. Such phases are convention dependent, but the fact that $\varepsilon'$ depends only on their difference, makes it an observable quantity. It is worthwhile noting the factor $\omega$ in Eq. (1.31), which suppresses the value of $\varepsilon'$ by a factor $1/2$. With the Wu-Yang phase convention, such that $\Im A_0 = 0$ (and $\Re A_0 > 0$), $\varepsilon'$ takes the form:

$$\varepsilon' = e^{i(\pi/2 + \delta_2 - \delta_0)} \frac{\omega}{\sqrt{2}} \frac{\Im A_2}{\Re A_2}. \quad (1.33)$$
Since $\phi' \approx \phi$, only two real quantities need be measured: the magnitude of $\varepsilon$ and the value of $\varepsilon'/\varepsilon$ including its sign. A suitable physical observable is defined from Eq. (1.23) as follows:

$$
\text{Re} \left( \frac{\varepsilon'}{\varepsilon} \right) = \frac{1}{6 (1 + \omega/\sqrt{2})} \left( 1 - \left| \frac{\eta_{00}}{\eta_{+-}} \right|^2 \right) \approx \frac{1}{6} \left( 1 - \left| \frac{\eta_{00}}{\eta_{+-}} \right|^2 \right),
$$

(1.34)

where a term $\omega/\sqrt{2}$ is usually neglected as being numerically small. The term $|\eta_{00}/\eta_{+-}|^2$ is referred to as the double ratio $R$:

$$
R = \left| \frac{\eta_{00}}{\eta_{+-}} \right|^2 = \frac{\Gamma(K_L \to \pi^0\pi^0)}{\Gamma(K_L \to \pi^+\pi^-)} \frac{\Gamma(K_S \to \pi^0\pi^0)}{\Gamma(K_S \to \pi^+\pi^-)} \approx 1 - 6 \text{Re} \left( \frac{\varepsilon'}{\varepsilon} \right).
$$

(1.35)

Experimentally $|\eta_{00}| \approx |\eta_{+-}|$, see Eq. (1.21), hence $\text{Re} \left( \varepsilon'/\varepsilon \right)$ is expected to be very small, of the order $O(10^{-3})$. Notice the factor 6 that enhances the contribution of $\varepsilon'/\varepsilon$ on the right-hand side of Eq. (1.35).

K mesons also decay semileptonically, into a charged pion with strangeness zero, and a lepton-neutrino pair. In the Standard Model, $K^0$ decays only to $\ell^+$ and $\bar{K}^0$ to $\ell^-$. This is commonly referred to as the $\Delta S = \Delta Q$ rule, and enables one to tag the strangeness of the decaying meson.

In the $K_{S,L} \to \pi \ell \nu_\ell$ case, the selection rule $\Delta S = \Delta Q$ allows for the existence of a single weak amplitude. In fact, defining $\langle \pi^-\ell^+\nu_\ell | H_W | K^0 \rangle \equiv A_\ell$, from $CPT$ it follows $\langle \pi^+\ell^-\bar{\nu}_\ell | H_W | K^0 \rangle = A_\ell^*$ and the decay amplitudes for $K_L$ and $K_S$ are:

$$
\begin{align*}
\langle \pi^+\ell^-\bar{\nu}_\ell | H_W | K_S \rangle &= - \langle \pi^+\ell^-\bar{\nu}_\ell | H_W | K_L \rangle = \frac{1 - \varepsilon}{\sqrt{2(1 + |\varepsilon|^2)}} A_\ell^*, \\
\langle \pi^-\ell^+\nu_\ell | H_W | K_S \rangle &= \langle \pi^-\ell^+\nu_\ell | H_W | K_L \rangle = \frac{1 + \varepsilon}{\sqrt{2(1 + |\varepsilon|^2)}} A_\ell.
\end{align*}
$$

(1.36)

In the decays $K_L \to \pi \ell \nu_\ell$, $CP$ violation can be observed via the charged asymmetry:

$$
\delta_L = \frac{\Gamma(K_L \to \pi^-\ell^+\nu_\ell) - \Gamma(K_L \to \pi^+\ell^-\bar{\nu}_\ell)}{\Gamma(K_L \to \pi^-\ell^+\nu_\ell) + \Gamma(K_L \to \pi^+\ell^-\bar{\nu}_\ell)} = \frac{2 \text{Re} \varepsilon}{1 + |\varepsilon|^2},
$$

(1.37)

where $\ell = e, \mu$. $\delta_L$ measures the difference between the phases of $M_{12}$ and $\Gamma_{12}$ and is sensitive to indirect $CP$ violation only. The direct measurements give [9]:

$$
\delta_L = (3.27 \pm 0.12) \times 10^{-3},
$$

(1.38)

that is in agreement with the numerical value of $\text{Re} \varepsilon$ obtained from Eqs. (1.29) and (1.30).
1.3 \( CP \) violation in the Standard Model

The Standard Model is a non-abelian gauge theory with three fermion generations, based on the \( SU(3)_C \times SU(2)_L \times U(1)_Y \) symmetry group, which describes the strong and electro-weak interactions of elementary particles. The \( SU(3)_C \) subgroup is the symmetry of color interactions and the \( SU(2)_L \times U(1)_Y \) symmetry governs the electro-weak interactions. \( SU(2)_L \times U(1)_Y \) is spontaneously broken to \( U(1)_Q \) via the Higgs mechanism in such a way that the photon remains massless, while the \( W^\pm \) and \( Z^0 \) gauge bosons acquire mass. Analogously, the quark and lepton masses are dynamically generated by their Yukawa couplings to the Higgs scalar field.

1.3.1 The weak charged currents and the CKM matrix

\( CP \) violation is naturally accommodated in the Standard Model framework, in the sector of the weak charged current interactions of quarks \([11]\). Such flavor changing interactions are mediated by the \( W^\pm \) bosons, and in the quark mass eigenstates basis they take the form:

\[
\mathcal{L}_W = -\frac{g}{2\sqrt{2}} \bar{u}_L^i \gamma^\mu V_{ij} d_L^j W^\mu + \text{h.c.},
\]

where \( g \) is the \( SU(2)_L \) weak coupling constant, \( u_L \) and \( d_L \) are the left-handed fields of the up-type and down-type quarks respectively, and \( i, j \) are the flavor indices. \( V_{ij} \) are the elements of the Cabibbo-Kobayashi-Maskawa matrix (CKM), which connects the weak eigenstates selected by the weak interaction, to the mass eigenstates having definite flavor (isospin, strangeness, charm, etc.) \([12, 13]\):

\[
V_{\text{CKM}} = \begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix}.
\]

\( V_{\text{CKM}} \) is a \( 3 \times 3 \) unitary matrix which in general depends on nine parameters: three can be chosen as real angles and six are phases. By properly rephasing the quark mass eigenstates, one can cancel out five of these phases. Only one irreducible phase remains: the Kobayashi-Maskawa phase \( \delta_{\text{KM}} \). Such a phase, which can induce complex couplings in the weak charged interactions of quarks in Eq. (1.39), is the only source of \( CP \) violation in the Standard Model.

For this reason, \( CP \) violation is expected to appear only in the processes involving the charged current interactions of the quarks. The neutral current interactions with the \( \gamma \)
1.3 CP violation in the Standard Model

and $Z^0$ are flavor diagonal by the GIM mechanism [14], while in the lepton sector there is no possibility of flavor mixing as long as the neutrinos are considered massless.

There are various useful ways to parameterize the CKM matrix. The standard choice is:

$$V_{\text{CKM}} = \begin{pmatrix}
    c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta_{\text{CKM}}} \\
    -s_{12} c_{23} - c_{12} s_{13} s_{23} e^{i\delta_{\text{CKM}}} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta_{\text{CKM}}} & s_{23} c_{13} \\
    s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta_{\text{CKM}}} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta_{\text{CKM}}} & c_{23} c_{13}
\end{pmatrix},$$

(1.41)

where $c_{ij} = \cos \vartheta_{ij}$ and $s_{ij} = \sin \vartheta_{ij}$. A particularly simple, and convenient, parameterization is due to Wolfenstein [15]:

$$V_{\text{CKM}} = \begin{pmatrix}
    1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\
    -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\
    A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1
\end{pmatrix} + O(\lambda^4).$$

(1.42)

This is an approximate parameterization, which exploits the fact that the entries of the CKM matrix exhibit an hierarchical structure, getting smaller away from the diagonal. Each element is expanded as a power of the sine of the Cabibbo angle $\lambda = |V_{us}| \simeq 0.22$. The other parameters are $A$, $\rho$ and $\eta$, the latter representing the $CP$ violating phase. To get more accurate theoretical estimates, it is usual to include corrections up to order $O(\lambda^5)$ and to define $\rho \rightarrow \bar{\rho} = \rho (1 - \lambda^2/2)$ and $\eta \rightarrow \bar{\eta} = \eta (1 - \lambda^2/2)$. Notice that, in the parameterization of Eq. (1.42), the $CP$ violating phase affects the heavy quark couplings $V_{ub}$ and $V_{td}$, showing the minimal number of three quark families for $CP$ violation to occur. In particular, loops with virtual top quark exchanges are needed to produce $CP$ violation in the kaon sector.

The fact that $\eta$ multiplies $\lambda^3$ in Eq. (1.42) clearly shows that $CP$ violation is naturally suppressed in the Standard Model.

1.3.2 The unitarity triangles

Direct and indirect information on the CKM matrix is neatly represented in terms of "unitarity triangles". The unitarity of the CKM matrix, $V_{\text{CKM}}^\dagger V_{\text{CKM}} = 1$, leads to a number of relationships among its entries, which require the sum of three complex quantities to vanish and hence can be geometrically represented by triangles in the complex plane. There are six such relationships. The most commonly studied is the following:

$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0,$$

(1.43)
because all the angles of the corresponding triangle are of the same order of magnitude. It is usual to draw the rescaled unitarity triangle in the plane $\bar{\rho} - \bar{\eta}$ (see Fig. 1.2). Choosing a phase convention such that $V_{cd}^* V_{cb}$ is real and dividing the lengths of all sides by $|V_{cd}^* V_{cb}|$, it follows that two vertices of the rescaled unitarity triangle are fixed at $(0, 0)$ and $(0, 1)$ and the coordinates of the remaining one correspond to $(\bar{\rho}, \bar{\eta})$. The lengths of the two complex sides are:

$$R_u = \frac{|V_{ud}^* V_{ub}|}{|V_{cd}^* V_{cb}|} = \frac{1 - \lambda^2/2}{\lambda} \left| \frac{V_{ub}}{V_{cb}} \right|, \quad R_d = \frac{|V_{td}^* V_{tb}|}{|V_{cd}^* V_{cb}|} = \frac{1}{\lambda} \left| \frac{V_{td}}{V_{cb}} \right|; \quad (1.44)$$

while the three angles are given by:

$$\alpha = \arg \left( \frac{V_{td}^* V_{tb}}{V_{ud}^* V_{ub}} \right), \quad \beta = \arg \left( \frac{V_{cd}^* V_{cb}}{V_{td}^* V_{tb}} \right), \quad \gamma = \arg \left( \frac{V_{ud}^* V_{ub}}{V_{cd}^* V_{cb}} \right). \quad (1.45)$$

The invariance of Eq. (1.43) under phase transformations implies that the corresponding triangle is rotated in the $\bar{\rho} - \bar{\eta}$ plane under such transformations. Since the angles and the sides (given by the moduli of the elements of the mixing matrix) in this triangle remain unchanged, they are phase convention independent and are physical observables. Consequently, they can be measured directly and independently in suitable experiments. In particular the angles $\beta$ and $\gamma$ are directly related to the complex phases of the CKM elements $V_{td}$ and $V_{ub}$, respectively.

Six of the nine absolute values of the CKM entries are measured directly [9]:

- $|V_{ud}| = 0.9736 \pm 0.0010$ (nuclear $\beta$-decays),
- $|V_{us}| = 0.2205 \pm 0.0018$ (semileptonic kaon and hyperon decays),
- $|V_{cd}| = 0.224 \pm 0.016$ (neutrino and antineutrino production of charm off valence $d$ quarks),
- $|V_{cs}| = 1.01 \pm 0.18$ (semileptonic D decays),
1.3 CP violation in the Standard Model

Figure 1.3: Constraints on the vertex $A$ of the unitarity triangle along with the physical processes, from which they are derived [16].

$$|V_{cb}| = 0.041 \pm 0.002$$  \hspace{1cm} (semileptonic exclusive and inclusive B decay),

$$|V_{ub}/V_{cb}| = 0.085 \pm 0.018$$  \hspace{1cm} (end-point spectrum in semileptonic B decays).

Indirect measurements, related to loop level processes (e.g., the mixing in the $B^0$-$\overline{B}^0$ system), give:

$$|V_{tb}^* V_{tb}| = 0.009 \pm 0.003$$

Using the CKM unitarity one can put constraints on $|V_{td}|$, $|V_{ts}|$, $|V_{tb}|$ and $|V_{cs}|$.

Being there a single source of CP violation, all CP violating observables are correlated with each other, in the sense that they are expressed in terms of just $\sin(\delta_{\text{KM}})$. A major aim of CP violation studies of K and B decays is to make enough independent measurements of the sides and angles in order that the unitarity triangle of Eq. (1.43) is overdetermined, and thereby check the validity of Standard Model predictions which relate various measurements to the triangle. In Fig. 1.3 we pictorially represent the kind of constraints on the triangle vertex, derived from the different physical observables. In Fig. 1.4 an example of the experimental constraints on the determination of the $A$ vertex is reported.
1.4 Theoretical estimates of $\varepsilon'/\varepsilon$

1.4.1 The theoretical framework

The study of kaon decays within the Standard Model is made complicated by the very large energy scale differences involved. Energies as far apart as the mass of the $t$ quark and the pion and kaon masses are relevant. The most appropriate framework for dealing with physical systems defined across different energy scales is that of effective theories. In an effective theory the transition amplitudes are assumed to be factorizable in high- and low-energy parts. The degrees of freedom at the higher scales are step-by-step integrated out, retaining only the effective operators made of the lighter degrees of freedom. In our case, the convenient technique is represented by the Operator Product Expansion (OPE).

The weak effective Hamiltonian for non-leptonic decays at the quark level, constructed using the OPE, has the following form:

$$H_W^{\text{eff}} = \sum_i C_i(\mu) Q_i(\mu) + \text{h.c.} .$$  \hspace{1cm} (1.46)

The $Q_i$'s are the relevant effective four-quark operators which govern the processes under consideration, in our case the $|\Delta S| = 1$ hadronic transitions. The $C_i$'s are numerical coefficients defined accordingly by:

$$C_i(\mu) = \frac{G_F}{\sqrt{2}} V_{ud} V^{*}_{us} \left[ z_i(\mu) + \tau y_i(\mu) \right] ,$$  \hspace{1cm} (1.47)

where $G_F$ is the Fermi coupling, $V_{ij}$ the CKM matrix elements, and $\tau = -V_{td}V^{*}_{ts}/V_{ud}V^{*}_{us}$. The functions $z_i(\mu)$ and $y_i(\mu)$ contain all the dependence on short-distance physics, and
1.4 Theoretical estimates of $\epsilon'/\epsilon$

Figure 1.5: Typical current-current diagrams for the $|\Delta S| = 1$ transitions with one-loop QCD corrections.

depend on the $t, W, b, c$ masses, the intrinsic QCD scale $\Lambda_{\text{QCD}}$, and a renormalization scale $\mu$. Also the operators $Q_i$ depend on $\mu$, but the products in Eq. (1.46) must be $\mu$-independent, being the matrix elements of $\mathcal{H}$ physically observable.

The effective $|\Delta S| = 1$ four-quark operators are obtained by integrating out the heavy degrees of freedom, namely the vector bosons and the heavy quarks $t, b$ and $c$. For $\mu < m_c$, only the light flavors are active ($q = u, d, s$) and the relevant quark operators are:

**Current–Current**:

\[
Q_1 = \bar{s}_a \gamma_\mu (1 - \gamma_5) u_\beta \bar{u}^\beta \gamma^\mu (1 - \gamma_5) d^\alpha \\
Q_2 = \bar{s} \gamma_\mu (1 - \gamma_5) u \bar{u} \gamma^\mu (1 - \gamma_5) d
\] (1.48)

**QCD–Penguins**:

\[
Q_{3,5} = \bar{s} \gamma_\mu (1 - \gamma_5) d \sum_q \bar{q} \gamma^\mu (1 \mp \gamma_5) q \\
Q_{4,6} = \bar{s}_a \gamma_\mu (1 - \gamma_5) d_\beta \sum_q \bar{q}^\beta \gamma^\mu (1 \mp \gamma_5) q^\alpha
\] (1.49)

**Electroweak–Penguins**:

\[
Q_{7,9} = \frac{3}{2} \bar{s} \gamma_\mu (1 - \gamma_5) d \sum_q \bar{e}_q \bar{q} \gamma^\mu (1 \pm \gamma_5) q \\
Q_{8,10} = \frac{3}{2} \bar{s}_a \gamma_\mu (1 - \gamma_5) d_\beta \sum_q \bar{e}_q \bar{q}^\beta \gamma^\mu (1 + \gamma_5) q^\alpha
\] (1.50)

where, $\alpha, \beta$ denote colours and $\bar{e}_q$ denotes the electric light quark charges reflecting the electroweak origin of $Q_7, Q_8, Q_9$ and $Q_{10}$. The various operators originate from different diagrams of the fundamental theory. First, at the tree level, we only have the current-current operator $Q_2$ induced by $W$-exchange, the so-called spectator diagram $s \rightarrow u \bar{u} d$. Basically, this would represent the physics at a scale $M_W$. Switching on QCD, one-loop gluon corrections to $W$-exchange correct $Q_2$, induce the new operator $Q_1$ and introduce the dependence on the scale $\mu$ through the running of the QCD coupling constant $\alpha_s$. 
(Fig. 1.5). Furthermore, QCD through the electroweak penguin loop [Fig. 1.6(a)] induces the gluonic penguin operators $Q_{3-6}$. Electroweak loop diagrams [Fig. 1.6(b)], where the penguin gluon is replaced by a photon or a Z boson, induce $Q_{7,9}$ and also a part of $Q_3$. The operators $Q_{8,10}$ are induced by the QCD renormalization of the electroweak loop operators $Q_{7,9}$. Penguin diagrams generate $s \rightarrow d q \bar{q}$ "neutral current" transition that, according to the GIM, requires higher orders. Notice that current-current operators control the $CP$ conserving transitions, while only the penguin operators are sensitive to the weak $CP$ phase because of the virtual top quark exchange.

The $|\Delta S| = 2$ processes, that induce $K^0 - \bar{K}^0$ mixing, occur through the box diagrams of Fig. 1.7. The corresponding, $\mu$-dependent effective four-fermion operator is given by:

$Q_{\text{box}}^{|\Delta S|=2} = \bar{s} \gamma^\mu (1 - \gamma_5) d \bar{s} \gamma_\mu (1 - \gamma_5) d + \text{h.c.}$,  \hspace{1cm} (1.51)

and the corresponding, $\mu$-dependent, short-distance coefficient can be expressed as:

$C_{\text{box}}(\mu) = \frac{G_F^2}{4 \pi^2} \frac{M_W^2}{F(\mu)}$,  \hspace{1cm} (1.52)

where $F(\mu)$ is the analogue of $C_1(\mu)$ in Eq. (1.47) and includes the dependence on CKM matrix elements, the ratios of the heavy quark and W masses, and the QCD coupling constant $\alpha_s(\mu)$.

The transition amplitudes for $K \rightarrow \pi \pi$ are given by taking matrix elements between initial and final states:

$\langle \pi \pi | \mathcal{H}_{W}^{\text{eff}} | K \rangle = \sum_i C_i(\mu) \langle Q_i(\mu) \rangle$,  \hspace{1cm} (1.53)

where $\langle Q_i(\mu) \rangle = \langle \pi \pi | Q_i(\mu) | K \rangle$ are the matrix elements of $Q_i$, evaluated at the renormal-
1.4 Theoretical estimates of $\varepsilon'/\varepsilon$

Theoretical estimates of $c_1/c_2$

Figure 1.7: $|\Delta S| = 2$ box diagrams.

The Wilson coefficients have been calculated including the complete next-to-leading order (NLO) corrections in $\alpha_s$ and $\alpha_e$ using standard perturbative methods. As an example, we report in Tab. 1.2 the numerical values of the coefficients $z_i$ and $y_i$ for different values of $\Lambda_{\text{MS}}^{(4)}$ at the leading order (LO) and at the next-to-leading order for two commonly used renormalization schemes: the naive dimensional regularization (NDR) and the t'Hooft-Veltman scheme (HV) [17]. As one can see, the coefficients $z_1$ and $z_2$, which correspond to current-current operators, are the largest ones, and dominate the $CP$ conserving $K \to \pi\pi$ amplitudes, while it turns out that the penguins $Q_6$ and $Q_8$ are dominant.
in the determination of the direct CP violation $\varepsilon'$.

The difficulty in estimating $\varepsilon$ and $\varepsilon'/\varepsilon$ resides in the hadronic matrix elements $\langle Q_i \rangle$. Since $\langle Q_i(\mu) \rangle$ involve long distance contributions, one is forced to use a non-perturbative approach, and there are different proposed ones that, unfortunately, lead to different numerical results specially in the case of $\varepsilon'/\varepsilon$. Consequently, the dominant theoretical uncertainty in the decay amplitudes is due to the matrix elements $\langle Q_i(\mu) \rangle$ and, therefore, is of genuine non-perturbative origin. One important problem is the compatibility (“matching”) of the calculated $\langle Q_i(\mu) \rangle$ with $C_i(\mu)$, that requires $\langle Q_i(\mu) \rangle$ to carry the correct $\mu$ and renormalization scheme dependence ensuring the $\mu$ and scheme independence of physical amplitudes. Some non-perturbative methods still have a problem of this kind. Clearly, this may also affect the choice of the optimal range of $\mu$-values and, correspondingly, of the basis of independent operators $Q_i$, such that the short distance OPE remains valid and relevant to the small scale of K decays as well.

Table 1.2: $|\Delta S| = 1$ Wilson coefficients at $\mu = 1$ GeV for $m_t = 170$ GeV. $y_1$ and $y_2$ are taken equal to zero. $\alpha$ is the fine structure constant.
1.4 Theoretical estimates of $\varepsilon'/\varepsilon$

proximation (VSA), which is based on the factorization of the four-fermion operators into products of currents or densities and the saturation of the completeness of the intermediate states by the vacuum state. In this case, experimentally known non-perturbative parameters can be used for the calculation, but there is the problem of identifying the value of $\mu$ at which such a procedure should work. This leads to a potentially large systematic uncertainty of the prediction.

