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XIX CICLO DEL
DOTTORATO DI RICERCA IN
GEOFISICA DELLA LITOSFERA E GEODINAMICA
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LITHOSPHERIC CHARACTERISTICS AND SEISMIC
SOURCES IN THE SCOTIA ARC THROUGH WAVEFORM
INVERSION

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To my parents Nery and Milton, my wife Elga, our children Angie, Sofía and Nicolás and my brothers Marianella and Dennis
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Abstract

La regione del Mare di Scotia si trova tra le placche Sudamericana e Antartica e costituisce un’area tettonica complessa, caratterizzata da numerosi processi attivi e cambiamenti nel moto e nella configurazione delle placche. Le caratteristiche tettoniche principali del Mare di Scotia sono stati oggetto di diversi studi, ciononostante alcuni dettagli delle interazioni tettoniche, dei margini di placca e del loro moto relativo sono ancora incerti. In questo senso, la determinazione delle caratteristiche della litosfera e lo studio dei meccanismi focali svolgono un ruolo importante per comprendere l’evoluzione geodinamica dell’area.

Questo studio si propone di utilizzare le attuali tecniche per l’inversione delle forme d’onda al fine di ottenere i meccanismi di sorgente per una serie di terremoti registrati in prossimità della Base Antartica Argentina Orcadas. L’applicazione del metodo è possibile grazie ai sismogrammi digitali registrati da una rete regionale installata a partire dal 1992 e che include la Antarctic Seismographic Argentinean Italian Network (ASAIN) ed altre tre stazioni sismografiche della Global Seismographic Network (GSN) operanti nella Penisola antartica, in Terra del Fuoco e nelle isole dell’Arco di Scotia.

Sono stati analizzati e riprodotti sette eventi che seguirono il terremoto di magnitudo 7.6 Mw verificatosi il 4 agosto 2003 e conosciuto in letteratura come Centenary Earthquake.

Il nucleo principale dello studio (Capitolo 3) è preceduto de due capitoli dedicati rispettivamente ad una dettagliata descrizione della situazione tettonica e della sismicita’ della regione del Mare di Scotia (Capitolo 1) ed alla rete sismografica Italo-Argentina ASAIN con particolare attenzione per le risposte in frequenza dei sismografi e la caratterizzazione dei livelli del rumore sismico (Capitolo 2).

Come complemento all’analisi dei meccanismi focali, applicando la legge di Omori, si è determinata la curva de decadimento temporale de la serie sismica registrata dalla stazione di Orcadas nei 60 giorni successivi al Terremoto del Centenario.
The Scotia Sea region is found between the south American and Antarctic plates and constitutes a complex area tectonics, characterized from numerous active processes and changes in the movement and in the configuration of the plates. The main tectonics characteristics of the Scotia Sea were object of different studies, nevertheless some details of the interactions tectonics, of the margin of plates and of their relative movement remain still uncertain. In this sense, the determination of the features of the lithosphere and the study of the focal mechanisms develop an important role to understand the geodynamic evolution of the area.

This study proposes to use the present technologies for the inversion of waveforms to the end of to obtain the source mechanisms for a series of earthquakes recorded in the proximity of the Antarctic Base Argentina Orcadas. The application of the method is possible thanks to the digital seismograms recorded from a regional network installed to leave from 1992 and that includes the Antarctic Seismographic Argentinean Italian Network (ASAIN) and other three seismographic stations of the Global Seismographic Network (GSN) operating in the antarctic Peninsula, Tierra del Fuego and in the islands of the Scotia arc.

Were analyzed and reproduced seven events that followed the earthquake of magnitude 7,6 Mw 4 August 2003 known in literature like Centenary Earthquake.

The main unit of the study (Chapter 3) is preceded of two chapters dedicated respectively to a detailed description of the situation tectonics and the seismicity of the Scotia Sea region (Chapter 1) and the Italian-Argentinean seismographic network ASAIN with special attention for the response in frequency of the seismograph and the characterization of the seismic noise levels (Chapter 2).

Like complement to the analysis of the focal mechanisms, applying the Omori’s law, it has determined the curve of temporal decay of the seismic sequence recorded from the Orcadas station in sixty days following to the Centenary Earthquake.
Acknowledgments

I am extremely grateful to the Istituto Nazionale di Oceanografia e di Geofisica Sperimentale - OGS for financing my PhD and to the Abdus Salam International Centre for Theoretical Physics (ICTP). The project "Broad Band Seismology in the Scotia Sea Region" is financially supported by the Italian Programma Nazionale di Ricerche in Antartide (PNRA) in the framework of the PNRA research area "Geodesia e Osservatori". Thanks are due to the Argentinean institutions cooperating to the ASAIN Network: Dirección Nacional del Antártico (DNA) - Instituto Antártico Argentino (IAA), Universidad Nacional de La Plata - Facultad de Ciencias Astronómicas y Geofísicas, Estación Astronómica Río Grande Tierra del Fuego (EARG) and the Centro Austral de Investigaciones Científicas (CADIC). Thanks are also due to the civilian and military personnel operating in the Argentinean bases Esperanza, Jubany, Orcadas, San Martín and Marambio for their contribution to the maintenance and the operation of the ASAIN field installations. I am also grateful to Martina Busetti for her suggestions in the geological interpretation of the results.
Chapter 1

The Scotia Sea Region

1.1 Introduction

The Scotia Sea region extends from about 75° to 25°W, 61° to 53°S, and the crust there is mainly of oceanic structure and origin. It is bounded on three sides by the Scotia Arc, islands and submarine ridges of the North and South Scotia Ridge and South Sandwich island arc that are a mixture of old continental fragments, arc volcanoes (active and dead), remnant arc and accretionary prism. At present the Scotia Sea region is composed by two small plates between the South American and the Antarctic plates: the Scotia and Sandwich plate. The boundaries of those plates lie mostly along the Scotia Arc, and the plate motions are mainly along the east-west direction. However, this relatively simple situation has not existed for long, its tectonic evolution over the past 40 Ma has been far more complex.

Such long-term study of the region has led to a general view of the major elements of its evolution. Fundamentally, the geology of the continental fragments now distributed around the Scotia Arc betrays an origin within a continuous continental connection between southermost South America and the Antarctic Peninsula, at a subducting Pacific margin. Scotia Arc evolution, however, has a second, separate subduction system.

The Scotia Sea formed by back-arc extension behind this second arc and trench, that developed close to the Pacific margin but migrated eastward and subducted South Atlantic oceanic lithosphere belonging to the South American plate (Barker, 2001).

Understanding the regional tectonic structure and evolution is very important for three main reasons. First of all, the continental connection between
southernmost South America and the Antarctic Peninsula was most probably the final barrier to continuous circum-Antarctic deep water circulation (Barker and Burrell, 1977). This situation greatly reduced north-south ocean circulation and heat transport, and may thus have had a profound effect not only on Antarctic climate by isolating the continent within a ring of cold water, but also at a more global scale. The second reason is that Alvarez (1982) proposed that the mass balance in the mantle, between a shrinking Pacific region and expanding Indian and Atlantic regions, has been achieved by shallow eastward mantle flow through the Caribbean and Scotia Sea region. Doglioni et al. (1999) have classified the Sandwich among the so called "west-directed subduction zones", which present peculiar characteristics, with strong differences particularly when compared with other subduction zones, that can be justified if the existence of a eastward mantle flow is accepted. They proposed that the Scotia Arc developed along the back-thrust belt of an earlier East-directed subduction zone, because there was oceanic lithosphere in the foreland of the related back-thrust belt. Therefore a better understanding of the Scotia and Sandwich tectonics can help in understanding other settings associated to west-directed subduction zones, such as the Caribbean, the Mediterranean and the Vrancea zone in Romania.

1.2 Tectonic setting and evolution

The Scotia Sea region is presently characterized by two main plates, the small Sandwich Plate in the extreme east, and the much larger Scotia Plate. In Figure (1.1) the main tectonic features of the Scotia Sea region are plotted. The seismicity concentrated along the bathymetric features, which surround the Scotia Sea, provides evidence for a kinematically distinct Scotia Plate (Pelayo and Wiens, 1989). The plate tectonic models developed for the Scotia Sea region in the last three decades invoked more micro plates to explain the different complex features in the region. Barker and Dalziel (1983), for example, show the existence of two additional micro plates, the Drake Plate and the South Shetland plate, between the Bransfield Strait and the South Shetland Trench. The South America-Scotia plate boundary runs through Tierra del Fuego and the North Scotia Ridge (Barker, 2001); the North Scotia Ridge, east of Burdwood Bank comprises three small-elevated blocks, then the Shag Rocks Block, and finally the South Georgia micro continent. Livermore et al. (1994) suggested the presence of significant underthrusting of the Scotia Plate beneath South Georgia. The Scotia-Antarctic boundary runs along the Shackleton Fracture
Figure 1.1: Tectonic boundaries of the Scotia Sea Region from University of Texas-Institute for Geophysics (UTIG) and bathymetric contour map (based on General Bathymetric Chart of the Oceans (GEBCO) 1-minute global bathymetric grid). Abbreviations are as follows: ARG, Argentine; TdF, Tierra del Fuego; SGI, South Georgia Island; SSI, South Sandwich Islands; SFZ, Shackleton Fracture Zone; SOM, South Orkney Microcontinent; ANP, Antarctic Peninsula; BrB, Bruce Bank; PoB, Powell Basin; PiB, Pirie Bank; BuB, Burdwood Bank; HeB, Herdman Bank; DiB, Discovery Bank.

Zone and South Scotia Ridge. Along the South Scotia Ridge, separated from the Antarctic Peninsula by the Powell Basin, there is the South Orkney micro continent, the largest fragment of continental crust in the South Scotia Ridge. The South Orkney Islands lie along the northern edge of the South-Orkney micro continent. Between Elephant Island and the South Orkney micro continent lies a pair of ridges, separated by the presently active Scotia-Antarctic plate boundary (Barker, 2001). In the eastern part of the South Scotia Ridge there are some elevations as the Jane Bank, Discovery Bank, and Herdman Bank.

Active spreading is present in the eastern part of the Scotia Sea, along the East Scotia Ridge, which is supposed to have created the small Sandwich Plate (Barker, 2001). In contrast to the other plates (South America, Scotia and Antarctica), the small Sandwich Plate is moving rapidly eastward, with respect to the major plates. The northern and southern Sandwich boundaries continue those of the Scotia plate, but there is rapid subduction of South America oceanic lithosphere (and rollback of the subduction hinge) at the eastern Sand-
wich margin, and rapid back-arc extension at the Sandwich-Scotia boundary (Barker and Dalziel, 1983; Barker, 2001). Most of the South Sandwich Islands are placed in an eastward-convex arc, on an oceanic floor formed during the present back-arc spreading. The Shackleton Fracture Zone is located in the Drake Passage, between South America and the Antarctic Peninsula and represents the western boundary of the Scotia Plate. The fracture zone intersects the West Scotia and Phoenix-Antarctica ridges, both extinct spreading centres, and developed a ridge-to-ridge transform zone in the central area of the Drake Passage (Galindo-Zaldívar et al., 2000). At present, the Shackleton Fracture Zone is a sinistral transpressive fault zone that connects the Chile Trench with the South Shetland Trench and the southern boundaries of the Scotia Plate (Cunningham et al., 1995). The southwest corner of the Scotia Plate is a geodynamic system undergoing rapid changes in plate motions and configuration. The Antarctic Peninsula Pacific margin is now passive except for the short section of the South Shetland Trench, the last surviving segment of a subduction zone that once extended all along the western margin of the Antarctic Peninsula (Larter, 2001). The Bransfield Strait is suggested to have an extensional origin; according to Barker and Dalziel (1983) it may represent back arc extension behind a still active South Shetland Trench. Galindo-Zaldívar et al. (1996) suggest that the extension of the Bransfield Strait could be partially explained by the westward continuation of the active fault system of the central South Scotia Ridge.

It has long been considered that southern South America and the Antarctic Peninsula were once joined, and that the islands and submarine ridges now distributed along the North and South Scotia Ridge originally formed a compact continental connection between them (Barker and Burrell, 1977). The Scotia Ridge is composed largely of continental fragments, whose onshore geology is incompatible with their present isolated situation, but indicates an original position close to a subducting continental margin (presumed to be the Pacific margin, between the Peninsula and Southern South America) (Barker and Dalziel, 1983). The geological similarities between South Georgia and southern Tierra del Fuego suggest that the original position of South Georgia was directly east of Cape Horn. Much of the South Scotia Ridge appears to be composed of continental fragments, which have moved eastward relative to their original positions adjacent to the northeastern end of the Antarctic Peninsula (Barker and Dalziel, 1983). Barker (1972) suggested that this connection had been probably broken by southeastward relative movement of the Antarctic Peninsula and the
South Scotia Ridge away from Tierra del Fuego and the North Scotia Ridge; Cunningham et al. (1995) showed that since 84 Ma there had been an east-west left-lateral strike-slip sense of relative motion with a lesser north-south divergent component between the southernmost South America and the Antarctic Peninsula. The continental separation between the southernmost South America and the Antarctic Peninsula eventually led to seafloor spreading in the Western Scotia Sea and development of the Scotia Arc. The West Scotia Ridge, an extinct spreading centre, formed the oceanic crust of the western Scotia Plate. The seafloor spreading in the Scotia Sea seems to have started approximately 30-40 Ma (Barker and Burrell, 1977; Barker and Dalziel, 1983).

1.3 Relative Plate Motions

Over most of the Scotia Sea the plate motions amount to an eastward movement of the Antarctic Plate with respect to the South American Plate (Barker and Dalziel, 1983; Pelayo and Wiens, 1989). The boundaries of the Scotia and Sandwich plates, the North and South Scotia ridges, are sub-parallel to the east-west direction of the slow, sinistral motion.

The relative motion with respect to South America is accommodated around the Scotia Sea by the North and South Scotia Ridge and by the Shackleton Fracture Zone (Barker and Dalziel, 1983; Livermore et al., 1994). Pelayo and Wiens (1989) found that there is a left-lateral strike-slip motion with a component of compression along the North Scotia Ridge, a left-lateral strike-slip with a component of extension along the South Scotia Ridge, and an east-west compression in the Drake Passage. In particular, the South Scotia Ridge presents combined extension and transcurrent motion accommodated by zones of oblique extension as well as discrete transform faults and extensional segments. The left-lateral movements of the North and South Scotia ridges induce a left lateral shear couple in the whole plate, with a maximum horizontal NE-SW shortening direction (Galindo-Zaldívar et al., 1996; Giner-Robles et al., 2003). The Shackleton Fracture Zone is characterized by a left-lateral motion with transpressive deformation (Pelayo and Wiens, 1989; Cunningham et al., 1995; Galindo-Zaldívar et al., 2000; Maldonado et al., 2000). Several studies show seismological evidences for active convergence and subduction along the South Shetland Trench (Pelayo and Wiens, 1989; Ibáñez et al., 1997; Robertson et al., 2003; Guidarelli and Panza, 2006).
1.4 Seismicity

In the Scotia Sea the seismicity is concentrated along the bathymetric features that surround the region (Pelayo and Wiens, 1989). Looking at the distribution of the epicentres in the period 1973-2003 (See Figure 1.2) it is evident that the South Sandwich Trench is the principal source of earthquakes in the region: there is an intense earthquake activity associated with the subduction. Sparse earthquake activity occurs along both North and South Scotia ridges, the Shackleton Fracture Zone and Bransfield Strait and on spreading centres in the Eastern Scotia Sea. Apart from the South Sandwich Arc, the most active seismic zone corresponds to the South Scotia Ridge, while lower seismicity has been recorded along the North Scotia Ridge. In the South Shetland Islands region there is a high level of local seismicity (Robertson et al., 2003) with a moderate number of intermediate-depth earthquakes (Ibáñez et al., 1997).

![Figure 1.2: Map of the Scotia Sea region with the 1973-2003 seismicity (epicenter location of the earthquakes are from the NEIC catalogue)](image)

The remote location of the Scotia Sea region has posed logistical problems for collection of seismological data. Only in the last 10-15 years there has been a major deployment of seismic instrumentation in the islands and continental regions surrounding the Scotia and Sandwich plates. Therefore just a few studies about the source mechanisms in the Scotia Arc had been published, the main works being those by Forsyth (1975) and by Pelayo and Wiens (1989). The paper by Robertson et al. (2003) is a study of the seismicity in the South
Shetland Islands and Bransfield Strait using the data provided by the SEPA Antarctic Project, that has been recently revised by Guidarelli and Panza (2006) by means of INPAR method (Šilený and Panza, 1991, Šilený et al., 1992) that will be widely used in this thesis. There are several tomographic studies of the Antarctica and surrounding oceans, but they are continental scale studies; there are not many studies on a regional scale for the Scotia Sea region; among these the works by Vuan et al. (1999, 2000) propose group velocity tomographic maps and shear wave velocity models for the Scotia Sea region.

Thanks to the installation of new broadband seismic stations in the Scotia Sea region, a large amount of good quality data is now available and it is possible to extend and refine the existing measurements, with the aim to produce structural models that permit to better constrain the present-day tectonics and lithospheric characteristics of the Scotia Sea. In this study we will use the broadband seismic data recorded in the Scotia Sea region to analyse the earthquake source mechanism and to determine the properties of the lithosphere in the region.
Chapter 2

The ASAIN Network

2.1 Introduction: Seismometry in Antarctica

An optimal global coverage of the Earth with broad-band seismographic stations is still an objective far from being reached. It has been remarked by many authors that this is especially true in the southern hemisphere, where relevant information on medium and low level seismicity is lost due to the lack of an appropriate density of stations in the oceanic regions and in the inhospital Antarctic areas. Such a situation existed, at least until about a decade ago, in the wide region formed by the continental areas of southernmost South America (Patagonia and Tierra del Fuego), the Antarctic Peninsula and the oceanic region limited by the islands and ridges of the Scotia Arc. No permanent broadband land station was operating there before 1992 and until today only a few temporary deployments of sea bottom seismographs on the oceanic floor of the Scotia Sea have been performed. Since then a relevant effort has been conducted by some national antarctic programs to install a broad-band regional network capable of providing the seismological data base necessary to afford the study of those areas. Seven land stations have been put into operation up to now. Five of them constitute the Antarctic Seismographic Argentinean Italian Network (ASAIN), while three broad band stations are operated at Palmer station (PMSA), on the Malvinas/Falkland Islands (EFI) and on South Georgia Islands (HOPE) by the Incorporated Research Institutions for Seismology (IRIS) consortium. In the following sections we will present in detail the state of the art of the ASAIN, providing also some information about the planned expansion of the network with new land installations in Tierra del Fuego and in the Antarctic Peninsula.
2.2 The ASAIN: History

Several permanent broad-band seismographic stations are operating today in the Scotia Sea area. Four of them constitute the ASAIN (Figure 2.1). The ASAIN project started on February 1992 when a temporary station based on a three component broad-band Teledyne Geotech BB-13 seismograph and PDAS-100 recorder was installed at the Argentinean Base Esperanza (ESPZ) by the Osservatorio Geofisico Sperimentale di Trieste (today Istituto Nazionale di Oceanografia e di Geofisica Sperimentale - OGS) and the Instituto Antártico Argentino (IAA) (Russi et al., 1996). During two years the station provided continuous monitoring of the Scotia Sea regional seismicity and its recordings were used to obtain significant new information on the large-scale structure of the lithosphere in the Scotia Sea area (Russi et al., 1994).

Based on the analysis of these results, a preliminary plan for the development of a Scotia Sea region seismographic network was prepared jointly by the OGS and the IAA. The network layout included five stations. Contacts were made with the British Antarctic Survey (BAS) and the United States Geological Survey (USGS) to check their opinion and to establish reciprocal cooperation.

Working ASAIN stations, after 15 years from the beginning of operation of
ESPZ station, are shown in Table 2.1. Two stations have been installed along the South Scotia Ridge at the Argentinean Antarctic bases Jubany (JUBA, 2002, South Shetland Islands) and Orcadas (ORCD, 1997, Laurie Island, South Orkney Islands), while one was installed at Parque Nacional Lapataia in Tierra del Fuego (Argentine), about 20 km west of Ushuaia city (USHU, 1995), and a fourth station is located at Estancia Despedida (DSPA, 2002), about 40 km west of Rio Grande city (Tierra del Fuego). ESPZ station, which has been the first broad-band station installed in the Scotia Sea region, has been temporarily closed at the beginning of 2002 and reopened in February 2005. The technology and capabilities of the seismological equipment available on the market underwent a kind of revolution along the last decade. Technological progress allowed us to pass from 1.44 Mb diskettes as recording media, allowing only three days storage autonomy, to three months local recording autonomy using RefTek 72A-08 systems, when ESPZ station were transformed into a permanent observatory (February 1995). The same kind of instruments was employed later when USHU and ORCD observatories started operation.

Today all ASAIN stations are equipped with Güralp broad-band seismograph: CMG-3T seismometer (0.01 s - 50 Hz), CMG-DM24 24 bit digitizer, GPS timing and great capacity storage media, allowing continuous recording of more than 18 months of three component seismic channels at 40, 20 and 2 samples/s. (See http://www.guralp.net).

At the beginning of 2005, the Antarctic stations ESPZ, ORCD and JUBA were connected to Internet and now they are sending seismological data to the ORFEUS (Observatories and Research Facilities for EUropean Seismology) data centre in real time. (See http://www.orfeus-eu.org/Data-info/vebsn.html). In October 2005, the USHU station was closed because of logistics problems, and a new station called TRVA (Termas del Río Valdés) began to operate the same month in Tierra del Fuego.

### 2.3 The ASAIN: On going developments

The seismic sequence initiated by the Centenary Earthquake, the 7.5 magnitude event which shook the South Orkney Islands on August 4th, 2003 when Orcadas base was preparing to celebrate its 100 years of continued operation, clearly delineated the guidelines to be followed in the short and medium term ASAIN development plans.

