

FREGE, COMPOSITIONALITY AND EXISTENCE

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Abstract: In this paper I show how Frege's treatment of existence in terms of quantification clashes against the principle of compositionality and I attempt to provide a possible solution to the problem. I expose some of the main innovations introduced by Frege in logic in order to show his good reasons in favor of compositionality and I show how such principle is at odds with the quantificational account of existence. In the final part, I propose a Meinongian solution, namely to abandon the idea that existence is expressed satisfactorily by the particular quantifier and that all terms that succeed to refer, refer to existent objects.

Key Words: Frege, Compositionality, Existence, Quantification, Meinong.

1. *Introduction*

The upshot of this paper¹ is to show that Frege's idea according to which existence is to be understood in terms of quantification is at odds with the principle of compositionality and that we should resolve the conflict abandoning the quantificational account of existence even though this commits us to a Meinongian ontology. In the first part of the paper I rehearse some fundamental features of Frege's logic and semantics, crucial for his acceptance of the principle of compositionality. I start by enunciating the principle in question and then I show how it is presupposed in Frege's work. After a brief exposition of Frege's motivations for formulating his Ideography, I analyze the Fregean treatment of predication and the introduction of the notions of function and argument in logic. I focus then on one of Frege's greatest achievements, namely his treatment of quantification involving second order functions. I show how Fregean logic presupposes the principle according to which, if a complex expression contains a term lacking denotation, the whole expression lacks denotation. My sympathetic reconstruction aims at persuading the reader that Frege's reasons are strong and can still have a strong grip on us. In the second part of the paper, I consider some

¹ I want to thank for their very useful comments all the participants to the workshop "Varieties of Existence" that took place at the philosophy department of the Otto-Friedrich University of Bamberg in June 2015 where the present material was originally presented. The research work for this essay was funded by the Department of Legal Sciences of the University of Udine, through the PRID project "Oggetti politici. La normatività e la metafisica delle comunità politiche", coordinated by Gabriele De Anna.

problematic issues concerning the principle of compositionality, in connection with the idea that existence is to be understood in terms of quantification. Subsequently I try to show how the appeal to non-existent objects can solve the problems affecting Frege's position, without giving up neither compositionality nor the intelligibility of some expressions of natural language.

2. *Compositionality*

The meaning of a complex expression is fully determined by its syntactic structure and by the meanings of its constituent parts. This, perhaps, is an obvious fact, but it plays a central role in the semantic inquiries of the last century. The most attractive quality of this principle is that it provides an explanation to the fact that we, as language users, are able to formulate and understand a virtually infinite number of complex expressions by knowing just a finite set of lexical terms and of syntactic rules. This means that we are able to formulate, according to the formation rules, new expressions by putting together those that we already know and that we can understand expressions never seen before because we grasp the meanings of the terms involved and the way they are linked together. This seems to suggest that there is a symmetry of some kind between syntax and semantics in languages. As Zoltán Szabó writes in the preface of *Problems of Compositionality*:

A sentence, a clause, or a phrase is a complex entity: it is built up from smaller parts – morphemes and words – in accordance with the rules of grammar. A sentence, a clause, or phrase is also a sign: it has some meaning, a role it plays in language and in human interactions. How are complexity and meaning related? The classic answer to this question is that the meaning of a complex expression is determined by the meanings of its constituents and its structure: once you fix what the parts mean and how they are put together, you have fixed the meaning of the whole. (Szabó 2000: vii)

An important thing Szabó's words tell us is the fact that not only complex linguistic expressions express a meaning, but that the meaning expressed is "built up" from the meanings expressed by their constituents. This is exactly a reformulation of the principle I stated at the beginning of the section: the principle of compositionality², namely the best explanation we have for our linguistic abilities. The first to postulate this principle was Gottlob Frege and this is why it is known also as Frege's principle even though the choice of one or another of its names is not perfectly equivalent. Although the principle here discussed bears Frege's name, the German philosopher never formulated it explicitly even though it was clearly presupposed when Frege conceived his formal language in his

² For an overview on compositionality see Szabó (2017).