The pie chart in Fig. 1.8 pictorially shows, as an indication, the relative importance of the effective operators defined above in the final determination of the value of $\varepsilon'/\varepsilon$, as obtained in the vacuum saturation approximation to the hadronic matrix elements [18]. It is evident that there is a crucial competition between the gluonic ($Q_6$) and electroweak ($Q_8$) penguins, which contribute with opposite signs. This leads to a partial cancellation among the different contributions that, actually, is a common feature of all numerical calculations of $\varepsilon'/\varepsilon$ performed so far. Clearly, the presence of such cancellation, that accidentaly might even lead to a vanishing $\varepsilon'/\varepsilon$ even in the presence of direct $CP$ violation at quark level, numerically emphasizes the value of the theoretical uncertainty and makes such a prediction with the desired accuracy a real challenge.

It is common to compare the hadronic matrix elements evaluated in the different non-perturbative approaches to the simple VSA results, by introducing the so-called $B$-factors.

Figure 1.8: Relative contributions to $\varepsilon'/\varepsilon$ of the effective hadronic operators as obtained in the vacuum saturation approximation. Operators giving a (negative) positive contribution are depicted in (green) yellow.
Chapter 1: CP violation in the $K^0 - \bar{K}^0$ system

(or bag-factors), as follows:

$$B_i(\mu) = \frac{\langle Q_i(\mu) \rangle}{\langle Q_i \rangle_{\text{VSA}}}.$$  \hspace{1cm} (1.55)

Thus, theoretical estimates quite often refer to the $B$-factors, rather than the hadronic matrix elements themselves. For example, in the case of $K^0 - \bar{K}^0$ mixing, see Eq. (1.54), one writes:

$$\langle K^0 | \bar{s} \gamma^\mu (1 - \gamma_5) d \bar{s} \gamma_\mu (1 - \gamma_5) d | K^0 \rangle \approx \frac{8}{3} f_K^2 M_K^2 \hat{B}_K [\alpha_s(\mu)]^{2/9},$$  \hspace{1cm} (1.56)

with $f_K \approx 1.2 f_\pi$ the kaon leptonic decay constant, and $\hat{B}_K$ a $\mu$-independent non-perturbative constant such that $\hat{B}_K = 1$ and $[\alpha_s(\mu)]^{2/9} \rightarrow 1$ in the VSA approximation. Current theoretical calculations indicate $\hat{B}_K = 0.85 \pm 0.15$, a value close to unity, but with a significant uncertainty.

Likewise, the $K \rightarrow \pi\pi$ decay amplitudes are parameterized in term of non-perturbative $B$-factors [ see Eq. (1.46) ] as:

$$\langle \pi\pi | H_W^{\text{eff}} | K^0 \rangle = \frac{G_F}{\sqrt{2}} V_{CKM} B_i \eta_i^{QCD} F_i^{SM},$$  \hspace{1cm} (1.57)

where $\eta_i^{QCD}$ and $F_i^{SM}$ are fully calculable in perturbation theory, and contain the result of vacuum saturation. For example:

$$\langle \langle \pi\pi \rangle_{I=0} | Q_6 | K^0 \rangle = -4 \sqrt{\frac{3}{2}} \left( \frac{M_K^2}{m_s(\mu)} \right)^2 (f_K - f_\pi) B_6(\mu),$$  \hspace{1cm} (1.58)

where $m_s(\mu)$ is the strange quark mass at the scale $\mu$ and $B_6(\mu)$ shows very weak $\mu$-dependence. New physics, beyond the Standard Model, can contribute additional terms, of analogous form, to the right-hand sides of Eqs. (1.56) and (1.57).

As mentioned previously, there are several non-perturbative methods that have been applied to the evaluation of the hadronic matrix elements $\langle Q_i \rangle$ or, alternatively, of the $B$-factors $B_i$. To just outline some of them, and referring to [18] and the references therein for a more detailed and critical description:

A) The $1/N_c$ approach: The estimate of the hadron matrix elements is based on a $1/N_c$ expansion, that includes chiral loop corrections. This approach shows problems with the scale stability and was eventually dropped in favor of the phenomenological one.
B) The phenomenological approach: Here, the matching scale is taken $\mu = m_c$, the charm quark mass, at which penguins are decoupled from the $CP$ conserving amplitudes, and some of the $B$-factors ($B_{1,2}^{(0,2)}$) and penguin matrix elements can be extracted from the measured amplitudes $A_0$ and $A_2$ and the knowledge of the $\Delta I = 1/2$ rule. Other penguin matrix elements ($B_6$ and $B_8^{(2)}$) are left as free input parameters, to be varied within given limits.

C) The lattice approach: In this case, the matching scale is taken at $\mu = 2$ GeV, above the charm quark mass. The regularization of QCD on a lattice and its numerical simulation is the most satisfactory theoretical approach to the computation of the hadronic matrix elements, because it is based purely on first principles. However, there still remain some technical difficulties and only some operators have been precisely determined on the lattice. Others, especially the crucial $B$-factors for the determination of the direct $CP$ violation, $B_6$ and $B_8$, are determined much less precisely, and refined lattice techniques are still under study to reduce the effective uncertainty.

D) The chiral quark Model: This approach formulates the theory at the rather low scale $\mu \simeq m_\rho \simeq 0.8$ GeV, essentially the chiral symmetry breaking scale. All the hadronic matrix elements are calculated by means of an effective chiral Lagrangian model derived in the spirit of Nambu-Jona-Lasinio models, that incorporates the notion of constituent quark mass in mesons, and up to one-loop (i.e., to order $p^4$) in the chiral expansion.

We now turn to a brief presentation of the situation concerning the theoretical calculation of $\varepsilon'/\varepsilon$ in the framework of the Standard Model.

### 1.4.2 Estimates of $\varepsilon'/\varepsilon$

Applying the effective non-leptonic weak Hamiltonian described above to $\varepsilon'/\varepsilon$, one gets the expression:

$$\frac{\varepsilon'}{\varepsilon} = e^{i\phi} \frac{G_F \omega}{2 |e| \Re A_0} \Im m \lambda_t \left[ \sum_i y_i (Q_i)_0 (1 - \Omega_{IB}) - \frac{1}{\omega} \sum_i y_i (Q_i)_2 \right],$$

(1.59)

where $\lambda_t = V_{td} V_{ts}^*$ and $\phi = \phi' - \phi_\varepsilon \simeq 0$. The subscripts 0 and 2 refer to the isospin eigenvalue of the $\pi\pi$ state. $\Omega_{IB}$ is a correcting factor that accounts for the isospin symmetry breaking that can generate a contribution to the amplitude $A_2$ proportional to $A_0$. 


Usually it is represented by $K \to (\eta \eta')\pi$ followed by mixing $\eta-\eta'-\pi^0$, plus electromagnetic corrections, and the calculated value is of the order of $\Omega_{IB} \simeq 0.20$, with a non-negligible uncertainty. The two sums in Eq. (1.59) run over all contributing operators: the first one is governed by QCD penguin contributions, the second one is fully dominated by electroweak penguin contributions. Notice, there, the minus sign signalling destructive interference and the enhancement factor $1/\omega \simeq 22.2$.

The main source of uncertainty in the calculation of $\epsilon'/\epsilon$ are the hadronic matrix elements $\langle Q_i \rangle$ that unfortunately cannot be directly inferred from the measured $K \to \pi\pi$ amplitudes, and $\Omega_{IB}$.

Making more explicit the analytic dependence of $\epsilon'/\epsilon$ on the values of short-distance coefficients and of the input parameters, one has the numerical expression:

$$\frac{\epsilon'}{\epsilon} = 10^{-4} \left[ \frac{\Im \lambda_t}{1.2 \times 10^{-4}} \right] F(m_t, B_6, B_8, \Omega_{IB}),$$

where, introducing the dominant $B$-factors:

$$F \simeq 16 \left[ \frac{110 \, \text{MeV}}{m_\pi(2 \, \text{GeV})} \right] B_6 \left(1 - \Omega_{IB}\right) - 0.4 \left( \frac{m_t(m_t)}{165 \, \text{GeV}} \right)^{2.5} \left( \frac{B_8}{340 \, \text{MeV}} \right).$$

Tab. 1.3 reports the typical values of the parameters used in the calculation of $\epsilon'/\epsilon$.

The short distance contributions and their $\mu$-dependence are governed by electroweak effects and by perturbative QCD and therefore depend on $\Im \lambda_t$ and $\Lambda_{\text{MS}}^{(4)}$. As such, their values are fully under control. The long distance contributions depend on $B_6, B_8, m_8$, final state interactions, and isospin breaking effect ($\Omega_{IB}$). The corresponding input parameters are not so well known. One should specially notice the dispersion in the estimates of $B_6$ and $B_8$, and in the range $\mu \simeq 1$ GeV their control is still poor.
<table>
<thead>
<tr>
<th>GROUP</th>
<th>$\Re (\varepsilon'/\varepsilon) [10^{-4}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trieste [18, 19]</td>
<td>$22 \pm 8$ (G) 9 $\rightarrow$ 48 (S)</td>
</tr>
<tr>
<td>Rome [20]</td>
<td>$8.1^{+10.3}_{-9.5}$ (G) $-13.0 \rightarrow 37.0$ (S)</td>
</tr>
<tr>
<td>Munich [21]</td>
<td>$9.2^{+6.8}_{-4.0}$ (G) $1.4 \rightarrow 32.7$ (S)</td>
</tr>
<tr>
<td>Dortmund [22]</td>
<td>$6.8 \rightarrow 63.9$ (S)</td>
</tr>
<tr>
<td>Montpellier [23]</td>
<td>$4.0 \pm 5.0$</td>
</tr>
<tr>
<td>Granada-Lund [24]</td>
<td>$34 \pm 18$</td>
</tr>
<tr>
<td>Dubna-DESY [25]</td>
<td>$-3.2 \rightarrow 3.3$ (S)</td>
</tr>
<tr>
<td>Taipei [26]</td>
<td>$7 \rightarrow 16$</td>
</tr>
<tr>
<td>Barcelona-Valencia [27]</td>
<td>$17 \pm 9$</td>
</tr>
<tr>
<td>Beijing [28]</td>
<td>$20 \pm 9$</td>
</tr>
</tbody>
</table>

Table 1.4: Results for $\Re (\varepsilon'/\varepsilon)$ in the Standard Model in units of $10^{-4}$. (G) refers to a Gaussian scan of the input parameters, whereas (S) to a flat scan.

Table 1.4 summarizes the results of the estimates of $\varepsilon'/\varepsilon$, obtained by different theoretical approaches. In Fig. 1.9 the same results are represented with superimposed the band of the experimental value. Clearly, the interpretation of the recent experimental results in $\varepsilon'$ requires a substantial improvement in the theoretical techniques.

1.5 The experimental status

Since the discovery in 1964 that $CP$ symmetry was violated in neutral kaon decays, several experiments have been taken place, aiming to understand the nature of the phenomenon, in particular to discriminate between the direct and indirect mechanisms. In this section a brief review of the experimental results on $CP$ violating parameters is done. In the first part, the $\Re (\varepsilon'/\varepsilon)$ measurements, performed by fixed target experiments at CERN and Fermilab, are reported. The final part is devoted to the CPLEAR experiment, which measured the magnitudes and phases of the amplitude ratios $\eta_{+-}$ and $\eta_{00}$, exploiting a different technique.
1.5.1 Measurements of $\Re \left( \varepsilon'/\varepsilon \right)$

The experimental status of $\Re \left( \varepsilon'/\varepsilon \right)$ measurements appears more defined than the theoretical estimates, summarized above. As explained in Sec. 1.2.2, the parameter $\Re \left( \varepsilon'/\varepsilon \right)$ is sensitive to direct CP violation: a non-vanishing value of $\Re \left( \varepsilon'/\varepsilon \right)$ means that the electroweak non-leptonic Hamiltonian must contain a CP violating term. The expected value of $\Re \left( \varepsilon'/\varepsilon \right)$ is of the order of $O(10^{-3})$. Therefore, in order to be significant, experiments must have a sensitivity of $10^{-4}$ on this parameter. The direct measurement of such a small quantity to the required precision represents a very delicate and challenging task. Dedicated experimental apparatus are needed in order to separate a very small signal from an overwhelming background and keep the systematic uncertainties at a level of $10^{-4}$. Fig. 1.10 shows the amazing improvement of the experimental precision in measuring $\Re \left( \varepsilon'/\varepsilon \right)$ during the past 25 years.

The double ratio method

The most convenient experimental method to determine $\Re \left( \varepsilon'/\varepsilon \right)$ is based on the double ratio $R$, defined in Eq. (1.35). The method consists in counting the number $N_{L,S}^{00}$ and $N_{L,S}^{+}$ of $K_{L,S} \rightarrow \pi^0\pi^0$ and $K_{L,S} \rightarrow \pi^+\pi^-$ decays, respectively, and computing the double ratio $R$, as follows:

$$R = \frac{N_{L}^{00} \ N_{S}^{+}}{N_{L}^{+} \ N_{S}^{00}} \simeq 1 - 6 \Re \left( \varepsilon'/\varepsilon \right). \tag{1.62}$$

$\Re \left( \varepsilon'/\varepsilon \right)$ is then extracted from (1.62). Two considerable features of this technique are:

- the factor 6 enhances the sensitivity of $R$ to $\Re \left( \varepsilon'/\varepsilon \right)$;
by observing all four decay modes simultaneously, or at least two at a time, the beam intensities and all losses associated with the detector (trigger and reconstruction efficiencies) and with the accidental beam activity cancel to first order in the double ratio, thus minimizing the systematic effects.

The statistical error on $R$ is dominated by the uncertainty on the $CP$ violating decay modes of the $K_L$, which are depressed by a factor $\varepsilon$ with respect to $K_S \rightarrow \pi\pi$ decays:

$$\frac{\Delta R}{R}_{\text{stat}} \approx \sqrt{\left(\frac{\Delta N_{L}^{00}}{N_{L}^{00}}\right)^2 + \left(\frac{\Delta N_{L}^{+-}}{N_{L}^{+-}}\right)^2} \approx \sqrt{\frac{3}{2} \frac{1}{N_{L}^{00}}} ,$$

(1.63)

since by isospin symmetry there are twice as many charged two pion decays as neutral two pion decays. The statistical error on $\Re (\varepsilon'\varepsilon)$ is given by:

$$\frac{\Delta \Re (\varepsilon'\varepsilon)}{\Re (\varepsilon'\varepsilon)}_{\text{stat}} \approx \frac{1}{6} \sqrt{\frac{3}{2} \frac{1}{N_{L}^{00}}} .$$

(1.64)

Therefore, in order to achieve a $10^{-4}$ precision, $\sim 4 \times 10^6 K_L \rightarrow \pi^0\pi^0$ events are needed.

As it will be shown in the following sections, the design of the experiments focus on making the inevitable systematic biases in the event counting symmetric between at least two of the four components of the double ratio. In this way, only the differences between two components need to be considered in detail in the analysis. The use of intense beams grants very high statistics and dedicated detectors allow to estimate reliably and reject the overwhelming backgrounds. In fact, a 1% shift in the double ratio (1.62) corresponds to a variation of $1.7 \times 10^{-3}$ in $\Re (\varepsilon'\varepsilon)$.
First experimental evidence of the direct \( CP \) violation: the NA31 and E731 experiments

The first evidence, although not conclusive, of the direct \( CP \) violation phenomenon came from two fixed target experiments in the Eighties: NA31 at CERN and E731 at Fermilab. The denomination “fixed target experiments” comes after the technique used to generate the neutral kaon beams: a \( K_S-K_L \) beam is produced by having an energetic proton beam impinging on a fixed target.

NA31 used the 450 GeV protons provided by the Super Proton Synchrotron (SPS). Intense beams of \( K_L \) and \( K_S \) mesons with energies around 100 GeV were produced alternately by the primary beam (\( 10^{11} \) and \( 10^7 \) protons per pulse, respectively) at two different targets. The \( \pi^0\pi^0 \) and \( \pi^+\pi^- \) decay modes were detected concurrently, however. The \( K_S \) data were taken with the corresponding target displaced in steps of 1.2 m, which resulted in uniform \( K_L \) and \( K_S \) decay distributions over a 48 m long decay region, despite the short \( K_S \) decay length (6 m on the average).

The decay region was evacuated and the space between two tracking wire chambers, placed 25 m apart, was filled with helium. Photons from \( \pi^0 \) decays were measured in a liquid-argon/lead calorimeter which was also used, together with an iron-scintillator calorimeter, to measure the energy of charged pions. There was no magnetic spectrometer. The \( K_{S,L} \rightarrow \pi^+\pi^- \) decays were reconstructed from the hits in the two wire chambers, and the \( K_{S,L} \rightarrow \pi^0\pi^0 \) decays from the measured positions and energies of the photons in the calorimeter.

E731 used Tevatron’s 800 GeV protons incident on a beryllium target to produce two parallel kaon beams, one pure \( K_L \) and one with coherently regenerated \( K_S \) mesons, obtaining this way \( K_L \) and \( K_S \) beams with almost identical momentum and spatial distributions. The regenerator alternated between the beams once every minute, thus essentially eliminating any small difference in beam intensity or detector acceptance for decays from the two beams. In this way \( K_L \) and \( K_S \) decays to \( \pi^0\pi^0 \) and \( \pi^+\pi^- \) final states were detected simultaneously.

A drift chamber spectrometer was employed to determine the \( \pi^+\pi^- \) momenta, mass and decay vertex. The energies and positions of the four photons from the \( \pi^0\pi^0 \) decays were measured with a lead-glass calorimeter. The \( K_{S,L} \rightarrow \pi^0\pi^0 \) decay position and \( \pi^0\pi^0 \) effective mass were obtained from the best pairing of photons into two pions.

The detector acceptance as a function of the decay vertex was determined by using a
highly detailed Monte Carlo simulation, which relied on $K_{e3}$ and $3\pi^0$ decays. Semileptonic events were removed from $\pi^+\pi^-$ sample using the ratio of shower energy to track momentum (for $K_{e3}$ decays) and a muon "hodoscope" (for $K_{\mu3}$ decays). The $3\pi^0$ background to the $\pi^0\pi^0$ data was estimated by Monte Carlo calculations. To obtain $\Re(\varepsilon'/\varepsilon)$, the ratio of vacuum to regenerator events was fitted in momentum and decay position bins.

The final results of NA31 and E731 were:

$$
\Re \left( \frac{\varepsilon'}{\varepsilon} \right) = \begin{cases} 
(23.0 \pm 6.5) \times 10^{-4} & \text{NA31 [29]} \\
(7.4 \pm 5.9) \times 10^{-4} & \text{E731 [30]} 
\end{cases}
$$

The NA31 measurement seems confirm the direct $CP$ violation, indicating a $3.5 \sigma$ effect, while the E731 value is compatible with no effect. To solve the dilemma a new generation of experiments was conceived: NA48 at CERN and KTeV at Fermilab, the successors of the former two, and KLOE at Laboratori Nazionali di Frascati, which exploits a completely different approach, as it will be shown in the sequel.

**Recent results on $\Re(\varepsilon'/\varepsilon)$: the NA48 and KTeV experiments**

The fixed target experiments of the new generation take particular care to collect the four decay modes at the same time and in the same decay volume, in order to exploit the cancellations in the double ratio. The layouts of the NA48 and KTeV experiments are shown in Fig. 1.11. They present a similar elongated geometry, which is optimized to catch the highly-boosted particles resulting from decays of the high-energy kaons, but they differ substantially in the technique they use to produce simultaneous $K_S$ and $K_L$ beams. In Tab. 1.5 some characteristic features of the NA48 and KTeV experiments are listed.

<table>
<thead>
<tr>
<th>Feature</th>
<th>NA48</th>
<th>KTeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean p momentum (GeV/c)</td>
<td>450</td>
<td>800</td>
</tr>
<tr>
<td>P beam intensity</td>
<td>$O(10^{11})$ protons/s</td>
<td>$O(10^{12})$ protons/s</td>
</tr>
<tr>
<td>Mean $K_L$ momentum (GeV/c)</td>
<td>110</td>
<td>70</td>
</tr>
<tr>
<td>$\lambda_S$ (m)</td>
<td>5.9</td>
<td>3.5</td>
</tr>
<tr>
<td>$\lambda_L$ (km)</td>
<td>3.4</td>
<td>2.2</td>
</tr>
</tbody>
</table>

*Table 1.5: Some characteristic features of the NA48 and KTeV experiments.*

In the NA48 experiment two targets, placed at different distances from the decay volume, are used. A schematic picture of the beam transfer lines is shown in Fig. 1.42. The primary
Chapter 1: CP violation in the $K^0 - \bar{K}^0$ system

SPS proton beam impinges on a beryllium rod, 400 mm long and 2 mm in diameter. The charged component of the outgoing particles is swept away by bending magnets. The neutral beam passes through three stages of collimation. It is dominated by long-lived kaons, neutrons and photons, and only a small fraction of the most energetic of the short-lived component ($K_S$ and $\Lambda$) survives. The non-interacting protons from the $K_L$ target are directed onto a mechanically bent mono-crystal of silicon. A small fraction of protons are deflected following the crystalline planes and pass through the tagging station, which precisely registers their time of passage. They are then deflected back onto the $K_L$ beam axis, transported through a series of quadrupoles, and finally directed to the $K_S$ target (same size as $K_L$) located 72 mm above the $K_L$ beam axis.

Seven annular counters surround the fiducial region to record photons escaping the acceptance of the main detector. Charged particles from decays are measured by a magnetic spectrometer. This is followed by a scintillator hodoscope which contributes to the trigger decision and also gives the precise time of charged decays. A liquid krypton calorimeter is used to trigger and reconstruct $K_{S,L} \to \pi^0\pi^0$ decays. It is also used, together with a
subsequent iron-scintillator calorimeter to measure the total visible energy for triggering purposes. Finally, at the end of the beam line, a series of muon counters are used to identify $K_L \rightarrow \pi \mu \nu$ ($K_{\mu 3}$) decays. Two beam counters are used to measure the intensity of the beams. One is located at the extreme end of the $K_L$ beam line ($K_L$ monitor) and the other ($K_S$ monitor) is at the $K_S$ target station.

The whole decay region is contained in a 90 m long evacuated tank. The kaon production angles are tuned to minimize the difference in the $K_L \rightarrow \pi \pi$ and $K_S \rightarrow \pi \pi$ decay spectra over a large range of kaon energies. The beam axes are almost collinear, both pointing to the center of the detector, so that the decay products hit the detector in a similar way. The small remaining differences in beam divergences and beam geometries are corrected using Monte Carlo simulation.