Orcadas base is only accessed during summer by the Argentinean Icebreaker
<table>
<thead>
<tr>
<th>Station</th>
<th>Latitude</th>
<th>Longitude</th>
<th>Height (m)</th>
<th>Operation Dates</th>
</tr>
</thead>
<tbody>
<tr>
<td>DSPA</td>
<td>-53.9536</td>
<td>-68.2668</td>
<td>150.7</td>
<td>11/2002-present</td>
</tr>
<tr>
<td>ESPZ</td>
<td>-63.3981</td>
<td>-56.9964</td>
<td>31</td>
<td>02/1992-present</td>
</tr>
<tr>
<td>JUBA</td>
<td>-62.2373</td>
<td>-58.6627</td>
<td>16</td>
<td>02/2002-present</td>
</tr>
<tr>
<td>ORCD</td>
<td>-60.7381</td>
<td>-44.7361</td>
<td>20</td>
<td>03/1997-present</td>
</tr>
<tr>
<td>SMAI</td>
<td>-68.1302</td>
<td>-67.1059</td>
<td>9</td>
<td>02/2007-present</td>
</tr>
<tr>
<td>USHU</td>
<td>-54.8433</td>
<td>-68.5569</td>
<td>12</td>
<td>12/1995-10/2005</td>
</tr>
</tbody>
</table>

Table 2.1: Coordinates and operation date of the ASAIN Network stations

A.R.A. Almirante Irizar, so serious instrumental faults may cause heavy data losses up to one year and more. Remote accessibility to the seismograph appears then to be an indispensable tool both to check its functioning and to recover the data. Almost the same is true for all the Antarctic ASAIN sites.

On February 2005, Esperanza station was reinstalled with a remote access link via Internet. Also, Internet connections are operating at DSPA and ORCD from this date. So, now remote access is possible and real-time monitoring of the seismicity at each ASAIN station site is possible.

On May 2005 the Seismological Communication Processor (SeisComP), (See http://www.gfz-potsdam.de/geofon/seiscomp/) was installed at OGS. This software permits the collection of the seismic data from Antarctic stations in real time, the conversion of the data to MiniSEED format and their transfer to the ORFEUS data centre.

During the summer Antarctic campaign 2007, a new digital seismograph was installed in the San Martín permanent Argentinean Antarctic base (68.13 S, 67.10 W) called SMAI. The Guralp instrumentation was used and connected to Internet in order to permit data transmission in real time and remote control.

The continued operation of ASAIN stations has provided a sizeable amount of broad-band seismograms that have been used to investigate the structural properties, the regional seismicity and the geodynamics of the Scotia Sea region (Guidarelli et al., 2003; Guidarelli and Panza, 2006). In turn, these results are of paramount importance to outline the evolution and the necessary improvements in the network geometry of ASAIN.

During 2006 and this year, significant progress was made in the development of the ASAIN Network, principally in the automated seismic data processing; incoming digital data in real time are injected in the Earthworm software suite
Primary functions of Earthworm are to serve as real time data server, auto detect and locate earthquakes.

2.4 Instrumentation

This and the following section 2.5 are taken from Havskov and Alguacil, 2004.

Introduction: Seismology cannot be a science without well-calibrated instruments. The real advances in seismology started around 1900 mainly due to improvements of the sensitivity of seismographs and of timing devices, so that earthquakes could be located with some accuracy. Later, the importance of accurate measurement of the true ground motion became evident for studying seismic wave attenuation, and the Richter magnitude scale depends on being able to calculate the ground displacement from our recorded seismogram. So what are the main topics of instrumental seismology? It all starts with being able to measure the ground motion, and this is the most important topic. We usually talk about ground displacement (formally measured in m) since this is what seismologists like to use. On the other hand, engineers, seem to think that the acceleration \( m/s^2 \) is the most natural unit, since it is directly related to force and the peak ground acceleration is an often quoted measure.

The range of amplitudes is very large. The natural background noise, highly frequency dependent, sets the limit for the smallest amplitudes we can measure, which is typically 1 nm displacement at 1 Hz, while the largest displacement is in the order of 1 m. This is a dynamic range of \( 10^9 \). The band of frequencies we are interested in, also has large range, from \( 10^{-5} \) to \( 10^3 \) Hz (See Table 2.2). The challenge is therefore to construct seismic instruments, both sensors and recorders, which cover at least part of these large frequency and dynamic ranges.

Seismic Sensors: A seismic sensor is an instrument to measure the ground motion when it is shaken by a perturbation. This motion is dynamic and the seismic sensor or seismometer also has to give a dynamic physical variable related to this motion. In the earlier seismometers, this variable was also a displacement of a stylus representing the amplified ground motion. In present day instruments, the physical output that a seismometer yields is always a voltage. Our objective is to measure the Earth motion at a point with respect to this same point undisturbed. Unfortunately it is not so easy to measure this.
motion and the seismic sensor is the most critical element of a seismograph (seismometer and recording unit). The main difficulties are: (1) The measurement is done in a moving reference frame, in other words, the sensor is moving with the ground and there is not any fixed undisturbed reference available. So displacement cannot be measured directly. According to the inertia principle, we can only observe the motion if it has an acceleration. Seismic waves cause transient motions and this implies that there must be acceleration. Velocity and displacement may be estimated, but inertial seismometers cannot detect any continuous component of them.

(2) The amplitude and frequency range of seismic signals is very large, see Table 2.2. These values are of course the extremes, but a good quality all round seismic station for local and global studies should at least cover the frequency band 0.01 to 100 Hz and Earth motions from 1 nm to 10 mm.

It is not possible to make one single instrument covering this range of values and instruments with different gain and frequency response are used for different ranges of frequency and amplitude. Sensors are labeled e.g. short period (SP), long period (LP) or strong motion. Today, it is possible make instruments with a relatively large dynamic and frequency range (so called broad band instruments (BB) or very broad band (VBB)) and the tendency go in the direction of increasing both the dynamic and frequency range. This means that earlier 2 or 3 sensors were used, one sensor can today often do the same job although still not covering the complete range.

**The standard inertia seismometer:** Since the measurements are done in a moving reference frame (the Earth’s surface), almost all seismic sensors are based on the inertia of a suspended mass, which will tend to remain stationary in response to external motion. The relative motion between the suspended

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>Type of measurements</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00001-0.0001</td>
<td>Earth tides</td>
</tr>
<tr>
<td>0.0001-0.001</td>
<td>Earth free oscillations, earthquakes</td>
</tr>
<tr>
<td>0.001-0.01</td>
<td>Surface waves, earthquakes</td>
</tr>
<tr>
<td>0.01-0.1</td>
<td>Surface waves, P and S waves, earthquakes with $M &gt; 6$</td>
</tr>
<tr>
<td>0.1-10</td>
<td>P and S waves, earthquakes with $M &gt; 2$</td>
</tr>
<tr>
<td>10-1000</td>
<td>P and S waves, earthquakes with $M &lt; 2$</td>
</tr>
</tbody>
</table>

Table 2.2: Typical frequencies generated by different seismic sources
mass and the ground will then be a function of the ground’s motion.

The Figure 2.2 shows a simple seismometer that will detect vertical ground motion. It consists of a mass suspended from a spring. The motion of the mass is damped using a *dash pot* so that the mass will not swing excessively near the resonance frequency of the system. A ruler is mounted on the side to measure the motion of the mass relative to the ground. If the system is at rest, what would happen to sudden (high frequency) impulse like the first swing of a P-wave? Intuitively one would expect that the mass initially remains stationary while the ground moves up. Thus the displacement of the ground can be obtained directly as the relative displacement between the mass and the Earth as read on the ruler. It is also seen that the ground moves up impulsively, the mass moves down relative to the frame, represented by the ruler, so there is a phase shift of $\pi$ in the measure of ground displacement. Similarly if the ground moves with a very fast sinusoidal motion, one would expect the mass to remain stationary and thus the ground sinusoidal motion could be measured directly. The amplitude of the measurement would also be the Earth’s amplitude and the seismometer would have a gain of 1. The seismometer thus measures relative displacement directly at high frequencies and we can say the seismometer response function (motion of mass relative to Earth motion) is flat at high frequencies. The phase is also $\pi$ in this case.

![Figure 2.2: A mechanical inertial seismometer. R is a dash pot.](image)

What about very low frequencies? With the ground moving very slowly,
the mass would have time to follow the ground motion, in other words, there would be little relative motion and less phase shift. Thus the gain would be low. Finally, what would happen at the resonance frequency? If the damping is low, the mass could get a new push at the exact right time, like pushing a swing at the right time, so the mass would move with a larger and larger amplitude, thus the gain would be larger than 1. For this to happen, the push from the Earth must occur when the mass is at an extreme position (top or bottom) and there must be a phase shift of \(-\pi/2\). The sign is negative since the maximum amplitude of the mass movement will be delayed relative to the ground motion.

Note that the phase shifts given are relative to the coordinate system as defined in Figure 2.2. For real seismographs, the convention is that a positive Earth motion gives a positive signal so the measuring scale will have to be inverted such that for high frequencies and/or an impulsive input, an upwards Earth motion gives a positive signal.

### 2.5 Seismometer frequency response

Let \(u(t)\) be the Earth’s vertical motion and \(z(t)\) the displacement of the mass relative to the Earth, both positive upwards. There are two real forces acting on the mass: the force of the deformed spring and the damping.

Spring force: \(-kz\), negative since the spring opposes the mass displacement, \(k\) is the spring constant. The resonant angular frequency of the mass-spring system is \(\omega_0 = \sqrt{k/m}\), where \(\omega_0 = 2\pi/T_0\) and \(T_0\) is the corresponding natural period.

Damping force: \(-d\dot{z}\), where \(d\) is the friction constant. Thus the damping force is proportional to the mass times the velocity and is negative since it also opposes the motion. The acceleration of the mass relative to an inertial reference frame, will be the sum of the acceleration \(\ddot{z}\) with respect to the frame (or the ground) and the ground acceleration \(\ddot{u}\).

Since the sum of forces must be equal to the mass times the acceleration, we write:

\[-kz - d\dot{z} = m\ddot{z} + m\ddot{u}\]  \(\text{(2.1)}\)

For practical reasons, it is convenient to use \(\omega_0\) and the seismometer damping constant, \(h = d/(2m\omega_0)\) instead of \(k\) and \(d\) since both parameters are directly related to measurable quantities. Equation (2.1) can then be written:

\[\ddot{z} + 2h\omega_0\dot{z} + \omega_0^2 = -\ddot{u}\]  \(\text{(2.2)}\)
The equation shows that the acceleration of the Earth can be obtained by measuring the relative displacement of the mass $z$, and its time derivatives. Before solving the equation (2.2), lets make a qualitative analysis of the equation. If the frequency is high, acceleration will be high compared to the velocity and displacement and the term $\ddot{z}$ will dominate. The equation can then be written approximately as

$$\ddot{z} = -\ddot{u}$$  \hspace{1cm} (2.3)

This shows that the motion of the mass is nearly the same as the motion of the Earth with reversed sign or phase shift of $\pi$. If the frequency is low, the $z$-term will dominate and the equation can be approximated as

$$\omega_0^2 z = -\ddot{u}$$  \hspace{1cm} (2.4)

Therefore, for small frequencies, the relative displacement of the mass is directly proportional to minus the ground acceleration, $z = -\ddot{u}/\omega_0^2$ and the sensitivity of the sensor to low frequency ground acceleration is inversely proportional to the squared natural frequency of the sensor.

In the general case, there is no simple relationship between the sensor motion and the ground motion and equation (2.2) will have to be solved so the input and output signals can be related. Ideally, we would like to know what the output signal is for any arbitrary input signal, however that is not easy to solve directly. Since an arbitrary signal can be described as a superposition of harmonics (Fourier Series), the simplest way to solve equation (2.2) is to assume an input of a harmonic Earth motion and solve for the solution in the frequency domain. Let's write the Earth motion as

$$u(t) = U(\omega)e^{i\omega t}$$  \hspace{1cm} (2.5)

where $U(\omega)$ is the complex amplitude and $\omega$ is the angular frequency. The equation (2.5) is written in complex form for simplicity of solving the equations and the real part represent the real Earth motion. Since a seismometer is assumed to represent a linear system, the seismometer motion is also a harmonic motion with the same frequency and amplitude is $Z(\omega)$:

$$z(t) = Z(\omega)e^{i\omega t}$$  \hspace{1cm} (2.6)

We then have

$$\ddot{u} = -\omega^2 U(\omega)e^{i\omega t}$$
$$\dot{z} = i\omega Z(\omega)e^{i\omega t}$$
$$\ddot{z} = -\omega^2 Z(\omega)e^{i\omega t}$$  \hspace{1cm} (2.7)
Inserting in equation (2.2) and dividing by the common factor $e^{i\omega t}$, we can calculate the relationship between the output and input as $T(\omega) = \frac{Z(\omega)}{U(\omega)}$, the so called displacement frequency response function:

$$T_d(\omega) = \frac{Z(\omega)}{U(\omega)} = \frac{\omega^2}{\omega_0^2 - \omega^2 + i2\omega_0h} = \frac{\omega^2((\omega_0^2 - \omega^2) - 2i\omega^2\omega_0)}{(\omega_0^2 - \omega^2)^2 + 4\omega^2\omega_0^2h^2} \quad (2.8)$$

From (2.8), the amplitude displacement response $A_d(\omega)$ and phase displacement response $\Phi_d(\omega)$ can be calculated as the modulus and phase of the complex amplitude response:

$$A_d(\omega) = |T_d(\omega)| = \frac{\omega^2}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4h^2\omega^2\omega_0^2}} \quad (2.9)$$

$$\Phi_d(\omega) = \arctan\left(\frac{\text{Im}(T_d(\omega))}{\text{Re}(T_d(\omega))}\right) = \arctan\left(\frac{-2h\omega^2\omega_0^2}{\omega_0^2 - \omega^2}\right) \quad (2.10)$$

and $T_d(\omega)$ can be written in polar form as

$$A_d(\omega) = A_d(\omega)e^{i\Phi_d(\omega)} \quad (2.11)$$

From equation (2.9), we can again see what happens in the extreme cases. For high frequencies, we get

$$A_d(\omega) \rightarrow 1 \quad (2.12)$$

This is a constant gain of one and the sensor behaves as a pure displacement sensor.

For low frequencies,

$$A_d(\omega) \rightarrow \frac{\omega^2}{\omega_0^2} \quad (2.13)$$

For a high damping,

$$A_d(\omega) \approx \frac{\omega}{2h\omega_0} \quad (2.14)$$

and we have a pure velocity sensor, however the gain is low since $h$ is high.

Figure 2.3 shows the amplitude and phase response of a sensor with a natural period of 1 s and damping from 0.25 to 4. As it can be seen, a low damping results in a peak in the response function which occur if $h < 1$. If $h = 1$, the seismometer mass will return to it rest position in the least possible time without overshooting, the response curve has no peak and the seismometer is said to be critically damped. From the shape of the curve and (2.9), we can see that the seismometer can be considered a second order high pass filter for the Earth
displacement. Seismometers perform optimally at damping close to critical. The most common value to use is $h = 1/\sqrt{2} = 0.707$. Inserting this value in (2.9), we obtain $A_d(\omega_0) = 0.707$. This is the amplitude value used to define the corner frequency of a filter or the -3 dB point. So using $h = 0.707$ means that we can describe our response as a second order high pass filter (actually the same as a Butterworth filter) with a corner frequency of $\omega_0$. When the damping increases above 1, the sensitivity decreases, as described in equation (2.9), and the response approaches that of a velocity sensor. From the Figure 2.3, we see that for $h = 4$, the response approaches a straight line indicating a pure velocity
response. Note that for an harmonic input signal with $\omega = \omega_0$, the phase shift is $\pi/2$ independent from damping.

**The velocity transducer - The electromagnetic sensor:** The majority of the seismometers uses a velocity transducer to measure the motion of the mass. The principle is to have a moving coil within a magnetic field. This can be implemented by having a fixed coil and a magnet that moves with the mass or a fixed magnet and the coil moves with the mass. The output from the coil is proportional to the velocity of the mass relative to the frame and we therefore call this kind of electromagnetic seismometer a velocity transducer.

In other words: For an electrodynamic system, the output voltage is - within the passband of the instrument - proportional to ground velocity (instead of displacement). Two new constants are brought into the system:

- **Generator constant $G$:** This constant relates the velocity of the ground to the output from the coil. It has a unit of $V/\text{ms}^{-1}$.
- **Generator coil resistance $R_g$:** The resistance of the generator coil (also called signal coil) in Ohms.

The signal coil makes it possible to damp the seismometer in a very simple way by loading the signal coil with a resistor. When current is generated by the signal coil, it will oppose to the mass motion with a proportional magnetic force.

The frequency response function for the velocity transducer is different from that of the mechanical sensor in (2.9). In (2.8), $Z(\omega)$ was the observed output signal. With the velocity transducer, the observed output signal is a voltage proportional to mass velocity $\dot{Z}(\omega) = i\omega Z(\omega)$ and $G$, instead of $Z(\omega)$. The displacement response for the velocity sensor is then

$$T_d^v(\omega) = G \frac{\dot{Z}(\omega)}{U(\omega)} = \frac{i\omega \omega^2 G}{\sqrt{\omega_0^2 - \omega^2}^2 + 4h^2 \omega^2 \omega_0^2}$$

(2.15)

and it is seen that the only difference compared to the mechanical sensor is the factor $G$ and $i\omega$.

For the response curve (2.15), the unit is $(\text{ms}^{-1}/\text{m})(\text{V}/\text{ms}^{-1}) = \text{V}/\text{m}$.

**Instrument response curves, different representation:** Usually, the standard displacement gain curve as shown in Figure 2.3 is used. Sometimes we also want to know the velocity and acceleration response functions look like for both the mechanical sensor and the electromagnetic sensor. Recall that the
displacement response function (2.8) was

\[
T_d(\omega) = \frac{Z(\omega)}{U(\omega)} = \frac{\omega^2}{(\omega_0^2 - \omega^2) + i2\omega\omega_0h} \quad (2.16)
\]

If we now replace \( U(\omega) \) with \( \dot{U}(\omega) = i\omega U(\omega) \), the velocity response function becomes

\[
T_v(\omega) = \frac{Z(\omega)}{\dot{U}(\omega)} = \frac{-i\omega}{(\omega_0^2 - \omega^2) + i2\omega\omega_0h} \quad (2.17)
\]

and similarly the acceleration response function will be

\[
T_a(\omega) = \frac{Z(\omega)}{\ddot{U}(\omega)} = \frac{-1}{(\omega_0^2 - \omega^2) + i2\omega\omega_0h} \quad (2.18)
\]

Similarly we get for the velocity sensor response function for velocity

\[
T_v^v(\omega) = \frac{\omega^2}{(\omega_0^2 - \omega^2) + i2\omega\omega_0h} \quad (2.19)
\]

and for acceleration

\[
T_a^v(\omega) = \frac{-i\omega}{(\omega_0^2 - \omega^2) + i2\omega\omega_0h} \quad (2.20)
\]

If the sensor is an accelerometer, the response curves are the same as for velocity sensor multiplied by \( i\omega \) like the velocity sensor curve was just the mechanical sensor curve multiplied by \( i\omega \). It is thus easy to convert from one type of curve to another since it is just a question of multiplying or dividing with \( \omega \) (or \( i\omega \) for the complex response). The Figure 2.4 gives a schematic overview of the different representations of standard sensors.

Strictly speaking, none of the sensors are linear-in the sense that an arbitrary waveform of ground motion be exactly reproduced at scale- for any kind of response. The mechanical sensor has a flat displacement response above \( \omega_0 \), but few such instruments are used e.g. Wood Anderson seismometer. The velocity sensor has a flat response for frequencies above \( \omega_0 \), so by making a BB sensor with corner frequency at e.g. 0.01 Hz, we have a sensor with a wide frequency band linear to velocity. However, the sensor is still used to measure signals at frequencies below \( \omega_0 \) so it is still only free of distortion in a limited frequency band. The accelerometer is the most broadband instrument available since it have a response flat for acceleration in the whole seismic frequency band. This is achieved by only making the accelerometer with a \( \omega_0 \) larger than the largest frequency of interest.
Figure 2.4: Schematically amplitude frequency response of a mechanical 1 Hz sensor (top), a electromagnetic 1 Hz sensor (middle) and an accelerometer 100 Hz (bottom). From left to right, the figures show the sensor output due to ground displacement, velocity and acceleration, respectively. The axes are logarithmic and the horizontal axes show frequency in Hz. The asymptotic slope for each segment is indicated. Note how one curve translates into another by just adding or subtracting one unit of the slope corresponding to a multiplication or division by the frequency.

**General representation of the frequency response function:** We have already seen an example of frequency response function like the standard inertial
seismometer. A seismic station has more elements which might not fit to it and a general description of the frequency response function is needed (hereafter called response function) which can cover any system used. So, the amplitude frequency response function for a mechanical seismometer is

\[ T_d(\omega) = \frac{\omega^2}{(\omega_0^2 - \omega^2) + i2\omega\omega_0 h} \]  

(2.21)

In general, the response function could be any complex function. It turns out that \( T(\omega) \) for all systems made from discrete mechanical or electrical components (masses, springs, coils, capacitors, resistors etc) can be represented exactly by rational functions of \( i\omega \) like

\[ T(\omega) = \frac{a_0 + a_1(\omega) + a_2(\omega) + ...}{b_0 + b_1(\omega) + b_1(\omega) + ...} \]  

(2.22)

where \( a_i \) and \( b_i \) are constants. The number of terms in the polynomials will depend on the complexity of the system. So, the mechanical seismometer response must be slightly rewritten to

\[ T_d(\omega) = \frac{-(i\omega)^2}{\omega_0^2 + 2i\omega\omega_0 h + (i\omega)^2} \]  

(2.23)

So for a seismometer \( a_0 = 0, a_1 = 0, a_2 = -1, b_0 = \omega^2, b_1 = \omega_0 h \) and \( b_2 = 1 \). This general representation is sometimes used and is one of the accepted ways of giving response in SEED (Standard for Exchange of Earthquake Data) format (http://www.iris.edu/manuals/SEEDManualV2.4.pdf), defined by the International Federation of Digital Seismograph Networks (FDSN), (http://www.fdsn.org). However, (2.23) can be written in an alternative and somewhat simpler way. Considering that a polynomial can be factorized, (2.23) can be written

\[ T(\omega) = c\frac{(i\omega - z_1)(i\omega - z_2)(i\omega - z_3)...}{(i\omega - p_1)(i\omega - p_2)(i\omega - p_3)...} \]  

(2.24)

where \( c \) is the combined normalization constant for nominator and denominator polynomials, \( z \) are the zeros (or roots) of the nominator polynomial while the zeros of the denominator polynomial are \( p \). Using (2.24) to represent \( T(\omega) \) is the so-called poles and zeros representation, which has become the most standard way. The representation of response curves in terms of poles and zeros requires to introduce the concept of Laplace transforms and complex \( s \)-plane (e.g. Sokolnikoff and Redheffer, 1958; Papoulis, 1962). For the seismometer,
the denominator polynomial must be factorized by finding the roots \( p_1 \) and \( p_2 \) of the second order equation in \( i\omega \)

\[
\omega_0^2 + 2i\omega_0 h + (i\omega)^2 = 0 \tag{2.25}
\]

where

\[
p_1 = -\omega_0(h + \sqrt{h^2 - 1}), \quad p_2 = -\omega_0(h - \sqrt{h^2 - 1})
\]

So \( T_d(\omega) \) can be written

\[
T_d(\omega) = -\frac{(i\omega - 0)(i\omega - 0)}{(i\omega - p_1)(i\omega - p_2)} \tag{2.26}
\]

So in addition to the poles \( p_1 \) and \( p_2 \) the seismometer response function has a double zero at \( z = 0 \) and the normalization constant is \(-1\). Note that since \( h \) is smaller than \( 1 \) (\( h = 0.707 \)), the poles are usually complex. Complex poles always appear as conjugate pairs.