Ideography (Begriffsschrift) (1960a). It is important then to understand the role compositionality plays in Frege's work, since I believe he had good reasons to presuppose such principle even though it clashes with other Fregean doctrines. I'll try to show that the conflict should be resolved in favor of the principle of compositionality.

3. Frege's Ideography

I'm going to focus now on the reasons that led Frege to elaborate his new formalism because, I believe, among them there is the idea that conceptual contents are formed compositionally and that a well-conceived language should be capable of making this explicit.

The formulation of a new symbolic language aimed at the realization of Frege's logicist project, namely the derivation of arithmetic from the basic laws of logic. In order to do so he needed a precise instrument of derivation capable of showing the real links between conceptual contents and that didn't allow for the intromission in conceptual thought of intuitive notions. Furthermore, Frege was diffident towards natural languages because, in his view, they were not trustworthy instruments when dealing with conceptual investigations and they tended to obscure the real form of thoughts.

In all events, it wouldn't be correct to assimilate the work of Frege to that of symbolic logicians such as Boole and Schroeder, mainly concerned with the elaboration of algorithms for determining whether a statement follows from a set of premises or not and if an argument is valid. Frege's interest, instead, is not purely formal, but rather and mainly semantic. The formulation of a formal language has to be seen, then, as a tool for the precise exposition of conceptual links and their compositional way of relating one to the other. With Alberto Coffa's words: «Traditional logicians were concerned basically with the problem of identifying mathematical algorithms aimed at solving traditional logical problems – what follows from what, what is valid and so on. Frege's goal went far beyond what we now call formal logic and into semantics, meanings, and contents, where he found the ultimate foundation of inference, validity, and much more» (Coffa 1991: 65).

From Coffa's words emerges a picture of Frege different from that of a logician merely interested in formal aspects, but rather that of a thinker who believed that conceptual links and inferences were deeply rooted in semantics. Frege's project was to develop a formal language capable of representing faithfully the structure of contents expressible by its means. «Given this "perfect language" derivability relations and conditions for validity would follow without any need to appeal to algebraic tricks extrinsic to the propositions under consideration, but simply by an analysis of the constituents of the claims involved

and their structural relations, as manifested perspicuously (i.e. syntactically) in their reformulation in the perfect language» (Coffa 199: 66).

Frege's way of conceiving the relation between syntax and semantics, as Coffa comments on it, seems to be consistent with what is expressed in Szabó's passage quoted at the beginning of this section but the former adds something to the latter, namely the idea that syntax is somehow grounded in semantics. From Frege's point of view then, given Coffa's interpretation, syntax is in some way embodied into semantics, but, in order to see it, we need the right formal tool, the language exposed in his Ideography (Frege, 1960a). The Ideography, Frege's formal language, was meant as a tool to represent on the linguistic level the links among contents on the semantic level: for that task, natural languages were not well suited and that is the reason why, Frege thought, they could not be used for conceptual work.

As we have seen, Frege never explicitly formulated the principle of compositionality. The same is not true of what can be seen as compositionality converse, namely the principle of context. This principle appears for the first time in *Die Grundlagen der Arithmetik*, where Frege tries to realize his logicist program of deriving arithmetic from logic. The principle of context says that it is in the context of a statement that terms receive their meaning. This can be seen as equivalent to affirm that Truth is the basic semantic notion in virtue of which all the others are to be interpreted. We use words to speak about things, to say something about them that can be either true or false (see Dummett 1973: 192-195). Even though compositionality and the principle of context may seem inconsistent, they are two sides of the same coin: on the one hand we grasp the meaning of a sentence because we understand the meanings of its constituent parts, on the other hand, words have a meaning just because we use them in sentences in order to express truths about the world. Compositionality and the principle of context taken together, as we will see, imply some important consequences about the meaningfulness of complex expressions.