$K_S$ decays are distinguished from $K_L$ decays by means of a tagging technique. Protons directed to the $K_S$ target pass through a high-rate tagging station. $K_S$ events can be identified by comparing the registered proton time to the event time. Since this method is used for both the $\pi^0\pi^0$ and the $\pi^+\pi^-$ samples, the double ratio depends only on the difference in the $K_S$ misidentification probabilities between the two decay modes, and not on their absolute values. Backgrounds affect differently each of the four modes in the double ratio. High resolution detectors are employed to achieve an efficient background rejection. Small remaining impurities due to three body $K_L$ decays are carefully subtracted.

In order to reduce the difference in acceptance, caused by the large difference in average decay lengths, only $K_L$ decays occurring in the region also populated by $K_S$ decays are used for the measurement of the double ratio. Furthermore, a cancellation of the residual difference is obtained by weighting the $K_L$ events used in the double ratio with a function of the proper lifetime $\tau$, proportional to the expected ratio of $K_S$ and $K_L$ decays at time $\tau$. In this way, the systematic accuracy of the result does not rely on a detailed Monte Carlo simulation of the experiment.

By combining the preliminary result from 1997 data set [31] and the value got from 1998-1999 statistics [32],

$$\text{Re} \left( \frac{\varepsilon'}{\varepsilon} \right) = \begin{cases} (18.5 \pm 7.3) \times 10^{-4} & '97, \\ (15.0 \pm 2.7) \times 10^{-4} & '98/'99, \end{cases}$$

the final result is

$$\text{Re} \left( \frac{\varepsilon'}{\varepsilon} \right) = (15.3 \pm 2.6) \times 10^{-4}.$$
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Figure 1.12: The beam transfer line of the NA48 experiment. AKS is the anti $K_S$ counter. To be noticed that the vertical scale is expanded.

Figure 1.13: $K_S$ and $K_L$ beams production in the KTeV experiment. To be noticed that the vertical scale is expanded.
KTeV uses two parallel kaon beams from a single target to enable the simultaneous collection of $K_L$ and $K_S$ decays (see Fig. 1.13). Protons accelerated to an energy of 800 GeV by Fermilab's Tevatron are directed onto a target, and a series of sweeping magnets and collimators are used to produce two side-by-side beams of neutral particles. The beams travel about 90 m before entering the KTeV decay region, allowing short-lived particles ($K_S$ and hyperons) to decay away and leaving mostly $K_L$ and neutrons in roughly equal numbers. The $K_S$ beam is produced by passing one of the $K_L$ beams through a regenerator, which coherently converts some $K_L$ to $K_S$. The regenerator is made of scintillator and is fully instrumented to reduce the background of the kaons scattered inelastically and assigned to the wrong beam. The Regenerator alternates sides between Tevatron beam-extraction cycles (about once per minute) to minimize the effect of any left-right beam or detector asymmetry. A large evacuated region allows the kaons and their decay products to undergo minimal interactions with matter.

The KTeV detector, located just downstream of the decay volume, is shown in Fig. 1.11. Its main components are a charged-particle spectrometer and an electromagnetic calorimeter. The spectrometer consists of four rectangular drift chambers, each with horizontal and vertical wires to measure the positions of charged particles passing through them, and a large dipole magnet. The spaces between the drift chambers are filled with helium gas to reduce scattering. The calorimeter, located downstream of the spectrometer, is built with cesium iodide (CsI) crystals with a geometry to measure both the particle energies and the positions. It is designed to have excellent resolution for the photons from $K_{S,L} \rightarrow \pi^0\pi^0$ decays, but it also is used to distinguish between charged pions and electrons based on fractional energy deposit. Some “photon veto” detectors are used to detect photons which miss the CsI calorimeter, in order to reject $K_L \rightarrow \pi^0\pi^0\pi^0$ decays, which otherwise would be a large source of background in the $\pi^0\pi^0$ samples. These photon veto detectors are placed at various points along the length of the decay volume, at the outer edges of the drift chambers and calorimeter, and behind the beam holes in the calorimeter. An additional detector called the “Mask Anti” (MA) determines the beginning of the decay volume, while the “Collar Anti” (CA) sharply defines the active area around the beam holes of the calorimeter. A scintillator hodoscope upstream of the calorimeter is used to trigger on charged particles. An additional hodoscope, located downstream behind 4 m of steel, is used to detect muons for veto purposes. A detailed Monte Carlo simulation, tested on a sample of $K_L$ three body decays, is used to determine the detector acceptance for the $\pi\pi$ modes and to evaluate backgrounds.

Most of the data were collected in 1996 and 1997. A preliminary analysis on the 20%
Figure 1.14: Summary of $\text{Re}(\varepsilon'/\varepsilon)$ measurements of NA31, E731, NA48, and KTeV. 97a and 97b, relative to KTeV values, refer to two statistically independent data sets. The weighted average has been superimposed.

of data set [33] and on a statistically independent sample of 1997 data gave the results:

$$\text{Re} \left( \frac{\varepsilon'}{\varepsilon} \right) = \begin{cases} (28.0 \pm 4.1) \times 10^{-4} & \text{96/97 (20\%)} \\ (19.8 \pm 2.9) \times 10^{-4} & \text{97} \end{cases}$$

(1.68)

The combined measurement gives:

$$\text{Re} \left( \frac{\varepsilon'}{\varepsilon} \right) = (20.7 \pm 2.8) \times 10^{-4}.$$  

(1.69)

The summary of all the experimental results on $\text{Re} \left( \frac{\varepsilon'}{\varepsilon} \right)$ is reported in Fig. 1.14. The weighted average of the final results of the four experiments is:

$$\text{Re} \left( \frac{\varepsilon'}{\varepsilon} \right) = (17.2 \pm 1.8) \times 10^{-4}.$$  

(1.70)

The averaging procedure gives a $\chi^2$ of 5.5 with 3 degrees of freedom, which corresponds to a confidence level for consistency of about 14%. Nevertheless, looking at the 9.5$\sigma$ deviation of the mean from zero, we can conclude that, 26 years after its formulation, the direct $CP$ violation in the $K^0-\bar{K}^0$ system is experimentally established. Moreover the result $\text{Re} \left( \frac{\varepsilon'}{\varepsilon} \right) \neq 0$ rules out the super-weak model, which foresees only indirect $CP$ violation.

The main concern of the fixed target experiments is to produce simultaneous $K_S$ and $K_L$ beams in the same decay volume. Above we have described the clever solutions
adopted by NA48 and KTeV. However, these introduce into the analysis a lot of systematic effects, which have to be carefully evaluated.

The KLOE and DAΦNE project [34] propose a completely different approach. In a $\phi$-factory kaons are produced in pairs with equal intensity, opposite directions and known energy, thus providing pure and monochromatic $K_S$ and $K_L$ beams. Many cancellations are intrinsic in this approach, on the other hand the intensity of the beams is lower and collecting the necessary statistics requires more time.

### 1.5.2 Measurement of the parameters $\eta^{+-}$ and $\eta^{00}$

An alternative and original approach to test $CP$ violation is that of CPLEAR experiment, which ran at CERN between 1992 and 1996. $CP$ violation is observed by measuring the time-dependent asymmetries between the rates of initially pure $K^0$ and $\bar{K}^0$ states decaying into two pions:

$$A_{\pi\pi} = \frac{\Gamma(K^0 \to \pi\pi) - \Gamma(\bar{K}^0 \to \pi\pi)}{\Gamma(K^0 \to \pi\pi) + \Gamma(\bar{K}^0 \to \pi\pi)}.$$  \hspace{1cm} (1.71)

By using (1.20), $A_{\pi\pi}(t)$ can be expressed as:

$$A_{\pi\pi}(t) \simeq 2 \Re \varepsilon - 2 \left| \eta_{\pi\pi} \right|^2 e^{\frac{1}{2}(\Gamma_S - \Gamma_L)t} \cos(\Delta m t - \phi_{\pi\pi}) \left( 1 + \left| \eta_{\pi\pi} \right|^2 e^{(\Gamma_S - \Gamma_L)t} \right),$$  \hspace{1cm} (1.72)

thereby isolating the $K_L$-$K_S$ interference term. Since $K^0 \to \pi\pi$ and $\bar{K}^0 \to \pi\pi$ are $CP$ conjugate processes, any difference in their rates is a clear sign of $CP$ violation. The asymmetries $A_{\pi^+\pi^-}(t)$ and $A_{\pi^0\pi^0}(t)$, resulting from the interference between the $K_S$ and $K_L$ decay amplitudes, have enabled both the magnitude and phase of the parameters $\eta^{+-}$ and $\eta^{00}$ to be measured.

Intense fluxes of neutral kaons are produced by stopping low-energy antiprotons (200 MeV/c, $10^6$ antiprotons/s) from the Low Energy Antiproton Ring (LEAR) in a high-pressure, gaseous hydrogen target via the reactions $p\bar{p} \to K^-\pi^+K^0$ and $p\bar{p} \to K^+\pi^-\bar{K}^0$.

The branching ratio of each of the above processes is about 0.2%, which means that $K^0$ and $\bar{K}^0$ mesons are produced in equal numbers. However the tagging efficiencies for $K^0$ and $\bar{K}^0$ are not identical because of the different cross sections for interactions of $K^+$ and $K^-$ mesons in the detector material.

The neutral kaon strangeness at production is tagged by the charge of the accompanying $K^\pm$. For each initial strangeness, the numbers of neutral kaons decaying into $\pi^+\pi^-$ and $\pi^0\pi^0$ are measured as a function of the decay time $t$. 

In Fig. 1.15 a longitudinal view of the CPLEAR detector is represented. The cylindrical detector is placed inside a solenoid of 1 m radius and 3.6 m length, which provides a magnetic field of 0.44 T. The tracking system consists of two proportional chambers, six drift chambers and two layers of streamer tubes. Fast kaon identification is provided by a threshold Čerenkov counter sandwiched between two scintillators, which also provide ionization and time-of-flight measurements. An electromagnetic calorimeter made of 18 layers of lead converters and high-gain tubes is used for photon detection.

The $CP$ violating parameter $\eta_{\pi\pi}$ was determined from the time-dependent asymmetry in the decay rates: Eq. (1.72) was fitted to the observed asymmetry, keeping $|\eta_{\pi\pi}|$ and $\phi_{\pi\pi}$ as free parameters.

The results for the charged [35] and neutral [36] pion decay modes are:

\[
|\eta_{+-}| = (2.264 \pm 0.035) \times 10^{-3} , \quad \phi_{+-} = 43.19^\circ \pm 0.73^\circ ; \\
|\eta_{00}| = (2.47 \pm 0.39) \times 10^{-3} , \quad \phi_{00} = 42.0^\circ \pm 5.9^\circ .
\] (1.73)

The values of $\eta_{+-}$ and $\eta_{00}$ different from zero represent a clear indication of $CP$ violation; however they are not adequate to infer the nature of the phenomenon.
Chapter 2

The DAΦNE collider and the KLOE experimental apparatus

In this chapter the DAΦNE collider and the KLOE experimental apparatus are outlined. In the first section, DAΦNE is briefly described. The second part is devoted to the KLOE detector: the structure of the electromagnetic calorimeter, of the drift chamber, and of the quadrupole tile calorimeter are explained. Then, the performance of each subdetector, as measured by using real data, is given.

2.1 The DAΦNE collider

The design of the DAΦNE (Double Annular Φ-Factory for Nice Experiments) electron-positron collider [37], working at the φ resonance energy of 1020 MeV, is strictly related to the main goal of the KLOE experiment: the measurement of the parameter $\Re(e'/e)$ with an accuracy of few $10^{-4}$.

As explained in Sec. 1.5.1, the requirement of having a statistical precision of $10^{-4}$ on $\Re(e'/e)$ implies a number of reconstructed $K_L \to \pi^0\pi^0$ of the order of $4 \times 10^6$. Considering the φ production cross section, the branching ratios, and the detection efficiencies, this corresponds to an integrated luminosity of $10^4$ pb$^{-1}$. Therefore, the collider was designed for a peak luminosity $\mathcal{L} = 5 \times 10^{32}$ cm$^{-2}$s$^{-1}$, which allows the needed luminosity to be collected in two years of data taking.

The single bunch luminosity is given by the product of the number of electrons $N_{e^-}$, the number of positrons $N_{e^+}$ and the crossing frequency $f$ divided by the transverse area
Chapter 2: The DAΦNE collider and the KLOE experimental apparatus

Figure 2.1: View of the intersection beam angle.

of the beams at the interaction point:

\[
\mathcal{L} = f \frac{N_{e^+}N_{e^-}}{4\pi \sigma_x \sigma_y}. \tag{2.1}
\]

This luminosity can be increased using the multi-bunch approach, in which \( n \) bunches circulate in the ring obtaining a luminosity \( n \) times bigger than the single bunch luminosity.

In the realization of a high luminosity collider it is very important the control of bunch-bunch interactions and intrabeam scattering between particles of the same bunch, effects that will decrease the beam stability and its lifetime. Small values of \( \sigma_x \) and \( \sigma_y \) can be obtained by means of the low-\( \beta \) technique, using quadrupoles for strong focusing of the beams near the interaction point. However these quadrupoles introduce perturbations in the particle motion that have to be corrected in the arcs of the rings. To reduce beam-beam interactions and to maximize the number of bunches, two separate rings are used for electrons and positrons. The beams collide in the two low-\( \beta \) interaction points, shown in Fig. 2.2, with a crossing angle in the horizontal \((x, z)\) plane of \( 2\theta_x = 25 \text{ mrad} \) (see Fig. 2.1). In Tab. 2.1 the DAΦNE characteristic parameters are summarized.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam energy</td>
<td>510 MeV</td>
</tr>
<tr>
<td>Current</td>
<td>1300 ÷ 5200 mA</td>
</tr>
<tr>
<td>Number of bunches</td>
<td>30 ÷ 120</td>
</tr>
<tr>
<td>Particles/bunch</td>
<td>( 8.9 \cdot 10^{10} )</td>
</tr>
<tr>
<td>Collision frequency</td>
<td>( 1/10.8\pm 1/2.7 ) ns</td>
</tr>
<tr>
<td>Half crossing angle</td>
<td>( 10\pm 15 ) mrad</td>
</tr>
<tr>
<td>Transversal dimension</td>
<td>( \sigma_x = 2.1 \text{ mm} )</td>
</tr>
<tr>
<td></td>
<td>( \sigma_y = 21 \mu\text{m} )</td>
</tr>
<tr>
<td>Bunch length</td>
<td>( \sigma_z = 3.0 \text{ cm} )</td>
</tr>
<tr>
<td>Luminosity</td>
<td>( (1.35\pm 5.40) \times 10^{32} )</td>
</tr>
</tbody>
</table>

Table 2.1: DAΦNE characteristic parameters.
Figure 2.2: View of the DAΦNE rings, the linear accelerator and the accumulation ring.
Figure 2.3: Typical DAΦNE performance during 2001 data taking.

Figure 2.4: Luminosity delivered to KLOE in three years of data taking.
2.1 The DAΦNE collider

<table>
<thead>
<tr>
<th>DECAY CHANNEL</th>
<th>BRANCHING RATIO $[\times 10^{-2}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^+K^-$</td>
<td>49.4 ± 0.7</td>
</tr>
<tr>
<td>$K_SK_L$</td>
<td>33.6 ± 0.6</td>
</tr>
<tr>
<td>$\rho\pi^0 + \pi^+\pi^-\pi^0$</td>
<td>15.5 ± 0.6</td>
</tr>
<tr>
<td>$\eta\gamma$</td>
<td>1.298 ± 0.029</td>
</tr>
<tr>
<td>$\pi^0\gamma$</td>
<td>0.124 ± 0.010</td>
</tr>
<tr>
<td>$e^+e^-$</td>
<td>0.0296 ± 0.0005</td>
</tr>
<tr>
<td>$\mu^+\mu^-$</td>
<td>0.029 ± 0.004</td>
</tr>
</tbody>
</table>

Table 2.2: Main $\phi$ decay channels [9].

The $\phi$ meson is produced in the $e^+e^-$ annihilation with a cross section $\sigma(e^+e^- \rightarrow \phi) \simeq 3.2 \mu b$. In Tab. 2.2 the main branching ratios of the $\phi$ are listed.

Since the beginning of the data taking in 1999, some problems in setting up the machine limited drastically the luminosity. Then, many of these problems were understood and partly solved. The standard DAΦNE performance during 2001 data taking is shown in Fig. 2.3. The first and second plots show the time dependence of the beam currents and of the beam lifetimes, respectively: the blue line represents the electrons, whereas the red line represents the positrons. The third plot shows the instantaneous luminosity: typical values of the peak luminosity are $3.5 \div 4 \times 10^{31}$. In Fig. 2.4 the total luminosity, delivered to KLOE in the three year activity of DAΦNE is shown.

2.1.1 Neutral kaons tagging in a $\phi$-factory

The main advantage of a $\phi$-factory is that kaons are produced in pairs with equal intensity, opposite directions and known energy. The observation of one kaon guarantees the identification of the other and the knowledge of its direction. In particular, the neutral kaons are produced as collinear pairs in the quantum state defined by $J^{PC} = 1^{--}$ and opposite momenta of about 110 MeV/c.

The process under study, shown in Fig. 2.5,

$$e^+e^- \rightarrow \gamma^* \rightarrow \phi \rightarrow K^0\bar{K}^0,$$

involves electromagnetic, in the $s\bar{s}$ production, and strong, in the hadronization, interactions, therefore it conserves $C$:

$$C(K^0\bar{K}^0) = C(\phi) = C(\gamma^*) = -1.$$
If we consider the final $|K^0\bar{K}^0\rangle$ at $t = 0$, it has $C = -1$ and can be written as:

$$|K^0\bar{K}^0\rangle = \frac{1}{\sqrt{2}} (|K^0,\bar{p}\rangle |\bar{K}^0, -\bar{p}\rangle - |\bar{K}^0, p\rangle |K^0, -p\rangle) .$$ \hspace{1cm} (2.4)

By using Eq. (1.14), Eq. (2.4) can be written as:

$$|K^0\bar{K}^0\rangle = \frac{1 + |\epsilon|^2}{\sqrt{2}(1 - \epsilon^2)} (|K_L,\bar{p}\rangle |K_S, -\bar{p}\rangle - |K_S, p\rangle |K_L, -p\rangle) ,$$ \hspace{1cm} (2.5)

so that the neutral kaon pair produced in $e^+e^-$ annihilation is a pure $K^0\bar{K}^0$ as well as a pure $K_SK_L$ for all times $t$, in vacuum. This means that, if at some time $t$ a $K_L$ ($K_S$, $K^0$, $\bar{K}^0$) is recognized, the other kaon, if still alive, is a $K_S$ ($K_L$, $K^0$, $\bar{K}^0$). A pure $K_S$ beam is a unique feature of $\phi$-factories.

### 2.2 The KLOE detector

KLOE ($K$-LOng Experiment) [38, 39] is a "general purpose" detector, designed to limit to $\sim 10^{-4}$ the effect of systematic errors in the measurement of $\Re e (\epsilon'/\epsilon)$. The design of KLOE is driven by the requirements of a large acceptance for $K_L$ decays into charged and neutral particles, precise location of the decay vertices, good invariant mass resolution, $\gamma$-e-$\pi$ identification and good self-calibrating capabilities. Kaon pairs are produced at

<table>
<thead>
<tr>
<th>DECAY</th>
<th>MOMENTUM [MeV/c]</th>
<th>DECAY LENGTH [cm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi \rightarrow K^+K^-$</td>
<td>$p_K = 127$</td>
<td>$\lambda^\pm = 95$</td>
</tr>
<tr>
<td>$\phi \rightarrow K_LK_S$</td>
<td>$p_K = 110$</td>
<td>$\lambda_L = 343$</td>
</tr>
<tr>
<td>$K_L, K_S \rightarrow \pi^+\pi^-$</td>
<td>$155 &lt; p_\pi &lt; 265$</td>
<td>$\lambda_S = 0.59$</td>
</tr>
<tr>
<td>$K_L, K_S \rightarrow \pi^0\pi^0$</td>
<td>$20 &lt; E_\gamma &lt; 280$</td>
<td></td>
</tr>
</tbody>
</table>

Table 2.3: Kinematics of decays at DAΦNE.
2.2 The KLOE detector

Figure 2.6: Schematic view of the KLOE detector: from the inside to the outside we have the quadrupoles with the associated calorimeters (QCAL), the drift chamber, the electromagnetic calorimeter, the superconducting coil, the iron yoke.

large polar angles relative to the beam axis: \( \frac{d\sigma}{d(\cos \vartheta)} \propto \sin^2 \vartheta \). The relevant features of their decays are listed in Tab. 2.3.

A schematic view of the detector is shown in Fig. 2.6. The decay volume around the interaction point is occupied by a large drift chamber (DC), 2 m radius and 3.2 m length. This is surrounded by a hermetic electromagnetic calorimeter (EmC) to measure the energy, the arrival time, and the impact point of photons. The tracking chamber and the calorimeter are immersed in the magnetic field of a superconducting coil of 2.5 m inner radius and 4.2 m length. The value of the magnetic field, 0.6 T, has been chosen after optimization of acceptances, pattern recognition and momentum resolution. Three permanent low-\( \beta \) quadrupoles are located between the vacuum pipe and the tracking chamber on both sides of the interaction point and are instrumented with two tile calorimeters (QCAL).

The interaction region is surrounded by a 10 cm radius spherical beampipe, made of a 60%Be-40%Al alloy and \( \sim 450 \mu \text{m} \) thick. Almost all \( K_S \)'s decay in vacuum inside the beampipe volume. The chosen low Z material minimizes the energy loss and the multiple scattering of charged particles crossing it as well as the regeneration of the \( K_L \) into \( K_S \).
Figure 2.7: Photograph of the KLOE detector before the insertion of the drift chamber showing the modules of the barrel and of one endcap.

2.2.1 The electromagnetic calorimeter

The main requirements for the electromagnetic calorimeter are:

- identification of neutral decays $K_{L,S} \rightarrow \pi^0\pi^0 \rightarrow 4\gamma$ with photons energy ranging from 20 to 280 MeV and reconstruction of the kaon decays paths with a precision of $\sim 1$ cm;

- rejection of the $K_L \rightarrow \pi^0\pi^0\pi^0$ events;

- particle identification to improve semileptonic decays rejection;

- provide a fast and unbiased first level trigger.