For the standard velocity transducer, the equation is

\[
T_v(\omega) = \frac{(i\omega - 0)(i\omega - 0)(i\omega - 0)}{(i\omega - p_1)(i\omega - p_2)} \tag{2.27}
\]

and there is thus one more zero and the normalization constant is now \( 1 \) instead of \(-1\) due to the polarity change caused by the velocity transducer.

In the Figure 2.5 we can see the amplitude and phase response plots, these correspond to a seismometer G"uralp CMG-3T that operates in the ASAIN network, these curves are obtained using the poles and zeros given by the constructor. The poles and zeros for a vertical and horizontal components and the negative normalizing factor for a velocity response output are:

Zeros (Hz): 0; 0; 150.5
Poles (Hz): \(-0.00707 \pm i \ast 0.00707; \quad -80.5 \pm i \ast 30.8\)
Normalizing factor at 1 Hz: \(-49.5\)

2.6 Instrument response removal

The removal of the instrument response is often a cause of much confusion. The following section presents an overview of how the correct parameters can be derived. However, you can be referred to a much more comprehensive discussion of the process in the volume "Of Poles and Zeros: Fundamentals of digital seismology", Scherbaum, F. (2001).
Figure 2.5: The amplitude and phase response plots for a seismometer Guralp CMG-3T (0.01 s - 50 Hz) with a natural frequency of 100 s.

### 2.6.1 Conversion of instrument response from Hz to Radian

To convert poles and zeros in Hz format to radian (omega) format:

\[
\text{Pole (rad)} = \text{Pole (Hz) \times (2 \times \pi)}
\]

\[
\text{Zero (rad)} = \text{Zero (Hz) \times (2 \times \pi)}
\]

To convert \( A_0 \) from Hz to radian

\[
A_0 (rad) = A_0 (Hz) \times (2 \times \pi)^{\text{number of Poles} - \text{number of Zeros}}
\]

\( A_0 \) is the normalisation constant which scales the amplitude of the transfer function to the unity. Therefore this constant must also be scaled to account for the difference in the number of \( 2 \times \pi \) factors on the numerator and denominator used in the conversion to radian. If you plots the amplitude response of the poles and zeros you will observe that without this scaling the response is not unity over the bandwidth of the instrument.
2.6.2 Conversion of instrument response from velocity to displacement

The frequency response of a system is given as the Fourier transform of the output divided by the Fourier transform of the input. In terms of velocity or displacement this is represented as:

\[ T_{\text{vel}}(i\omega) = \frac{\text{Output}(i\omega)}{\text{Input}_{\text{vel}}(i\omega)} \quad \text{or} \quad T_{\text{disp}}(i\omega) = \frac{\text{Output}(i\omega)}{\text{Input}_{\text{disp}}(i\omega)} \]

The velocity input is obtained by differentiation of the displacement input in the time domain, equivalent to multiplication of the displacement input spectrum with \( i\omega \) in the frequency domain.

\[ \text{Input}_{\text{vel}}(i\omega) = i\omega \text{Input}_{\text{disp}}(i\omega) \quad \text{giving} \quad T_{\text{vel}}(i\omega) = \frac{\text{Output}(i\omega)}{i\omega \text{Input}_{\text{disp}}(i\omega)} \]

hence \( T_{\text{vel}}(i\omega) = \frac{T_{\text{disp}}(i\omega)}{i\omega} \quad \text{or} \quad T_{\text{disp}}(i\omega) = T_{\text{vel}}(i\omega) i\omega \)

That is, to convert the velocity response to the displacement response in the frequency domain, multiplication with \( i\omega \) is used. This is equivalent to one more zero in the pole-zero representation.

With an output signal amplitude of \( A_0 \) and an input ground motion of amplitude \( A_{\text{disp}} \), the calibration gain, \( g_d \), which would scale the output to unity at the calibration frequency would be calculated as:

\[ g_d = \frac{A_{\text{input}}}{A_{\text{output}}} = \frac{A_{\text{disp}}}{A_0} \left[ \frac{\text{m}}{\text{counts}} \right] \quad \text{or in terms of velocity} \quad g_v = \frac{A_{\text{vel}}}{A_0} \left[ \frac{\text{m s}^{-1}}{\text{counts}} \right]. \]

For a harmonic signal at \( \omega_{\text{cal}} \), \( A_{\text{disp}} = \omega_{\text{cal}} A_{\text{disp}} \) and hence \( g_d = g_v/\omega_{\text{cal}} \).

Poles and zeros with positive normalization factors for all G"uralp instruments are obtained by the frequency response of the instrument. They are measured using the HP spectrum analyzer and a curve is then fitted to this data. All poles and zeroes supplied with G"uralp equipment are derived from actual measured data and not generated from examination of theoretical calculations. The system that G"uralp Systems uses produces the lowest order transfer function that fits the data. For the response of the instruments this function always has a negative normalization factor. Whilst negative normalization factors are mathematically correct and give the correct results, G"uralp Systems has been made
aware that the SEED convention does not allow for them and that the standard rdseed conversion program does not handle negative normalization factors properly. By increasing the order of the transfer function, we have been able to provide an alternative fit to the data with a positive normalization factor. Because the instruments are true feedback instruments, their response is entirely defined by the parameters in the feedback path. The frequency response of instruments is within 0.1% at the long period end and within 2% at the high frequency end.

Güralp CMG-3T Pole-Zero Instrument Response

<table>
<thead>
<tr>
<th>Hz</th>
<th>VELOCITY</th>
<th>DISPLACEMENT</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Radian</td>
<td>Radian</td>
</tr>
<tr>
<td>$A_0=2304000$</td>
<td>$A_0=571507690$</td>
<td>$A_0=90958274$</td>
</tr>
<tr>
<td>Zeros (2)</td>
<td>Zeros (2)</td>
<td>Zeros (3)</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Poles (5)</td>
<td>Poles (5)</td>
<td>Poles (5)</td>
</tr>
<tr>
<td>$-80$</td>
<td>$-502.654824$</td>
<td>$-502.654824$</td>
</tr>
<tr>
<td>$-160$</td>
<td>$-1005.309648$</td>
<td>$-1005.309648$</td>
</tr>
<tr>
<td>$-180$</td>
<td>$-1130.973354$</td>
<td>$-1130.973354$</td>
</tr>
<tr>
<td>$-0.00707 + i 0.00707$</td>
<td>$-0.044422 + i 0.044422$</td>
<td>$-0.044422 + i 0.044422$</td>
</tr>
<tr>
<td>$-0.00707 - i 0.00707$</td>
<td>$-0.044422 - i 0.044422$</td>
<td>$-0.044422 - i 0.044422$</td>
</tr>
</tbody>
</table>

2.6.3 Instrument response removal in SAC

In order to remove the instrument response of the data or applying the instrument response to the Green functions, we use the Seismic Analysis Code (SAC) (http://www.llnl.gov/sac) SAC format pole-zero files may be exported from rdseed when the SAC output option is selected. Alternatively, we can create pole-zero files observing the following format:
ZEROS 3
POLES 5
−502.6548246 0.0
−1005.309649 0.0
−1130.973355 0.0
−0.04442212 +0.04442212
−0.04442212 −0.04442212
CONSTANT 2.44931867 E+17 du/m

SAC pole-zero files are always in radian (omega) format. Zeros of value 0 need not be represented in the file. SAC pole-zero files are exported from rdseed as displacement responses.

For a Guralp CMG-3T seismometer and Guralp CMG-DM24 24 bit digitizer with gain values of 1500 V/m s and 3.5 μV/bit = 285714.285714 du/V (for all bandwidth) respectively, and the normalisation constant A0(Vel-Hz)=2304000, the SAC constant is calculated as:

\[ CONSTANT = A_0 \times \text{Sensor Gain} \times \text{Digitizer Gain} \times (2 \times \pi) \]

\[ CONSTANT = 2304000 \times (2 \times \pi)^2 \times 1500 \times 285714.285714 \times (2 \times \pi) \]

the units are \([s^5/s^3] \times s^{-2} \times (V \times s)/m \times (du/V) \times s^{-1}] = du/m\]

We have considered that bits=counts=digital units=du.

If you want to express the response file for each specific component you have to change the sensitivity of seismometer and digitizer by the corresponding value of the sensibility of the component how is listed in the documents of the calibration for each instrument. In SAC, the response is removed with the command line:

SAC> transfer from polezero subtype < filepolezero > to none

If using the pole-zero file in the above format to remove the transfer function, output data will be displacement in units of meters.

### 2.7 Seismic Noise

Electronic seismographs can be designed for any desired magnification of the ground motion. A practical limit is however imposed by the presence of undesired signals which must not be magnified so strongly that they obscure the record. Such signals are usually referred to as noise and may be of seismic, environmental, or instrumental origin; the instrumental self-noise may come from mechanical or electronic sources. The following discussion of noise is taken from
Seismic and environmental noise. Seismic noise has many different causes. Short-period noise is at most sites predominantly man-made and somewhat larger in the horizontal components than in the vertical. At intermediate periods (2 to 20 s), marine microseisms dominate. They have similar amplitudes in the horizontal and vertical directions. At long periods, horizontal noise may be larger than vertical noise by a factor up to 300, the factor increasing with period. This is mainly due to tilt which couples gravity into the horizontal components but not into the vertical. Tilt may be caused by traffic, wind, or local fluctuations of the barometric pressure. Large tilt noise is sometimes observed on concrete floors when an unventilated cavity exists underneath; the floor then acts like a membrane. Such noise can be identified by its linear polarization and its correlation with the barometric pressure. Even on an apparently solid foundation, the long-period noise often correlates with the barometric pressure (Beauduin et al. 1996). If the situation cannot be remedied otherwise, the barometric pressure should be recorded with the seismic signal and used for a correction.

Self noise. The USGS New Low Noise Model (Peterson, 1993) summarizes the lowest observed vertical seismic noise levels throughout the seismic frequency band. It is extremely useful as a reference for assessing the quality of seismic stations, for predicting the detectability of small signals, and for the design of seismic sensors. Stationary seismic noise is normally measured in terms of acceleration power density. This density must be integrated over the passband of a low-pass or band-pass filter to obtain the power (the mean square of the amplitude) at the output of the filter. The square-root of the power is then the rms (or effective) amplitude. To determine absolute signal levels from a diagram of noise power density is thus a little inconvenient; also, power densities cannot directly be compared to the levels of transient signals such as earthquakes, so earthquake signals cannot be included in a diagram of power density. (see Figure 2.6)
Figure 2.6: The Peterson noise curves and noise spectral level for the ASAIN station ESPZ. The noise level is in dB relative to $1 \text{ (ms}^{-2}\text{)}^2/\text{Hz}$. The Peterson high and low models are shown with dashed lines. The noise spectra are shown for all 3 components.
Chapter 3

Moment Tensor Inversion

3.1 The Seismic Source

A very important topic in Seismology is the full description of the physical processes that occur in the seismic source and neighbouring area. A common approach is the approximation of seismic sources by a model of equivalent forces that correspond to the linear wave equations neglecting non-linear effects in the near source region (Geller, 1976; Kennett, 1983; Aki and Richards, 2002). Equivalent forces are defined as producing displacements at the Earth’s surface that are identical to those from the actual forces at the source (Burridge and Knopoff, 1964). The equivalent forces are determined from observed seismograms that contain information about the source, path and includes also the recording instrumental influence.

For describing seismic sources having small dimensions compared to the wavelengths of interest (point source approximation) we considered their strength (magnitudes, seismic moment) and their fault plane solutions (Herrmann, 1975). The seismic moment tensor is a valuable tool in the evaluation of seismic source mechanisms because it completely describes in a first order approximation the equivalent forces of a general seismic point source. The equivalent forces can be correlated to physical source models such as sudden relative displacement at a fault surface (elastic rebound model by H. F. Reid, 1910), rapidly propagating metastable phase transition (Evison, 1963), sudden volume collapse due to phase transitions, or sudden volume increase due to explosions (Kennett, 1983; Vasco and Johnson, 1988). The equivalent forces representing a sudden displacement on a fault plane form the double couple. The equivalent forces representing a sudden change in shear modulus in presence of axial strain are represented by
a linear vector dipole (Knopoff and Randall, 1970). In conclusion a seismic moment tensor is a general concept, describing a variety of seismic source models, the shear dislocation (double couple source) being just one of them (Jost and Herrmann, 1989).

Earthquakes and underground explosions are sources of seismic waves internal to the Earth; the analytical framework to analyse the internal sources is not easy to develop because the equations governing the elastic motion do not hold throughout the solid Earth. We can distinguish two different categories among internal sources: faulting sources and volume sources. A faulting source is an event associated with an internal surface, such a slip across a fracture plane. A volume source is an event associated with an internal volume, such as a sudden expansion throughout a volumetric source region. A unified treatment of both source types is possible, the common link being the concept of an internal surface across which discontinuities can occur in displacement (for the faulting source) or in strain (for the volume source) (Aki and Richards, 2002).

The equivalent forces can be determined from an analysis of the eigenvalues and eigenvectors of the moment tensor. The sum of eigenvalues of the moment tensor describes the volume change in the source (isotropic component of the moment tensor). If the sum is positive, the isotropic component is due to an explosion. The source has an implosive component if the sum is negative. If the sum of the eigenvalues vanishes, then the moment tensor has only deviatoric components. The deviatoric moment tensor represents a pure double couple source if one eigenvalue equals zero. If none of the eigenvalues vanishes and their sum still equals zero, the moment tensor can be decomposed into a major and minor double couple and a compensated linear vector dipole (CLVD) (Knopoff and Randall, 1970). A CLVD is a dipole that is corrected for the effect of volume change, describing seismic sources which have no volume change, net force, or net moment. In general, a complete moment tensor can be the superposition of an isotropic component and three vector dipoles (or three CLVD’s, or three double couples, Ben-Menahem and Singh, 1981).

The moment tensor is analogous to the strain tensor, which represents deformation of a body. If all the deformation within the source region occurs seismically, then the moment tensor is proportional to the tensor representing the released strain. Just as we completely specify a strain tensor by describing the orientations of its principal axes (eigenvectors) and its three principal strains (eigenvalues), we completely specify a moment tensor by determining the trend and plunge of its P, T and B axes (eigenvectors), and the values of its three
principal moments (eigenvalues). When material motions near an earthquake source are especially simple, the moment tensor have simple forms described as isotropic, deviatoric, or double couple (Figure 3.1), analogous to simple strain fields which are isotropic, deviatoric, or pure shear (Frohlich and Apperson, 1992).

**Moment Tensor Decomposition**

The equivalent forces of a seismic point source can be determined by the analysis of the eigenvalues and eigenvectors of the moment tensor.

A common way to decompose the moment tensor is in terms of ISOTROPIC (V), DOUBLE COUPLE (DC) and COMPENSATED VECTOR LINEAR DIPOLE (CLVD) components.

![Diagram of moment tensor decomposition](image)

Figure 3.1: Depiction of ”Pure” earthquake source models. The figure shows isotropic, double couple and Compensated linear vector dipole (CLVD) source models as equal area projections of P wave first motions, and near-source particle motion.
3.2 Seismic source representation

In a Cartesian system, the motion equations in an elastic, anisotropic and inhomogeneous solid (Burridge and Knopoff, 1964) are:

\[(c_{ijpq}(x) u_{p,q}(x,t))_j - \rho(x) \ddot{u}_i(x,t) = -f_i(x,t)\] (3.1)

where \(u_i(x,t)\) is the \(i\)-component of the displacement vector, \(c_{ijpq}(x)\) are the elastic constants of the medium and \(f_i\) the \(i\)-component of the body force in the point of coordinates \(x = (x_1, x_2, x_3)\) at time \(t\). The summation convention applies to letter subscripts. Since the stress tensor is symmetric and the strain tensor is likewise symmetric, 

\[c_{ijpq} = c_{jipq} = c_{ijqp} = c_{jiqp},\]

reducing the number of components from 81 (3\(^4\)) to 36. It may further be shown that 

\[c_{ijpq} = c_{pqij},\]

which further reduces the number of independent components to 21, and for an isotropic solid 

\[c_{ijpq} = \lambda \delta_{ij}\delta_{pq} + \mu(\delta_{ip}\delta_{jq} + \delta_{iq}\delta_{jp})\] where \(\lambda\) and \(\mu\) are the Lamé constants of the medium and \(\delta_{ij}\) the Kronecker delta.

If \(v_i(x,t)\) is another motion due to body forces \(g_i(x,t)\), then

\[(c_{ijpq} v_{p,q})_j - \rho \ddot{v}_i = -g_i\] (3.2)

Equations (3.1) and (3.2) are satisfied in a volume \(V\) bounded by a surface \(S\), and for all time. We assume that \(f_i\) and \(g_i\) vanish for \(t < -T\), \(T\) a constant, and that \(u_i\) and \(v_i\) also vanish for \(t < -T\). This last is a causality condition which guarantees that the disturbance does not start before the force which causes it.

If we replace \(t\) by \(-t\) in equation (3.2) we have

\[[c_{ijpq} \bar{v}_{p,q}]_j - \rho \ddot{\bar{v}}_i = -\bar{g}_i\] (3.3)

where

\[\bar{v}_p(x,t) = v_p(x,-t), \bar{v}_i(x,t) = g_i(x,-t)\]

and

\[\bar{v}_p = 0, t > T.\]

We now multiply (3.1) by \(\bar{v}_i\), (3.3) by \(\bar{u}_i\), subtract, integrate over all of \(V\), and further integrate with respect to \(t\) from \(-\infty\) to \(\infty\). This yields

\[
\int_{-\infty}^{\infty} dt \int_V dV \left\{ (\bar{v}_i c_{ijpq} u_{p,q} - u_i c_{ijpq} \bar{v}_{p,q})_j - \rho \frac{\partial}{\partial t}(\bar{v}_i \bar{u}_i - u_i \bar{\dot{v}}_i) \right. \\
- c_{ijpq}(\bar{v}_{i,j} u_{p,q} - u_{i,j} \bar{v}_{p,q}) \right\} = \int_{-\infty}^{\infty} dt \int_V dV (u_i \bar{g}_i - \bar{v}_i \bar{f}_i)\] (3.4)
The integrals over \( t \) are in fact finite; the integrands vanish outside the interval \((-T, T)\).

The last term on the left-hand side of (3.4) vanishes since \( c_{ijpq} = c_{pqij} \). Since \( \rho \) is independent of \( t \), we integrate the second term with respect to \( t \) and find that the result vanishes since \( u_i \) vanishes for \( t < -T \) and \( \bar{v}_i \) and \( \dot{\bar{v}}_i \) vanish for \( t > T \). The first term on the left-hand side may be transformed by means of the divergence theorem so that we have the identity

\[
\int_{-\infty}^{\infty} dt \int_v dV \left( u_i \dot{\bar{g}}_i - \bar{v}_i f_i \right) = \int_{-\infty}^{\infty} dt \int_s dSn_j \left( \bar{v}_i c_{ijpq} u_{p,q} - u_i c_{ijpq} \bar{v}_{p,q} \right), \tag{3.5}
\]

where \( n_i \) is an outward drawn unit vector normal to \( S \).

If we now set \( g_i(x, t) = \delta_{ni} \delta(x, t; y, -s) \) where \( \delta(x, t; y, -s) = \delta(x_1 - y_1) \delta(x_2 - y_2) \delta(x_3 - y_3) \), \( y \) is a point in \( V \), and \( \delta \) is the Dirac delta function, then \( \bar{g}_i(x, t) = \delta_{ni} \delta(x, t; y, s) \) and (3.5) becomes

\[
u_n(y, s) = \int_{-\infty}^{\infty} dt \int_v G_{in}(x, -t; y, -s) f_i(x, t) dV_x + \int_{-\infty}^{\infty} dt \int_s n_j \left\{ G_{in}(x, -t; y, -s) c_{ijpq}(x) u_{p,q}(x, t) - u_i(x, t) c_{ijpq}(x) G_{pm,q}(x, -t; y, -s) \right\} dS_x, \tag{3.6}\]

where \( G_{in}(x, t; y, s) \) is the displacement in the \( i \)-direction at \( (x, t) \) due to an instantaneous point force of unit impulse in the \( n \)-direction at \( (y, s) \).

If, in (3.6), \( u_i \) and \( G_{in} \) satisfy the same homogeneous boundary conditions on \( S \) then the surface integral will vanish. If, in addition, \( f_i(x, t) = \delta_{im} \delta(x, t; y', -s') \), then (3.6) gives

\[
G_{nm}(y, s; y', s') = G_{mn}(y', -s'; y, -s) \tag{3.7}
\]

here \( G_{nm}(x, t; y', s') \) satisfies the same boundary conditions as \( G_{mn}(x, t; y, s) \). Therefore (3.6) may be rewritten as

\[
u_n(y, s) = \int_{-\infty}^{\infty} dt \int_v G_{ni}(y, s; x, t) f_i(x, t) dV_x + \int_{-\infty}^{\infty} dt \int_s n_j \left\{ G_{ni}(y, s; x, t) c_{ijpq}(x) u_{p,q}(x, t) - u_i(x, t) c_{ijpq}(x) G_{pn,q}(y, s; x, t) \right\} dS_x. \tag{3.8}\]

Here

\[
G_{pn,q}(y, s; x, t) = \frac{\partial}{\partial x_q} G_{np}(y, s; x, t)
\]
where \( q' \) indicates that the subscript refers to the second set of arguments of \( G_{np} \) and the summation convention still applies.