Even though Frege's aim was to formulate a language well suited for the task of expressing precisely any kind of content, his attention was particularly addressed to mathematics. With Peter Geach's words: «One of the main goals Frege set before himself in his intellectual career was to devise an adequate and perspicuous symbolism to express mathematical propositions and deductions. It might perhaps be supposed that the ordinary symbolism of mathematics was already pretty well adequate; but Frege, for sufficient reasons did not think so [...]» (Geach and Anscombe 1961: 131).

Frege's main interest in formulating a precise formal language then, was to obtain a better instrument capable of clearly expressing mathematical propositions and their relations with one another. This, it is reasonable to assume, in order to realize his logicist program of deriving arithmetic from purely logical

concepts. To reach this goal Frege introduced mathematical notions in logic and by doing that he gave birth to what it is now known as classical logic.

4. Function and Argument

As said before, Frege was diffident towards natural languages, since he thought that they were not able to render the real links between contents at the semantic level. One of the main sources of confusion was, for Frege, the couple of notions subject/predicate that logicians before him had widely employed. One of the greatest Fregean ideas was to replace such notions with the couple function/argument (Frege 1960a: 2-4). This was not just a technical expedient, but rather a way to mimic, on the linguistic level, the kind of relations subsisting at the semantic level. Traditionally, in the sentence 'Bob is sad', 'Bob' was considered the subject whereas 'is sad' the predicate. Sadness is predicated here of Bob. Whereas 'Bob' can never be predicated of anything, sadness can play the double role both of subject and of predicate. Let us consider for instance the sentence 'Sadness is an emotion'. This kind of ambivalence was seen by Frege as a possible source of confusion and a distortion of the real structure of predication. The distinction between function and argument was such that it didn't allow such distortion.

In Frege's formal language, we can find two kinds of signs, namely signs of functions and signs of objects. Signs of functions are always distinct from signs of objects and their proper feature is that of having an empty place that needs to be filled by an object sign. Object symbols, on the other hand, are never incomplete. In this way the compositional nature of the conceptual links is made explicit at the symbolic level. By filling a function empty sign with an object symbol, we obtain an object symbol. To be precise though, I said before that the couple introduced in logic by Frege was that of function and argument. Yet now I'm talking of objects. So it is worthwhile to specify that any symbol that fills the empty space of a function is an argument symbol, but not any argument symbol is an object symbol. So it is true that for any symbol in Frege's language it is either a function symbol or an object symbol, but this distinction is not equivalent with that between argument and function. In fact, functions too can be arguments of other kinds of functions. Let us consider our sentence 'Bob is sad', in Frege's language its real form is something like 'Being sad(Bob)'. 'Bob' is the object sign that also is the argument sign, whereas 'Being sad()' is the function sign with its empty place. Once the function sign is completed with the object sign, what we have is a complex object sign.

4.1. *Frege's semantic categories*

So far, I have focused on signs and the way they can be combined in Frege's language, but as I said before, the Ideography was a means conceived to make explicit the real links between contents at the semantic level. It is now time to introduce Frege's semantic categories. The two kinds of entities signified by Frege's symbols are objects and functions. Objects are somehow difficult to define as Frege recognizes: «I regard a regular definition as impossible, since we have here something too simple to admit of logical analysis» (Frege 1960b: 32). The easiest way, if not the only, to characterize objects then, is to consider them as opposed to functions. Intuitively, though, objects are the common things we find in the world like chairs, tables and abstract entities like numbers. Functions are easiest to describe since they are in some way incomplete and this incompleteness must always be represented on the linguistic level: «Only an incomplete or 'unsaturated' expression – one with an argument-place – could stand for a function or a concept; hence a function or concept-word ought never to appear without its argument-place» (Dummett 1991: 90).