The solution to fulfill these requirements is a fine sampling calorimeter [40], made of lead and scintillating fibers, covering $\sim 98\%$ of $4\pi$ and consisting of a central part, the barrel, and two endcaps, as shown in Fig. 2.6. The barrel is made of 24 modules with trapezoidal cross section, 5.1 m long, 60 cm wide and 23 cm thick. The modules are arranged to form a cylindrical shell with an inner diameter of 4 m, covering the polar region $40^\circ < \vartheta < 140^\circ$, and have the fibers parallel to the beams. Each module is divided in 5 layers and 12 columns. The endcaps are made of 32 modules with different lengths, 23 cm thick and C-shaped, to hermetically close the tracking volume occupied by the drift chamber, and cover the polar regions $26^\circ < \vartheta < 40^\circ$ and $140^\circ < \vartheta < 154^\circ$. 
Each module is obtained by gluing 0.5 mm thick lead foils machined for the housing of the 1 mm diameter fibers, Fig. 2.8. The resulting structure has fiber:lead:glue volume ratio of 48:42:10, an average density of 5 g/cm³, a mean radiation length of 1.6 cm, and a sampling fraction of ~15% for minimum ionizing particles.

The read-out granularity is ~ (4.4 x 4.4) cm² (cell), with one photomultiplier on each side of the cell, making a total number of 4880 read-out channels. The signal of each photomultiplier is read out with an ADC and a TDC. The x and y coordinates of the cell are given by the x and y coordinates of its center, whereas the coordinate along the fiber is measured using the relation:

\[
z = v_f \frac{\Delta T}{2},
\]

with \(\Delta T\) being the time difference at the module ends and \(v_f\) the light propagation speed in the fibers. The measured light propagation speed is \(v_f = 17.2\) cm/ns, which is in good agreement with the refractive index of the fiber core (\(n = 1.6\)) and the bounce angle of the light travelling in the fiber (\(\theta \approx 21°\)).

Energy clusters are reconstructed by summing the ADC response of adjacent cells. The time of the cluster is measured by the time response of the TDC weighed for the energy of the cells constituting the cluster. The detection efficiency for photons with energy larger than 20 MeV is ~ 99%. Energy resolution and linearity of the calorimeter are measured by using radiative "Bhabha" events: \(e^+e^- \rightarrow e^+e^-\gamma\). Event reconstruction from tracking information determines the \(\gamma\) direction and energy with good accuracy. Fig. 2.9(a), top, shows \((E_{\text{clus}} - E_\gamma)/E_\gamma\) versus \(E_\gamma\). Linearity is better than 1% for \(E_\gamma > 75\) MeV, deviations from linearity at the 4-5% level are observed below 75 MeV. The energy resolution as a
Figure 2.9: Energy resolution and linearity for $e^+e^- \rightarrow e^+e^-\gamma$ events (a) and time resolution (b) of the electromagnetic calorimeter.

function of $E_\gamma$ is shown in Fig. 2.9(a), bottom; the fit gives:

$$\frac{\sigma_E}{E} = \frac{5.7\%}{\sqrt{E \text{[GeV]}}} \quad (2.7)$$

In Fig. 2.9(b) the timing resolution for photons from radiative $\phi$ decays is given as a function of $E_\gamma$. Good agreement between the different measurements is observed down to 100 MeV. The curve in the plot is a fit yielding:

$$\sigma_T = \frac{54 \text{ ps}}{\sqrt{E \text{[GeV]}}} \oplus 140 \text{ ps} \quad (2.8)$$

where the constant term is due partly to residual calorimeter miscalibrations ($\sim$50 ps), but mostly to the intrinsic spread due to the finite length in the $z$ direction of the luminous region ($\sim$125 ps).

2.2.2 The drift chamber

The requirements for the KLOE tracking subdetector are [42]:

- homogeneous and isotropic tracking volume;

- large number of track samplings, in order to have a high reconstruction efficiency;
2.2 The KLOE detector

Figure 2.10: The drift chamber completely strung.

- spatial resolution $\sigma_{pc} \sim 200 \mu m$ in the transverse plane and $\sigma_z \sim 5$ mm along the $z$ axis;
- charged vertex resolution $\sigma_{vtx} \leq 3$ mm;
- transverse momentum resolution $\sigma_{pt}/p_T \simeq 10^{-3}$ for a good rejection of the background.

The tracking is done with a drift chamber made of almost square drift cells, placed along circular coronas, with dimensions $(3 \times \pi)$ cm$^2$ in the outer region and $(2 \times 2\pi/3)$ cm$^2$ in the inner region, which is closer to the interaction point, where a higher granularity is needed. In order to achieve the required resolution along the $z$ axis, these cells are arranged in a stereo configuration. Each sense wire layer lies between two field wire layers, the inner layer has the same stereo angle as the sense wires one, while the outer layer has the opposite stereo angle. The ratio between field and sense wires 3:1, is a compromise between high granularity, good spatial resolution and the minimization of the number of wires, to reduce the multiple scattering, and of the load on the endplates.

The drift chamber has to be transparent for multiple scattering and to low energy photons. This has led to the choice of a helium based gas mixture 90\% He – 10\% iC$_4$H$_{10}$ that has a radiation length $X_0 \simeq 1300$ cm.

The whole mechanical supporting structure is made of carbon fiber and epoxy resin. The endplates have a spherical shape of radius $\sim 9.76$ m and 8 mm thickness. The surface is covered by a thin copper foil 30 $\mu$m thick to provide ground connection and electrostatic screen. The two endplates are connected together by 12 longitudinal carbon fiber struts.
Chapter 2: The DAΦNE collider and the KLOE experimental apparatus

Figure 2.11: KLOE drift chamber geometry: the z axis is defined along the beams direction. The stereo angle $\varepsilon$ is fixed in such a way that the ‘stereo drop’ $\delta = R_p - R_0$ is constant.

The rings surrounding the external flanges allow the recovering of the deformations caused by the wires load on the endplates. The outer wall is made of 12 panels of honeycomb and carbon fiber 2 cm thick. The internal tube is made of carbon fiber 0.7 mm thick. The wire materials were chosen with particular care to ensure good transparency: 80 $\mu$m diameter silver plated aluminium field wires and 25 $\mu$m diameter gold plated tungsten sense wires.

In Fig. 2.12 the hardware and software efficiencies of the DC cells, calculated by using cosmic rays tracks, are shown as a function of the drift distance. The “hardware efficiency” is defined as the ratio of the number of hits found in a cell to the number of tracks crossing it. This efficiency is $\sim$99.6% both for big and small cells and is constant over the whole chamber volume. The “software efficiency” is defined by requiring the hit found in the cell to be used by the track fit, and was found to be 97%.

In Fig. 2.13 the spatial resolution for small and big cells is shown as a function of the drift distance. The spatial resolution averaged over the whole detector is smaller than 200 $\mu$m for a large part of the drift cell and is almost independent of the track direction and cell shape.

Dealing with charged particles of momenta ranging from 50 to 300 MeV/c, the main contribution to the momentum resolution comes from multiple scattering. The momentum resolution is given by combining in quadrature the following terms:

\[
\begin{align*}
\left( \frac{\Delta p_T}{p_T} \right)_{\text{ris}} & = \frac{\sigma_{p_T}}{B L \beta} \left( \frac{90}{\sqrt{n+5}} \right), \\
\left( \frac{\Delta p_T}{p_T} \right)_{\text{ms}} & = 0.053 \left( \frac{\sqrt{L}}{X_0} \right).
\end{align*}
\]
2.2 The KLOE detector

Figure 2.12: Hardware (a) and software (b) cell efficiencies as a function of the drift distance.

Figure 2.13: Spatial resolution for small (a) and big (b) cells.
Figure 2.14: Momentum resolution for 510 MeV/c electrons and positrons as a function of the polar angle.

where $p_T$ is the transverse momentum expressed in GeV/c, $L$ is the total track length expressed in meters, $L_{\rho\phi}$ its projection on the $\rho$-$\phi$ plane, $B$ the module of the magnetic field in Tesla, $n$ the sampling number of the track and $X_0$ the material radiation length. In Fig. 2.14 the momentum resolution for 510 MeV/c electrons and positrons is plotted as a function of the polar angle. In the range $50^\circ < \vartheta < 130^\circ$, where the projected track length is constant, the resolution is 1.3 MeV/c.

2.2.3 The quadrupole tile calorimeter

Photons hitting the low-$\beta$ quadrupoles have a high probability of being absorbed. In order to reduce the number of undetected photons, the quadrupoles have been instrumented

Figure 2.15: View of upper half of one QCAL calorimeter.
with compact tile calorimeters (QCAL) \[41\], consisting of a sampling structure made of 1.9 mm lead plates and 1 mm scintillator tiles, as shown in Fig. 2.15. The chosen materials and dimensions take account of mechanical and geometrical constraints, weighting less than 350 Kg and having a thickness of less than 5.5 cm. This choice is related to the insertion of these calorimeters inside the drift chamber and to their positioning on the beam pipe supports.

Average detection efficiencies of ~98% and ~92% have been measured for cosmic rays and photons from \( K_L \rightarrow \pi^0\pi^0\pi^0 \) events, respectively. The measured time resolution is:

\[
\sigma_T = \frac{240 \, \text{ps}}{\sqrt{E \, [\text{GeV}]}}.
\]

In Fig. 2.16 a picture of the QCAL calorimeters with the spherical beampipe is shown.

### 2.2.4 The trigger

The efficiencies for the \( \phi \) decay channels related to the \( CP \) violation measurement of \( \Re (\epsilon' / \epsilon) \) have to be known at a level of \( 10^{-4} \) and trigger efficiencies for these channels must be almost one, if we do not want to introduce systematic effects. The easiest approach could be that of collecting everything and then select offline the interesting channels; however background events, namely machine background (Touschek effect, beam-gas interaction), cosmic rays and Bhabha scattering have very high rates (see Tab. 2.4) as compared to the \( \phi \) production rate.

KLOE has implemented a two level trigger \[43\] based on calorimeter and drift chamber information. A scheme of the trigger logic is given in Fig. 2.17. The identification of
Table 2.4: Monte Carlo rates estimations of the trigger rates at $L = 5 \times 10^{32}$ cm$^{-2}$s$^{-1}$.

<table>
<thead>
<tr>
<th>EVENT TYPE</th>
<th>RATE [kHz]</th>
<th>LEVEL 1 [kHz]</th>
<th>LEVEL 2 [kHz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>all $\phi$ decays</td>
<td>$\sim 5$</td>
<td>$\sim 5$</td>
<td>$\sim 5$</td>
</tr>
<tr>
<td>Bhabha</td>
<td>50</td>
<td>7.22</td>
<td>3.10</td>
</tr>
<tr>
<td>Machine background</td>
<td>$\mathcal{O}(10^3)$</td>
<td>1.30</td>
<td>0.98</td>
</tr>
<tr>
<td>Cosmic</td>
<td>$4\div5$</td>
<td>1.55</td>
<td>1.55</td>
</tr>
<tr>
<td>TOTAL</td>
<td>$\mathcal{O}(10^3)$</td>
<td>15.07</td>
<td>10.63</td>
</tr>
</tbody>
</table>

Table 2.4: Monte Carlo rates estimations of the trigger rates at $L = 5 \times 10^{32}$ cm$^{-2}$s$^{-1}$.

physics channels and backgrounds is done looking at local calorimeter energy deposits and drift chamber multiplicity, considering the characteristics of the signal and background:

- background calorimeter clusters are concentrated in the endcaps;

- background drift chamber hits are mainly distributed near the beampipe while for the signal they are all over the tracking volume;

- background events have low multiplicity in the calorimeter and in the drift chamber.

The calorimeter readout is grouped in 200 overlapping “sectors”, each made of 20-30 channels and with two programmable thresholds: the high threshold is for Bhabha events while the low threshold is for detecting $\phi$ decays. The drift chamber channels are grouped in concentric rings, the “superlayers”, in which the hit multiplicity is evaluated. The first trigger level is given during the first 150 ns from the first signal over threshold and enables the calorimeter front-end electronics. However this time is not sufficient to collect more than 25% of the drift chamber hits so a second level trigger is given after 850 ns allowing the arrival of about 80% of the drift chamber hits.

The time evolution of the physical process in the detector and the propagation time of the trigger signals are much slower than the period of the bunch crossing in DAΦNE, which is 2.7 ns. For this reason the time of the crossing that had generated an event can only be determined offline when reconstructing the whole event.

### 2.2.5 The data acquisition

The data acquisition (DAQ) system [44] has been devised and tested to manage a data flow larger than 80 MBytes/s. The main characteristics of the KLOE data acquisition are:
short dead time, which has to be independent of the event configuration, in order not to introduce systematic effects in the double ratio measurement;

- asynchrony with respect to the trigger signal, due to the small time interval of the bunch crossing;

- low channel occupancy, related to the expected low channel multiplicity.

The acquisition process can be summarized by the three steps shown in the scheme of Fig. 2.18:

1. Each standard VME crate of the front-end electronics (FEE) is provided with a controller, the Read Out Controller for KLOE (ROCK), that reads the FEE boards when asked by the trigger. Whenever a trigger signal is received, the FEE and ROCK boards increment a local trigger number. The communication occurs via a custom data bus, the AUX-bus, which has been optimized to read a low number of channels per crate and with an average reading time of $\sim 35 \mu s$, guaranteeing 10 kHz of trigger frequency. There are 5000 FEE channels of the calorimeter ADCs, 5000 of the calorimeter TDCs and 13000 of the drift chamber TDCs, giving a total of 23000 FEE channels. The FEE crates are grouped in ten chains, each ending with a crate that houses the chain controller, named ROCK-Manager (ROCKM), a CPU, and an FDDI interface.
2. Each ROCKM collects part of the event, associated to a given trigger number (sub-event) from the ROCKs belonging to its chain. The communication between ROCKM and ROCKs is done via the custom chain bus (C-bus). Each C-bus has a maximum data flow of 5 MBytes/s.

3. The CPU sends via FDDI the sub-event to a switch (GIGASWITCH), which distributes the sub-events among the online farm processors. At this point the so-called Data Flow Controller (DFC) guarantees all the sub-events with the same trigger number to be sent to the same processor, which reconstructs and pre-processes the event after having verified its integrity.

Every step of the DAQ is controlled by the Run Control process. Regarding the dead time, the calorimeter ADC and TDC boards have the input stage locked for a fixed time interval of 2.2 $\mu$s after the trigger signal. For the drift chamber TDC boards this time interval is fixed to 2.6 $\mu$s, as required by the drift time. This means that at the estimated rate of 10 kHz the dead time is at the negligible level of 2%.
2.2 The KLOE detector

2.2.6 The slow control

The slow control [45] is a special hardware/software system devoted to the monitoring and control of:

- low voltages of the VME crates and other power supplies;
- high voltages of the EmC photomultipliers and of the DC wires;
- threshold settings;
- temperature and fan speed of the FEE crates.

The slow control system communicates via dedicated Ethernet lines with the process that monitors the gas system of the drift chamber, retrieving the gas flow, its pressure and its composition, and with the process that surveys the magnet, getting the temperature and cryogenics information. The slow control provides also general information on the DAΦNE status, e.g. the number of colliding bunches, the electron and positron currents, the luminosity.

The whole slow control is implemented in VME standard and the CAENet serial transmission protocol is used for the communication with the several controllers, which are housed in the low and high voltage crates of the calorimeter and the drift chamber.
Each subdetector monitoring item has its own V288 CAENet VME interface and they are all accommodated in the slow control VME crate. They are controlled and readout by a local dedicated VME CPU, on which six subdetector slow control processes run: calorimeter and drift chamber high voltages, calorimeter and drift chamber low voltages, and status of the DAQ crates. This CPU is connected via optical fibres to the Slow Control Workstation (KLOESLOW) that circulates information and, eventually in case of anomalies, provides warning or alarm messages. In Fig. 2.19 a sketch of the slow control architecture is shown.
Chapter 3

The $K_L$-tagging and the neutral vertex reconstruction

Three fundamental and related aspects of the analysis are briefly described in this chapter. In the first section, the measurement of the boost and the vertex position of the $\phi$ meson is treated. In the second section, the method used to identify the $\phi \rightarrow K_S K_L$ events is explained. In the last section, the kinematical reconstruction of the $K_L$ vertex is described for the case when the $K_L$ decays into fully neutral pionic states.

3.1 Measurement of the boost and vertex position of the $\phi$ meson

The boost and the decay vertex position of the $\phi$ meson are measured run by run, by using the “Bhabha” events [46]. The electron and positron beams collide in KLOE with a crossing mean angle of 25 mrad and the produced $\phi$ mesons are boosted of about $\sim 13$ MeV/c along the $x$ coordinate. Depending on the running conditions of DAΦNE, the boost may vary run by run. Moreover, due to the perturbations between the bunches belonging to the same beam, the $z$ coordinate of the $\phi$ decay vertex oscillates forward and backward during the collisions.

The $\phi$ boost $P_{\phi}$ is obtained by exploiting the total momentum conservation in the elastic scattering process $e^+e^- \rightarrow e^+e^-$, i.e. $P_{\phi} = p_{e^+} + p_{e^-}$. In Fig. 3.1 the measured components of the $\phi$ momentum are plotted. $P_{\phi}^x$ and $P_{\phi}^z$ are centered around zero, whereas
Chapter 3: The $K_L$-tagging and the neutral vertex reconstruction

$P^z_\phi$ is about $-12.75$ MeV/c on average.

The position and shape of the interaction region is measured by fitting the equation $d = x \sin \varphi - y \cos \varphi$ to the Bhabha track pairs at the point of closest approach to the $z$ axis; $x$ and $y$ are the estimated average coordinates of the Bhabha vertex, $d$ is the signed impact parameter and $\varphi$ is the azimuthal angle of the tracks. $\bar{z}$ is the average of the $z$ coordinates of the electron and positron trajectories at the point of closest approach to $(\bar{x}, \bar{y})$. In Fig. 3.2 the position of the interaction vertex $\vec{V}_\phi = (x_\phi, y_\phi, z_\phi)$ is shown. The spread of 1.4 cm of the $z$ coordinate is mainly due to the bunch length and to the interbunch displacements. In order to perform the measurement to the precision shown in Figs. 3.1 and 3.2, a statistics of about 5000 Bhabha events ($\sim 600 \mu b^{-1}$) is needed.

3.2 The $K_L$-tagging

One of the major advantages of studying $K$ mesons at a $\phi$-factory, is that they are produced in a well defined quantum state, allowing to select pure samples of monochromatic kaons.
3.2 The $K_L$-tagging

3.2.1 $K_L$-tagging methods

The tagging methods, used in $\phi$-factories, exploit the clean and simple topology of the events to select pure $K_L$ or $K_S$ samples, in principle, independently from their decay channels.

At DA$\Phi$NE energy, i.e. the $\phi$-resonance ($\sqrt{s} = M_\phi \simeq 1020$ MeV), the $K_S$ decay length is 5.9 mm, thus the $K_S$ decays on average after a few centimeters. In Tab. 3.1 the most abundant $K_S$ decay modes into pions are reported. Both can be used to tag the $K_L$ in the opposite hemisphere [47]. In Fig. 3.3 a schematic representation of the decay $\phi \rightarrow K_S K_L$ inside the KLOE detector is shown.

<table>
<thead>
<tr>
<th>DECAY MODE</th>
<th>BR [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^+\pi^-$</td>
<td>68.60 ± 0.27</td>
</tr>
<tr>
<td>$\pi^0\pi^0$</td>
<td>31.40 ± 0.27</td>
</tr>
</tbody>
</table>

Table 3.1: Main $K_S$ decay modes into pions with the corresponding branching ratios, as quoted by PDG [9].
Chapter 3: The $K_L$-tagging and the neutral vertex reconstruction

Figure 3.3: A schematic representation (not in scale) of the decay $\phi \to K_SK_L$ inside the KLOE detector. The $K_S$ and $K_L$ momenta are not back-to-back because of the $\phi$ boost.

The $K_S \to \pi^+\pi^-$ decay

In this case, the $K_L$ is tagged when two tracks of opposite charge, coming from a vertex close to the interaction region and kinematically compatible with a $K_S$ decay, are detected. The track momenta of the pions, together with a good determination of the $\phi$ boost provide a very accurate estimate of the $K_L$ direction.

In Fig. 3.4 an image of the KLOE event display shows an example of $K_L \to \pi\ell\nu_\ell$ decay, tagged by the clear signature of the $K_S$ decay into $\pi^+\pi^-$. The magenta dots represent the hits in the drift chamber, the white lines show the fitted tracks, the crossed red squares (⊗) represent the fitted vertices, and the blue circled pluses (⊕) are the centroid positions of the clusters in the electromagnetic calorimeter.

The $K_S \to \pi^0\pi^0$ decay

The $K_S$ decay into two neutral pions is characterized by four prompt clusters, i.e. clusters which are compatible with photons coming from the primary vertex. Prompt clusters are identified by exploiting the very good time resolution of the calorimeter. In this case the $K_L$ is tagged if four prompt clusters are found. In Fig. 3.5 an image of the KLOE event display shows a $K_L \to \pi^+\pi^-\pi^0$ event, tagged by using the $K_S$ decay into $\pi^0\pi^0$. The red circles indicate the four clusters produced by the $K_S$ photons.

In the present analysis the $K_S \to \pi^0\pi^0$ tagging will not be used: it needs different techniques (e.g. a global event fit) still under development, the $K_S \to \pi^0\pi^0$ branching ratio is half the $K_S \to \pi^+\pi^-$, and it gives a worse estimate of the $K_L$ direction, needed
3.2 The $K_L$-tagging

Figure 3.4: An image of the KLOE event display, which shows an example of the $K_L$-tagging by exploiting the clear signature of $K_S \rightarrow \pi^+\pi^-$. 

Figure 3.5: An example of a $K_L \rightarrow \pi^+\pi^-\pi^0$ event, tagged by using the $K_S$ decay into $\pi^0\pi^0$. 
Figure 3.6: Distribution of the $K_S$ decay vertices in the $\rho_S$-$z_S$ plane (a) and distribution of the $K_S$ decay length $\lambda_S = |\vec{V}_S - \vec{V}_\phi|$ (b).

by the neutral vertex algorithm.

3.2.2 $K_L$-tagging by $K_S \rightarrow \pi^+\pi^-$

As discussed in the previous section, the $K_S \rightarrow \pi^+\pi^-$ events have a clean signature: two tracks coming from a vertex close to the interaction region, with opposite charge and total momentum compatible with the $K_S$ one: $\vec{p}_1 + \vec{p}_2 = \vec{P}_S$.