Equation (3.7) is the reciprocal theorem. It was examined by Knopoff and Gangi (1959). It is a special case of Helmholtz’s reciprocal theorem in generalized mechanics (see Whittaker, 1944; Lamb, 1889). Equation (3.5) is Betti’s reciprocal theorem applied to \( u_i \) and \( \dot{v}_i \) and integrated with respect to \( t \). Equation (3.8) is the representation theorem which we shall use in the next section. De Hoop (1958) derives a similar theorem for an isotropic homogeneous medium and our notation has followed this. The first representation theorem of this type was derived by Knopoff (1956).

Let it be assumed that we wish to find the radiation from prescribed discontinuities in the displacement and its derivatives across a surface \( \Sigma \) imbedded in \( V \) (Figure 3.2). Let \( \nu \) be the unit normal to \( \Sigma \) and let \([u_i](x, t)\) be the discontinuities in \( u_i \) and \( u_{p,q} \) across \( \Sigma \) and in the direction of \( \nu \) at the point \( x \) at time \( t \). Assume \( u_i \) and \( G_{ij} \) satisfy the same homogeneous boundary conditions on \( S \) and apply equation (3.8) to the region bounded internally by \( \Sigma \) and externally by \( S \). \( G_{mi} \) does not have prescribed discontinuities on \( \Sigma \). Then the surface integral over \( S \) vanishes and we are left with the surface integral over \( \Sigma \) only:

\[
\begin{align*}
\int_{-\infty}^{\infty} dtd \int_{V} G_{mi}(y, s; x, t)f_i(x, t)dV_x + \\
\int_{-\infty}^{\infty} dt \int_{\Sigma} \nu_j \{ [u_i](\xi, t)c_{ijpq}(\xi)G_{mp,q'}(y, s; \xi, t) - \\
G_{mi}(y, s; \xi, t) c_{ijpq}(\xi) [u_{p,q}](\xi, t) \} d\Sigma_{\xi}.
\end{align*}
\]

Equation (3.9) is a representation of \( [u_m] \) in terms of the prescribed discontinuities in \( u \) and its derivatives across \( \Sigma \).
3.3 Equivalent Forces

By means of the properties of the delta function and its derivatives we shall introduce volume integrals into the second term on the right hand side of (3.9).

We note that
\[ G_{mi}(y, s; \xi, t) = \int_V \delta(x, \xi)G_{mi}(y, s; x, t)dV_x \]
and
\[ -G_{mi,q}(y, s; \xi, t) = \int_V \delta_q(x, \xi)G_{mi}(y, s; x, t)dV_x, \]
where
\[ \delta(x; \xi) = \delta(x_1 - \xi_1)\delta(x_2 - \xi_2)\delta(x_3 - \xi_3) \]
and
\[ \delta_q(x; \xi) = \frac{\partial}{\partial x_q}\delta(x; \xi) \]
If these expressions are substituted in (3.9) we get
\[ u_m(y, s) = \int_{-\infty}^{\infty} dt \int_V G_{mp}(y, s; x, t) \left[ f_p(x, t) - \int_\Sigma d\Sigma \nu_j \left[ [u_i](\xi, t)c_{ijpq}(\xi)\delta_q(x; \xi) + [u_{i,q}](\xi, t)c_{pjiq}(\xi)\delta(x; \xi) \right] \right] dV_x \]
But \( f_p(x, t) \) represents a body force; since the surface integral within the square bracket is involved in the same way as \( f_p \), we conclude that the effect of the prescribed discontinuities across \( \Sigma \) is the same as introducing extra body forces \( e_p(x, t) \), given by
\[ e_p(x, t) = -\int_\Sigma d\Sigma \nu_j \left[ [u_i](\xi, t)c_{ijpq}(\xi)\delta_q(x; \xi) + [u_{i,q}](\xi, t)c_{pjiq}(\xi)\delta(x; \xi) \right] \] (3.10)
into an unfaulted medium.

Equation (3.10) holds for any inhomogeneity and anisotropy of the elastic medium provided only that the equations of motion are given by (3.1), and \( c_{ijpq} = c_{pqij} \) at each point of \( V \).
3.4 Mathematically consistent discontinuities

We cannot assign values to \([u_i]\) and \([u_{p,q}]\) completely arbitrarily. For instance, if \(\Sigma\) is part of the plane \(x_3 = 0\), then \([u_{p,1}] = [u_p]_1\) and \([u_{p,2}] = [u_p]_2\) so that the discontinuities in these tangential derivatives are determined when \([u_p]\) is specified. The discontinuities in the normal derivatives \([u_{p,2}]\) however, may be specified independently. This feature was not considered by Knopoff and Gilbert (1960).

The discontinuity in the normal traction across \(x_3 = 0\) is given by
\[
[T_i] = [\tau_{3i}] = c_{3ipq}[u_{p,q}] = [\mathcal{F}_i] + c_{3ip3}[u_{p,3}]
\]
where \(\tau_{ij}\) is the stress tensor and \([\mathcal{F}_i]\) is the part of the discontinuity \([T_i]\) that is determined when \([u_i]\) is specified. Clearly \([T_i]\) may be given arbitrary values by specifying \([u_{p,3}]\) suitably if \(\text{det } c_{3ip3} \neq 0\). This condition is certainly satisfied by an isotropic solid since only the diagonal terms are non-zero, and \(c_{3113} = c_{3223} = \mu, c_{3333} = \lambda + 2\mu\) where \(\lambda\) and \(\mu\) are the Lamé constants of the medium. It is not satisfied, however, by a liquid since \(\mu = 0\).

From now on we shall assume that we may choose the six quantities \([u_i]\) and \([T_i] = \nu_j[\tau_{ij}]\) independently. We rewrite (3.10), accordingly, as
\[
e_p(x, t) = - \int \{[u_i](\xi, t)\nu_j c_{ijpq}(\xi)\delta(x, \xi) + [T_p](\xi, t)\delta(x, \xi)\}d\Sigma_{\xi}
\]
The quantities \([T_i]\) measure the failure of the two sides of the discontinuity to obey the law of equal and opposite action and reaction, that is that sources of stress are introduced on \(\Sigma\).

We note that the equivalent force depends only on the local properties of the medium in the immediate vicinity of the surface \(\Sigma\).

3.5 General elastodynamic source

By using the representation theorem for seismic sources (3.8), (Aki and Richards, 2002), the observed displacement \(d_n\) at an arbitrary position \(x\) at the time \(t\) due to a distribution of equivalent body force densities, \(f_k\), in a source region is
\[
d_n(x, t) = \int_{-\infty}^{\infty} dt \int_V G_{nk}(x, t; r, \tilde{t})f_k(r, \tilde{t})dV(r)d\tilde{t}
\]
where $G_{nk}$ are the components of the Green’s function containing the propagation effects, and $V$ is the source volume where $f_k$ are non-zero. The subscript $n$ indicates the component of the displacement. We will use the following coordinate system (Figure 3.3): the $x$-axis points towards north, the $y$-axis towards east, and the $z$-axis down (right hand system). By assuming that the Green’s functions vary smoothly within the source volume in the range of moderate frequencies, the Green’s functions can be expanded into a Taylor series around a reference point ($r = \xi$) to facilitate the spatial integration in (3.12).

The physical source region is characterized by the existence of the equivalent forces. These forces arise due to differences between the model stress and the actual physical stress (stress glut, Backus and Mulcahy, 1976). Outside the source region, the stress glut vanishes as do the equivalent forces. The centroid of the stress glut is then a weighted mean position of the physical source region (Backus, 1977; Aki and Richards, 2002; Dziewonski and Woodhouse, 1983). While the centroid of the stress glut gives a better position for the equivalent point source of an earthquake than the hypocenter which describes just the position of rupture initialization. The Taylor expansion of the components of the Green’s functions around this new reference point is

$$G_{nk}(x, t; r, \bar{t}) = \sum_{m=0}^{\infty} \frac{1}{m!} (r_{j_1} - \xi_{j_1}) \cdots (r_{j_m} - \xi_{j_m}) G_{nk,j_1 \cdots j_m}(x, t; \xi, \bar{t}) \quad (3.13)$$

The comma between indices in (3.13) describes partial derivatives with respect to the coordinates after comma. We define the components of the time dependent force moment tensor as:

$$M_{k,j_1 \cdots j_m}(\xi, \bar{t}) = \int_V (r_{j_1} - \xi_{j_1}) \cdots (r_{j_m} - \xi_{j_m}) f_k(r, \bar{t})dV \quad (3.14)$$
If conservation of linear momentum applies, such as for a source in the interior of a body, then a term in $M_k$ does not exist in (3.13). The Taylor expansion (3.13) and the definition of the time dependent moment tensor (3.14), the displacement (3.11) can be written as a sum of terms which resolve additional details of the source (multipole expansion) (Backus and Mulcahy, 1976):

$$d_n(x, t) = \sum_{m=0}^{\infty} \frac{1}{m!} G_{nk,j_1, \ldots, j_m}(x, t; \xi, \bar{t}) \ast M_{kj_1}(\xi, \bar{t})$$  

(3.15)

where $\ast$ denotes the temporal convolution. By using a seismic signal that has much longer wavelengths than the dimensions of the source (point source approximation), we need to consider only the first term in (3.15) (Backus and Mulcahy, 1976). We can see, that single forces will not be present in (3.15) if there are no externally applied forces (indigenous source). The total force, linear and angular momentum must vanish for the equivalent forces of an indigenous source (Backus and Mulcahy, 1976). The conservation of angular momentum for the equivalent forces leads to the symmetry of the seismic moment tensor (Gilbert, 1970).

We assume that all components of the time dependent seismic moment tensor in (3.15) have the same time dependence $s(\bar{t})$ (synchronous source, Silver and Jordan, 1982). Neglecting higher order terms, we get (Stump and Johnson, 1977)

$$d_n(x, t) = M_{kj} [G_{nk,j} \ast s(\bar{t})]$$  

(3.16)

$M_{kj}$ are constants representing the components of the second order seismic moment tensor $M$, usually termed the moment tensor. We can see that the displacement $d_n$ is a linear function of the moment tensor elements and terms in the square brackets. If the source time function $s(\bar{t})$ is a delta function, the only term left in the square brackets is $G_{nk,j}$ describing nine generalized couples. The derivatives of a Green’s function component with respect to the source coordinate $\xi_j$ is equivalent to a single couple with arm in the $\xi_j$ direction. For $k = j$, i.e. force in the same direction as the arm, the generalized couples are vector dipoles, (Figure 3.4). Thus, the moment tensor component $M_{kj}$ gives the excitation of the generalized $(k, j)$ couple.
3.6 General seismic point sources

Let us assume that the seismic source cannot be described by a pure double couple mechanism. The moment tensor is represented as sum of an isotropic part, which is a scalar times the identity matrix, and a deviatoric part. To decompose the seismic moment tensor we need to compute its eigenvalues and eigenvectors. Let $m_i$ be the eigenvalue corresponding to the eigenvector $\mathbf{a}_i = (a_{ix}, a_{iy}, a_{iz})^T$. Using the orthonormality of the eigenvectors, we can express the moment tensor in terms of eigenvalues and eigenvectors (Jost and Herrmann, 1989).

$$
\mathbf{M} = [\mathbf{a}_1 \ \mathbf{a}_2 \ \mathbf{a}_3] \mathbf{m} \begin{bmatrix}
\mathbf{a}_1^T \\
\mathbf{a}_2^T \\
\mathbf{a}_3^T
\end{bmatrix} =
$$
\[
\begin{bmatrix}
a_{1x} & a_{2x} & a_{3x} \\
a_{1y} & a_{2y} & a_{3y} \\
a_{1z} & a_{2z} & a_{3z}
\end{bmatrix}
\begin{bmatrix}
m_1 & 0 & 0 \\
0 & m_2 & 0 \\
0 & 0 & m_3
\end{bmatrix}
\begin{bmatrix}
a_{1x} & a_{1y} & a_{1z} \\
a_{2x} & a_{2y} & a_{2z} \\
a_{3x} & a_{3y} & a_{3z}
\end{bmatrix}
\]

(3.17)

The relations between the eigenvectors components and the moment tensor elements are

\[
M_{xx} = m_1 a_{1x}^2 + m_2 a_{2x}^2 + m_3 a_{3x}^2 \\
M_{yy} = m_1 a_{1y}^2 + m_2 a_{2y}^2 + m_3 a_{3y}^2 \\
M_{zz} = m_1 a_{1z}^2 + m_2 a_{2z}^2 + m_3 a_{3z}^2 \\
M_{xy} = m_1 a_{1x} a_{1y} + m_2 a_{2x} a_{2y} + m_3 a_{3x} a_{3y} \\
M_{xz} = m_1 a_{1x} a_{1z} + m_2 a_{2x} a_{2z} + m_3 a_{3x} a_{3z} \\
M_{yz} = m_1 a_{1y} a_{1z} + m_2 a_{2y} a_{2z} + m_3 a_{3y} a_{3z}
\]

(3.18)

\(\mathbf{m}\) in (3.17) is the diagonalized moment tensor. The elements of \(\mathbf{m}\) are the eigenvalues of \(\mathbf{M}\). We now define the general moment tensor decomposition by rewriting \(\mathbf{m}\) as

\[
\mathbf{m} = \frac{1}{3} \begin{bmatrix}
tr(\mathbf{M}) & 0 & 0 \\
0 & tr(\mathbf{M}) & 0 \\
0 & 0 & tr(\mathbf{M})
\end{bmatrix}
+ \begin{bmatrix}
m_1^* & 0 & 0 \\
0 & m_2^* & 0 \\
0 & 0 & m_3^*
\end{bmatrix}
= \frac{1}{3} \begin{bmatrix}
tr(\mathbf{M}) & 0 & 0 \\
0 & tr(\mathbf{M}) & 0 \\
0 & 0 & tr(\mathbf{M})
\end{bmatrix}
+ \sum_{i=1}^{N} \mathbf{m}_i
\]

(3.19)

where \(tr(\mathbf{M}) = m_1 + m_2 + m_3\) is the trace of the moment tensor and \(\mathbf{m}_i\) is a set of diagonal matrices whose sum yields the second term in (3.19). The purely deviatoric eigenvalues \(m_i^*\) of the moment tensor are

\[
m_i^* = m_i - \frac{m_1 + m_2 + m_3}{3} = m_i - \frac{1}{3} tr(\mathbf{M})
\]

(3.20)

The first term on the right hand side of (3.20) describes the isotropic part of the moment tensor and quantifies a volume change in the source. The second term describes the deviatoric part of the moment tensor that can be decomposed into different components: vector dipoles, double couples, CLVDs, major and minor couples.
**Vector Dipoles:** A moment tensor can be decomposed into an isotropic part and three vector dipoles. In \(3.19\) let \(N = 3\) and

\[
\mathbf{m}_1 = \begin{bmatrix} m_1^* & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{m}_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & m_2^* & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{m}_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & m_3^* \end{bmatrix}
\]

(3.21)

Applying (3.17) to \(m_1\), we get for the first deviatoric term \((i = 1)\) in the decomposition

\[
m_1^* \begin{bmatrix} a_{1x}^2 & a_{1x}a_{1y} & a_{1x}a_{1z} \\ a_{1x}a_{1y} & a_{1y}^2 & a_{1y}a_{1z} \\ a_{1x}a_{1z} & a_{1y}a_{1z} & a_{1z}^2 \end{bmatrix} = m_1^* \mathbf{a}_1 \mathbf{a}_1
\]

(3.22)

where we identified the matrix as the dyadic \(\mathbf{a}_1 \mathbf{a}_1\). The dyadic \(\mathbf{a}_1 \mathbf{a}_1\) describes a dipole in the direction of the eigenvector \(\mathbf{a}_1\). By applying (3.17) to \(m_2\) and \(m_3\) in (3.21), we get similar expressions involving \(\mathbf{a}_2 \mathbf{a}_2\) and \(\mathbf{a}_3 \mathbf{a}_3\), describing the second and third deviatoric terms in the decomposition. So, (3.17) can be written for the decomposition into three linear vector dipoles along the directions of the eigenvectors of \(\mathbf{M}\) as

\[
\mathbf{M} = \frac{1}{3} (\mathbf{m}_1 + \mathbf{m}_2 + \mathbf{m}_3) \mathbf{I} + m_1^* \mathbf{a}_1 \mathbf{a}_1 + m_2^* \mathbf{a}_2 \mathbf{a}_2 + m_3^* \mathbf{a}_3 \mathbf{a}_3
\]

(3.23)

which is identical to (3.18).

**Double Couples:** We decompose a moment tensor into an isotropic part and three double couples. For the deviatoric part in (3.19) let \(N = 6\) and

\[
\mathbf{m}_1 = \frac{1}{3} \begin{bmatrix} m_1^* & 0 & 0 \\ 0 & -m_1^* & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{m}_2 = \frac{1}{3} \begin{bmatrix} m_2^* & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -m_2^* \end{bmatrix}
\]

\[
\mathbf{m}_3 = \frac{1}{3} \begin{bmatrix} m_3^* & 0 & 0 \\ 0 & m_3^* & 0 \\ 0 & 0 & -m_3^* \end{bmatrix}, \quad \mathbf{m}_4 = \frac{1}{3} \begin{bmatrix} -m_4^* & 0 & 0 \\ 0 & m_4^* & 0 \\ 0 & 0 & 0 \end{bmatrix}
\]

\[
\mathbf{m}_5 = \frac{1}{3} \begin{bmatrix} m_5^* & 0 & 0 \\ 0 & -m_5^* & 0 \\ 0 & 0 & m_5^* \end{bmatrix}, \quad \mathbf{m}_6 = \frac{1}{3} \begin{bmatrix} -m_6^* & 0 & 0 \\ 0 & m_6^* & 0 \\ 0 & 0 & m_6^* \end{bmatrix}
\]

(3.24)

where each \(\mathbf{m}_i\) is equivalent to a pure double couple source. Note that each double couple consists of two linear vector dipoles. Each dipole consists of two
forces of equal strength but opposite direction. So, the double couple can be seen this way: the first couple is formed by one force of each dipole, one force pointing in the positive $a_1$, the other in the negative $a_2$ direction. The corresponding other couple is constructed by the complementary force of each dipole, pointing toward the negative $a_1$ and positive $a_2$ direction. Using (3.17) with (3.19) and (3.24), we get the result that a moment tensor can be decomposed into an isotropic part and three double couples.

$$M = \frac{1}{3}(m_1 + m_2 + m_3)I + \frac{1}{3}(m_1 - m_2)(a_1a_1 - a_2a_2) + \frac{1}{3}(m_2 - m_3)(a_2a_2 - a_3a_3) + \frac{1}{3}(m_3 - m_1)(a_3a_3 - a_1a_1)$$

(3.25)

which is identical to equation (4.57) in Ben-Menahem and Singh (1981).

**CLVD:** A moment tensor can also be decomposed into an isotropic part and three compensated linear vector dipoles. Adding terms like $m_1$ and $m_2$ in (3.24) gives a CLVD, $2a_1a_1 - a_2a_2 - a_3a_3$. This CLVD represents a dipole of strength 2 in direction of the eigenvectors $a_1$, and two dipoles of unit strength in the direction of the eigenvectors $a_2$ and $a_3$, respectively. The decomposition can then be expressed as:

$$M = \frac{1}{3}(m_1 + m_2 + m_3)I + \frac{1}{3}m_1(2a_1a_1 - a_2a_2 - a_3a_3) + \frac{1}{3}m_2(2a_2a_2 - a_1a_1 - a_3a_3) + \frac{1}{3}m_3(2a_3a_3 - a_1a_1 - a_2a_2)$$

(3.26)

which is identical to equation (4.56) in Ben-Menahem and Singh (1981).

**Major and Minor Couple:** We decompose a moment tensor into an isotropic component, a major and minor double couple. The major couple seems to be the best approximation of a general seismic source by a double couple, since the directions of the principal axes of the moment tensor remain unchanged. The major double couple is constructed in the following way (Kanamori and Given, 1981; Wallace, 1985): The eigenvector of the smallest eigenvalue is taken as the null-axis. Let’s assume that $|m_3^*| \geq |m_2^*| \geq |m_1^*|$ in (3.19). In (3.19), let $N = 2$ and use the deviatoric condition $m_1^* + m_2^* + m_3^* = 0$ to obtain

$$\bar{m}_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -m_3^* & 0 \\ 0 & 0 & m_3^* \end{bmatrix}, \quad \bar{m}_2 = \begin{bmatrix} m_1^* & 0 & 0 \\ 0 & -m_1^* & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(3.27)

Applying (3.17) to $\bar{m}_1$, we get the first deviatoric term in the decomposition.
which corresponds to a pure double couple termed *major couple.*

\[
M^{\text{MAJ}} = \begin{bmatrix} a_1 & a_2 & a_3 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & -m_3^* & 0 \\ 0 & 0 & m_3^* \end{bmatrix} \begin{bmatrix} a_1^T \\ a_2^T \\ a_3^T \end{bmatrix}
\] (3.28)

Instead of the *major* double couple, a best double couple can be constructed similarly by replacing \(m_3^*\) in (3.28) by the average of the largest two eigenvalues. Applying (3.17) to \(\overline{m}_2\) gives the second deviatoric term in the decomposition which also corresponds to a pure double couple termed *minor couple.*

\[
M^{\text{MIN}} = \begin{bmatrix} a_1 & a_2 & a_3 \end{bmatrix} \begin{bmatrix} a_1^T \\ a_2^T \\ a_3^T \end{bmatrix}
\] (3.29)

The complete decomposition is then:

\[
M = \frac{1}{3} (m_1 + m_2 + m_3) I + m_3 (a_3 a_3 - a_2 a_2) + m_1 (a_1 a_1 - a_2 a_2)
\] (3.30)

**Double Couple - CLVD:** We consider the decomposition of the moment tensor into an isotropic part, a double couple and a compensated linear vector dipole, as proposed by Knopoff and Randall (1970) and Fitch et al. (1980).

