MOTOR SPEED TRANSFER FUNCTION FOR WIND VECTOR ESTIMATION ON MULTIROTOR AIRCRAFT

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1. Introduction

Ambient wind plays an important role in aviation because it can affect the flight characteristics significantly. Especially small unmanned aircraft (UA) are very sensitive to wind due to their low inertia. Hence, it can be crucial for several applications to measure the wind vector, i.e. both speed and direction, in real-time. Since the wind field varies locally, the measurements have to be performed on board the UA. In case of fixed-wing aircraft, typically pitot tubes are used for this purpose, as shown, for instance, in [1]. However, this method cannot be easily adopted to rotary-wing aircraft like multicopters because their rotor wash causes strong perturbations of the aerodynamic field surrounding the aircraft. Instead, the state variables of the aircraft can be used to estimate the wind field. Neumann et al. inferred the 2D wind vector from simple pose measurements of an UA [2]. In this paper, this approach is extended to the 3D case, which requires knowledge of the rotor speeds. Therefore, the motor speed transfer function is analysed exemplary for a DJI S1000 octocopter.

2. Methods

The relation between ground speed $\mathbf{v}_{\text{gnd}}$, wind speed $\mathbf{v}_{\text{wind}}$ and airspeed $\mathbf{v}_{\text{air}}$ is given by the wind triangle [2]:

$$\mathbf{v}_{\text{gnd}} = \mathbf{v}_{\text{air}} + \mathbf{v}_{\text{wind}}$$  \(1\)

According to the Newton-Euler equations the translational dynamics of a rigid body can be described by

$$m \frac{d}{dt} \mathbf{v}_{\text{air}} + B \mathbf{\omega}_{IB} \times m \mathbf{v}_{\text{air}} = Bf^e$$  \(2\)

where $m$ denotes the mass, $B \mathbf{v}_{\text{air}}$ is the velocity expressed in the body coordinate frame $B$, $B \mathbf{\omega}_{IB}$ is the angular velocity of $B$ with respect to the inertial coordinate frame $I$ expressed in $B$ and $Bf^e$ are the external forces acting on the body.

Mainly, the thrust and drag (or friction) force contribute to $f^e$. Let $f_i$ denote the thrust force generated by rotor $i$, the total thrust becomes [3]

$$T = \sum f_i = C_T \rho A_R \sum (r_R \omega_i)^2$$  \(3\)

where $C_T$ is an aircraft specific thrust constant, $\rho$ is the air density, $A_R$ the area swept out by the rotor, $r_R$ the radius of the rotors and $\omega_i$ the angular velocity of rotor $i$. In the body coordinate frame $B$ the thrust force is defined as

$$Bf_T = [0 \ 0 \ T]^T.$$  \(4\)

With $\otimes$ denoting the Hadamard (or Schur) product, the drag force can be written as

$$Bf_D = \frac{1}{2} \rho \left( c_D \otimes a_{\text{proj}} \otimes \mathbf{v}_{\text{air}} \otimes \mathbf{v}_{\text{air}} \right)$$  \(5\)

where $c_D$ is the vector of drag coefficients and $a_{\text{proj}}$ is the vector of projected surface areas (cf. [2]). Finally, the gravitational force is given by:

$$Bf_g = [0 \ 0 \ -g]^T.$$  \(6\)

The total external force becomes

$$Bf^e = Bf_T + R_{BI} \left( Bf_g + Bf_D \right).$$  \(7\)

where $R_{BI}$ is the rotation matrix from $I$ to $B$. By inserting (7) into (2) a solution for $\mathbf{v}_{\text{air}}$ can be computed, which gives $\mathbf{v}_{\text{wind}}$ by using (1).

An UA equipped with typical sensors like an inertial measurement unit (IMU), global positioning system (GPS) and compass can compute or measure $\mathbf{v}_{\text{gnd}}$, $B \mathbf{\omega}_{IB}$ and $R_{BI}$. The values for $m, \rho, C_T, A_R, \omega_i, c_D, a_{\text{proj}}(R_{BI})$ and $g$ are given or can be determined experimentally. In contrast, the rotor speeds $\omega_i$ have to be measured online.
In case of an open flight controller, it is straight-forward to get the set points of the motor controllers (electronics speed control, ESC), which can be used as an approximation for $\omega_i$. However, this does not apply to most commercial products like the DJI S1000. Since the motor controller and the motor itself form a fixed unit, the rotor speed cannot be measured, for example, by tapping the counter-electromotive force from the brushless motors. Instead, the signal from the flight controller to the ESCs is analysed and related to the actual rotor speed. The latter is measured using a digital, non-contact tachometer (UNI-T UT372), which offers a sampling frequency of approximately 6.25 Hz.

![Image](image.png)

Fig. 2. Measuring the angular velocity of the motors.

3. Results

A simple signal analysis showed that DJI uses a PWM signal with 400 Hz carrier frequency to control the ESCs. After turning the copter on, the duty cycle is set to 37.6% to signal the presence of the flight controller. In this state the rotors are not turning. If the rotors are set to idle speed the duty cycle is increased to 48%. The flight controller never sets the duty cycle above 74%, which defines the value for top speed.

To obtain the transfer function describing the relation between PWM duty cycle and motor speed a test signal was generated using a microcontroller. The duty cycle was increased steadily from 47% to 74% within 80 seconds, which is shown in figure 3. Basically, the function looks exponentially damped, i.e. similar to

$$f_1(x) = a + b(1 - e^{-c x}). \quad (8)$$

This function was fitted to the curve showing the revolutions per minute (rpm), but there was an oscillating portion with decreasing frequency and damped amplitude left resulting in a standard deviation of ca. 70 rpm. Therefore, the approach was extended by

$$f_2(x) = d \cdot e^{-e x} \sin(1 - e^{-g x}) + e^{h x}. \quad (9)$$

The remaining difference between the model, $f(x) = f_1(x) + f_2(x)$, and the measurement is shown in figure 4. Its standard deviation is 14 rpm, which is in the range of the measurement accuracy of the used tachometer.

![Image](image.png)

Fig. 3. PWM duty cycle sent to the ESC and the resulting angular velocity of the motor.

Fig. 4. Difference between model and measurement.

References