The $K_L$ mesons are tagged by requiring in the selection algorithm:

- One charged vertex inside the "$K_S$ fiducial volume", defined as a cylinder of radius $\rho_S$ and length $2|z_S|$, so that:

$$\rho_S = \sqrt{x_S^2 + y_S^2} \leq 4 \text{ cm} ,$$
$$|z_S| \leq 8 \text{ cm} .$$  \hspace{1cm} (3.1)

This volume contains 99.8% of the vertices of the events $K_S \rightarrow \pi^+\pi^-$. The choice of the shape (cylinder) and of the dimensions of the $K_S$ fiducial volume is made to account for the displacements of the beam-spot along the $z$ coordinate during the collisions, as shown in Fig. 3.6(a). In Fig. 3.6(b) the raw distribution of the $K_S$ decay length is shown for data and Monte Carlo. The spread of the $\phi$ vertex displacements (Fig. 3.2) is of the same order of magnitude as the $K_S$ decay length $\lambda_S$ [Fig. 3.6(b)].
3.2 The $K_L$-tagging

- The two tracks of opposite charge connected to the $K_S$ vertex to have:
  - Total momentum $\vec{p}_1^* + \vec{p}_2^*$ in the $\phi$ rest frame in the range:
    \[ 103.7 \leq |\vec{p}_1^* + \vec{p}_2^*| \leq 116.4 \text{ MeV}/c \, , \]
    \[ (3.2) \]
    which represents a $\pm 3\sigma$ window in the distribution of Fig. 3.7(b). In Fig. 3.7(a) the $K_S$ momentum in the laboratory frame is plotted; the concavity in the histogram is due to the $\phi$ boost, the difference between data and Monte Carlo depends on the fact that in Monte Carlo $P_\phi$ is fixed at the nominal value of $-12.75 \text{ MeV}/c$, while in real data it varies with the machine running conditions.

- Invariant mass $M_{\pi\pi}$ of the two tracks, in the pion hypothesis, in the range:
  \[ 494.7 \leq M_{\pi\pi} \leq 500.5 \text{ MeV}/c^2 \, , \]
  \[ (3.3) \]
  which represents a $\pm 3\sigma$ cut in the $M_{\pi\pi}$ distribution, shown in Fig. 3.8; it is evident that data and Monte Carlo have different resolutions.

- At least one track associated to an electromagnetic cluster in the calorimeter to get the correct timing of the event, being the KLOE trigger asynchronous with respect to the beam crossing and based on the calorimeter TDC's information. To first
approximation, the timing is given by the fastest particle, hypothesized to be a photon; then, off-line corrections are applied to select the correct bunch-crossing of the event. In order to correctly account for the spiralization of the charged pions in the magnetic field and assign the right timing to the $K_S \rightarrow \pi^+\pi^-$ events, it is necessary that at least one pion reaches the calorimeter.

The contamination in the tagged sample, mainly due to the decay $\phi \rightarrow K^+K^-$, has been estimated from Monte Carlo to be below 1%.

In principle, this tagging should be independent of the decay mode of the $K_L$. However, Monte Carlo studies indicate the contrary, as shown in Tab. 3.2: the $K_L$-tagging efficiencies change depending on the $K_L$ decay mode. In particular, for the charged decay channels of the $K_L$, the efficiencies become worse and worse as the average track momentum decreases. This effect is mainly due to the spiralization of tracks in the magnetic field, which causes a superposition of the hits of the $K_L$ tracks with the hits of the $K_S$ pions. The problem originates at the pattern recognition level, where the two drift chamber stereo views are first treated separately and the longitudinal coordinate is not yet known.

### 3.3 The neutral vertex reconstruction

The longlived $K_L$, produced in the $\phi$ decay at the origin, can decay everywhere inside the KLOE detector, being the decay length $\sim 343 \text{ cm}$ in the laboratory-frame. The
3.3 The neutral vertex reconstruction

<table>
<thead>
<tr>
<th>$K_L$ DECAY CHANNEL</th>
<th>EFFICIENCY</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^0\pi^0\pi^0$</td>
<td>0.6606 ± 0.0008</td>
</tr>
<tr>
<td>$\pi^0\pi^0$</td>
<td>0.6622 ± 0.0008</td>
</tr>
<tr>
<td>$K_L$ interacting in EmC</td>
<td>0.6628 ± 0.0008</td>
</tr>
<tr>
<td>$\pi e\nu_e$</td>
<td>0.6345 ± 0.0010</td>
</tr>
<tr>
<td>$\pi\mu\nu_{\mu}$</td>
<td>0.6355 ± 0.0012</td>
</tr>
<tr>
<td>$\pi^+\pi^-\pi^0$</td>
<td>0.6275 ± 0.0017</td>
</tr>
<tr>
<td>$\pi^+\pi^-$</td>
<td>0.6377 ± 0.0010</td>
</tr>
</tbody>
</table>

Table 3.2: $K_L$-tagging efficiencies for different $K_L$ decay mode, calculated in Monte Carlo.

$K_L$ decay modes into neutral pions are considerably more difficult to be investigated experimentally than their charged counterparts. In fact, the neutral pions decay within $10^{-16}$ s, mainly into two photons, and cannot be observed directly. They have to be reconstructed kinematically from the final state photons. In this section the algorithm for the reconstruction of the $K_L$ neutral vertex is described.

3.3.1 The “time-of-flight” algorithm

The vertex of the $K_L$ decay into neutral pions is reconstructed in KLOE by means of a “time-of-flight” algorithm, which relies on the very good time and spatial resolutions of the electromagnetic calorimeter.

The input information, needed by the algorithm, is:

- the position of the decay vertex $\vec{V}_\phi$ and the boost $\vec{P}_\phi$ of the $\phi$ meson,
- the $K_L$ flight direction,
- the impinging points $\vec{r}_{\text{chu}}^i$ of photons onto the calorimeter and their times $T_{\text{chu}}^i$.

The $x_\phi$ and $y_\phi$ coordinates of the $\phi$ decay vertex and the boost $\vec{P}_\phi$ are measured by using Bhabha events, as explained in Sec. 3.3, whereas the $z_\phi$ coordinate of the vertex, which has a worse resolution, is obtained by extrapolating the $K_S$ momentum back to the planes $x = x_\phi$ or $y = y_\phi$, according to the track direction. The $K_L$ momentum, i.e. the $K_L$ direction, is given by $\vec{P}_L = \vec{P}_\phi - \vec{P}_S$, where $\vec{P}_S$ is the $K_S$ momentum, calculated as the sum of the momenta of the charged pions. By using the tagging cuts, described in Sec. 3.2.2, the angular precision on the $K_L$ direction is of the order of $\pm^\circ$—both in polar
and azimuthal angle. The impact points $r_{\text{clu}}^i$ and the arrival times $T_{\text{clu}}^i$ of photons are measured by means of the calorimeter information.

The time $T_{\text{clu}}^i$ of each cluster can be written as the sum of the $K_L$ and the photon times of flight:

$$T_{\text{clu}}^i = \frac{L_k^i}{\beta K c} + \frac{L_\gamma^i}{c}, \quad (3.4)$$

where $L_k^i$ is the $K_L$ decay length, as given by the $i$-th photon, $L_\gamma^i$ is the path length of the $i$-th photon, and $\beta_K = c |\vec{P}_L|/E_L \simeq 0.216$ is the $K_L$ velocity. A determined system of equations, in the unknowns $L_k^i$ and $L_\gamma^i$, is obtained by pairing Eq. (3.4) to Eq. (3.5), which is a geometrical relation connecting the $K_L$ decay vertex to the primary vertex and the $\gamma$ impact point onto the calorimeter:

$$L_\gamma^{i2} = L_k^{i2} + d_i^2 - 2 d_i L_k^i \cos \alpha_i. \quad (3.5)$$

In Eq. (3.5) $d_i = |\vec{r}_{\text{clu}}^i - \vec{V}_\phi|$ is the distance between the decay point of the $\phi$ meson and the impinging point of the photon onto the calorimeter and $\alpha_i$ is the angle between the vectors $\vec{r}_{\text{clu}}^i - \vec{V}_\phi$ and $\vec{P}_L$, as shown in Fig. 3.9. The system of Eqs. (3.4) and (3.5), being of second order, has two solutions, but only one is physically correct.

The time-of-flight is applied for each cluster not associated to any track with $E_{\text{clu}} > 7$ MeV, and for each selected cluster the value $L_k^i$ of the $K_L$ decay length is computed by solving the system of Eqs. (3.4) and (3.5). By definition all the $L_k^i$'s lie along the $K_L$ flight direction. Then, an isolation cut along the flight direction is applied to remove clusters believed not to be associated to photons from the same decay vertex. The neutral vertex $\vec{V}_L$ is, therefore, calculated by using only the closest values of $L_k^i$ (details of the cluster

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**Figure 3.9:** Geometrical quantities used by the time-of-flight algorithm.
3.3 The neutral vertex reconstruction

![Graphs showing differences between neutral and charged vertex coordinates](image)

**Figure 3.10**: Difference between the coordinates of the neutral and the charged vertices in \( K_L \to \pi^+\pi^-\pi^0 \) events.

selection algorithm are given in the next subsection, while details on the efficiency of the algorithm are given in Sec. 4.3.2).

The resolution \( \sigma_L \) on \( L_K^i \) depends on the uncertainty on the cluster time and on the error on the \( K_L \) flight direction. The latter contribution is dominant for large decay lengths. \( \sigma_L \) ranges from ~1.5 cm at \( L_K^i \approx 30 \) cm to ~3.5 cm at \( L_K^i \approx 200 \) cm.

### 3.3.2 Determination of the neutral vertex

The decay lengths \( L_K^i \), calculated by applying the time of flight algorithm to the neutral clusters in the event, are distributed along the \( K_L \) flight direction. A selection criterion must be used to reject the clusters which give bad decay lengths, i.e. the clusters which are not compatible with photons coming from the same decay vertex.

Starting from the three closest \( L_K^i \)'s, their mean is calculated. Then, other clusters are joined to those, if their distance from the mean is smaller than \( 6 \sigma \), where \( \sigma \) is the resolution of the decay length at a distance equal to the mean. The \( K_L \) decay length \( L_K \)
Figure 3.11: Two examples of the reconstruction of the $K_L$ neutral vertex: $K_L \rightarrow \pi^0\pi^0\pi^0$ (above) and $K_L \rightarrow \pi^0\pi^0$ (below) with six and four photons in the final state, respectively.
is calculated as the average of the good $L_K^i$'s, weighted with the cluster energies:

$$L_K = \frac{\sum_i E_{\text{clu}}^i L_K^i}{\sum_i E_{\text{clu}}^i}.$$  \hspace{1cm} (3.6)

Finally, the neutral vertex coordinates are calculated by projecting $L_K$ onto the $x$, $y$, and $z$ axes:

$$\vec{V}_L = \vec{V}_\phi + L_K \frac{\vec{p}_L}{|\vec{p}_L|}.$$ \hspace{1cm} (3.7)

In Fig. 3.10 the positions of the neutral and charged vertices in a sample of $K_L \rightarrow \pi^+\pi^-\pi^0$ events are compared. The charged vertex is obtained from a fit of the $\pi^\pm$ tracks in the chamber, while the neutral vertex is reconstructed from the two photons, produced in the $\pi^0$ decay. Being the resolution of the charged vertex $\sim 3$ mm, the standard deviations of the three superimposed Gaussians (of the order of centimeters) represent the mean resolution of the neutral vertex.

In Fig. 3.11 two examples of reconstructed neutral vertices are shown. The red dashed lines connect the neutral vertex to the clusters used for its determination.
Chapter 4

Measurement of BR(\(K_L \rightarrow 3\pi^0\))

In this chapter the measurement of the branching ratio of the \(K_L\) meson into three neutral pions is presented. The method used to perform the measurement is explained in the first part, then the selection of the sample and the calculation of the efficiencies and the geometrical acceptances are described. In the last section, the result is given.

4.1 The method

The decays of the \(\phi\) meson into neutral kaon pairs always produce a system of \(K_S-K_L\) mass eigenstates. The certain identification of the \(K_S\) allows to select a pure \(K_L\) sample. If the tagging algorithm is unbiased, e.g., it is independent from the \(K_L\) decay channel, it preserves the relative ratios between the \(K_L\) decay modes and the absolute branching ratios can be measured in this tagged sample.

As said in Sec. 3.2.2, the most convenient method to tag the \(K_L\) is to look at the \(K_S\) decays into \(\pi^+\pi^-\). On the other hand, it has already been pointed out that such a tagging algorithm is not completely democratic between charged and neutral decays of the \(K_L\), therefore this bias of the \(K_L\) sample must be taken into account in the measurement of the branching ratios.

The method used in this analysis is explained in the sequel. If the number \(N_{3\pi^0}^{PV}\) of \(K_L \rightarrow \pi^0\pi^0\pi^0\) decays in a given fiducial volume and the number \(N_{K_L}^{tag}\) of \(K_L\)-taggings are
known, the $K_L$ branching ratio into $\pi^0\pi^0\pi^0$ is given by:

$$\text{BR}(K_L \to \pi^0\pi^0\pi^0) = \frac{\Gamma(K_L \to \pi^0\pi^0\pi^0)}{\Gamma_{\text{total}}} = \frac{1}{A_{K_L}^{\text{FV}} A_{K_L}^{\text{tag}}} N_{3\pi^0}^{\text{FV}},$$  \hspace{1cm} (4.1)$$

where $A_{K_L}^{\text{FV}}$ is the geometrical acceptance of the chosen fiducial volume for the $K_L$ meson.

$N_{3\pi^0}^{\text{FV}}$ can be determined from the observed number $N_{\text{obs},n}^{\text{FV}}$ of reconstructed events inside the fiducial volume with $n$ clusters associated to the neutral vertex:

$$N_{3\pi^0}^{\text{FV}} = N_{\text{obs},n}^{\text{FV}} - N_{\text{bkg},n}^{\text{FV}},$$  \hspace{1cm} (4.2)$$

where $N_{\text{bkg},n}^{\text{FV}}$ is the number of background events and the total efficiency $\epsilon_n^{\text{tot}}$ is given by

$$\epsilon_n^{\text{tot}} = \epsilon_n^{\text{NV}} \epsilon_n,$$  \hspace{1cm} (4.3)$$

where $\epsilon_n^{\text{NV}}$ is the efficiency for reconstructing the neutral vertex inside the fiducial volume with $n$ clusters and $\epsilon_n$ is the efficiency of counting $n$ clusters, including the cluster reconstruction efficiency and the calorimeter geometrical acceptance.

In this analysis, only the events with six and seven clusters connected to the neutral vertex are considered.

### 4.2 Sample selection

The typology of the events of interest is represented by the cases in which the $K_S$ decays into $\pi^+\pi^-$, tagging the event, whereas the $K_L$ decays into $\pi^0\pi^0\pi^0$. The $\pi^0$ lifetime is of the order of $10^{-16}$ s, hence it decays immediately after its production, mostly into $\gamma\gamma$, and six photons are present in the final state. The charged pion trajectories are tracked by the drift chamber, while the electromagnetic calorimeter is used to measure the energy, the time, and the impact points of the photons.

Since the result is dominated by the systematic uncertainty, only $4.1\text{ pb}^{-1}$ ($2.807 \times 10^6$ events), collected with very stable running conditions in November 2000, are analyzed. Events are selected if the following requirements are satisfied:

- they pass the $K_L$-tagging;
- a neutral vertex is reconstructed inside the fiducial volume defined by:

$$FV_L = \begin{cases} 
35 < \rho_L < 155 \text{ cm} \\
|z_L| < 155 \text{ cm}
\end{cases},$$  \hspace{1cm} (4.4)$$
4.2 Sample selection

Figure 4.1: Distribution in the $\rho_L$-$z_L$ plane of the reconstructed neutral vertices of the $K_L$ (a). The red box indicates the chosen fiducial volume. Picture (b) shows the distribution of the raw decay length $\lambda_L = |\vec{V}_L - \vec{V}_\phi|$.

- six clusters are associated to the neutral vertex, all of them having:

$$E_{clu} > 20 \text{ MeV} ,$$
$$\rho_{clu} > 65 \text{ cm} .$$

(4.5)

The distribution in the $\rho_L$-$z_L$ plane of the reconstructed neutral vertices and the chosen fiducial volume (red box) are shown in Fig. 4.1(a). The choice of the lower boundary is made to avoid the regeneration of the $K_L$ at the inner wall of the drift chamber, placed at a distance of 25 cm from the detector axis. The upper boundary has been chosen in order to exclude all those events in which the $K_L$ interacts in the calorimeter and a fake vertex is reconstructed, as showing Fig. 4.1(a), where vertices accumulate in the proximity of the calorimeter wall. In the endcaps the problem is less striking, since kaons are produced with a polar angle distribution proportional to $\sin^3 \vartheta$. In Fig. 4.1(b) the $K_L$ decay length

<table>
<thead>
<tr>
<th>CUT</th>
<th>NUMBER OF EVENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_L$-tagging</td>
<td>$1.551 \times 10^6$</td>
</tr>
<tr>
<td>neutral vertex in $FV_L$</td>
<td>$4.967 \times 10^5$</td>
</tr>
<tr>
<td>$N_{clu} = 6$</td>
<td>$4.089 \times 10^4$</td>
</tr>
</tbody>
</table>

Table 4.1: Number of events after the selection cuts.
\begin{table}[h]
\centering
\begin{tabular}{|c|c|}
\hline
NO. OF CLUSTERS & NO. OF EVENTS \\
\hline
3 & 8782 \\
4 & 12450 \\
5 & 32300 \\
6 & 40890 \\
7 & 2074 \\
\hline
\end{tabular}
\caption{Number of events in the selected sample.}
\end{table}

\( \lambda_L \) is shown. The two peaks at low \( \lambda_L \) are due to \( K_L \) regeneration in the beam-pipe and in the DC inner wall, whereas the peak at 200 cm is produced by the fake vertices, reconstructed when the \( K_L \) interacts in the calorimeter.

The energy spectrum of the photons produced in \( \pi^0 \) decays, shown in Fig. 4.3(a), ranges from 20 MeV up to \( \sim 250 \) MeV. The cut at 20 MeV on the cluster energy is applied to reduce the spurious clusters, mainly due to machine background and \( \pi^\pm \) interacting hadronically in the calorimeter.

The numbers of selected events with 3, 4, 5, 6, and 7 clusters associated to the neutral vertex are listed in Tab. 4.2. The same numbers are plotted in Fig. 4.2 and compared to the Monte Carlo predictions: the data and Monte Carlo samples are in good agreement for \( N_{\text{clu}} \geq 4 \). In Tab. 4.3 the sample composition is shown for events with 3, 4, 5, 6, ...

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure42.png}
\caption{Selected sample.}
\end{figure}
and 7 clusters associated to the neutral vertex, as given by the complete Monte Carlo simulation. When a number \( N_{\text{clu}} \geq 6 \) of clusters connected to the neutral vertex is required, there are no other \( \phi \) decays which present the same topology and the selected sample is pure. Therefore, we will consider in this analysis only events with six and seven clusters associated to the neutral vertex, for which there is no background. Background is negligible even in the \( N_{\gamma} = 5 \) subsample, but in this case the systematics related to photon counting are much more difficult to estimate. In principle the QCAL calorimeter could be used to recover part of the \( N_{\gamma} = 5 \) sample, by looking for events with five clusters in the central calorimeter and one hit in this subdetector. Nevertheless, it was chosen not to use it, because it is subject to a high rate of spurious hits, due to machine background, and its Monte Carlo simulation is not yet completely reliable.

The Monte Carlo prediction has been checked by comparing the distributions of the most relevant kinematical variables of the event to the corresponding data distributions. Being the background negligible, the used Monte Carlo data set consists only of \( K_L \rightarrow 3\pi^0 \) events. In Fig. 4.3 the distributions of the single photon energies \( E_{\gamma}^i \), given by the cluster energies \( E_{\text{clu}}^i \), and the photon total energy, defined as \( E_{\text{tot}} = \sum_i E_{\gamma}^i \), are shown. By knowing the neutral vertex position \( \vec{V}_L \) and the impact point \( \vec{r}_{\text{clu}}^i \) of \( \gamma \)'s on the calorimeter the photon momenta are reconstructed, as follows:

\[
\vec{p}_{\gamma}^i = E_{\text{clu}}^i \frac{\vec{r}_{\text{clu}}^i - \vec{V}_L}{|\vec{r}_{\text{clu}}^i - \vec{V}_L|}.
\]

In Fig. 4.4(a) the total photon invariant mass \( M_{\gamma}^{\text{inv}} = \sqrt{(\sum_i E_{\gamma}^i)^2 - |\sum_i \vec{p}_{\gamma}^i|^2} \) is plotted, whereas in Fig. 4.3(b) the \( \pi^0 \) invariant mass is shown. The neutral pions momenta have been reconstructed by considering the 15 combinations of three pairs of photons and choosing the one, which gives the best three \( \pi^0 \) masses. Although no constraint is applied, the particle mass central values differ from the nominal ones by less than 1\%.

<table>
<thead>
<tr>
<th>Sample composition</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi^0\pi^0\pi^0 )</td>
<td>0.335</td>
<td>0.940</td>
<td>0.999</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>( \pi^0\pi^0 )</td>
<td>0.032</td>
<td>0.023</td>
<td>&lt; 10(^{-3} )</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( \pi^+\pi^-\pi^0 )</td>
<td>0.557</td>
<td>0.028</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( \pi \ell \nu_\ell )</td>
<td>0.037</td>
<td>&lt; 10(^{-3} )</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( K_L ) interacting in EmC</td>
<td>0.039</td>
<td>0.008</td>
<td>&lt; 10(^{-3} )</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

**Table 4.3:** Relative sample composition for events with 3, 4, 5, 6, and 7 clusters associated to the neutral vertex (Monte Carlo).
Figure 4.3: The energy spectrum of photons (a) and their total energy (b); solid line is Monte Carlo, points are data.

Figure 4.4: The photons invariant mass (a) and the invariant mass of the reconstructed $\pi^0$'s (b); solid line is Monte Carlo, points are data.
4.3 Efficiencies measurement

In this section, the measurement of the acceptances and of the efficiencies for \( N_\gamma = 6 \), introduced in Sec. 4.1, is described. Real data are used in the method as far as possible; when the use of the Monte Carlo is forced, the agreement of the simulation with the data is carefully checked by tuning, if necessary, the kinematical cuts to the resolution in the Monte Carlo.