We assume that \(|m_3^*| \geq |m_2^*| \geq |m_1^*|\) in (3.20). The deviatoric part can be written as:

\[
\overline{m}_1 = m_3^* \begin{bmatrix} m_1^*/m_3^* & 0 & 0 \\ 0 & m_2^*/m_3^* & 0 \\ 0 & 0 & 1 \end{bmatrix}
\] (3.31)

We can decompose the (3.21) into two parts representing a double couple and CLVD:

\[
\overline{m}_1 = m_3^* \left( 1 + \frac{m_1^*}{m_3^*} \right) \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - m_3^* \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}
\]

The complete decomposition is then

\[
M = \frac{1}{3} (m_1 + m_2 + m_3) I + m_3^* \left( 1 + \frac{m_1^*}{m_3^*} \right) (a_3 a_3 - a_2 a_2) - m_3^* \left( 2 a_3 a_3 - a_2 a_2 - a_1 a_1 \right)
\]
To estimate the deviation of the seismic source from the model of a pure double couple we can use the parameter (Dziewonski et al., 1981)

\[ \epsilon = \frac{|m^*_{\text{min}}|}{|m^*_{\text{max}}|} \]

where \( m^*_{\text{min}} \) is the smallest eigenvalue and \( m^*_{\text{max}} \) is the largest. For a pure double couple source, \( m^*_{\text{min}} = 0 \) and \( \epsilon = 0 \); for a pure CLVD, \( \epsilon = 0.5 \).

The moment tensor components in an isotropic medium for a double couple of equivalent forces are given by (Aki and Richards, 2002)

\[ a_{kj} = \mu A(u_k \nu_j + u_j \nu_k) \] (3.32)

where \( \mu \) is the shear modulus, \( A \) is the fault plane area, \( u \) is the slip vector on the fault surface and \( \nu \) is the vector normal to the fault plane. The term in brackets in (3.32) forms a tensor describing a double couple, whose eigenvalues are proportional to (1, 0, -1) (Jost and Herrmann, 1989). The eigenvectors to the above eigenvalues correspond to the principal axes vectors and are related to the vectors \( u \) and \( \nu \) (Udías, 1999)

\[ t = \frac{1}{\sqrt{2}}(\nu + u) \]

\[ p = \frac{1}{\sqrt{2}}(\nu - u) \]

\[ b = \nu \times u \]

The vector \( b \) corresponds to the null eigenvalue and it is the unit vector normal to the plane of the forces. This vector is known as the null axis, since there is no component of forces in its direction. The eigenvector \( t \), which corresponds to the positive eigenvalue, gives the tension axis T, and the eigenvector \( p \), corresponding to the negative eigenvalue, gives the pressure axis. The P axis is in the direction of maximum compressive motion on the fault surface while the T axis is in the direction of maximum tensitional motion (or better minimum compressive motion). The vectors \( u \) and \( \nu \) can be expressed in terms of the principal axes vectors

\[ u = \frac{1}{\sqrt{2}}(t + p) \]

\[ \nu = \frac{1}{\sqrt{2}}(t - p) \]
A shear fracture can also be specified in terms of the angles $\phi$, strike, $\delta$, dip, and $\lambda$, slip or rake, then the vectors $\mathbf{u}$ and $\mathbf{v}$ can be expressed as
\[
\mathbf{u} = \bar{u}(\cos \lambda \cos \phi + \cos \delta \sin \lambda \sin \phi)e_x \\
+ \bar{u}(\cos \lambda \sin \phi - \cos \delta \sin \lambda \cos \phi)e_y - \bar{u} \sin \delta \sin \lambda e_z
\]
\[
\mathbf{v} = -\sin \delta \sin \phi e_x + \sin \delta \cos \phi e_y - \cos \delta e_z
\]
where $\bar{u}$ is the mean displacement on the fault plane. Then the Cartesian components of the moment tensor in terms of the strike, dip and slip are
\[
M_{xx} = -M_0 (\sin \delta \cos \lambda \sin 2\phi + \sin 2\delta \sin \lambda \sin^2 \phi)
\]
\[
M_{yy} = M_0 (\sin \delta \cos \lambda \sin 2\phi - \sin 2\delta \sin \lambda \cos^2 \phi)
\]
\[
M_{zz} = M_0 (\sin 2\delta \sin \lambda)
\]
\[
M_{xy} = M_0 (\sin \delta \cos \lambda \cos 2\phi + 0.5 \sin 2\delta \sin \lambda \sin 2\phi)
\]
\[
M_{xz} = -M_0 (\cos \delta \cos \lambda \cos \phi + \cos 2\delta \sin \lambda \sin \phi)
\]
\[
M_{yz} = -M_0 (\cos \delta \cos \lambda \sin \phi - \cos 2\delta \sin \lambda \cos \phi)
\]
where the factor $M_0$ is the scalar seismic moment.

Seismic moment, $M_0$, of an earthquake is defined in terms of source parameters as the product of average shear modulus $\mu$ of the crustal rock around the earthquake, area $A$ of the fault ruptured in the earthquake, and $D$, the average slip during the earthquake (Brune, 1968), that is,
\[
M_0 = \mu A D
\]
The units for $M_0$ in the CMT-Harvard Catalogs is expressed in $\text{dyne-cm}$, values for $\mu$ varies between 32 GPa = $3.2 \times 10^{11} \text{dyne/cm}^2$ in the crust to $\sim 75 \text{GPa}$ in the mantle (See [http://earthquake.usgs.gov/learning/glossary.php?term=seismic\%20moment](http://earthquake.usgs.gov/learning/glossary.php?term=seismic\%20moment)).

### 3.7 Radiation pattern of seismic surface waves from buried dipolar point sources in a flat stratified Earth

The inverse problem of reconstructing the seismic source from amplitude measurements at stations encircling the epicenter was mainly restricted to first-motion studies. In spite of the fact that certain amounts of information could be
obtained from a tiny portion of the seismogram, seismologists soon realized that the complexity of earthquake origins in space and time could not be adequately understood and uniquely determined unless ways and means were found to extract the source parameters from main body of the seismograms.

The present availability of long-period records from the worldwide net of standardized stations will undoubtedly put source mechanism studies on a new level. To prepare the ground for studies of this kind, it is necessary to know first the theoretical radiation pattern of surface waves from various sources. Yanovskaya (1958) calculated the response of a layer over a half-space to love and Rayleigh waves due to singlets and couples. Ben-Menahem (1961) calculated radiation pattern for Rayleigh waves in a half-space due to moving faults. Harkrider (1964) calculated the response of a multilayered half-space to Love and Rayleigh waves due to buried horizontal and vertical point sources.

The present work supplies the general theory of Rayleigh and Love wave radiation from dipolar point sources with arbitrary elements in a multilayered Earth.

List of symbols

\( A_{R_m}, A_{L_m} \) The Rayleigh and Love product matrices of \( m \) layers.
\( A_R, A_L \) The Rayleigh and Love amplitude factors.
\( A(h) \) Rayleigh wave ellipticity function.
\( c(\omega) \) Phase velocity.
\( c_R, c_L \) Rayleigh and Love phase velocities.
\( d_i \) Radiation pattern coefficients.
\( e_1, e_2, e_3 \) Unit vectors at the source along strike, dip and the vertical directions respectively.
\( e_r, e_\theta, e_z \) Unit vectors along the radial, azimuthal and vertical axes at the recording station.
\( (F_r, F_\theta, F_z) \) A three vector representation of the total far displacement field at the recording station.
\( G(h, \omega) \) Love wave dipolar transfer function.
\( h \) Source depth.
\( H_0^{(2)}(k_n r), H_1^{(2)}(k_n r) \) Hankel functions.
\( k_n \) Wave number of \( n \)th mode.
\( k_R, k_L \) Rayleigh and Love wave numbers.
\( M_{R_m}, M_{L_m} \) Rayleigh and Love matrices of the \( m \)th layer.
\( \mathbf{n} \) Normal vector to plane of motion.
\(N_\theta(h, \omega)\) Love wave singlet transfer function.

\(N_{rr}(h, \omega)\) Rayleigh wave first singlet transfer function.

\(N_{rz}(h, \omega)\) Rayleigh wave second singlet transfer function.

\(R(\omega)\) The time transformed displacement vector in plane of motion.

\(t\) time.

\[
\frac{\ddot{u}_S(h)}{\dot{w}_0}, \frac{\ddot{v}_S(h)}{\dot{v}_0}, \frac{\ddot{w}_S(h)}{\dot{w}_0}
\]

Haskell’s plane wave particle velocity (displ.) ratios.

\(\dot{u}_0^* = -i\dot{u}_0\).

\(U(\omega)\) The time transformed vector displacement field.

\(U^S, U^C, U^{DC}\) The time transformed vector displacement fields due to singlet, couple, and double couple, respectively.

\(U_r, U_\theta, U_z\) Components of time transformed displacement field in cylindrical coordinates.

\(z\) Depth of receiver (station).

\(\alpha\) Compressional velocity.

\(\alpha_0\) Compressional velocity at depth \(h = 0\).

\(\alpha_S\) Compressional velocity in the source layer.

\(\beta\) Shear velocity.

\(\beta_0\) Shear velocity at depth \(h = 0\).

\(\beta_S\) Shear velocity in the source layer.

\(\epsilon_0\) Rayleigh wave surface ellipticity.

\(\mu\) Rigidity.

\(\mu_S\) Rigidity of the source layer.

\(\rho\) Density.

\(\sigma_{RS}(h)\) Depth dependent factors of normal stress associated with the Rayleigh waves.

\(\sigma_0\) Poisson ratio at depth \(h = 0\).

\(\tau_{RS}(h), \tau_{LS}(h)\) Depth dependent factors of tangential stresses associated with the Rayleigh and Love waves.

\(\chi(\theta)\) The radiation pattern function.

\(\omega\) Angular frequency.

We consider a shear-type fault with arbitrary dip (\(\delta\)) and slip (\(\lambda\)) in a layered Earth. We replace the physical fault by an equivalent force system. This force system is generated from a single force, which is directed along the fault’s motion and has a magnitude proportional to the amount of displacement in the plane of motion. We can furthermore assume, without loss of generality, that the time
dependence of the force of the Dirac delta function. The Dirac delta function, often referred to as the unit impulse function, can usually be informally thought of as a function $\delta(x)$ that has the value of infinity for $x = 0$, the value zero elsewhere.

The geometry of the situation is displayed in Figure 3.5.

Following Haskell (1963) we choose the source coordinates system to be cartesian with the $x_1$ axis in the strike direction, the $x_2$ axis in the dip direction, and the $x_3$ axis in the upward vertical direction. The convention is followed that all three force components are positive on hanging-wall (moving block) side of a reverse left-lateral fault.

Hence the vector $\mathbf{R}(\omega)$ will be represented by

$$
\mathbf{R} = |\mathbf{R}|(\cos \lambda \mathbf{e}_1 + \sin \lambda \cos \delta \mathbf{e}_2 + \sin \lambda \sin \delta \mathbf{e}_3)
$$

Likewise, the normal is given by

$$
\mathbf{n} = |\mathbf{n}|(-\sin \delta \mathbf{e}_2 + \cos \delta \mathbf{e}_3)
$$

where $0 \leq \delta \leq \pi$ and $0 \leq \lambda \leq 2\pi$.

### 3.7.1 Single force in a multilayered medium

It has been shown by Ben-Menahem and Toksöz (1963) that the displacement field of an arbitrary force system in a multilayered medium can be
generated from the response of the medium to three mutual perpendicular unit forces.

The position of the recording station is given by \((\theta, r, z = 0)\) in the source cylindrical system. \(\theta\) is measured positively anticlockwise from the positive strike direction \((x_1 = 0)\) when viewed from above the half-space downward. \(r\) is measured positively outward from the source. Thus we can write the far displacement field \(U^s\) due to a single force \(R\) at the source as

\[
U^s = (R \cdot F_\theta) e_\theta + (R \cdot F_r) e_r + (R \cdot F_z) e_z
\]

(3.36)

where \(F_i\) are obtained from Table 3.1 in the form

\[
F_\theta = (\sin \theta e^{i3\pi/4} N_\theta, \cos \theta e^{-i\pi/4} N_\theta, 0)
\]

(3.37)

\[
F_r = (\cos \theta e^{-i\pi/4} N_{rr}, \sin \theta e^{-i\pi/4} N_{rr}, e^{i\pi/4} N_{rz})
\]

(3.38)

\[
F_z = (\cos \theta e^{-i3\pi/4} N_{zr}, \sin \theta e^{-i3\pi/4} N_{zr}, e^{-i\pi/4} N_{zz})
\]

(3.39)

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Symbol</th>
<th>(U_\theta)</th>
<th>(U_r)</th>
<th>(U_z)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single force up (+)</td>
<td>↑</td>
<td>0</td>
<td>(e^{i\frac{\pi}{4}})</td>
<td>(e^{-i\frac{\pi}{4}})</td>
</tr>
<tr>
<td>Single force down (−)</td>
<td>↓</td>
<td>0</td>
<td>(e^{-i\frac{3\pi}{4}})</td>
<td>(e^{i\frac{3\pi}{4}})</td>
</tr>
<tr>
<td>Single force east (+)</td>
<td>→</td>
<td>(\sin \theta e^{i\frac{\pi}{4}})</td>
<td>(\cos \theta e^{-i\frac{\pi}{4}})</td>
<td>(\cos \theta e^{-i\frac{3\pi}{4}})</td>
</tr>
<tr>
<td>Single force west (−)</td>
<td>←</td>
<td>(\sin \theta e^{-i\frac{\pi}{4}})</td>
<td>(\cos \theta e^{i\frac{3\pi}{4}})</td>
<td>(\cos \theta e^{i\frac{\pi}{4}})</td>
</tr>
<tr>
<td>Left lateral couple (+)</td>
<td>⇋</td>
<td>(\sin^2 \theta e^{i\frac{\pi}{4}})</td>
<td>(2\theta e^{-i\frac{\pi}{4}})</td>
<td>(2\theta e^{i\frac{\pi}{4}})</td>
</tr>
<tr>
<td>Right lateral couple (−)</td>
<td>⇋</td>
<td>(\sin^2 \theta e^{-i\frac{\pi}{4}})</td>
<td>(2\theta e^{i\frac{\pi}{4}})</td>
<td>(2\theta e^{-i\frac{\pi}{4}})</td>
</tr>
<tr>
<td>Right double couple</td>
<td>⇆</td>
<td>(\cos 2\theta e^{i\frac{\pi}{4}})</td>
<td>(2\theta e^{i\frac{\pi}{4}})</td>
<td>(2\theta e^{-i\frac{\pi}{4}})</td>
</tr>
<tr>
<td>Left double couple</td>
<td>⇇</td>
<td>(\cos 2\theta e^{-i\frac{\pi}{4}})</td>
<td>(2\theta e^{-i\frac{\pi}{4}})</td>
<td>(2\theta e^{i\frac{\pi}{4}})</td>
</tr>
</tbody>
</table>

Table 3.1: Spatial factors of basic force configuration for the leading terms of Love and Rayleigh displacements

Using the representation of \(R\) as given in (3.34) we obtain the far displacement field in terms of the elements of the fault \((\lambda, \delta)\) and the azimuth angle \(\theta\).

\[
U^s_\theta = |R| N_\theta e^{i3\pi/4}(\sin \theta \cos \lambda - \cos \theta \sin \lambda \cos \delta)
\]

(3.40)

\[
U^s_r = |R| e^{-i\pi/4}[N_{rr}(\cos \theta \cos \lambda + \sin \theta \sin \lambda \cos \delta) + i N_{rz} \sin \lambda \sin \delta]
\]

(3.41)

\[
\frac{U_r}{U_z} = \epsilon_0 e^{i\pi/2}
\]

(3.42)
In these expressions the factor \( (2\pi r)^{-1/2} e^{i(\omega t-k_n r)} \) has been suppressed. All other constants are incorporated in the definition of \( N_\theta \), \( N_{rr} \), and \( N_{rz} \). The symbol \( N_{ij} \) which appears in the (3.40) and (3.41) stands for the component of the amplitude response of the medium in the \( i \) direction (horizontal or vertical) due to a unit force acting along the \( j \) direction (horizontal or vertical). Thus, for example, we write \( N_{rz} \) for the horizontal response due to a vertical force. These functions are obtained directly from the exact solution to the boundary value problem. They depend on the frequency, the depth of the source, and the constants of the layered medium. \( N_\theta \) is the Love wave response of the medium to a horizontal force. The factor \( k_n^{-1/2} \) which arises from the asymptotic expansion approximation of \( H_{0,1}^{(2)}(k_n r) \) is absorbed in \( N_{ij} \).

Comparing equations (3.36) to (3.42) with the solutions given by Harkrider (1964) for a vertical and horizontal force at depth in a multilayered half space, we can express the functions \( N_{ij} \) at \( z = 0 \) as

\[
N_{zr}(h) = -[\dot{u}_S(h)/\dot{w}_0] A_R k_R^{-1/2} \tag{3.43}
\]

\[
N_{rr}(h) = -[\dot{u}_S(h)/\dot{w}_0] N_{zr}(h) = \epsilon_0 N_{zr}(h) \tag{3.44}
\]

\[
N_{zz}(h) = -[\dot{w}_S(h)/\dot{w}_0] A_R k_R^{-1/2} \tag{3.45}
\]

\[
N_{rz}(h) = \epsilon_0 N_{zz}(h) \tag{3.46}
\]

\[
N_{\theta}(h) = -[\dot{v}_S(h)/\dot{v}_0] A_L k_L^{-1/2} \tag{3.47}
\]

The amplitude factors \( A_L \) and \( A_R \) are functions of frequency and the elastic properties of the multilayered array. They are independent of the source type and depth. This is also true for the ellipticity. The Rayleigh wave amplitude factor \( A_R \) also occurs in the solution to an explosive source at depth. The properties of \( A_R \) and \( A_L \) are given in more detail by Harkrider (1964).

The depth dependent factor in equations (3.43) to (3.47) can be represented in terms of the Thomson-Haskell layer matrices (Haskell, 1953) as

\[
[\dot{u}_S(h)/\dot{w}_0] = [A_{Rs}(h)]_{12}^* - \epsilon_0 [A_{Rs}(h)]_{11} \tag{3.48}
\]

\[
[\dot{w}_S(h)/\dot{w}_0] = [A_{Rs}(h)]_{22} + \epsilon_0 [A_{Rs}(h)]_{21}^* \tag{3.49}
\]

\[
[\dot{v}_S(h)/\dot{v}_0] = [A_{Ls}(h)]_{11} \tag{3.50}
\]

\[
A_{Rs}(h) = M_{Rs}(h) M_{Rs-1} \ldots M_{R1} \tag{3.51}
\]

\[
A_{Ls}(h) = M_{Ls}(h) M_{Ls-1} \ldots M_{L1} \tag{3.52}
\]
and $M_{Rm}$, $M_{Lm}$ the Rayleigh and Love matrices of the $m$th layer. The source layer is indicated by $m = S$ and the surface layer by $m = 1$. The matrices $M_{Rs}(h)$ and $M_{LS}(h)$ are layer matrices for a sublayer in the source layer, the thickness of which is equal to the penetration of the source depth $h$ into the source layer. From equations (3.43) to (3.47) we see that

$$
\epsilon_0 = N_{rz}(h)/N_{zz}(h) = N_{rr}(h)/N_{zr}(h) \quad (3.53)
$$

for all source depths. Since, by definition, $\dot{u}_0 \equiv \dot{u}_1(0)$ and $\dot{w}_0 \equiv \dot{w}_1(0)$, it follows that for a surface source, $h = 0$,

$$
N_{rz}(0) = N_{zr}(0) \quad (3.54)
$$

3.7.2 Generation of dipolar source

The couple displacements are obtained by the application of a differential operator to the displacement vector $U^s$ due to a single force. Thus

$$
U^c = -(n \cdot \nabla) U^s = \left( \sin \theta \sin \delta \frac{\partial}{\partial r} - \cos \delta \frac{\partial}{\partial h} \right) U^s + O(r^{-3/2}) \quad (3.55)
$$

We must choose here the negative sign for the gradient because the derivative is taken at the source, the station remaining fixed. Since we are interested only in the far field, we have discarded terms of higher order than $r^{-1/2}$. The vector $n$ is a unit vector with dimensions of length.

The double couple source is obtained by the superposition of two equal couples at right angles so that the total moment of the system is zero. To obtain the far displacement field for this system we add to the operational representation of $U^c$ as given by (3.55) the displacement field due to a second couple which is formed by the interchange of $R$ with $n$. Hence

$$
U^{DC} = -(n \cdot \nabla) U^s(R) - \left( \frac{R}{|R|} \cdot \nabla \right) U^s(|R|, n) \quad (3.56)
$$

Performing the operations indicated (3.56) we obtain general formulas for the far field of Love and Rayleigh waves. The general form of this field may be written as

$$
U = |R| |n| k_n e^{-i3\pi/4} N(h) \chi(\theta) \quad (3.57)
$$

where $k_n$ is either $k_R$ or $k_L$, $N(h)$ is either $N_\theta(h)$ or $N_{rz}(h)$, and $\chi(\theta)$ is the complex function

$$
\chi(\theta) = d_0 + i(d_1 \sin \theta + d_2 \cos \theta) + d_3 \sin 2\theta + d_4 \cos 2\theta \quad (3.58)
$$

52
The coefficients $d_i$ are given in Table 3.2 and Table 3.3. Where $\lambda$ is measured counterclockwise from the positive strike direction, and $\delta$ is measured downward from the negative dip direction. The dimensionless entities $A$, $B$, $C$, and $G$ are given by

$$A(h) = \frac{N_{rr}(h)}{N_{rz}(h)}$$

(3.59)

$$B(h) = \left( N_{rr}(h) + \frac{2}{kR} \frac{\partial N_{rz}(h)}{\partial h} \right) / N_{rz}(h)$$

(3.60)

$$C(h) = \left[ N_{rz}(h) + \frac{1}{kR} \frac{\partial N_{rr}(h)}{\partial h} \right] / N_{rz}(h)$$

(3.61)

$$G(h) = \frac{\partial N_\theta(h)}{\partial h} / kLN_\theta(h)$$

(3.62)

The common phase constant $e^{-i3\pi/4}$, appearing in (3.57) has been chosen so as to render the functions $A$, $B$, $C$, and $G$ non negative at $h = 0$. The signs in these functions are adjusted in such a way that $h$ is increasing positively downward.

The complex function $\chi(\theta)$ defines two real functions of the real variable $\theta$. Its modulus $|\chi(\theta)|$ gives the amplitude radiation pattern, and its argument $\text{arg} \chi(\theta)$ gives the spatial phase (Ben-Menahem and Toksöz, 1963) of the source. From equations 3.43 to 3.52 and the definition of the elements of the layer.

### Table 3.2: Radiation pattern coefficients for a couple system force

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Love</th>
<th>Rayleigh</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_0$</td>
<td>$-0.5 \cos \lambda \sin \delta$</td>
<td>$0.25 \sin \lambda \sin 2\delta B(h)$</td>
</tr>
<tr>
<td>$d_1$</td>
<td>$\cos \lambda \cos \delta G(h)$</td>
<td>$\sin \lambda [1 - C(h) \cos^2 \delta]$</td>
</tr>
<tr>
<td>$d_2$</td>
<td>$-\sin \lambda \cos^2 \delta G(h)$</td>
<td>$\cos \lambda \cos \delta [1 - C(h)]$</td>
</tr>
<tr>
<td>$d_3$</td>
<td>$0.25 \sin \lambda \sin 2\delta$</td>
<td>$0.5 \cos \lambda \sin \delta A(h)$</td>
</tr>
<tr>
<td>$d_4$</td>
<td>$0.5 \cos \lambda \sin \delta$</td>
<td>$-0.25 \sin \lambda \sin 2\delta A(h)$</td>
</tr>
</tbody>
</table>

### Table 3.3: Radiation pattern coefficients for a double couple system force

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Love</th>
<th>Rayleigh</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_0$</td>
<td>0</td>
<td>$0.5 \sin \lambda \sin 2\delta B(h)$</td>
</tr>
<tr>
<td>$d_1$</td>
<td>$\cos \lambda \cos \delta G(h)$</td>
<td>$-\sin \lambda \sin 2\delta C(h)$</td>
</tr>
<tr>
<td>$d_2$</td>
<td>$-\sin \lambda \sin 2\delta G(h)$</td>
<td>$-\cos \lambda \cos \delta C(h)$</td>
</tr>
<tr>
<td>$d_3$</td>
<td>$0.5 \sin \lambda \sin 2\delta$</td>
<td>$\cos \lambda \sin \delta A(h)$</td>
</tr>
<tr>
<td>$d_4$</td>
<td>$\cos \lambda \sin \delta$</td>
<td>$-0.5 \sin \lambda \sin 2\delta A(h)$</td>
</tr>
</tbody>
</table>
matrices (Haskell, 1953), the derivatives appearing in the expressions $B(h)$, $C(h)$, and $G(h)$ can be given as

$$G(h) = \frac{1}{\mu_S} \frac{\{[A_{LS}(h)]_{21}\} \hat{\nu_S}(h)}{\hat{v}_0} \quad (3.63)$$

$$A(h) = -\left[\frac{\hat{u}_S^*(h)}{\hat{w}(h)}\right] \quad (3.64)$$

$$B(h) = -\left(3 - 4\frac{\beta^2_2}{\alpha^2_0}\right) \left[\frac{\hat{u}_S^*(h)}{\hat{w}(h)}\right] - \frac{2\beta^2_3}{\mu_S \alpha^2_0} \left[\frac{\hat{w}_0}{\hat{w}(h)}\right] \{[A_{RS}(h)]_{32}^* - \epsilon_0 [A_{RS}(h)]_{31}\} \quad (3.65)$$

$$C(h) = -\left[\frac{\hat{A}_{RS}(h)]_{42}^* + \epsilon_0 [A_{RS}(h)]_{41}^*}{\mu_S \frac{\hat{w}(h)}{\hat{w}_0}}\right] \quad (3.66)$$

where the functions inside the braces have the physical meaning of stress ratios at depth $h$; in the original paper by Ben-Menahem and Harkrider (1964) there is a sign missprint thus (3.63) has been corrected accordingly.

$$[A_{LS}(h)]_{21}^* = \left[\frac{\tau_{LS}^*(h)}{\hat{w}_0/c_L}\right] \quad (3.67)$$

$$[A_{RS}(h)]_{32}^* - \epsilon_0 [A_{RS}(h)]_{31}^* = \left[\frac{\sigma_{RS}^*(h)}{\hat{w}_0/c_R}\right] \quad (3.68)$$

$$[A_{RS}(h)]_{42}^* + \epsilon_0 [A_{RS}(h)]_{41}^* = \left[\frac{\tau_{RS}^*(h)}{\hat{w}_0/c_R}\right] \quad (3.69)$$

It can be shown from (3.64) that $A(h)$ is the ellipticity that a receiver at depth $z = h$ would measure for all source depths.

$$A(h) = \epsilon(z = h) \quad A(0) = \epsilon(0) = \epsilon_0 \quad (3.70)$$

Since the ellipticity is independent of the source depth, we can write

$$\epsilon_0 = \frac{\partial N_{rr}(h)/\partial(h)}{\partial N_{zz}(h)/\partial(h)} = \frac{\partial N_{rz}(h)/\partial(h)}{\partial N_{zz}(h)/\partial(h)} \quad (3.71)$$

For a surface source, $h = 0$, the stress ratios defined by equations (3.67) to (3.69) vanish, and we obtain

$$B(0) = \epsilon_0 \left(3 - 4\frac{\beta^2_2}{\alpha^2_0}\right) = \epsilon_0 \left(\frac{1 + \sigma_0}{1 - \sigma_0}\right); \quad C(0) = G(0) \equiv 0 \quad (3.72)$$
3.7.3 Radiation Pattern in function of the moment tensor elements

The seismic source can be represented by different force systems. Whatever type of representation comes chosen, it is however always possible, to consider the motion of the ground recorded in a station like the convolution:

1) Of the function that represents the seismic source;
2) Of the function that represents the response of the model to an elementary source.
3) Of the function that represents the instrument response (transfer function).

To describe the displacement field, this is the ground displacement, in all point, it is suitable to consider two coordinates systems. The first system is cylindrical \((r, \theta, z = 0)\), with the origin in the recording station, the axes forming a right-handed system and orientated in the radial direction \(r\) (epicenter-station direction), transversal \((\theta)\), perpendicular to radial and coplanar with it, and \(z\) axis in the downward vertical direction. The second system is right-handed cartesian system with origin in the epicenter and the axes orientated to the North \((x\) axis), East \((y\) axis) and \(z\) axis in the downward vertical direction. (See Figure 3.5).

The \(\theta\) (Azimuth) is the angle between the radial axis of the cylindrical system and the \(x\) axis (North) of the cartesian system.

We can write the unit vectors of the cylindrical system in the cartesian system components:

\[
\begin{align*}
e_r &= (\cos \theta, \sin \theta, 0) \\
e_\theta &= (-\sin \theta, \cos \theta, 0) \\
e_z &= (0, 0, 1)
\end{align*}
\]  

The generic displacement field can be calculated like a linear combination of three vectors that describe the ground displacements generated by elementary sources of unit module and a duration represented by the Dirac delta function, applied in the three fundamental directions of the system of reference adopted.

It is observed that the action of a unit force in a direction, produce a vector displacement in the same direction if the structure in which they propagate themselves is homogeneous.

We will now show to construct the far displacement field for an arbitrary force system. Consider how an orthogonal coordinate system and let a unit
force \( \mathbf{R} \) be applied to the origin of this system. The components of \( \mathbf{R} \) in the cartesian system are:

\[
R_x = R \sin \theta_R \cos \varphi_R \\
R_y = R \sin \theta_R \sin \varphi_R \\
R_z = R \cos \theta_R
\]  \hspace{1cm} (3.74)

where \( \theta_R \) is the angle between \( \mathbf{R} \) vector and the vertical (plunge), \( \varphi_R \) is the angle between \( \mathbf{R} \) vector and North axis (azimuth, measured counterclockwise).

The components of the unit vector \( \mathbf{n} \) (couple arm) in the cylindrical system are:

\[
n_r = \sin \theta_n \cos \varphi_n \cos \theta + \sin \theta_n \cos \varphi_n \sin \theta \\
n_\theta = -\sin \theta_n \cos \varphi_n \sin \theta + \sin \theta_n \sin \varphi_n \cos \theta \\
n_z = \cos \theta_n
\]  \hspace{1cm} (3.75)

Take into account (3.37) to (3.39), (3.36) and performing the operations indicated by (3.55), we obtain:

\[
U^C_r = n_r (ik_r) R \left[ \sin \theta_R N_{rr} e^{-i\pi/4} (\cos \varphi_R \cos \theta + \sin \varphi_R \sin \theta) + \cos \theta_R N_{rz} e^{i\pi/4} \right] \\
- n_z R \left[ \sin \theta_R \frac{\partial N_{rr}}{\partial z} e^{-i\pi/4} (\cos \varphi_R \cos \theta + \sin \varphi_R \sin \theta) + \cos \theta_R \frac{\partial N_{rz}}{\partial z} e^{i\pi/4} \right] \]  \hspace{1cm} (3.76)

\[
U^C_\theta = n_r (ik_\theta) R \left[ \sin \theta_R N_{\theta\theta} e^{i3\pi/4} (\cos \varphi_R \sin \theta - \sin \varphi_R \cos \theta) \right] \\
- n_z R \left[ \sin \theta_R (\cos \varphi_R \sin \theta - \sin \varphi_R \cos \theta) \frac{\partial N_{\theta\theta}}{\partial z} e^{i3\pi/4} \right] \]  \hspace{1cm} (3.77)

\[
U^C_z = n_r (ik_z) R \left[ \sin \theta_R N_{zz} e^{-i3\pi/4} (\cos \varphi_R \cos \theta + \sin \varphi_R \sin \theta) + \cos \theta_R N_{rz} e^{-i\pi/4} \right] \\
- n_z R \left[ \sin \theta_R \frac{\partial N_{zz}}{\partial z} e^{-i3\pi/4} (\cos \varphi_R \cos \theta + \sin \varphi_R \sin \theta) + \cos \theta_R \frac{\partial N_{rz}}{\partial z} e^{-i\pi/4} \right] \]  \hspace{1cm} (3.78)

**Radial component:**

We consider the equation (3.76) and when the seismic source is represented by the element of the moment tensor \( M_{11} \), the couple and the arm have the same orientation in the cartesian system, the \( x \) direction in this case. The azimuth and the plunge angles associate to the \( M_{11} \) and the unit vector \( \mathbf{n} \) are:

\( \varphi_R = 0, \ \theta_R = \pi/2 \Rightarrow \cos \varphi_R = 1, \ \sin \varphi_R = 0; \ \cos \theta_R = 0, \ \sin \theta_R = 1 \)

\( \varphi_n = 0, \ \theta_n = \pi/2 \Rightarrow \cos \varphi_n = 1, \ \sin \varphi_n = 0; \ \cos \theta_n = 0, \ \sin \theta_n = 1 \)
and substituting these values in the equations (3.74) and (3.75) we obtain:

\[ R_x = R, \quad R_y = 0, \quad R_z = 0 \quad \text{and} \quad n_r = \cos \theta, \quad n_\varphi = -\sin \theta, \quad n_z = 0, \]

The radial component of the far-displacement field due to the \( M_{11} \) can be written:

\[ U_r^{M_{11}} = (ik_r) e^{-i\pi/4} N_{rr} \cos^2 \theta \]

Applying the same procedure for other components of the moment tensor, we obtain:

\[
\begin{align*}
n_r &= \cos \theta \quad \Rightarrow \quad U_r^{M_{11}} &= (ik_r) e^{-i\pi/4} N_{rr} \cos^2 \theta \\
n_r &= \sin \theta \quad \Rightarrow \quad U_r^{M_{22}} &= (ik_r) e^{-i\pi/4} N_{rr} \sin^2 \theta \\
n_z &= 1 \quad \Rightarrow \quad U_r^{M_{33}} &= -e^{i\pi/4} \frac{\partial N_{rz}}{\partial z} \\
n_r &= \sin \theta \quad \Rightarrow \quad U_r^{M_{12}} &= (ik_r) e^{-i\pi/4} N_{rr} \sin \theta \cos \theta \\
n_r &= \cos \theta \quad \Rightarrow \quad U_r^{M_{21}} &= (ik_r) e^{-i\pi/4} N_{rr} \sin \theta \cos \theta \\
n_z &= 1 \quad \Rightarrow \quad U_r^{M_{13}} &= -e^{-i\pi/4} \frac{\partial N_{rr}}{\partial z} \cos \theta \\
n_r &= \cos \theta \quad \Rightarrow \quad U_r^{M_{31}} &= (ik_r) e^{i\pi/4} N_{rz} \cos \theta \\
n_z &= 1 \quad \Rightarrow \quad U_r^{M_{23}} &= -e^{-i\pi/4} \frac{\partial N_{rr}}{\partial z} \sin \theta \\
n_r &= \sin \theta \quad \Rightarrow \quad U_r^{M_{32}} &= (ik_r) e^{i\pi/4} N_{rz} \sin \theta 
\end{align*}
\]

The generic seismic source is a superposition of the nine components of the moment tensor, and considering that \( M_{ij} = M_{ji} \), the radial component of the displacement field can be now expressed:

\[
U_r = -Rk_r e^{-i3\pi/4} \left\{ N_{rr} \left( M_{11} \cos^2 \theta + M_{22} \sin^2 \theta + M_{33} \frac{\partial N_{rz}}{k_r N_{rr} \partial z} + M_{12} \sin 2\theta \right) \\
+ i k_r \left[ \left( N_{rz} + \frac{\partial N_{rr}}{k_r \partial z} \right) M_{13} \cos \theta + \left( N_{rz} + \frac{\partial N_{rr}}{k_r \partial z} \right) M_{23} \sin \theta \right] \right\} 
\]

(3.79)

and using the following trigonometric relations

\[
\begin{align*}
2 \cos^2 \theta &= 1 + \cos 2\theta \\
2 \sin^2 \theta &= 1 - \cos 2\theta
\end{align*}
\]

(3.80)

(3.79) can be rewritten

\[
U_r = Rk_r e^{-i3\pi/4} N_{rz} \left\{ -\frac{1}{2} \frac{N_{rr}}{N_{rz}} (M_{11} + M_{22}) + M_{33} \frac{1}{k_r N_{rz}} \frac{\partial N_{rz}}{\partial z} - \right\}
\]

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\[ M_{12} \frac{N_{rr}}{N_{rz}} \sin 2\theta - \frac{1}{2} (M_{11} - M_{22}) \cos 2\theta - \] 
\[ i \left( M_{13} \left( \frac{N_{rz}}{N_{rz}} + \frac{1}{k_r N_{rz}} \frac{\partial N_{rr}}{\partial z} \right) \cos \theta + M_{23} \left( \frac{N_{rz}}{N_{rz}} + \frac{1}{k_r N_{rz}} \frac{\partial N_{rr}}{\partial z} \right) \sin \theta \right) \] 
\[ \} \quad (3.81) \]

Confronting (3.57) with (3.81), we obtain the expressions for the five elements of radiation pattern \( d_i \) in function of six linear independent components of the seismic moment tensor

\[ d_0' = -\frac{1}{2} \left[ \frac{N_{rr}}{N_{rz}} (M_{11} + M_{22}) - 2M_{33} \frac{1}{k_r N_{rz}} \frac{\partial N_{rr}}{\partial z} \right] \]
\[ d_1' = -M_{23} \left( \frac{N_{rz}}{N_{rz}} + \frac{1}{k_r} \frac{\partial N_{rr}}{\partial z} \right) \frac{1}{N_{rz}} \]
\[ d_2' = -M_{13} \left( \frac{N_{rz}}{N_{rz}} + \frac{1}{k_r} \frac{\partial N_{rr}}{\partial z} \right) \frac{1}{N_{rz}} \]
\[ d_3' = -M_{12} \frac{N_{rr}}{N_{rz}} \]
\[ d_4' = -\frac{1}{2} \frac{N_{rr}}{N_{rz}} (M_{11} - M_{22}) \]  \( \) (3.82)

Considering (3.59) to (3.61) we can write (3.82) with a same criterion to that used in Ben-Menahem and Harkrider (1964)

\[ d_0^r = -\frac{1}{2} (M_{11} + M_{22}) A(h) + \frac{1}{2} [B(h) - A(h)] M_{33} \]
\[ d_1^r = -M_{23} C(h) \]
\[ d_2^r = -M_{13} C(h) \]
\[ d_3^r = -M_{12} A(h) \]
\[ d_4^r = -\frac{1}{2} (M_{11} - M_{22}) A(h) \]  \( \) (3.83)

**Transversal component:**

We consider the equation (3.77) and when the seismic source is represented by the element of the moment tensor \( M_{12} \) the couple is parallel to \( x \) direction and the couple arm is parallel to \( y \) direction. The azimuth and the plunge angles associate to the \( M_{12} \) and the unit vector \( \mathbf{n} \) are:

\[ \varphi_R = 0, \; \theta_R = \pi/2 \Rightarrow \cos \varphi_R = 1, \; \sin \varphi_R = 0; \cos \theta_R = 0, \sin \theta_R = 1 \]
\[ \varphi_n = \pi/2, \; \theta_n = \pi/2 \Rightarrow \cos \varphi_n = 0, \; \sin \varphi_n = 1; \cos \theta_n = 0, \sin \theta_n = 1 \]

and substituting these values in the equations (3.74) and (3.75) we obtain:
\( R_x = R, \ R_y = 0, \ R_z = 0 \) and \( n_r = \sin \theta, \ n_\phi = \cos \theta, \ n_z = 0, \)

The transversal component of the far-displacement field due to the \( M_{12} \) can be written:

\[ U_{\varphi}^{M_{12}} = -(ik_\varphi)e^{-i\pi/4}N_\varphi \sin^2 \theta \]

Applying the same procedure for other components of the moment tensor, we obtain:

\[
\begin{align*}
n_r = \cos \theta & \Rightarrow U_{\varphi}^{M_{11}} = (ik_\varphi)e^{i3\pi/4}N_\varphi \sin \theta \cos \theta \\
n_r = \sin \theta & \Rightarrow U_{\varphi}^{M_{22}} = -(ik_\varphi)e^{i3\pi/4}N_\varphi \sin \theta \cos \theta \\
n_z = 1 & \Rightarrow U_{\varphi}^{M_{33}} = 0 \\
n_r = \sin \theta & \Rightarrow U_{\varphi}^{M_{12}} = (ik_\varphi)e^{i3\pi/4}N_\varphi \sin^2 \theta \\
n_r = \cos \theta & \Rightarrow U_{\varphi}^{M_{21}} = -(ik_\varphi)e^{i3\pi/4}N_\varphi \cos^2 \theta \\
n_z = 1 & \Rightarrow U_{\varphi}^{M_{32}} = -e^{i3\pi/4} \frac{\partial N_\varphi}{\partial z} \sin \theta \\
n_r = \cos \theta & \Rightarrow U_{\varphi}^{M_{13}} = 0 \\
n_z = 1 & \Rightarrow U_{\varphi}^{M_{23}} = e^{i3\pi/4} \frac{\partial N_\varphi}{\partial z} \cos \theta \\
n_r = \sin \theta & \Rightarrow U_{\varphi}^{M_{31}} = 0
\end{align*}
\]

The generic seismic source is a superposition of the nine components of the moment tensor, and considering that \( M_{ij} = M_{ji} \), the transversal component of the displacement field can be now expressed:

\[ U_\varphi = Rk_\varphi N_\varphi e^{-i3\pi/4} \left\{ M_{11} \sin \theta \cos \theta - M_{22} \sin \theta \cos \theta + M_{12} \sin^2 \theta - M_{21} \cos^2 \theta \\
+ i \left( \frac{1}{k_\varphi N_\varphi} \frac{\partial N_\varphi}{\partial z} \right) M_{13} \sin \theta - i \left( \frac{1}{k_\varphi N_\varphi} \frac{\partial N_\varphi}{\partial z} \right) M_{23} \cos \theta \right\} \quad (3.84) \]

and using the trigonometric relations given by (3.80)

\[ U_\varphi = Rk_\varphi N_\varphi e^{-i3\pi/4} \left\{ \frac{1}{2} (M_{11} - M_{22}) \sin 2\theta - M_{12} \cos 2\theta \\
+ i \left( \frac{1}{k_\varphi N_\varphi} \frac{\partial N_\varphi}{\partial z} \right) M_{13} \sin \theta - i \left( \frac{1}{k_\varphi N_\varphi} \frac{\partial N_\varphi}{\partial z} \right) M_{23} \cos \theta \right\} \quad (3.85) \]

Confronting (3.57) with (3.85), we obtain the expressions for the five elements of radiation pattern \( d_i \) in function of six linear independent components of the
seismic moment tensor

\[ d_0^\varphi = 0 \]
\[ d_1^\varphi = M_{13} \left( \frac{1}{k_\phi} N_\varphi \frac{\partial N_\phi}{\partial z} \right) \]
\[ d_2^\varphi = -M_{23} \left( \frac{1}{k_\phi} N_\varphi \frac{\partial N_\phi}{\partial z} \right) \]
\[ d_3^\varphi = \frac{1}{2} (M_{11} - M_{22}) \]
\[ d_4^\varphi = -M_{12} \]

(3.86)

Considering (3.62) we can write (3.86) with a same criterion to that used in Ben-Menahem and Harkrider (1964)

\[ d_0^\varphi = 0 \]
\[ d_1^\varphi = M_{13} G(h) \]
\[ d_2^\varphi = -M_{23} G(h) \]
\[ d_3^\varphi = \frac{1}{2} (M_{11} - M_{22}) \]
\[ d_4^\varphi = -M_{12} \]

(3.87)

**Vertical component:** Following the same steps for the other components, we consider the equation (3.78) and when the seismic source is represented by the element of the moment tensor $M_{13}$ the couple is parallel to $x$ direction and the couple arm is parallel to $z$ direction. The azimuth and the plunge angles associate to the $M_{13}$ and the unit vector $n$ are:

$\varphi_R = 0, \theta_R = \pi/2 \Rightarrow \cos \varphi_R = 1, \sin \varphi_R = 0; \cos \theta_R = 0, \sin \theta_R = 1$

$\theta_n = \pi/2 \Rightarrow \cos \theta_n = 0, \sin \theta_n = 1$

and substituting these values in the equations (3.74) and (3.75) we obtain:

$R_x = R, R_y = 0, R_z = 0$ and $n_x = 0, n_\phi = 0, n_z = 1,$

The vertical component of the far-displacement field due to the $M_{13}$ can be written:

\[ U_z^{M_{13}} = -e^{-i3\pi/4} \frac{\partial N_{rz}}{\partial z} \cos \theta \]
Applying the same procedure for other components of the moment tensor, we obtain:

\[ n_r = \cos \theta \Rightarrow U_z^{M1} = (ik_z)e^{-i\frac{3\pi}{4}}N_{zr} \cos^2 \theta \]
\[ n_r = \sin \theta \Rightarrow U_z^{M2} = (ik_z)e^{-i\frac{3\pi}{4}}N_{zr} \sin^2 \theta \]
\[ n_z = 1 \Rightarrow U_z^{M3} = i e^{-i\frac{3\pi}{4}} \frac{\partial N_{zz}}{\partial z} \]
\[ n_r = \sin \theta \Rightarrow U_z^{M4} = (ik_z)e^{-i\frac{3\pi}{4}}N_{zr} \sin \theta \cos \theta \]
\[ n_r = \cos \theta \Rightarrow U_z^{M5} = (ik_z)e^{-i\frac{3\pi}{4}}N_{zr} \sin \theta \cos \theta \]
\[ n_z = 1 \Rightarrow U_z^{M6} = -e^{-i\frac{3\pi}{4}} \frac{\partial N_{rz}}{\partial z} \sin \theta \]

The generic seismic source is a superposition of the nine components of the moment tensor, and considering that \( M_{ij} = M_{ji} \) and \( i = e^{i\pi/2} \), the vertical component of the displacement field can be now expressed:

\[ U_z = R k_z N_{zz} e^{-i\frac{3\pi}{4}} e^{i\pi/2} \left\{ M_{11} \frac{N_{zr}}{N_{zz}} \cos^2 \theta + M_{22} \frac{N_{zr}}{N_{zz}} \sin^2 \theta + M_{33} \frac{1}{k_z N_{zz}} \frac{\partial N_{zz}}{\partial z} + 2M_{12} \frac{N_{zr}}{N_{zz}} \sin \theta \cos \theta + \right. \]
\[ \left. i \left[ M_{13} \left( \frac{1}{k_z N_{zz}} \frac{\partial N_{zr}}{\partial z} \right) \cos \theta + M_{23} \left( \frac{1}{k_z N_{zz}} \frac{\partial N_{zr}}{\partial z} \right) \sin \theta \right] \right\} \] (3.88)

and using the trigonometric relations given by (3.80)

\[ U_z = R k_z N_{zz} e^{-i\frac{3\pi}{4}} e^{i\pi/2} \left\{ \frac{1}{2} \frac{N_{zr}}{N_{zz}} (M_{11} + M_{22}) + M_{33} \frac{1}{k_z N_{zz}} \frac{\partial N_{zz}}{\partial z} + \right. \]
\[ \left. M_{12} \frac{N_{zr}}{N_{zz}} \sin 2\theta + \frac{1}{2} \frac{N_{zr}}{N_{zz}} (M_{11} - M_{22}) \cos 2\theta + \right. \]
\[ \left. i \left[ M_{13} \left( \frac{N_{zz}}{N_{zz}} + \frac{1}{k_z N_{zz}} \frac{\partial N_{zr}}{\partial z} \right) \cos \theta + M_{23} \left( \frac{N_{zz}}{N_{zz}} + \frac{1}{k_z N_{zz}} \frac{\partial N_{zr}}{\partial z} \right) \sin \theta \right] \right\} \] (3.89)

Confronting (3.57) with (3.89) and taking into account (3.58), we obtain the expressions for the five elements of the radiation pattern \( d_i \) in function of the
six linear independent components of the seismic moment tensor

\[
\begin{align*}
\delta^0_0 &= e^{i\pi/2} \left\{ \frac{1}{2} \left[ \frac{N_{zr}}{N_{zz}} (M_{11} + M_{22}) + 2M_{33} \frac{1}{k_z} \frac{1}{N_{zz}} \frac{\partial N_{zr}}{\partial z} \right] \right\} \\
\delta^1_1 &= e^{i\pi/2} \left\{ \frac{1}{N_{zz}} \left( N_{zz} + \frac{1}{k_z} \frac{\partial N_{zr}}{\partial z} \right) M_{23} \right\} \\
\delta^2_2 &= e^{i\pi/2} \left\{ \frac{1}{N_{zz}} \left( N_{zz} + \frac{1}{k_z} \frac{\partial N_{zr}}{\partial z} \right) M_{13} \right\} \\
\delta^3_3 &= e^{i\pi/2} \frac{N_{zr}}{N_{zz}} M_{12} \\
\delta^4_4 &= e^{i\pi/2} \left\{ \frac{1}{2} \frac{N_{zr}}{N_{zz}} (M_{11} - M_{22}) \right\}
\end{align*}
\]

(3.90)

Considering (3.53) and (3.71) rewritten the equations (3.59), (3.60) and (3.61)

\[
\begin{align*}
A(h) &= \frac{N_{rr}}{N_{rz}} = \frac{\epsilon_0 N_{zr}}{\epsilon_0 N_{zz}} = \frac{N_{zr}}{N_{zz}} \\
B(h) &= \frac{1}{N_{rz}} \left( N_{rr} + \frac{2}{k_r} \frac{\partial N_{zr}}{\partial z} \right) = \frac{1}{N_{zz}} \left( N_{zr} + \frac{2}{k_r} \frac{\partial N_{zz}}{\partial z} \right) \\
C(h) &= \frac{1}{N_{rz}} \left( N_{rz} + \frac{1}{k_r} \frac{\partial N_{rr}}{\partial z} \right) = \frac{1}{N_{zz}} \left( N_{zz} + \frac{1}{k_r} \frac{\partial N_{zr}}{\partial z} \right)
\end{align*}
\]

Now we can write (3.90) with a same criterion to that used in Ben-Menahem and Harkrider (1964)

\[
\begin{align*}
\delta^0_0 &= e^{i\pi/2} \left\{ \frac{1}{2} (M_{11} + M_{22}) A(h) - \frac{1}{2} [B(h) - A(h)] M_{33} \right\} \\
\delta^1_1 &= e^{i\pi/2} M_{23} C(h) \\
\delta^2_2 &= e^{i\pi/2} M_{13} C(h) \\
\delta^3_3 &= e^{i\pi/2} M_{12} A(h) \\
\delta^4_4 &= e^{i\pi/2} \left\{ \frac{1}{2} (M_{11} - M_{22}) A(h) \right\}
\end{align*}
\]

(3.91)

We can see that the displacement field radial and vertical are differentiated in amplitude by a scale factor ($\epsilon_0$) and in phase by $e^{i\pi/2}$.

### 3.7.4 General properties of the radiation patterns

**Symmetry relations.** It is useful to look for symmetry with respect to the variable $\theta$ and the parameters $\lambda$ and $\delta$ in order to minimize the tabulation of the
azimuthal distribution of the modulus $|\chi(\theta)|$ and the spatial phase $\chi(\theta)$. We denote by $m_R(\theta)e^{i\phi_R}$ and $m_L(\theta)e^{i\phi_L}$ the respective polar representations of the radiation patterns for Rayleigh and Love waves. It then follows directly from (3.57), (3.58), Table (3.2) and Table (3.3) that

$$U(\theta + \pi; \lambda, \delta, \omega, h) = \overline{U}(\theta; \lambda, \delta, \omega, h) \tag{3.92}$$

for all the couple and double couple displacements.

Equation (3.92) can also be written as

$$m_{R,L}(\theta + \pi) = m_{R,L}(\theta)$$

$$\phi_R(\theta + \pi) + \phi_R(\theta) = \phi_L(\theta + \pi) + \phi_L(\theta) = \pi/2 \tag{3.93}$$

The relations given in (3.93) are due to Aki (1964) and are useful for checking the calculations. It is assumed here that the source time dependence is a Dirac delta function. The antisymmetry between right-reverse and left-normal faultings yields

$$U(\lambda + \pi) = -U(\lambda) \tag{3.94}$$

$$U(-\delta) = -U(\delta) \tag{3.95}$$

$$U_r(\pi - \lambda, \pi - \theta) = U_r(\lambda, \theta) \tag{3.96}$$

$$U_\theta(\pi - \lambda, \pi - \theta) = -U_\theta(\lambda, \theta) \tag{3.97}$$

which can be established from the nature of the coefficients in Tables (3.2) and (3.3). The foregoing relations show that it is sufficient to know the radiation pattern for $0 \leq \theta \leq \pi$, $0 \leq \delta \leq \pi/2$, and $0 \leq \lambda \leq \pi/2$. The values for the supplementary angles can be obtained from these by reflections.

**Dependence of patterns on the source elements.** The modulus $|\chi(\theta)|$ describes in the general case an obliquely symmetric, closed curve of the sixth order, otherwise known as the hypotrochoid. Its shape is governed by the coefficients $d_i$ (Tables 3.2 and 3.3) which in turn are composite functions of the source constants $(\lambda, \delta, h)$, the period $T$, and the physical constants of the Earth model.
3.8 Moment Tensor Inversion

Many approaches have been proposed to invert waveforms with the aim of studying the earthquake source. The inversion can be done in the time or frequency domain, and use different type of data (body waves, surface waves, free oscillations, dominant part of the records, etc.).

The application of waveform inversions to the study of earthquake source can be helpful and preferable to standard first arrivals techniques in case of seismic signal contaminated by strong noise. A high level of noise is typical of oceanic environments. In this case, the onsets may be significantly distorted or completely hidden in the noise.

Šilený et al., 1992) proposed a method to retrieve the complete seismic moment tensor by waveform inversion of the dominant part of the records. The method named INPAR (INdirect PARameterisation) determines simultaneously all moment tensor components and the source time function by comparing synthetic and observed seismograms recorded in a seismic network.

The method is based on the assumption that the seismic focus is described, in the point source approximation, by 6 independent components of the moment tensor \( M_{ij} \), \( i = 1...3, j = 1...3 \). Considering (3.16) we can see that the \( k \)-th component of the displacement of the radiated seismic waves can be expressed as the convolution of the moment tensor with the space derivatives of the Green’s functions \( G_{ki,j} \)

\[
  u_k(t) = \sum_{i,j=1}^{3} M_{ij}(t) * G_{ki,j}(t) \tag{3.98}
\]

The \( G_{ki,j} \) are the responses of the medium to sources represented by elementary single force, with the time dependence given by a δ-function. For a seismic source it is reasonable to consider an increasing time function \( M_{ij}(t) \) with a constant asymptote, i.e., the time derivatives of this function are non-zero in a limited interval only. The time derivatives of the moment tensor components, \( \dot{M}_{ij}(t) \), are thus more convenient for the parameterisation than the functions \( M_{ij}(t) \), therefore it is advantageous to write

\[
  u_k(t) = \sum_{i,j=1}^{3} \dot{M}_{ij}(t) * H_{ki,j}(t) \tag{3.99}
\]

where \( H_{ki,j}(t) \) are the responses of the medium to sources represented by elementary single force, with the time dependence given by a Heaviside function.
These functions $H_{ki,j}(t)$ are called by Šilený and Panza (1991) "base functions" and can be conveniently computed using the modal summation method (Panza, 1985; Florsch et al., 1991; Panza et al., 2001).

Introducing the notation
\[
\dot{M}_{ij}(t) \to F_m \quad i, j = 1...6
\]
\[
H_{ki,j}(t) \to \Phi_{km} \quad i, j, k = 1...6
\]

Then (3.99) can be rewritten as
\[
u_k(t) = \sum_{m=1}^{6} F_m(t) \ast \Phi_{km}(t)
\tag{3.100}
\]

The derivatives of the moment tensor components are parameterised by means of overlapping triangles delayed in time (Nabelek, 1984). Each triangle of half width $\Delta \tau$ and of unitary height, with time delay equal to $\Delta \tau$, is obtained as a convolution of two box functions of length $\Delta \tau$
\[
T_{\Delta \tau}(t) = \frac{1}{\Delta \tau} B_{\Delta \tau}(t) \ast B_{\Delta \tau}(t)
\tag{3.101}
\]

where
\[
\begin{cases}
B_{\Delta \tau}(t) = 1 & \text{for } 0 \leq t \leq \Delta \tau \\
B_{\Delta \tau}(t) = 0 & \text{elsewhere}
\end{cases}
\]

Then
\[
F_m(t) = \sum_{n=1}^{N_t} F_{nm} T_{\Delta \tau}[t - (n - 1)\Delta \tau]
\tag{3.102}
\]

where $F_{nm}$ is the weight of the $n$-th triangle in the parameterisation of the $m$-th moment tensor component and $N_t$ the number of triangles. Then (3.100) can be written
\[
u_k(t) = \sum_{n=1}^{6} \sum_{m=1}^{N_t} F_{nm} T_{\Delta \tau}[t - (n - 1)\Delta \tau] \ast \Phi_{km}(t)
\tag{3.103}
\]

Introducing an index $p = m + 6(n - 1)$, (3.103) becomes
\[
u_k(t) = \sum_{p=1}^{6N_t} w_p A_{kp}(t)
\tag{3.104}
\]

where $w_p$ correspond to $F_{nm}$, and $A_{kp}(t)$ corresponds to $T_{\Delta \tau}[t - (n - 1)\Delta \tau] \ast \Phi_{km}(t)$. Each set of weights $(F_{n1}...F_{n6})$ represents the geometrical part of the
moment tensor corresponding to the \( n \)-th triangle. Now, we can introduce the components of the observed records \( d_k(t) \); therefore the unknown weights are determined by solving the equations

\[
d_k(t) = \sum_{p=1}^{6N_t} w_p A_{kp}(t)
\] (3.105)

concatenated for all the components of all the stations used in the analysis. This system of linear equations can be written in the compact form

\[
A w = d
\]

where \( A \) is the matrix of the functions \( A_{kp}(t) \), \( d \) is the vector of the concatenated sampled observed records, and \( w \) contains the parameterisation weights

\[
w = (F_{11}, ..., F_{16}, F_{21}, ..., F_{26}, ..., F_{N_t,1}, ..., F_{N_t,6})
\]

The system of equations can be solved by the damped least squares method, therefore solving the normal equations

\[
(A^T A) w = A^T d
\]

which is a system of \( 6N_t \) equations for \( 6N_t \) unknown weights \( w_p \). The matrix \( A^T A \) has the special form of a 'block-Toeplitz' matrix and it is composed of \( N_t(6x6) \) sub matrices; for that reason the system of equations can be solved by means of recursive techniques. Once the weights \( w_p \) have been determined, the synthetic seismograms (3.104) are computed and matched with the observed records. The measure of the similarity between synthetic and observed seismograms is defined with the \( L_2 \)-norm of the difference. For each station it is possible to multiply the equations (3.105) by different weights, in order to enhance stations with small, but reliable, recorded amplitudes or to decrease the weight of the signals with low quality.

Differences in source depth influence the relative excitation of normal modes, causing systematic errors in the inversion. Systematic errors in the inversion are also due to deviations of the Earth model from the actual properties of the Earth, affecting the synthetic Green’s functions. The base functions are thus computed (e.g. using the modal summation method) from a set of values of the source depth lying between two extremes, and two structural models A and B, assumed to represent the range of variability of possible models of the study.
area. Indicating the different values of the source depth with the variable $X$ and representing the different structural models with the parameter $Y(0 \leq Y \leq 1)$, then the structure A used to compute the base functions corresponds to $Y = 0$, while the structure B corresponds to $Y = 1$. The structural models corresponding to the values of the variable $Y$ in the range $[0,1]$ are more similar to A if $Y < 0.5$, and more similar to B if $Y > 0.5$. During the inversion intermediate values of the parameters $X$ and $Y$ are computed, incrementing the initial values with steps chosen a priori. The base functions corresponding to intermediate values of depth and structure are computed with linear interpolation of the base functions at the assumed set of depths and structures. The difference between the observed records and the synthetic seismograms, corresponding to a given source depth and structural model, is computed using a $L_2$-norm. Therefore the norm can be considered as a function of the parameters $X$ and $Y$, and the values of $X$ and $Y$ are searched that minimize the norm.

The first step of the inversion is a linear inversion and the six moment tensor rate functions (MTRF) $\dot{M}_{ij}(t) (i,j = 1,2,3)$, are retrieved. The MTRF are six linearly independent functions of time, but if, in the second inversion step, we assume that the time dependence is the same for all the six components, i.e. the mechanism of the source does not change during the focal process, we can then write

$$\dot{M}_{ij}(t) = \dot{m}_{ij} f(t)$$

where $f(t)$ is a function called the source time function, which contains information about the energy release in time at the source and the $\dot{m}_{ij}$ describe an average source mechanism. The second step of the inversion is then a non-linear one, and the retrieved MTRFs, describing a source mechanism varying in time, are reduced to a constant moment tensor and the corresponding source time function, taking only the correlated part from each MTRF. The full moment tensor can be decomposed into a volumetric part (V) representing volume changes, a CLVD, and a double couple (DC) part due to dislocation movements.

The reliability of the solution is estimated with a modified inversion procedure that makes use of the genetic algorithm (Šilený, 1998). The genetic algorithm is applied to find the point where the misfit function is a minimum and at the same time to map the model space in its vicinity with the aim of estimating the confidence regions of the model parameters.

To retrieve information about the error of the solution we use the posterior probability density function to mark confidence zones of the model parameters. From the size and shape of the confidence areas we can decide about the
reliability of the solution.

The MTRFs retrieved from the waveform inversion, and then the average mechanism and source time function, are considered to be affected by three types of errors, generated respectively by the following: (1) the noise present in the data; (2) the mislocation of the hypocenter adopted to compute the Green’s function in the depth grid used in the inversion; (3) the improper structural models used to construct the Green’s functions (Šilený et al., 1996).

3.9 Features of the INPAR method

The INPAR method has been applied in different tectonic scenarios, several synthetics tests have been performed in order to test the reliability and the applicability of the method. The method has successfully been applied within tectonic (Campus et al., 1996; Radulian et al., 1996; Vuan et al., 2001; Chimera et al., 2003), volcanic, and geothermal environments (Campus et al., 1993; Cesspuglio et al., 1996; Guidarelli et al., 2000; Kravanja et al., 2000; Saraó et al., 2001; Guidarelli et al., 2003). Campus and Fäh (1997) have confirmed the capability of the method to retrieve reliable isotropic components and to distinguish between tectonic earthquakes and man-made explosions.

Šilený and Panza (1991) and Kravanja et al., (1999) showed that the inversion of low-pass filtered data does not depend on the detail of the structural model used, therefore, if we can constrain our data to low frequencies, the wavelength is large compared with the details of the structure which may be poorly known or roughly modeled, and the recovered source parameters are not distorted significantly. The good fit between ‘observed’ and synthetic seismograms constructed for an improper structural model is paid by complexities in the source time function that do not reflect real source characteristics. However, the length of the source time function seems to be less affected, thus providing a good estimate of the source process duration. Beside that, the effect due to the use of records computed for a structure not contained in the interpolation range of the base functions are mainly expressed by an apparent volumetric component of the source.

Kravanja et al., (1999) considered that the INPAR inversion method takes into account the fact that the structural model might not be appropriate. Thanks to the ‘overparameterization’ of the rupture process, by means of independent MTRFs and their subsequent reduction to a positive source time function, it has the capacity to absorb in the MTRFs artifacts introduced by
the inappropriate modeling of the medium, and then to minimize them during the factorisation of the MTRFs.

The second step of the inversion reduces the six independent MTRFs, \( \dot{M}_{ij}(t) \), into a constant moment tensor and a source time function. This is a non-linear inverse problem that is solved iteratively and in which constraints are imposed on the source parameters to be determined. The theoretical values of the MTRFs, \( \dot{M}_{ij}(t) = \dot{m}_{ij} f(t) \), are compared with the ‘observed’ MTRFs; that is to those obtained as the output of the first step of the procedure. Thus, by introducing the MTRFs in the first step of the procedure, the problem of matching the seismograms is transformed into the problem of matching the MTRFs. The advantage is that there are always six MTRFs at most (or five in case of deviatoric sources), and their sampling can be modified with respect to the sampling of the observed seismograms, which allows us to reduce the system of equations for the searched source parameters. However the principal advantage of the method is the capacity of the MTRFs, which are ‘overparameterized’ in the first step, being considered as independent functions, to capture spurious signals originating from the deconvolution from the observed records of Green’s functions that only approximate reality (Kravanja et al., 1999). The effects of poor modelling of the structure are partly absorbed in the MTRFs, thanks to the high degree of freedom of the inversion in the first step, and then, in the second part of the inversion, reduced in the MTRFs by searching for their correlated part, with the constraint of the reduction of back slips in the source time function.

The inversion can be performed even for low signal to noise ratios, provided that the structural model is known in sufficient detail. The unresolved effects due to the noise are transformed mainly into the source time function, which prevents it from being extracted by the inversion. However, its length seems to be less affected, thus providing a good estimate of the source process duration. Also the source mechanism retrieved from the noisy data seems to be fairly insensitive to the details of the structural model used in the inversion (Šilený and Panza, 1991; Šilený et al., 1992).

The fit of synthetic seismograms to noisy data is quite nice for all experiments performed, even if the CLVD parts are not determined correctly (Šilený et al., 1992).

Šilený (1998) showed that the resolution of the source mechanism is quite high and the orientation of the double couple component is very stable, even in presence of noise. Šilený (1998) also demonstrated that the confidence regions
are smaller when the sampling of the source time function is finer.

The retrieval of the geometrical part of the MTRFs is dependent on the distribution of stations with respect to the hypocenter. The normal equations for the retrieval of the six components of the moment tensor allow their unique determination in the presence of at least six independent phases on the focal sphere. If only a single phase is recorded by each station, records from at least six stations must be taken into account; if we use three components stations, i.e., P, SV and SH phases are known, two stations are, in principle, capable of determining the complete moment tensor. However, the normal equations may be well- or ill-posed due to regular or sparse distribution of the stations on the focal sphere, even if their number is sufficient. Šilený (1996) performed several synthetic tests to investigate the dependence of the source mechanism resolution on the focal sphere coverage. Even if three stations (3-components) with a configuration of tightly clustered station projections on the focal sphere has, in principle, the resolving power to determine the geometrical part of the MTRFs within a 1 per cent of error.

Šilený et al. (1996) have shown that the solutions are robust until the noise in the data is less than 20% of the maximum amplitude and that the main features of the source process are retrieved even with a rough knowledge of the structural model. Vuan et al. (2001) proved the capability of the method to retrieve useful information even when using data from a limited number of stations.