4.3.1 Fiducial volume acceptance

As explained in Sec. 4.1, only the events with a neutral vertex reconstructed inside a fiducial volume \((FV_L)\) are counted. In Fig. 4.5 the chosen fiducial volume is depicted. The acceptance \( A_{K_L}^{FV} \) of the fiducial volume is only a geometrical factor which can be calculated analytically, given the \( K_L \) angular distribution \( N_{K_L}(\theta) \) and the \( K_L \) decay length.

Referring to Fig. 4.5, the fraction of \( K_L \) decays expected to occur inside the \( FV_L \) can be written, as follows:

\[
A_{K_L}^{FV} = \frac{\int_{\theta_{\text{min}}}^{\pi - \theta_{\text{min}}} d\theta N_{K_L}(\theta) \int_{L_1(\theta)}^{L_2(\theta)} d\ell f(\ell)}{\int_0^{\pi} d\theta N_{K_L}(\theta) \int_0^\infty d\ell f(\ell)},
\]  

where \( f(\ell) \) is the exponential decay function:

\[
f(\ell) = \frac{1}{\lambda} e^{-\ell/\lambda},
\]
with \( \lambda = c \tau_L \beta_K \gamma_K = 342.9 \pm 2.7 \) cm. \( N_K(\vartheta) \) is the \( K_L \) polar angle distribution, which is given by:

\[
\frac{dN_K(\vartheta)}{d\vartheta} \propto \sin^3 \vartheta .
\]  

(4.9)

To a first approximation, the \( \phi \) boost has been neglected in this calculation, hence the system is symmetric with respect to the \( \phi \) coordinate. By solving Eq. (4.7) for the values in Eq. (4.4), the following result is obtained:

\[
A_{KL}^{E^V} = 0.2807 \pm 0.0015 .
\]  

(4.10)

The error is due to the uncertainties on the parameters, used in the calculation of \( A_{KL}^{E^V} \), namely the \( K_L \) lifetime, and the \( K \) and \( \phi \) masses, as reported in in Tab. 4.4. By using a dedicated Monte Carlo, the effect of the \( \phi \) boost has been estimated to be less than 0.0001. The systematic effect due to the neutral vertex resolution is accounted for in the definition of the reconstruction efficiency.

**4.3.2 Reconstruction efficiency of the neutral vertex**

The reconstruction efficiency of the neutral vertex has been estimated by using the Monte Carlo simulation. Then, the Monte Carlo value has been corrected in order to account for the difference between the Monte Carlo and the real data.

The reconstruction efficiency of the neutral vertex inside the fiducial volume has been defined as the ratio of the number of events with a reconstructed vertex inside the fiducial volume with six clusters, having \( E_{clu} > 20 \) MeV and \( \rho_{clu} > 65 \) cm, to the number of events with a vertex generated inside the fiducial volume and six clusters in the calorimeter. Such a definition gives the following Monte Carlo efficiency:

\[
\epsilon_{MC,6}^{NV} = 0.9292 \pm 0.0018 .
\]  

(4.11)
4.3 Efficiencies measurement

A subsample of $K_L$ decays into fully neutral final states has been selected both in data and Monte Carlo, by asking for events with no other charge vertex in DC than the $K_S$ vertex and no other track than the charged pion tracks. Then, the total efficiencies for reconstructing a neutral vertex inside the $K_L$ fiducial volume have been calculated as the fraction of events with a neutral vertex in the fiducial volume with respect to the expected events, i.e. the number of total selected events times the geometrical acceptance $A_{K_L}^{PV}$ of the chosen fiducial volume. By comparing the data and Monte Carlo values, the correction factor is estimated to be:

$$\eta_{corr}^{NV} = 0.9782 \pm 0.0090 . \quad (4.12)$$

When this correction is applied, the reconstruction efficiency of the neutral vertex with six clusters becomes:

$$\epsilon_6^{NV} = \eta_{corr}^{NV} \epsilon_{MC,6}^{NV} = 0.9089 \pm 0.0085 . \quad (4.13)$$

By evaluating $\epsilon_6^{NV}$ inside the fiducial volume, we account also for the cases when the timing of the event is wrong. Since the trigger is asynchronous with respect to the bunch crossing, the correct timing of the events must be calculated offline for each event. It may happen that the event is associated to a wrong bunch crossing, moving all the times by multiples of 2.715 ns. In this case, also the neutral vertex results moved by multiples of 15 cm along the flight direction of $K_L$ and it may fall outside the fiducial volume. In the definition of $\epsilon_6^{NV}$ given above, this is seen as an inefficiency of the neutral vertex algorithm.

4.3.3 Total efficiency for photon detection

When counting the photons produced in $\pi^0$ decays, the main systematic effects are due to the not ideal structure of the electromagnetic calorimeter, to some limits in the reconstruction offline software, as well as to external sources, which produce spurious photons.

The hardware inefficiency of the electromagnetic calorimeter is due to the fact that it is not completely hermetic and some photons may escape from its acceptance; moreover, in some regions, the EmC is less efficient in collecting the energy released by photons. The main problems related to the clustering algorithm are the “cluster splitting,” when two clusters are reconstructed from one photon, and the “merging,” when the algorithm is not able to resolve two different photons, close in space and time, and reconstructs only one cluster. Spurious photons, coming from the beams (“accidentals”) or produced in the
hadronic interaction of charged pions in the EmC (“fragments”), can be in the right time window and be associated to the neutral vertex, biasing the $N_{\gamma} = 6$ sample. Another subtler problem is the so-called “pile-up,” when two photons impinge the calorimeter in the same point, releasing a cluster which does not carry anymore a useful information.

The number of events with six observed clusters connected to the neutral vertex have different contributions. All second order effects, namely the cases in which there are two cluster splittings, or one splitting and one merging, or one splitting and one accidental and so on, are neglected. With this assumption the cases to take into account are enumerated in Tab. 4.5. In the $K_L \to 3\pi^0$ final state one can have either six photons (when all $\pi^0$ decay into $\gamma\gamma$) or five photons (when a $\pi^0$ decays into $e^+e^-\gamma$), thus six clusters, associated to the neutral vertex, are counted when:

- there are six photons in the final state, all of them are in the geometrical acceptance of the EmC, and all of them are reconstructed;

- there are six photons in the final state, all of them are in the geometrical acceptance of the EmC, but only five of them are reconstructed and a split cluster, or a fragment, or an accidental is associated to the neutral vertex;

- there are six photons in the final state, five in geometrical acceptance and reconstructed, plus one split cluster, or a fragment, or an accidental;

<table>
<thead>
<tr>
<th>NO. OF $\gamma$'s FROM $\pi^0$'s</th>
<th>$\gamma$'s FROM $\pi^0$'s IN GEOM. ACCEPT.</th>
<th>REC. CLUSTERS</th>
<th>SPLIT CLUSTERS</th>
<th>FRAGMENT CLUSTERS</th>
<th>ACCIDENTAL CLUSTERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>6</td>
<td>6</td>
<td>1</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>5</td>
<td>-</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>5</td>
<td>-</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>5</td>
<td>1</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 4.5: Components of the event sample with six clusters connected to the neutral vertex, when second order effects are neglected (see the text).
4.3 Efficiencies measurement

<table>
<thead>
<tr>
<th>NO. OF γ's FROM π^0's</th>
<th>γ's FROM π^0's IN GEOM. ACCEPT.</th>
<th>REC. CLUS.</th>
<th>SPLIT CLUSTERS</th>
<th>FRAGMENT CLUSTERS</th>
<th>ACCIDENTAL CLUSTERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>BR^3_γ</td>
<td>A_{6,6}</td>
<td>(ε_{clus})^6</td>
<td>(1 - P_s)^6</td>
<td>(1 - P_{frag})^6</td>
<td>(1 - P_{acc})</td>
</tr>
<tr>
<td>BR^3_γ</td>
<td>A_{6,6}</td>
<td>(ε_{clus})^6</td>
<td>(1 - P_s)^5</td>
<td>(1 - P_{frag})^5</td>
<td>(1 - P_{acc})</td>
</tr>
<tr>
<td>BR^3_γ</td>
<td>A_{6,6}</td>
<td>(ε_{clus})^6</td>
<td>(1 - P_s)^5</td>
<td>(1 - P_{frag})^5</td>
<td>(1 - P_{acc})</td>
</tr>
<tr>
<td>3 BR^2_γ BR_e</td>
<td>A_{5,5}</td>
<td>(ε_{clus})^5</td>
<td>(1 - P_s)^4</td>
<td>(1 - P_{frag})^4</td>
<td>(1 - P_{acc})</td>
</tr>
</tbody>
</table>

Table 4.6: Factors for the observed sample with six clusters associated to the neutral vertex, neglecting second order effects; the factors of each row refer to the event topology of the corresponding row in Tab. 4.5.

- there are five photons in the final state (a π^0 decays into e^+e^-γ), all reconstructed in the EmC geometrical acceptance, one of them splits, or a fragment or an accidental is wrongly associated to the neutral vertex.

In order to consider all these contributions and to factorize their efficiencies, the following coefficients are introduced:

BR_γ: the π^0 branching ratio into γγ;

BR_e: the π^0 branching ratio into e^+e^-γ;

A_{6,6}: the fraction of events with six clusters in geometrical acceptance out of six generated;

A_{5,6}: the fraction of events with five clusters in geometrical acceptance out of six generated;

A_{5,5}: the fraction of events with five clusters in geometrical acceptance out of five generated;

⟨ε_{clus}⟩: the cluster reconstruction efficiency averaged in energy;
Figure 4.6: EmC geometrical acceptance.

$P_s$: the probability that a cluster splits;

$P_{\text{frag}}$: the probability for a cluster associated to the neutral vertex to be a fragment;

$P_{\text{acc}}$: the probability to have an event with at least one accidental cluster associated to the neutral vertex.

$P_s$ and $P_{\text{frag}}$ are assumed to be binomial probabilities. If all second order terms of $BR_e$, $P_s$, $P_{\text{frag}}$ and $P_{\text{acc}}$ ($BR_e^2$, $P_s^2$, $P_{\text{frag}}^2$, $P_{\text{acc}}^2$, and their products) are neglected, each entry of Tab. 4.5 can be associated to a product of factors, as shown in Tab. 4.6.

From Tab. 4.6 it is straightforward to express the following formula for the efficiency of detecting six clusters:

$$
\epsilon_6 \approx BR_\gamma A_{6,6} \langle \epsilon_{\text{clu}} \rangle^6 (1 - P_s)^6 (1 - P_{\text{frag}})^6 (1 - P_{\text{acc}})+ \\
+ \left[ BR_\gamma^3 A_{6,6} \langle \epsilon_{\text{clu}} \rangle^5 + BR_\gamma^3 A_{5,6} \langle \epsilon_{\text{clu}} \rangle^5 + 3 BR_\gamma^2 BR_e A_{5,5} \langle \epsilon_{\text{clu}} \rangle^5 \right] \times \\
\times \left[ 5 P_s (1 - P_s)^4 (1 - P_{\text{frag}})^5 (1 - P_{\text{acc}}) + (1 - P_s)^5 \right] \\
5 P_{\text{frag}} (1 - P_{\text{frag}})^4 (1 - P_{\text{acc}}) + (1 - P_s)^5 (1 - P_{\text{frag}})^5 P_{\text{acc}} \right] .
$$

(4.14)

In the following, the quantities, which appear in this formula, will be separately measured and, then, used in Eq. (4.14) to get the value of $\epsilon_6$.

**Photon geometrical acceptances**

$A_{6,6}$, $A_{5,6}$, and $A_{5,5}$ of Eq. (4.14) are pure geometrical factors, which must be computed from Monte Carlo. As represented in Fig. 4.6, the geometrical acceptance of the calorime-
4.3 Efficiencies measurement

Figure 4.7: Distribution of the angle between photon and $K_L$ directions for data (points) and Monte Carlo (solid line).

The acceptance has been defined as a cylindrical surface, delimited by the calorimeter inner wall, with two circular holes of 65 cm radius in the bases. All photon losses inside this acceptance are accounted for in the calculation of the cluster reconstruction efficiency.

The acceptance factors have been determined in the following way. The directions of the photon momenta, generated in Monte Carlo, have been projected onto the internal surface of the calorimeter and for each event the number of photons inside the defined acceptance have been counted. Then, $A_{6,6}$, $A_{5,5}$, and $A_{5,5}$ have been calculated as the fraction of events with six photons out of six, five photons out of six, five photons out of six in acceptance, respectively. The values obtained for the acceptance coefficients are reported in Tab. 4.7 with the corresponding errors. In order to check the Monte Carlo description of the photon emission angles, in Fig. 4.7 the distribution of the cosine of the angle between the photons and the $K_L$ directions $\cos \alpha = \vec{p}_i^\gamma \cdot \vec{P}_L / (|\vec{p}_i^\gamma| |\vec{P}_L|)$ has been

<table>
<thead>
<tr>
<th>ACCEPTANCE FACTOR</th>
<th>VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{6,6}$</td>
<td>0.7543 ± 0.0007</td>
</tr>
<tr>
<td>$A_{5,5}$</td>
<td>0.2138 ± 0.0007</td>
</tr>
<tr>
<td>$A_{5,5}$</td>
<td>0.784 ± 0.004</td>
</tr>
</tbody>
</table>

Table 4.7: Photon acceptance coefficients calculated in Monte Carlo.
Cluster reconstruction efficiency

The reconstruction efficiency of clusters has been calculated in the KLOE Collaboration by using different physical channels, for instance, $e^+e^- \rightarrow e^+e^-\gamma$ and $\phi \rightarrow \pi^+\pi^-\pi^0$. The tracking system and the momentum conservation allow to determine the photon direction. The efficiency is derived by opening a cone around the estimated direction of the photon and looking for a cluster inside the cone. Then, the efficiency is given as a function of the photon energy and polar angle. However, the photons of the considered events come from the origin and it is easy to determine their impact angle on the calorimeter. The case of $K_L \rightarrow 3\pi^0 \rightarrow 6\gamma$ is quite different, because the photons may come from any point inside the fiducial volume and may impinge the EmC with different angles.

For this reason, it has been chosen to calculate the cluster reconstruction efficiency in a $K_L \rightarrow 3\pi^0$ data set, produced with the complete Monte Carlo simulation of the KLOE apparatus. Then, a sample of $K_L \rightarrow \pi^+\pi^-\pi^0$ events has been used to calculate a correction coefficient, which accounts for the difference between data and Monte Carlo [48]. In Fig. 4.8 the Monte Carlo efficiency of reconstructing a cluster with $E_{\text{clus}} > 20$ MeV and $\rho_{\text{clus}} > 65$ cm is plotted as a function of the cluster energy. The average efficiency is given by:

$$\langle e_{\text{clus}}^{\text{MC}} \rangle = 0.9428 \pm 0.0008 \quad (4.15)$$
4.3 Efficiencies measurement

Data and Monte Carlo are compared in the $K_L \rightarrow \pi^+\pi^-\pi^0$ samples, where the number of events with one and two clusters are counted. The energy and the spatial distribution of the selected clusters are the same as the $K_L \rightarrow 3\pi^0$ sample, as well as the fiducial volumes; the cluster times must be compatible with the $K_L$ vertex hypothesis. The following correction factor has been calculated:

$$\eta_{\text{corr}}^{\text{clu}} = 0.9918 \pm 0.0055 .$$

By applying this correction to the Monte Carlo efficiency, it results:

$$\langle \epsilon_{\text{clu}} \rangle = \eta_{\text{corr}}^{\text{clu}} \langle \epsilon_{\text{clu}}^{MC} \rangle = 0.9351 \pm 0.0055 .$$

**Accidental cluster probability**

The accidental clusters are mainly due to the machine background, e.g. photons emitted by electrons and positrons of the beams. They may be emitted at any moment and, therefore, are characterized by a uniform time distribution. In particular, they are independent of the timing of particles produced in $\phi$ decays. It may happen that an accidental cluster is in time with the $K_L$ photons and is wrongly associated to the neutral vertex.

As shown in Fig. 4.10, the accidental clusters accumulate in the endcaps around the beam-pipe. Their energy spectrum, in Fig. 4.9, presents a $1/E_\gamma$ behavior and this is
Figure 4.10: Distribution in the $x$-$y$ plane (a) and in the $p$-$z$ plane (b) of the centroid positions of accidental clusters, whose times are in the time window $[-60, -40]$ ns.

typical of radiated photons. It is evident that the energy cut at 20 MeV is not sufficient to eliminate completely their contamination.

The probability to have an event with at least one accidental cluster can be determined from the data. In order to estimate the amount of accidental clusters, the distribution of the variable $T_{\text{clu}} - R_{\text{clu}}/c$ is considered, where $T_{\text{clu}}$ is the cluster time and $R_{\text{clu}} = |\vec{r}_{\text{clu}} - \vec{V}_\phi|$ is the distance of the cluster centroid from the $\phi$ vertex. Referring to Fig. 4.11(a), the bump at 0 is due to the photons coming from the interaction region, the peak around +2.5 ns is due to the charged pions, whereas the broad shoulder on the right of the pion peak depends on the photons produced in $K_L$ decay, which reach the calorimeter at a later time. The multipeak structure of the tails reflects the multibunch structure of the beams.

The probability $P_{\text{acc}}$ to have an accidental cluster is estimated by counting the fraction of events with at least one cluster in the time window $[-60, -40]$ ns and, then, rescaling this number for the time window of the photons associated to the neutral vertex. To define the latter time window the distribution of $T_{\text{clu}} - D_{\text{clu}}/c$ has been fitted with two Gaussians and the interval $[-3\sigma, 3\sigma]$ has been taken, where $\sigma$ is the standard deviation of the broader Gaussian. $D_{\text{clu}} = |\vec{r}_{\text{clu}} - \vec{V}_L|$ is the distance between the cluster centroid and the neutral vertex.
4.3 Efficiencies measurement

In conclusion, the probability to have an event with at least one accidental cluster is:

\[ P_{\text{acc}} = 0.0035 \pm 0.0001 \]  \hspace{1cm} (4.18)

**Probability for fragments**

When the charged pions reach the calorimeter, they interact hadronically with the calorimeter material, producing neutral pions which decay into photons. The most energetic photons may escape from the calorimeter and generate other clusters in a different calorimeter region. Such clusters are called “fragments” since, although they can travel faraway, they are part of the hadronic clusters of the charged pions.

The probability \( P_{\text{frag}} \) for a cluster to be a fragment has been estimated in Monte Carlo, by calculating the fraction of clusters produced by any other source than a \( \pi^0 \) photon and associated to the neutral vertex:

\[ P_{\text{frag}} = 0.0038 \pm 0.0003 \]  \hspace{1cm} (4.19)

The check of this Monte Carlo prediction in the real data represents a difficult task that is under study. However, it has been seen that a 10% variation of \( P_{\text{frag}} \) affects the final result by less than 0.1%.
<table>
<thead>
<tr>
<th>DECAY MODE</th>
<th>BR [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma\gamma$</td>
<td>98.798 ± 0.0032</td>
</tr>
<tr>
<td>$e^+e^-\gamma$</td>
<td>1.198 ± 0.0032</td>
</tr>
</tbody>
</table>

Table 4.8: Main $\pi^0$ decay modes with the corresponding branching ratios, as quoted by PDG [9].

**Splitting probability**

The probability $P_s$ that a cluster with $E_{clu} > 20$ MeV and $\rho_{clu} > 65$ cm splits has been estimated from the data sample by comparing the number of events with six and seven clusters associated to the neutral vertex. It has been shown that the subsamples with $N_\gamma = 6$ and $N_\gamma = 7$ have no background. Therefore, the seventh cluster can only be a split cluster, a fragment, or an accidental.

The number $N_{6}^{FV}$ of the events in the fiducial volume with six clusters associated to the neutral vertex is given by $\varepsilon_6 N_{3\pi^0}^{FV}$, whereas the number of events with seven clusters can be written as $N_{7}^{FV} = \varepsilon_7 N_{3\pi^0}^{FV}$, where

$$
\varepsilon_7 \simeq BR_\gamma^3 A_{6,6} (\varepsilon_{clu})^6 \left[ 6 P_s (1 - P_s)^5 (1 - P_{frag})^6 (1 - P_{acc}) + 
+ 6 (1 - P_s)^6 P_{frag} (1 - P_{frag})^5 (1 - P_{acc}) + 
+ (1 - P_s)^6 (1 - P_{frag})^6 P_{acc} \right].
$$

The ratio of the number of events $N_{7}^{FV}/N_{6}^{FV}$ is given by:

$$
\frac{N_{7}^{FV}}{N_{6}^{FV}} = 0.050 \pm 0.001 \simeq \frac{\varepsilon_7(P_s)}{\varepsilon_6(P_s)}. \quad (4.21)
$$

If $P_{acc}$ and $P_{frag}$ are taken from Eqs. (4.18) and (4.19), respectively, this equation has only one unknown, i.e. the splitting probability $P_s$. By solving Eq. (4.21) with numerical methods, the result is:

$$
P_s = 0.0046 \pm 0.0002. \quad (4.22)
$$

The uncertainty on $P_s$ has been estimated by varying the ratio $N_{7}^{FV}/N_{6}^{FV}$ within its error.

**Calculation of the photon counting efficiency**

At this point, it is possible to calculate $\varepsilon_6$. The values of the $\pi^0$ branching ratios into $\gamma\gamma$ and $e^+e^-\gamma$ are taken from PDG and are reported in Tab. 4.8. All the other coefficients
4.4 Measurement of $\text{BR}(K_L \rightarrow 3\pi^0)$

The measured value of the $K_L$ branching ratio into three neutral pions, before the correction for the tagging biases, is:

$$BR(K_L \rightarrow \pi^0 \pi^0 \pi^0)_{\text{raw}} = 0.2105 \pm 0.0011 \text{ (stat.)} \pm 0.0077 \text{ (syst.)}.$$ (4.25)

Taking into account the bias in the $K_L$-tagging algorithm, due to the different efficiencies listed in Tab. 3.2, the observed raw branching ratio $BR_i^\text{raw}$ of the $i$-th decay channel of the $K_L$ is actually connected to all the measured branching ratios:

$$BR_i^\text{raw} = \frac{\epsilon_i^\text{tag} \cdot BR_i^\text{meas}}{(1 - F_{\text{crash}}) \sum_j \epsilon_j^\text{tag} \cdot BR_j^\text{meas} + \epsilon_{\text{crash}}^\text{tag} \cdot F_{\text{crash}}},$$ (4.26)
Table 4.10: Contributions to the statistical and systematic error on $BR(K_L \to 3\pi^0)$, when using the sample of events with six clusters connected to the neutral vertex.

<table>
<thead>
<tr>
<th>SOURCE</th>
<th>STAT. ERROR [$\times 10^{-4}$]</th>
<th>SYST. ERROR [$\times 10^{-4}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N^F_6$</td>
<td>10.4</td>
<td>-</td>
</tr>
<tr>
<td>$N^F_{KL}$</td>
<td>1.7</td>
<td>-</td>
</tr>
<tr>
<td>$\epsilon_6$</td>
<td>-</td>
<td>73.6</td>
</tr>
<tr>
<td>$A^F_{KL}$</td>
<td>-</td>
<td>11.2</td>
</tr>
<tr>
<td>$\epsilon^N_6$</td>
<td>-</td>
<td>19.7</td>
</tr>
<tr>
<td>TOTAL</td>
<td>10.5</td>
<td>77.0</td>
</tr>
</tbody>
</table>

where $\epsilon_i^{tag}$ is the tagging efficiency for the $i$-th decay channel, $F_{crash}$ is the fraction of $K_L$'s, which reach the calorimeter wall, and $\epsilon_i^{tag}$ is the tagging efficiency for this kind of events.

Eq. (4.26) can be inverted to extract the measured branching ratio $BR_i^{meas}$:

$$BR_i^{meas} = \frac{BR_i^{raw}}{1 - (1 - F_{crash}) BR_i^{raw}} \left(1 - F_{crash}\right) \sum_{j\neq i} \frac{\epsilon_j^{tag} BR_j^{meas}}{\epsilon_j^{tag}} + \epsilon_j^{tag} F_{crash} F_{crash}. \quad (4.27)$$

By using in Eq. (4.27) the value $F_{crash} = 0.5330 \pm 0.0026$, calculated with Eq. (4.7), the tagging efficiencies reported in Tab. 3.2, and taking the other branching ratios from PDG, the corrected value of $BR(K_L \to \pi^0\pi^0\pi^0)_{meas}$ is:

$$BR(K_L \to \pi^0\pi^0\pi^0)_{meas} = 0.2073 \pm 0.0011 \text{ (stat.)} \pm 0.0084 \text{ (syst.)}, \quad (4.28)$$

to be compared with the PDG value [9]:

$$BR(K_L \to \pi^0\pi^0\pi^0)_{PDG} = 0.2111 \pm 0.0027. \quad (4.29)$$

In Eq. (4.28) the systematic error accounts also for the contribution from the applied correction.

The correction of Eq. (4.27) has been calculated assuming that the trigger efficiencies are well above 99% for all the $K_L$ decay channels, as actually is if both the calorimeter and the drift chamber triggers are used. It remains to carefully evaluate the effect of the trigger, when the $K_L$ passes through the calorimeter without interacting. Nevertheless, it is worthwhile to outline that such a correction is needed only to measure the absolute $K_L \to \pi^0\pi^0\pi^0$ branching ratio, whereas in the ratio $\Gamma(K_L \to 2\pi^0)/\Gamma(K_L \to 3\pi^0)$ this effect cancels between the numerator and the denominator.
4.4 Measurement of $BR(K_L \to 3\pi^0)$

<table>
<thead>
<tr>
<th>SOURCE</th>
<th>ERROR [$\times 10^{-4}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$BR_\gamma$</td>
<td>0.2</td>
</tr>
<tr>
<td>$\langle \epsilon_{clu} \rangle$</td>
<td>8.8</td>
</tr>
<tr>
<td>$A_{6,6}$</td>
<td>0.2</td>
</tr>
<tr>
<td>$P_s$</td>
<td>5.3</td>
</tr>
<tr>
<td>$P_{\text{frag}}$</td>
<td>7.9</td>
</tr>
<tr>
<td>$P_{\text{acc}}$</td>
<td>0.4</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td><strong>13.0</strong></td>
</tr>
</tbody>
</table>

Table 4.11: Contributions to $\epsilon_7$ uncertainty.

As a check, the same method has been used for the data set with seven clusters connected to the neutral vertex. As in the case of six clusters, the branching ratio is given by:

$$BR(K_L \to \pi^0\pi^0\pi^0) = \frac{N_{FV}^{FV}}{N_{K_L}^{tag} A_{K_L}^{FV} \epsilon_{6}^{NV} \epsilon_{7}} ,$$

(4.30)

where $\epsilon_7$ is defined in Eq. (4.20). Its value is:

$$\epsilon_7 = 0.0249 \pm 0.0013 .$$

(4.31)

In Tab. 4.11 the error sources are summarized. In this case the raw branching ratio is:

$$BR(K_L \to \pi^0\pi^0\pi^0)_{\text{raw}} = 0.210 \pm 0.005 \ (\text{stat.}) \pm 0.011 \ (\text{syst.}) .$$

(4.32)

In Tab. 4.12 the contributions to the statistical and systematic uncertainties are listed. The corrected value and the corresponding error are:

$$BR(K_L \to \pi^0\pi^0\pi^0)_{\text{meas}} = 0.207 \pm 0.005 \ (\text{stat.}) \pm 0.012 \ (\text{syst.}) .$$

(4.33)

<table>
<thead>
<tr>
<th>SOURCE</th>
<th>STAT. ERROR [$\times 10^{-4}$]</th>
<th>SYST. ERROR [$\times 10^{-4}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{FV}^{FV}$</td>
<td>46</td>
<td>-</td>
</tr>
<tr>
<td>$N_{K_L}^{tag}$</td>
<td>2</td>
<td>-</td>
</tr>
<tr>
<td>$\epsilon_{6}$</td>
<td>-</td>
<td>109</td>
</tr>
<tr>
<td>$A_{K_L}^{FV}$</td>
<td>-</td>
<td>11</td>
</tr>
<tr>
<td>$\epsilon_{6}^{NV}$</td>
<td>-</td>
<td>20</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td>46</td>
<td>112</td>
</tr>
</tbody>
</table>

Table 4.12: Contributions to the statistical and systematic error on $BR(K_L \to 3\pi^0)$, when using the event sample with seven clusters associated to the neutral vertex.
The results of Eqs. (4.28) and (4.33) are in very good agreement between each other and, within a standard deviation, with the PDG value.
Chapter 5

Measurement of BR($K_L \rightarrow 2\pi^0$)

In this chapter the measurement of the $K_L$ branching ratio into the CP violating final state with two neutral pions is treated. In the first part, after a brief description of the method, the event selection is described. In particular, the kinematical cuts to reduce the background and the fit procedure to estimate the signal are explained. Then, the determination of the geometrical acceptances and the measurement of the efficiencies are described. Finally, the result is given.

5.1 The method

In order to measure the branching ratio of the $K_L$ decay into two neutral pions, the same technique used to measure $BR(K_L \rightarrow 3\pi^0)$ and explained in the previous chapter was followed. However, in this case, the searched signal is about 230 times smaller and is immersed in an overwhelming background, mainly due to the $K_L$ decays into three $\pi^0$s. Most of the background events can be easily rejected by exploiting the peculiar kinematical features of a two body decay. For this purpose, the energies and the momenta of the neutral pions must be reconstructed from the final state photons, which are detected in the calorimeter. The remaining irreducible background has to be subtracted by means of a fit procedure.

The subsample with four clusters connected to the neutral vertex is the most populated by the signal events and is the simpler to deal with. Therefore, it was decided to take into account only this case. The number of events $N_{obs,4}^{PV}$, which have a neutral vertex inside
Chapter 5: Measurement of \(BR(K_L \rightarrow 2\pi^0)\)

the chosen fiducial volume with four connected clusters, is counted. By estimating the number of background events \(N^{FV}_{\text{bkg},4}\) and measuring the total efficiency \(\epsilon_4^{\text{tot}}\), the number of \(K_L \rightarrow \pi^0\pi^0\) decays in the \(FV_L\) is given by:

\[
N^{FV}_{2\pi^0} = \frac{N^{FV}_{\text{obs},4} - N^{FV}_{\text{bkg},4}}{\epsilon_4^{\text{tot}}}.
\]  

(5.1)

The total efficiency \(\epsilon_4^{\text{tot}}\) is defined as:

\[
\epsilon_4^{\text{tot}} = \epsilon_4^{\text{NV}} \epsilon_4^{\text{sel}},
\]  

(5.2)

where \(\epsilon_4\) is the efficiency of detecting four photons, \(\epsilon_4^{\text{NV}}\) is the reconstruction efficiency of the neutral vertex with four clusters, and \(\epsilon_4^{\text{sel}}\) is the selection efficiency of the applied kinematical cuts. Then, the branching ratio is given by:

\[
BR(K_L \rightarrow \pi^0\pi^0) = \frac{\Gamma(K_L \rightarrow \pi^0\pi^0)}{\Gamma_{\text{total}}} = \frac{1}{A_{KL}^{FV}} \frac{N^{FV}_{2\pi^0}}{N_{KL}^{\text{tag}}},
\]  

(5.3)

where \(A_{KL}^{FV}\) is the geometrical acceptance of the fiducial volume and \(N_{KL}^{\text{tag}}\) is the number of \(K_L\)-tagged events.

5.2 Sample selection

The data sample collected in 2000, 23.3 pb\(^{-1}\), was used in this analysis. A preliminary selection, common to the analysis of \(3\pi^0\) decays, is applied by requiring:

- the \(K_L\)-tagging;
- a neutral vertex in the fiducial volume, defined by:

\[
FV_L = \begin{cases} 
35 < \rho_L < 155 \text{ cm} \\
|z_L| < 155 \text{ cm} 
\end{cases}
\]  

(5.4)
- four clusters associated to the neutral vertex, all of them having \(E_{\text{clu}} > 20\) MeV and \(\rho_{\text{clu}} > 65\) cm.

It was previously remarked that the sample selected in this way has an overwhelming background. As reported in Tab. 4.3, the main contamination is due to the \(CP\) conserving \(K_L \rightarrow \pi^0\pi^0\pi^0 \rightarrow 6\gamma\) decays, about 230 times more frequent than the \(CP\) violating \(K_L \rightarrow 2\pi^0 \rightarrow 4\gamma\) decays, and, to a smaller amount, to \(K_L \rightarrow \pi^+\pi^-\pi^0\) decays. Since \(3\pi^0\) decays can simulate \(4\gamma\) events if two photons are not detected, one has to rely on distinct kinematical features of the \(2\pi^0\) decay mode to reduce this background.
Table 5.1: Number of events after the preselection cuts.

<table>
<thead>
<tr>
<th>CUT</th>
<th>NUMBER OF EVENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_L$-tagging</td>
<td>$8.806 \times 10^6$</td>
</tr>
<tr>
<td>neutral vertex in $FV_L$</td>
<td>$2.798 \times 10^6$</td>
</tr>
<tr>
<td>$N_{\text{clu}} = 4$</td>
<td>$3.428 \times 10^4$</td>
</tr>
</tbody>
</table>

Cut on missing energy and momentum

If no photons have been lost, due to the four-momentum conservation law, the total energy and momentum of photons are expected to be those of the $K_L$. Therefore, the missing energy $E_{\text{miss}}$ and the missing momentum $P_{\text{miss}}$ are defined as:

$$E_{\text{miss}} = E_L - \sum_i E_i, \quad P_{\text{miss}} = \left| \vec{P}_L - \sum_i \vec{P}_i \right|,$$

where $\vec{P}_L = \vec{P}_\phi - \vec{P}_S$ is measured by the drift chamber and $E_L = \sqrt{P_L^2 + M_K^2}$. The photons' energies $E_i$ are given by the cluster energies $E_{\text{clu}}$, whereas the photon momenta are estimated by knowing the $K_L$ vertex position $\vec{V}_L$ and the impact points $\vec{r}_i$ of $\gamma$'s on the calorimeter in the following way:

$$\vec{P}_i = E_{\text{clu}} \frac{\vec{r}_{\text{clu}} - \vec{V}_L}{|\vec{r}_{\text{clu}} - \vec{V}_L|}.$$

In Fig. 5.1 the variable $E_{\text{miss}}$ is plotted versus $P_{\text{miss}}$ in the real data sample for events with 3, 4, 5, and 6 photons connected to the neutral vertex. In the case of $N_{\gamma} = 6$, the most populated sample, the signal of $\pi^0\pi^0\pi^0$ decays is expected to accumulate around $(0,0)$, as actually is in Fig. 5.1(d). Therefore, as illustrated in Fig. 5.1(b), a rectangular cut in the $E_{\text{miss}}$-$P_{\text{miss}}$ plane is set also for the $N_{\gamma} = 4$ sample:

$$|E_{\text{miss}}| < 150 \text{ MeV},$$
$$P_{\text{miss}} < 100 \text{ MeV}/c.$$

Even though it is very loose, this cut rejects about $\sim 70\%$ of the background, retaining $97\%$ of the signal, as estimated from the Monte Carlo simulation. The distributions of $E_{\text{miss}}$ and $P_{\text{miss}}$ for data and Monte Carlo are shown in Fig. 5.2 in the case $N_{\gamma} = 4$. Due to the different energy resolutions in the data and in Monte Carlo samples, the histograms of Figs. 5.2(a) and 5.2(b) differ slightly. In order to overcome this problem, a fit procedure, described below, was introduced. The uncertainty on the efficiency of the cut (5.7), due to this difference, has been estimated by comparing the distributions of $E_{\text{miss}}$ and $P_{\text{miss}}$ in the control sample with six clusters, shown in Fig. 5.3.
Figure 5.1: $E_{\text{miss}}$ vs $P_{\text{miss}}$ for events with 3, 4, 5, and 6 clusters associated to the neutral vertex in the real data sample. In plot (b) the applied cut is shown.
5.2 Sample selection

Figure 5.2: $E_{\text{miss}}$ (a) and $P_{\text{miss}}$ (b) distributions for events with four clusters connected to the neutral vertex.

Figure 5.3: $E_{\text{miss}}$ (a) and $P_{\text{miss}}$ (b) distributions for events with six clusters connected to the neutral vertex.
Figure 5.4: The energy spectrum of the photons after the energy adjustment (a) and the pull distribution (b) in the subsample with six photons.

Constrained fit of photon energies

Because of their low values, the energies of the photons, produced in the π⁰ decays, are badly measured by the calorimeter. Their mean value is 85 MeV and the corresponding relative error is ~20%. Therefore, a constrained fit with the least squares method is performed in order to vary the photon energies within their errors by imposing the four-momentum conservation:

\[ \sum_i E_{\gamma_i} = E_L, \]
\[ \sum_i \vec{p}_{\gamma_i} = \vec{P}_L. \]  
(5.8)

The fit is performed following a standard procedure described in detail in the reference [49]. After the fit, the photon momenta are recalculated like in Eq. (5.6) with the corrected energies. Finally, only events with all the energies in the interval \(0 < E_{\gamma_i} < 500\) MeV are kept. This way, 92% of signal survives, while 45% of background is rejected.

Since for \(N_{\gamma} = 4\) the presence of the background distorts the distributions, the fit procedure has been tested in the clean sample with six photons. In this case, the mean value of the \(\chi^2\) distribution is 5.47 with four degrees of freedom. In Fig. 5.4(a) the energy spectrum of photons after the fit is plotted for data and Monte Carlo. Here a nice agreement between data and Monte Carlo is seen. In Fig. 5.4(b) the distribution of the pull of the energies is shown. The pull variable is defined as the difference between the
5.2 Sample selection

Figure 5.5: $\pi^0$ invariant mass (a) and momentum in the $K_L$ rest frame (b) calculated before and after the adjustment of photon energies in the case $N_\gamma = 4$.

direct measurement of the variable and its value as obtained from the least squares fit, normalized by dividing by the estimated error of this difference:

$$pull_i = \frac{E_\gamma^i - E_\gamma^{Fit,i}}{\sigma_i} .$$

(5.9)

Under the assumption of Gaussian errors, pulls should exhibit a standard normal distribution, i.e. a normal distribution centered at zero with unitary standard deviation. The histogram of Fig. 5.4(b) has been fitted to a Gaussian.

Considering the subsample with four photons, the effect of the constrained fit on the $\pi^0$ invariant mass and the $\pi^0$ momentum in the $K_L$ rest frame are shown in Fig. 5.5. The estimation of the $\pi^0$ invariant mass and momentum is discussed below.

Cut on pion masses

After the constrained fit, the energies and momenta of the two neutral pions are reconstructed in order to exploit the peculiar kinematical features of the two body decay into $\pi^0\pi^0$. All the possible couples of pairings of the four photons are considered and the one giving the best two invariant masses for the pions is chosen.

In Fig. 5.6, the invariant masses of the two reconstructed pions are plotted. In the three-dimensional view, the peak of the signal at $(135, 135)$ MeV/c$^2$ is clearly visible.
Chapter 5: Measurement of $\text{BR}(K_L \to 2\pi^0)$

![Figure 5.6: $M_{\pi^0}$ vs $M_{\pi_1}$. In plot (a) the circular cut is shown. In the tridimensional view (b) the peak of the signal is clearly visible.](image)

Therefore, we apply a 65 MeV/c² radius circular cut around this point. Such a cut keeps 96% of signal and rejects 65% of background.

**Cut on pion momenta**

The modulus of the pion momentum is also studied in order to reject the background. Being $K_L \to \pi^0\pi^0$ a two body decay, the two pions have a fixed momentum of 209 MeV/c in the $K_L$ rest frame. The $P_{\pi^0}^*$ distribution is plotted in Fig. 5.7. A cut around the signal peak is applied:

$$195 < P_{\pi^0}^* < 220 \text{ MeV/c}.$$  \hspace{1cm} (5.10)

After this cut 96% of signal survives, whereas 50% of background is rejected.

Relying on the good agreement between the data and Monte Carlo distributions after the constrained fit, the selection efficiencies of the signal for the $M_{\pi^0}-M_{\pi_1}$ and $P_{\pi^0}^*$ cuts are taken from Monte Carlo. The selection efficiencies of the cut on $E_{\text{miss}}-P_{\text{miss}}$ and on photon energies after the fit are estimated by using Monte Carlo and, then, corrected by a factor calculated in the control sample with six photons. The selection efficiencies are summarized in Tab. 5.2 with their errors. The total selection efficiency is:

$$\epsilon_{4}^{sel} = 0.7821 \pm 0.0058.$$  \hspace{1cm} (5.11)
5.2 Sample selection

Figure 5.7: Modulus of the $\pi^0$ momentum in the $K_L$ rest frame. The vertical arrows show the boundaries of the applied cut.

In Fig. 5.8 the variable $\overline{M}_{\pi^0} = (M_{\pi_1^0} + M_{\pi_2^0})/2$ is plotted at different steps of the sample selection. The Monte Carlo histograms fit very well to the data distributions. The data and Monte Carlo histograms have been normalized to the same number of entries in Fig. 5.8(a). Then, the selection cuts have been applied separately in data and in Monte Carlo. The agreement is improved by the constrained fit of photon energies, which imposes the conservation of the $K_L$ four-momentum, being the energy resolution of clusters different for data and for Monte Carlo.

This series of kinematical cuts completely rejects all background coming from the $K_L$

<table>
<thead>
<tr>
<th>CUT</th>
<th>EFFICIENCY</th>
<th>STAT. ERROR</th>
<th>SYST. ERROR</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{\text{miss}}-P_{\text{miss}}$</td>
<td>0.9414</td>
<td>0.0018</td>
<td>0.0028</td>
</tr>
<tr>
<td>Kinematic fit</td>
<td>0.9079</td>
<td>0.0031</td>
<td>0.0015</td>
</tr>
<tr>
<td>$M_{\pi_1^0}-M_{\pi_2^0}$</td>
<td>0.9574</td>
<td>0.0024</td>
<td>0.0024</td>
</tr>
<tr>
<td>$P_{\pi^0}$</td>
<td>0.9557</td>
<td>0.0025</td>
<td>0.0027</td>
</tr>
<tr>
<td>TOTAL</td>
<td>0.7821</td>
<td>0.0042</td>
<td>0.0040</td>
</tr>
</tbody>
</table>

Table 5.2: Selection efficiencies for the signal with the error due to the Monte Carlo statistics and the systematic error, due to the difference between data and Monte Carlo and to the resolution effect on the cut boundaries.
Figure 5.8: Distributions of the average $\pi^0$ mass after the fit (a), after the circular cut in the $M_{\pi^0} - M_{\pi^0}$ plane (b), and after the cut on $P_{\pi^0}$ (c).
5.3 Background subtraction

KL decay channels different from $\pi^0\pi^0\pi^0$. Nevertheless, as is evident in Fig. 5.8(c), an irreducible background remains mainly due to $K_L \rightarrow \pi^0\pi^0\pi^0$ events in which a low energy photon has been lost and two photons have been merged into a unique cluster by the clustering algorithm. This kind of events are very similar to the case of $K_L \rightarrow \pi^0\pi^0$, since there are no energy losses and the merged clusters give a momentum, which is roughly the sum of the two photons momenta.

**5.3 Background subtraction**

In order to estimate the signal and the remaining background in the histogram of Fig. 5.8(c) a fit procedure was set up. First, the shapes of the signal and the background have been estimated by using $8 \times 10^5 K_L \rightarrow \pi^0\pi^0$ events and $7.3 \times 10^6 K_L \rightarrow \pi^0\pi^0\pi^0$ events, respectively. The two distributions are shown in Fig. 5.9. Then, these shapes were used in the histogram fit with the maximum likelihood technique, as implemented in a routine of the HBOOK package [50], which uses Poisson statistics and accounts for the error due to the finite size of the Monte Carlo sample. The result of the fit is:

$$N_{4}^{FV} = 1014 \pm 55, \quad N_{bkg,4}^{FV} = 1580 \pm 63.$$ (5.12)

The quoted errors account for a negative correlation of 56% between the number of events of the signal $N_{4}^{FV}$ and the background $N_{bkg,4}^{FV}$. In Fig. 5.10(a) the fitted histogram is shown
with the estimated background (hatched area), while in Fig. 5.10(b) the signal peak is shown after the background subtraction and the Monte Carlo shape is superimposed. The stability of the fit has been tested by varying the binning of the histogram: the effect is smaller than 1%.

### 5.4 Efficiencies measurement

At this point, after the background subtraction, the analysis follows the same scheme as the previous chapter analysis. Most of the quantities, calculated in Sec. 4.3 and common to both channels, will be used: the geometrical acceptance of the fiducial volume, the cluster reconstruction efficiency, the probability of splittings, fragments, and accidentals.

**Reconstruction efficiency of the neutral vertex**

The Monte Carlo efficiency of reconstructing a neutral vertex with four clusters, having $E_{cu} > 20\,\text{MeV}$ and $\rho_{cu} > 65\,\text{cm}$, has been estimated to be:

$$\epsilon_{MC,4}^{NV} = 0.9431 \pm 0.0019 .$$  \hspace{1cm} (5.13)
Table 5.3: Components of the event sample with four clusters connected to the neutral vertex, if second order effects are neglected.