It follows from such considerations that the INPAR inversion scheme represents a suitable method to analyse earthquakes in an environment such as the Scotia Sea region, where, (a) the high level of seismic noise, typical of the oceanic environment, makes it difficult to determine the fault plane solutions using standard techniques, based on first arrivals polarities, (b) the logistic does not allow the operation of a large number of stations, (c) the structural properties are not known with large detail. All this fully justify an intensive use of the methodology in the Scotia Sea region to study main regional earthquakes, and low magnitude, local events, as well.
3.10 Analysis of aftershocks sequence of the Centenary earthquake

3.10.1 Introduction

On August 4th, 2003 04:37:19 GMT a major earthquake shook the southeastern Scotia Sea along the South Scotia Ridge. Its magnitude was evaluated as 7.6 $M_w$ (Harvard Centroid Moment Tensor-CMT) and its epicentre located at Latitude: $-60.55$, Longitude: $-43.49$ (about 100 km East from the Argentinean Base Orcadas (Laurie Island, South Orkney Island) where the ASAIN station ORCD is operating since 1997 (Russo and Febrer, 2001).

It represents the strongest known event in the neighbouring of the South Orkney Microcontinent region. It has been nicknamed the "Orcadas Centenary Earthquake" because the Orcadas Base was founded exactly one hundred years before and has been continuously open since then.

Recordings of the main shock from ASAIN network are shown in Figure (3.6). Only the records of 1 sps at ORCD seismograph station are not clipped, while those of 20 sps are clipped, See Figures (3.7) and (3.8).

![Figure 3.6: 3-components records of Centenary Earthquake at JUBA station, ASAIN Network.](image-url)
Figure 3.7: *Centenary* Earthquake recorded at ORCD station, 1 sps (LH) channels are not clipped.

Figure 3.8: *Centenary* Earthquake recorded at ORCD station, 20 sps (BH) channels are clipped.
Immediately following the *Centenary* Earthquake a notable aftershock sequence started. Within three hours from the main shock about twenty events exceeding magnitude 4.0 occurred and this number equals the historical seismicity extracted of NEIC database for the period July 1974 to July 2003. (See Table 3.4). 24 hours later the number of events between magnitude 3.6 and 5.6 exceeded 50. See Table 3.5.

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Table 3.4: Historical seismicity by NEIC from July 1974 to July 2003.

In spite of the development of a permanent seismographic network in the Scotia Sea, the application of seismological methodologies to the study of the South Orkney region was not possible before the *Centenary* earthquake due to the lack of significant seismicity as evident from Figure 3.9. This gap in the seismicity of the South Orkney area was filled after the *Centenary* earthquake aftershock sequence and two years later a low level seismicity had yet remained.
3.10.2 Spatial and temporal distribution

The geographical distribution of the aftershocks with magnitude larger than 3.6, occurred from August-December 2003, covers an ellipsoidal area with the major axis extended for more than 150 km in the east-west direction along the main trend of the border between the Antarctic and the Scotia plates, and about 60 km in the north-south direction. A estimate of slip displacement for the Centenary Earthquake using the equation (3.33) and considering the scalar moment $Mo = 2.73 E + 27 $ dyne-cm from CMT-Harvard, shear modulus for the crust $\mu = 3.2 E + 11 $ dyne/cm$^2$ and a rupture area $150 \times 60 \text{ km}^2$, is about 1 m.

![Figure 3.9](image_url)

Figure 3.9: Historical seismicity in red squares, small red squares are events without magnitude and seismicity after the Centenary Earthquake in green circles from NEIC Catalogue. ORCD station in blue triangle.

From the analysis of the spatial and temporal distribution of aftershocks we can observe an antisymmetrical bilateral rupture that initiates in the middle and propagates to both ends of a fault, but with a predominance towards the west.

During an earthquake, the slip distribution, the speed of rupture front propagation and the segmentation of the fault contribute to change the distribution of the stress on a complex network of rough and irregular fractures. Aftershocks can be described as a part of a relaxation process of the zones of high stress...
Almost all crustal earthquakes, regardless of size, appear to be followed by a series of smaller shocks, or aftershocks. Aftershocks are usually strongly related to the source region of the main shock, but the spatial patterns within the aftershock region are quite varied for different sequences. In contrast, the time sequence of aftershocks is consistently similar for all sequences, suggesting that aftershock sequences may be produced by a well-defined gross process (Scholz, 1968).

Laboratory investigation of microfracturing in brittle rock has revealed that microfracturing events can be detected after brittle fracture of rock in compression, provided the specimen remains intact. If the sample isolated after fracture, microfracturing activity decays hyperbolically in a manner similar to typical earthquake aftershock sequence (Scholz, 1968).

Davis and Frohlich (1991), using single-link cluster analysis, they investigated how various properties of aftershock sequences depend on their tectonic regime and focal depth. For the International Seismological Centre (ISC) earthquakes of \( m_b \geq 4.8 \), they found that earthquakes deeper than 70 km have the fewest and smallest aftershock sequences. Even after accounting for differences in detectability and maximum magnitude, they found that ridge-transform earthquakes have smaller aftershock sequence than shallow subduction zone earthquakes. Among different subduction zones, they found that zones with high moment release rates possess larger aftershock sequences. Comparing ridge-transform zones, they found those with slower spreading rates possess larger aftershock sequence.

The temporal behavior of the aftershocks in the sixty days following this major earthquake is shown in Figure (3.10). The stress energy following a main shock is slowly released as a sequence of aftershocks. According to Omori (1894), the rate of aftershocks decays with time as \( 1/t \), where \( t \) is the time from the main shock. A generalization of Omori’s Law was proposed by Utsu (1961), in which the rate of aftershocks decays with time as \( 1/(t + t_0)^n \), where \( t_0 \) is a constant and \( n \) is the exponent. From this generalized Omori’s relation, the number of aftershocks, \( N \), occurring per unit time is proportional to \( 1/t^n \), when \( t \gg t_0 \). The aftershocks activity of the California earthquakes gave the value of \( n \) lying between 0.5 and 1.5 (Reasenberg and Jones, 1989), which is also observed in most of the aftershock activity in the rest of the world.
Figure 3.10: Number of aftershocks per hour after the *Centenary* Earthquake, observed at ORCD station, for sixty days following the main shock.

Dashed and dash-dotted traces in the Figure (3.11) indicate the hyperbolic fit, $77.58 \ t^{-1.1}$, and the exponential fit, $41.41 \ e^{-0.17t}$, respectively.

The superimposed oscillations present in the rate of decay of aftershocks are analytically extracted by subtracting the exponential and hyperbolic fits from the observed decay curve. Then we calculated the amplitude and phase spectra of the resulting curves. In order to find the nature of aftershocks it is necessary to examine the phase spectra, whether these oscillations correspond to random time series or deterministic series.

The result of this test is called $z$ score or test statistic value (Bradley, 1968). If $|z|$ score is greater than 1.96, then the data under consideration are not random at a confidence level of 95%. Using the test, the $z$ score of the phase spectra of hyperbolic and exponential cases are found to be $-7.52$ and $-6.97$ respectively. Since the $|z|\text{score}$ for the phase spectra are greater than 1.96, the phase spectra of two fits are not random at a confidence level of 95%. Hence the oscillations
Figure 3.11: Continuous trace shows the plot of rate of aftershocks of *Centenary* Earthquake, 2003, against time, dash trace shows the hyperbolic fit and dash-dotted shows the exponential fit.

present in the aftershocks activity are not random. (See Appendix A).

### 3.10.3 Moment tensor Inversion

For the study of the seismic source tensor, we used the waveforms provided by the Antarctic Seismograph Argentinean Italian Network (ASAIN) network. We also got waveforms from Incorporated Research Institutions for Seismology (IRIS) network database (http://www.iris.edu), for the Antarctic stations that belong to the IRIS/IDA (II) network, PMSA Palmer station and IRIS/USGS (IU) network, EFI Islas Malvinas and HOPE South Georgia Island stations.

We selected 7 events from the NEIC Catalogue, among those with magnitude greater than 4.3 $m_b$, which have sufficient signal to noise ratio. See Table 3.5.
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<td>-45.03</td>
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Continued on next page
Table 3.5 – Continued from previous page

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<th>LONG</th>
<th>DEPTH</th>
<th>MAGNITUDE</th>
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Table 3.5: List of events located by NEIC from August 4 to December 3, 2003

The August 4, 2003, Centenary Earthquake occurred on the boundary between the Scotia plate and the Antarctic plate. In the epicentral region, the Scotia Sea plate is moving to the west-northwest with respect to the Antarctic plate. The relative velocity between the two plates is not well determined but is likely to be about 1 cm/y. The overall boundary is a transform-fault boundary, involving predominantly strike-slip faulting, although prior normal-faulting earthquakes have also occurred. The focal mechanism for the main shock was obtained by the CMT-Harvard, the results are summarized below:

Date: 2003/08/04  Centroid Time: 04:37:42.5 GMT  
Lat: −60.80  Lon: −43.21  
Depth: 15.0  Half Duration: 14.7  
Moment Tensor: Expo:27  −1.040 1.550 −0.514 0.647 1.960 1.150  
Mw=7.6  mb=6.2  Ms=7.5  Scalar Moment=2.73 E+27  
Fault Plane: strike=101  dip=36  slip=−23  
Fault Plane: strike=210  dip=76  slip=−124

The earthquake focal mechanism for the main shock derived from teleseismic
observations indicates a normal faulting with nodal planes striking about east-west and dipping south. This is consistent with north-south contraction within the Scotia Plate. (See Figure 3.12).

The moment tensor inversion for the aftershocks is performed through the following processing steps. After data acquisition and pre-processing, linear trends are identified and removed. High frequency noise in the data is removed by low-pass filtering. We low-pass filter the data at the frequency of 0.07 Hz for all events. This filtering frequency was chosen because it enables us to make use of the point source approximation when studying the source in that frequency range. To consider the instrument effect we apply the instrument response to the synthetic Green’s functions and compare them with the observed data. We do not deconvolve the instrument response from the data. After mean removing, tapering and filtering, we select the temporal window of the seismograms to be inverted for the retrieval of the moment tensor components.

The synthetic Green’s functions are calculated using the structural models obtained by Vuan et al. (2000) from group velocity tomography (see Figure 3.13). As we are dealing with tectonic earthquakes, the inversion has been performed with the volumetric component constrained to zero. With this constraint, we reduce the number of degrees of freedom in the inversion to five, obtaining a gain in stability during the inversion step. The list of the events analyzed in this study is reported in Table 3.6. The locations and the origin time are taken from the USGS-PDE database. We selected the aftershocks with magnitude $M \geq 4.3$ in the period 08/04/2003 up to 03/12/2003. The results of the inversions are reported in Table 3.7, the moment magnitude, $M_w$, is computed.
Figure 3.13: The structural models used to compute the synthetic Green’s functions. The models are taken from Vuan et al., (2000).

<table>
<thead>
<tr>
<th>Event</th>
<th>Date</th>
<th>Origin Time</th>
<th>Latitude</th>
<th>Longitude</th>
<th>Mag</th>
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<td>-42.701</td>
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<td>18:18:29.65</td>
<td>-60.586</td>
<td>-43.042</td>
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</tr>
<tr>
<td>3</td>
<td>2003/08/06</td>
<td>17:01:54.43</td>
<td>-60.337</td>
<td>-45.033</td>
<td>5.1</td>
</tr>
<tr>
<td>4</td>
<td>2003/08/14</td>
<td>08:41:31.68</td>
<td>-60.543</td>
<td>-43.891</td>
<td>5.3</td>
</tr>
<tr>
<td>5</td>
<td>2003/08/29</td>
<td>14:50:34.81</td>
<td>-60.557</td>
<td>-43.205</td>
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</tr>
<tr>
<td>6</td>
<td>2003/10/09</td>
<td>18:49:33.57</td>
<td>-60.236</td>
<td>-44.546</td>
<td>4.3</td>
</tr>
<tr>
<td>7</td>
<td>2003/12/03</td>
<td>03:17:24.81</td>
<td>-60.747</td>
<td>-42.573</td>
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</tbody>
</table>

Table 3.6: List of aftershocks analyzed in this study according to Kanamori (1997).

\[ M_w = \left(\frac{2}{3}\right) \log_{10} M_0 - 10.7 \]
Table 3.7: The results of inversions for the 7 aftershocks analyzed. Events are numbering according to Table 3.6.

where \( M_0 \) is given in units of dyne-cm. Thus a scalar moment of \( 10^{27} \) dyne-cm correspond to a moment magnitude \( M_w \) of 7.3.

In Figure 3.14, we have reported the fit between the observed and synthetic data (a), the full and deviatoric moment rates functions (b), the source time function (c), the fault plane solution and the scalar moment (e). For the source time function, the fault plane solution and the scalar moment the confidence regions are plotted. Our results are compared with the solutions taken from other studies, i.e. Harvard-CMT, which are reported in Table 3.8.

### 3.11 Aftershocks Source Mechanisms

The Antarctic-Scotia Plate boundary, located at the northern margin of the South Scotia Ridge is characterized by an active sinistral transform fault (Forsyth, 1975). Geophysical studies westward of the South Orkney Microcontinent recognised complex geodynamic setting of this boundary. Besides the strike slip kinematic also a convergent component producing the subduction of
Figure 3.14: Solution obtained from the moment tensor inversion of event 1. (a) Waveform fit for each waveform we report the epicentral distance and starting time (left), whereas the peak values of amplitudes and the correlation value (Cor.) are reported on the right. (b) Full and deviatoric moment tensor rate functions ($Nm/s$). (c) Source time functions; the peak value is normalized to 1. (d) Fault plane solution with confidence error areas. (e) Scalar moment.
the Scotia Plate below the Antarctic Plate has been proposed by Kavoun and Vinnikovskaya (1994), Lodolo et al. (1997), Maldonado et al. (1998).

Taking into account the tectonic setting previously delineated, we will describe in detail the solutions obtained for each event analyzed, these solutions were obtained in the first step with constrained the epicenter location, thus alone the depth could vary. See Table 3.9, for events 1 to 4 we obtained normal source mechanisms varying from pure slip to slightly strike-slip, the depth varies from 2 km up to 49 km, the differences among the values of magnitude that we have obtained and those of CMT-Harvard vary between 0.2 and 0.5, the results for the events 2, 3 and 4 agree with those obtained by CMT-Harvard. For the event 5 we have discrepancies with CMT-Harvard, while we obtained an thrust mechanism, CMT-harvard obtained one normal, also for this event we have the greater difference in the magnitude (0.7). For events 6 and 7 we obtained thrust mechanisms, event 6 is a pure slip while the event 7 observes a strike-slip component. The depths for the events 5, 6 and 7 vary between 2 and 24 km.

In the second step solutions were obtained with not constrained hypocenter location, thus latitude, longitude and depth could vary freely, in this case, the inversions obviously carry a lot of computational time. See Table 3.10

The resulting mechanisms for all events are normal faulting mechanisms, the event 1 is a pure slip mechanism, the depth is 7 km and the magnitude differ
<table>
<thead>
<tr>
<th>Date</th>
<th>Depth (km)</th>
<th>Half Dur. (s)</th>
<th>Scalar Mom. dyne-cm</th>
<th>$M_w$</th>
<th>Fault Plane Solution ($^\circ$)</th>
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</thead>
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<td>5</td>
<td>1.2</td>
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<td>5.8</td>
<td>160 11 -76 / 326 80 -93</td>
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<tr>
<td></td>
<td>15 (CMT)</td>
<td>1.1</td>
<td>1.12E+24</td>
<td>5.3</td>
<td>6 23 -81 / 176 68 -94</td>
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<td>49</td>
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<tr>
<td></td>
<td>15 (CMT)</td>
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<td>6.88E+23</td>
<td>5.2</td>
<td>268 18 -128 / 128 76 -78</td>
</tr>
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<td>61 12 -179 / 330 90 -78</td>
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<td>30 47 -171 / 295 84 -43</td>
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<td>17 54 24 / 272 71 141</td>
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Table 3.9: Fault plane solutions with constraint epicentral locations.
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<th>Half Dur. (s)</th>
<th>Scalar Mom. (dyne-cm)</th>
<th>$M_w$</th>
<th>Fault Plane Solution (°)</th>
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<td>6 23 -81 / 176 68 -94</td>
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<tr>
<td>06/08/2003 17:01:54.4</td>
<td>18 -60.23 -44.73</td>
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<td>292 22 -152 / 176 80 -70</td>
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<td>5.2</td>
<td>268 18 -128 / 128 76 -78</td>
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<td>9.28E+23</td>
<td>5.2</td>
<td>190 40 -143 / 70 67 -56</td>
</tr>
</tbody>
</table>

Table 3.10: Fault plane solutions without constraint hypocentral locations.
slightly from those calculated considering the epicentral constraint. The event 2 is also pure slip mechanism, do not there was change in the depth (2 km) and the magnitudes differ slightly with the CMT-Harvard solution. Our results For the event 3, the focal mechanisms, depth and magnitude are very similar to those obtained by CMT-Harvard. For the event 4 we obtained a pure slip mechanism while CMT-Harvard obtained a mostly strike-slip solution, the depth and the magnitude has a small difference. Event 5 is characterized by a pure slip mechanism and it present a total agreement with the CMT-Harvard solution, the depth are practically the same, the magnitudes present some differences.
Events 6 and 7 have a small strike-slip component, the depth for both events is 6 km.

The possibility of relocation of hypocentral coordinates even for shallow events is one of the main features of the INPAR method, since it permits a refinement of focal depth mainly when depth values are fixed (15.0 km) a priori in Harvard-CMT inversion scheme. Take into account that the main shock is a normal mechanism with a strike-slip component and that all the aftershocks analyzed present normal faulting we can conclude that after the main shock has been initiated a relaxation process in the area.

Maps with the fault plane solution of events are reported in Figure 3.15 and Figure 3.16.
Figure 3.16: Bathymetric map with the focal mechanisms obtained considering no constraints hypocentral locations, black star: Main shock location
Chapter 4

Conclusions

We have applied the method of waveform inversion developed by Šilený and Panza (1991) and Šilený et al. (1992) to data recording in Antarctica. The present inversion method also have been applied with success in volcanic and geothermal areas.

The source characteristics of the most relevant aftershocks have been investigated by means of the INPAR method, which is a suitable tool to study medium energy sources in areas where a small number of stations is available to record the regional seismicity and the standard procedures employing first arrival data are not effective to analyze situations where a relevant noise level is present (Kravanja et al., 1999).

To retrieve the source parameters from waveform inversion it is necessary to model the medium as well as possible, so a crucial prerequisite of the success of the waveform inversion is the availability of a reliable structural model of the crust in the region under study. In comparison with already existing similar approaches, it has two advantages: (1) it allows to vary the structural model during the inversion; this is accomplished by interpolation between the base functions a priori computed for two structural models which can be assumed as acceptable extremes for the region under study; (2) it allows to change the focal depth in the course of the inversion, by means of a linear interpolation, with variable spacing, between the base functions a priori computed for a set of different depths.

Still few information is available on the structural and tectonic setting of the SOM. Most of the existing information on the structural and geodynamics setting comes from some seismic profiles performed by Italian, Spanish and Russian vessels during the late decade of the twentieth century, and the of
average structural parameters for the lithosphere of the Scotia Sea region by surface waves tomography (Vuan et al., 2001).

According to the interpretation of this information plus bathymetry data (1 minute grid cell = 1.851 km) a structural map for the SOM area has been constructed, this map is displayed in Figure 3.16 together with the focal mechanisms of the events analyzed. The strike-slip kinematics of the fault system on the northern border, which is also part of the Scotia Plate - Antarctic Plate margin along the South Scotia Ridge, and the existence of a convergent component responsible for the subduction of the Scotia Plate below the Antarctic Plate are put into evidence.

Besides allowing us to fill a relevant gap in the seismicity map of the South Scotia Ridge the analysis of Centenary earthquake and its aftershock sequence waveform data in an increased seismological information about the characteristics of the seismic sources acting in the area which confirm the structural evidences collected by seismic experiments and gravimetry and the hypothesis of the existence both of transpresive and transtensive areas along the northern border of the SOM found in the literature.
Appendix A

Runs test

Runs test (Bradley, 1968) is performed to check whether a given sequence of data is random or not. According to this test, the average value of the data is computed. Each member of the data whose value is above this average is assigned as $+1$, and those are below are assigned as $-1$. The total number of $+1$ is taken as $n_1$ and that of $-1$ is taken as $n_2$. The number of consecutive changes of $+1$ to $-1$ and vice versa is taken as the length of the run $R$. The compute

$$\alpha = 1 + 2 \times n_1 \times n_2 / (n_1 + n_2)$$

$$\sigma^2 = 2 \times n_1 \times n_2 \times (2 \times n_1 \times n_2 - n_1 - n_2) / ((n_1 + n_2)^2 \times (n_1 + n_2 - 1))$$

$$z = (R - \alpha) / \sigma$$

This $z$ value is called the $z$ score or test statistic value. It is compared with the Cumulative Standard Normal Distribution curve given by

$$\phi(Z) = P(Z \leq z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{u^2}{2}\right) \, du$$

A $z$ score with absolute value greater than 1.96 indicates non-randomness at the confidence level of 95%.

Matlab code for calculate the zscore

```matlab
function z=runtest(x)
%
% PURPOSE: check if the positive and negative runs in the vector x is random or not.
%-----------------------------
```

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%USAGE: z=runtest(x)
%where: z is the z stat for a run
%      x is a column vector variable (nobs x 1)
%-------------------------------

%The function could be used on data with a binomial distribution.
%But the user need to assign positivity and negativity to the
%two states in the data.

if size(x,2)>1;error('Input data must be a column vector');end;
if size(x,1)<=20; error('Too few observations'); end;

logic=(x>0); % extract only + and - sign, 1 positive, 0 negative
run=diff(logic); % assign 1 or -1 to the beginning of the run.
run=abs(run); % turn the negative run count into positive
run=[1;run]; % add a starting run, complete the run count.
n=size(logic, 1); n1=sum(logic); n2=n-n1; r=sum(run);
u_r=(2*n1*n2)/(n1+n2)+1;
std_r=sqrt((2*n1*n2*(2*n1*n2-n1-n2))/(((n1+n2)^2)*(n1+n2-1)));
z=(r-u_r)/std_r;
Bibliography