<table>
<thead>
<tr>
<th>NO. OF $\gamma$'s FROM $\pi^0$'s</th>
<th>$\gamma$'s FROM $\pi^0$'s IN GEOM. ACCEPT.</th>
<th>REC. CLUSTERS</th>
<th>SPLIT CLUSTERS</th>
<th>FRAGMENT CLUSTERS</th>
<th>ACCIDENTAL CLUSTERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>4</td>
<td>4</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>3</td>
<td>-</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>-</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 5.4: Factors for the observed sample with four clusters associated to the neutral vertex, neglecting second order effects; the factors of each row refer to the event topology of the corresponding row of Tab. 5.3.

| NO. OF $\gamma$'s FROM $\pi^0$'s | $\gamma$'s FROM $\pi^0$'s IN GEOM. ACC. | REC. CLUS. | SPLIT CLUS. | FRAG. CLUS. | ACC. CLUS. |
|-----|---------------------------------|------------|-------------|-------------|-------------|------------|
| $BR^2_\gamma$ | $A_{4,4}$ | $\langle e_{\text{clus}} \rangle^4$ | $(1 - P_s)^4$ | $(1 - P_{\text{frag}})^4$ | $(1 - P_{\text{acc}})$ |
| $BR^2_\gamma$ | $A_{4,4}$ | $\langle e_{\text{clus}} \rangle^3$ | $3 P_s (1 - P_s)^2$ | $(1 - P_{\text{frag}})^3$ | $(1 - P_{\text{acc}})$ |
| $BR^2_\gamma$ | $A_{3,4}$ | $\langle e_{\text{clus}} \rangle^3$ | $3 P_s (1 - P_s)^3$ | $3 P_{\text{frag}} (1 - P_{\text{frag}})^2$ | $(1 - P_{\text{acc}})$ |
| $2 BR_\gamma BR_e$ | $A_{3,3}$ | $\langle e_{\text{clus}} \rangle^3$ | $3 P_s (1 - P_s)^2$ | $(1 - P_{\text{frag}})^3$ | $(1 - P_{\text{acc}})$ |
Table 5.5: Photon acceptance coefficients calculated in Monte Carlo.

<table>
<thead>
<tr>
<th>ACCEPTANCE FACTOR</th>
<th>VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{4,4}$</td>
<td>$0.8245 \pm 0.0006$</td>
</tr>
<tr>
<td>$A_{3,4}$</td>
<td>$0.1646 \pm 0.0006$</td>
</tr>
<tr>
<td>$A_{3,3}$</td>
<td>$0.866 \pm 0.004$</td>
</tr>
</tbody>
</table>

By applying the correction factor of Eq. (4.12), it becomes:

$$\epsilon_4^{NV} = \eta_{corr}^{NV} \epsilon_{MC,4}^{NV} = 0.9225 \pm 0.0087.$$  \hspace{1cm} (5.14)

**Total detection efficiency of photons**

Following the method used in the case of six photons in the final state, the efficiency for detecting four photons is given by:

$$\epsilon_4 \simeq BR_\gamma^2 A_{4,4} (\epsilon_{clu})^4 (1 - P_s)^4 (1 - P_{frag})^4 (1 - P_{acc}) +$$

$$+ \left[ BR_\gamma^2 A_{4,4} (\epsilon_{clu})^3 + BR_\gamma^2 A_{3,4} (\epsilon_{clu})^3 + 2 BR_\gamma BR_e A_{3,3} (\epsilon_{clu})^3 \right] \times$$

$$\times \left[ 3 P_s (1 - P_s)^2 (1 - P_{frag})^3 (1 - P_{acc}) + (1 - P_s)^3 + 3 P_{frag} (1 - P_{frag})^2 (1 - P_{acc}) + (1 - P_s)^3 (1 - P_{frag})^3 P_{acc} \right].$$  \hspace{1cm} (5.15)

The formula was written according to Tabs. 5.3 and 5.4, which summarize the different cases when four photons are counted in $K_L \to \pi^0 \pi^0$ decays.

<table>
<thead>
<tr>
<th>SOURCE</th>
<th>ERROR [$\times 10^{-4}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$BR_\gamma$</td>
<td>4.0</td>
</tr>
<tr>
<td>$BR_e$</td>
<td>0.1</td>
</tr>
<tr>
<td>$\langle \epsilon_{clu} \rangle$</td>
<td>143.5</td>
</tr>
<tr>
<td>$A_{4,4}$</td>
<td>4.7</td>
</tr>
<tr>
<td>$A_{3,4}$</td>
<td>0.1</td>
</tr>
<tr>
<td>$A_{3,3}$</td>
<td>&lt; 0.1</td>
</tr>
<tr>
<td>$P_s$</td>
<td>0.1</td>
</tr>
<tr>
<td>$P_{frag}$</td>
<td>0.2</td>
</tr>
<tr>
<td>$P_{acc}$</td>
<td>0.2</td>
</tr>
<tr>
<td>TOTAL</td>
<td>143.6</td>
</tr>
</tbody>
</table>

Table 5.6: Contributions to $\epsilon_4$ uncertainty.
The photon acceptances $A_{4,4}$, $A_{3,4}$, and $A_{3,3}$ have been calculated as in Sec. 4.3.3 and are reported in Tab. 5.5. The mean cluster reconstruction efficiency and the probabilities of splittings, fragments, and accidentals have been assumed to be the same as in the three pion decays. The resulting efficiency is:

$$\epsilon_4 = 0.616 \pm 0.014 .$$  \hspace{1cm} (5.16)

The contributions to the uncertainty on $\epsilon_4$ are reported in Tab. 5.6.

### 5.5 Measurement of $BR(K_L \rightarrow 2\pi^0)$

The branching ratio of the $K_L$ meson into the $\pi^0\pi^0$ final state is given by:

$$BR(K_L \rightarrow \pi^0\pi^0) = \frac{N_{4}^{FV}}{N_{K_L}^{tag} A_{K_L}^{FV} \epsilon_{4}^{NV} \epsilon_{4}^{sel} ,}$$  \hspace{1cm} (5.17)

where all the quantities are defined in Sec. 5.1.

Without the correction for the tagging biases, the measured value is:

$$BR(K_L \rightarrow \pi^0\pi^0)_{raw} = (9.24 \pm 0.50 \text{ (stat.)} \pm 0.25 \text{ (syst.)}) \times 10^{-4} .$$  \hspace{1cm} (5.18)

The contributions to the statistical and systematic uncertainties on the branching ratio $BR(K_L \rightarrow \pi^0\pi^0)$ are listed in Tab. 5.7.

When the correction of Eq. (4.27) is applied, the value of the branching ratio becomes:

$$BR(K_L \rightarrow \pi^0\pi^0)_{meas} = (9.09 \pm 0.50 \text{ (stat.)} \pm 0.25 \text{ (syst.)}) \times 10^{-4} ,$$  \hspace{1cm} (5.19)

<table>
<thead>
<tr>
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<th>STAT. ERROR [$\times 10^{-6}$]</th>
<th>SYST. ERROR [$\times 10^{-6}$]</th>
</tr>
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<tbody>
<tr>
<td>$N_{4}^{FV}$</td>
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<td>-</td>
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<tr>
<td>$N_{K_L}^{tag}$</td>
<td>0.3</td>
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</tr>
<tr>
<td>$\epsilon_{4}^{sel}$</td>
<td>-</td>
<td>6.9</td>
</tr>
<tr>
<td>$\epsilon_{4}$</td>
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</tr>
<tr>
<td>$A_{K_L}^{FV}$</td>
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<tr>
<td>$N_{K_L}^{NV}$</td>
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<td>8.7</td>
</tr>
<tr>
<td>TOTAL</td>
<td>50.1</td>
<td>24.7</td>
</tr>
</tbody>
</table>

*Table 5.7:* Contributions to the statistical and systematic error on $BR(K_L \rightarrow 2\pi^0)$. 
where the systematic error accounts also for the contribution coming from the correction. Concerning this correction, the same considerations as in Sec. 4.4 hold. The result is in agreement within the error with the PDG value [9]:

\[
BR(K_L \rightarrow \pi^0\pi^0)_{PDG} = (9.28 \pm 0.19) \times 10^{-4}.
\] (5.20)
Chapter 6
Measurement of \( \Gamma(K_L \rightarrow 2\pi^0)/\Gamma(K_L \rightarrow 3\pi^0) \)

In this chapter the analyses of the previous two chapters are combined to give the ratio of the decay amplitude \( \Gamma(K_L \rightarrow 2\pi^0) \) to \( \Gamma(K_L \rightarrow 3\pi^0) \). In this case, some of the acceptances and efficiencies calculated for the measurements of the absolute branching ratios simplify either completely or partially.

6.1 The method

By using the acceptances and the efficiencies calculated in the previous chapters, it is straightforward to get \( \Gamma(K_L \rightarrow 2\pi^0)/\Gamma(K_L \rightarrow 3\pi^0) \). The ratio of Eq. (5.17) to Eq. (4.24) is given by:

\[
\frac{\Gamma(K_L \rightarrow 2\pi^0)}{\Gamma(K_L \rightarrow 3\pi^0)} = \frac{N_{\text{tag},6}^{FV} N_4^{PV}}{N_{\text{tag},4}^{FV} N_6^{PV}} \frac{\epsilon_6^{NV} \epsilon_4^{MC,6}}{\epsilon_4^{MC,4} \epsilon^{sel}_{2\pi^0}} .
\]

(6.1)

In this case, the geometrical acceptance \( A_{KL}^{PV} \) and the correction coefficient of the reconstruction efficiency of the neutral vertex cancel in the ratio.

The term \( \epsilon_6/\epsilon_4 \) can be further simplified. The ratio of the photon detection efficiencies, defined in Eqs. (4.14) and (5.15), can be written as:

\[
\frac{\epsilon_6}{\epsilon_4} \sim BR(\pi^0 \rightarrow \gamma\gamma) \left( \epsilon_{6\ell} \right)^2 \frac{C_1^{(6)} + C_2^{(6)}}{C_1^{(4)} + C_2^{(4)}} .
\]

(6.2)
where the coefficients $C_{1,2}^{(6)}$ and $C_{1,2}^{(4)}$, introduced for simplicity, are given by:

$$C_1^{(6)} = A_{6,6} BR_{\gamma} \langle \epsilon_{\text{clu}} \rangle (1 - P_s)^6 (1 - P_{\text{frag}})^6 (1 - P_{\text{acc}}),$$

$$C_2^{(6)} = [3 A_{5,5} BR_{\epsilon} + (A_{5,6} + A_{6,6}) BR_{\gamma}] [1 - P_{\text{frag}} (1 - P_s) - P_s]^4$$

$$[P_{\text{frag}} (5 - 10 P_s) + 5 P_s + P_{\text{acc}} (1 - 6 P_s - P_{\text{frag}} (6 - 11 P_s))],$$

$$C_1^{(4)} = A_{4,4} BR_{\gamma} \langle \epsilon_{\text{clu}} \rangle (1 - P_s)^4 (1 - P_{\text{frag}})^4 (1 - P_{\text{acc}}),$$

$$C_2^{(4)} = [2 A_{3,3} BR_{\epsilon} + (A_{3,4} + A_{4,4}) BR_{\gamma}] [1 - P_{\text{frag}} (1 - P_s) - P_s]^2$$

$$[P_{\text{frag}} (3 - 6 P_s) + 3 P_s + P_{\text{acc}} (1 - 4 P_s - P_{\text{frag}} (4 - 7 P_s))].$$

By using in Eq. (6.2) the values calculated in the previous chapters, the ratio is:

$$\frac{\epsilon_6}{\epsilon_4} = 0.7938 \pm 0.0093.$$  \hspace{1cm} (6.4)

The contributions to the uncertainty on $\epsilon_6/\epsilon_4$ are listed in Tab. 6.1. The main contribution to the error on $\epsilon_6$ and $\epsilon_4$ come from the uncertainty on the cluster reconstruction efficiency, because of the sixth and fourth powers in Eqs. (4.14) and (5.15), respectively. In the ratio of Eq. (6.2), the dominant term is $\langle \epsilon_{\text{clu}} \rangle$ raised to the second power.
6.2 Measurement of $\Gamma(K_L \to 2\pi^0)/\Gamma(K_L \to 3\pi^0)$

If the bias on $K_L$ tagging is neglected, the following result is obtained:

$$\frac{\Gamma(K_L \to 2\pi^0)}{\Gamma(K_L \to 3\pi^0)} \bigg|_{\text{raw}} = (4.37 \pm 0.24 \text{ (stat.)} \pm 0.06 \text{ (syst.)}) \times 10^{-3}. \quad (6.5)$$

In Tab. 6.2 the sources of the statistical and systematic errors are reported with their contributions. In order to correct for the tagging bias, the raw measurement has to be multiplied by the following factor:

$$\frac{\varepsilon_{\text{2\pi^0}}^{\text{tag}}}{\varepsilon_{\text{3\pi^0}}^{\text{tag}}} = 0.9976 \pm 0.0017, \quad (6.6)$$

where the values of $\varepsilon_{\text{3\pi^0}}^{\text{tag}}$ and $\varepsilon_{\text{2\pi^0}}^{\text{tag}}$ are taken from Tab. 3.2. The final result is:

$$\frac{\Gamma(K_L \to 2\pi^0)}{\Gamma(K_L \to 3\pi^0)} \bigg|_{\text{meas}} = (4.36 \pm 0.24 \text{ (stat.)} \pm 0.06 \text{ (syst.)}) \times 10^{-3}, \quad (6.7)$$

to be compared to the PDG value [9]:

$$\frac{\Gamma(K_L \to 2\pi^0)}{\Gamma(K_L \to 3\pi^0)} \bigg|_{\text{PDG}} = (4.39 \pm 0.11) \times 10^{-3}. \quad (6.8)$$

The measured value is dominated by the statistical uncertainty, whereas the relative systematic uncertainty is of 1.4%. During 2001, data taking almost 200 pb$^{-1}$ will be collected. When all these data will be available for the analysis, the statistical error on $\Gamma(K_L \to 2\pi^0)/\Gamma(K_L \to 3\pi^0)$ should be reduced by a factor $\sim$3, making the KLOE measurement competitive with the PDG value.

<table>
<thead>
<tr>
<th>SOURCE</th>
<th>STAT. ERROR [x10$^{-3}$]</th>
<th>SYST. ERROR [x10$^{-3}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_4^{\text{FV}}$</td>
<td>0.237</td>
<td>-</td>
</tr>
<tr>
<td>$N_6^{\text{FV}}$</td>
<td>0.022</td>
<td>-</td>
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</tr>
<tr>
<td>$\varepsilon_6/\varepsilon_4$</td>
<td>-</td>
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</tr>
<tr>
<td>$\varepsilon^{e\ell}$</td>
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</tr>
<tr>
<td>$\varepsilon_4^{\text{NV}}$</td>
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<tr>
<td>$\varepsilon_6^{\text{NV}}$</td>
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<td>0.008</td>
</tr>
<tr>
<td>TOTAL</td>
<td>0.238</td>
<td>0.062</td>
</tr>
</tbody>
</table>

Table 6.2: Contributions to the statistical and systematic error on $\Gamma(K_L \to 2\pi^0)/\Gamma(K_L \to 3\pi^0)$.
Conclusions

The main goal of the KLOE experiment is the measurement of the parameter $\Re (\varepsilon'/\varepsilon)$ to a precision of $O(10^{-4})$. Due to the unexpected difficulty in setting up the DAΦNE collider to the design luminosity, plans for the $CP$ violation measurement have been momentarily delayed.

Two fixed target experiments, NA48 at CERN and KTeV at FNAL, have recently produced their results, proving beyond any doubt that the $CP$ symmetry is violated also via the direct mechanism. However, the measured values show a $2\sigma$ discrepancy. In order to exploit the cancellations in the double ratio, the four decay modes $K_{S,L} \rightarrow \pi^+\pi^-$ and $K_{S,L} \rightarrow \pi^0\pi^0$ must be collected simultaneously and in the same decay region. This requires very complex techniques, affected by several systematic effects which have to be carefully kept under control. On the contrary, the $\phi$-factories face the problem from a completely different point of view. $K_S$ and $K_L$ mesons, produced in $\phi$ decays, are well separated and may be tagged in a very powerful and clean manner. Many cancellations are intrinsic in this approach; nevertheless, the main drawback is represented by the difficulty in getting high beam intensities. A measurement of $\Re (\varepsilon'/\varepsilon)$ from KLOE would provide a completely independent confirmation to the previous results.

Since the beginning of data taking in 1999, the KLOE detector has been operating according to the design specifications. The continuous improvements of DAΦNE allowed almost 200 pb$^{-1}$ to be delivered during this year; the collected data sample will be available on disk for analysis purposes by next months. This is still a factor ten below the statistics needed for the measurement of $CP$ violation; on the other hand, many interesting intermediate measurements may be performed and results, which can improve the accuracy of PDG values, are expected. Moreover, in the meantime, the study of several systematics,
common to the CP measurement, are in progress.

This is the case of this thesis. The ratio $\Gamma(K_L \rightarrow 2\pi^0)/\Gamma(K_L \rightarrow 3\pi^0)$ has been measured by using $\sim 23$ \text{pb}^{-1} collected in the year 2000. The result is in agreement, within the error, with the PDG value. The relative error of 5.7\% is dominated by the limited statistics, whereas the systematic uncertainty is at a level of 1.5\%. When the whole data set is available for the analysis, the total error will be of the order of 2\%.

The results of Chapter 5, although very preliminary, can give a first insight into which would be, at the present time, the contribution of the channel $K_L \rightarrow \pi^0\pi^0$ to the systematic uncertainty on the double ratio, defined in Eq. (1.62), and hence to $\Re(e'/\varepsilon)$. In order to measure $\Re(e'/\varepsilon)$ to a precision of $O(10^{-4})$, the systematic uncertainty in counting the $K_L$ decays to two neutral pions must be kept at the level of $10^{-3}$. The number $N_{L}^{00}$ of $K_L$ decays into $\pi^0\pi^0$ in the fiducial volume is given by:

$$N_{L}^{00} = \frac{N_{\text{obs,4}}^{\text{EV}} - N_{\text{bkg,4}}^{\text{EV}}}{\epsilon_{4}^{\text{NV}} \epsilon_{4}^{\text{sel}} \epsilon_{4}},$$  \hspace{1cm} (6.9)$$

where the quantities $N_{\text{obs,4}}^{\text{EV}}$ and $N_{\text{bkg,4}}^{\text{EV}}$, and the efficiencies $\epsilon_{4}^{\text{NV}}$, $\epsilon_{4}^{\text{sel}}$, and $\epsilon_{4}$ were introduced in Sec. 5.1. The numerator of Eq. (6.9) contributes to the statistical uncertainty, whereas the relative systematic uncertainty on $N_{L}^{00}$ is given by:

$$\left. \frac{\Delta N_{L}^{00}}{N_{L}^{00}} \right|_{\text{syst}} = \sqrt{\left( \frac{\Delta \epsilon_{4}^{\text{NV}}}{\epsilon_{4}^{\text{NV}}} \right)^2 + \left( \frac{\Delta \epsilon_{4}^{\text{sel}}}{\epsilon_{4}^{\text{sel}}} \right)^2 + \left( \frac{\Delta \epsilon_{4}}{\epsilon_{4}} \right)^2}.$$  \hspace{1cm} (6.10)$$

The three terms were estimated in Chapter 5: they contribute uncertainties of 0.9\%, 0.7\%, and 2.3\%, respectively, giving an overall relative error of 2.6\%. However, a large part of these uncertainties comes from the limited statistics both of the data and Monte Carlo samples. An upper limit on the current systematic uncertainty on $\Re(e'/\varepsilon)$ can be estimated by disentangling the non-statistical from the statistical contributions.

The main contribution to the error on $\epsilon_{4}^{\text{NV}}$ is due to the uncertainty on the $K_L$ lifetime: 0.5\%. It is worthwhile to notice that in the double ratio this systematic effect is compensated by the $K_L$ charged decays. The knowledge of $\epsilon_{4}^{\text{sel}}$ is limited by the error due to the resolution effect on the boundaries of the selection cuts: 0.4\%. The error on $\epsilon_{4}$ is dominated by the uncertainty on the cluster reconstruction efficiency, which in turn has a contribution due to the data and Monte Carlo statistics, whereas the remaining term is 0.4\%.

By considering only these contributions, the relative error on $N_{L}^{00}$ becomes 0.8\%. Since $R \approx 1$, the contribution to its error coming from $N_{L}^{00}$ is of the order of $8 \times 10^{-3}$. Reminding
of the factor six in Eq. (1.62), the error on $\Re (\varepsilon' / \varepsilon)$ due to the counting of $K_L$ neutral decays is of the order of $10^{-3}$. Therefore, KLOE has currently a sensitivity on $\Re (\varepsilon' / \varepsilon)$ equal to or lower than the $\Re (\varepsilon' / \varepsilon)$ value. As might be expected, this represents a very preliminary estimate, which can be much improved by refining the clustering algorithm and tuning more accurately the Monte Carlo response.
Acknowledgments

At the end of this work, I want to thank my supervisor Prof. Nello Paver who, although an esteemed theorist, has dared to soil his hands with this experimental stuff, and my co-supervisor Dr. Fabrizio Scuri; without their valuable support in the past three months I would not have succeeded in completing this thesis.

I feel sincerely grateful to many other people who never denied me their help: first of all, to Caterina Bloise and Stefano Miscetti for the precious advice they constantly gave me, for all the things they taught me, and for the patience they explained those things again; to Stefano Di Falco for his collaboration in filtering the 2000 data; to Matt Moulson for having introduced me to the dark side of PAW, *Midnight*; to Fabio Bossi and Matteo Palutan for having removed any doubt – licit or illicit – about the trigger; to Erika De Lucia for her help for the second chapter; to Paolo Valente and Luigi Benussi for their technical advice on Linux; to Giovanni Mazzitelli for the nice plot of the KLOE integrated luminosity; and, last but not least, to Claudio Piemonte for his private lessons on Latex.

I ought to show my solidarity with Camilla Di Donato, Barbara Sciascia, and Tommaso Spadaro, who shared this PhD experience with me and spent the past weeks writing their theses.

Finally, I’d like to thank all the members of the KLOE Collaboration I dealt with, in particular the EmC Group, whose names I cannot mention all without writing another thesis, which nobody wants.
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