The best way to understand what “being incomplete” means is considering some examples: x^2 is the function squaring. It can be paraphrased as ‘The result of multiplying a number for itself’. x^2 isn't any particular number, as we said, in fact, numbers are objects. What is it then? Using an example similar to that proposed by Frege in “Function and Concept” (1960b: 24), it can be considered as what there is in common among 2^2 , 3^2 , 4^2 etc. If we eliminate the 2, 3 and 4 what remains is the function squaring. In this case x^2 becomes, once completed, a complex object sign, a sign, respectively, of 4, 9 and 12. In other terms we say that the function x^2 gives the value 4 for the argument 2, the value 9 for the argument 3 and the value 12 for the argument 4. It seems quite reasonable to suppose that the fact that functions signs are incomplete is due to the fact that, somehow, they are obtained by a process of disintegration of complex object signs (see Coffa 1991: 68). As Frege himself writes: «In the proposition ‘Cato killed Cato’ shows the same thing. If we imagine ‘Cato’ as replaceable at its first occurrence, then ‘killing Cato’ is the function; if we imagine ‘Cato’ as replaceable at its second occurrence, then ‘being killed by Cato’ is the function; finally, if we imagine ‘Cato’ as replaceable at both occurrences, then ‘killing oneself’ is the function» (Frege 1960a: 13).

4.2. *Statements*

Let now consider the function $x < 7$, as I said, incomplete signs, when completed, become complex objects signs. What kind of object could ‘ $6 < 7$ ’ stand for then? In Frege's view it stands for a truth-value, in this specific case the True. This implies that for Frege the True and False are objects, and statements are complex objects signs (Frege 1960b: 31). Statements then are what we obtain when putting

together an object sign and those functions the values of which are the True or the False. Such functions are what, traditionally, were called predicates and Frege calls them concepts (Frege 1960b: 30). So in Frege's terms, to predicate something of an individual means to take it as the argument of a function whose values are the True or the False. All simple statements then are complex objects signs.

As Richard Heck and Robert May write: «For Frege, then, the concept denoted by the predicate 'is mortal' is a function. Its arguments are objects, such as Socrates. But what are its values? By the principle of compositionality, the denotation of 'Socrates is mortal' is the result of applying the function denoted by 'is mortal' to the object denoted by 'Socrates'. So the question what the values of concept-functions are is equivalent to the question what the denotations of sentences are. Frege's answer to that question, famously, is that sentences denote truth-values» (Richard and Heck 2006: 7). Simple statements are not the only ones though, for instance it is possible to combine them together in order to obtain complex statements. Let us consider the statement 'Bob is sad and lonely'. It is composed by the simple statements 'Bob is sad' and 'Bob is lonely' and the function sign 'and'. This function sign denotes a function that gives as value the true only when its arguments are names of the true. Since in Frege's system all names of truth-values are statements, it follows that the function sign 'and' becomes a name of the truth only when completed with statements³.

4.3. *Quantification*

«Frege recognized that logical rigor depends, first and foremost, upon our being able to articulate the structure of sentences, and what Frege understood profoundly was that how sentences containing words of generality compose is fundamentally different from how sentences containing no such words compose» (Richard and Heck 2006: 2-3).

As said before, not all the argument signs are object signs and this means that some functions signs allow non-object signs as argument signs. Since all signs in Frege's language are either function signs or object signs, it follows that some function signs allow as argument signs of other function signs (Frege 1960a: 16-20). This is exactly what Richard and Heck are referring to in the quoted text at the beginning of the paragraph. In other words: there are functions that take other functions as their arguments. To better explain this, it is necessary to mention another great novelty introduced by Frege, what Dummett describes as «[...]the deepest single technical advance ever made in logic» (Dummett 1973: xv), namely the quantifiers. Quantifiers in natural language are words like 'all', 'some'

³ To be precise, in Frege's language the conjunction is not taken as a primitive sign, but it is rather defined in terms of the material implication and negation, so 'Bob is sad and lonely', in primitive language, becomes 'not-(if Bob is sad then not-Bob is lonely).'

and ‘most of’. In the Fregean system, they are taken to be second level functions, by that meaning that they take only first level functions as their arguments, namely functions whose arguments are objects. To better understand what this means consider the statement ‘There are dogs’: according to Frege, this statement says that the property of being a dog is instantiated, namely that at least one object is a dog, which means that the function being a dog assumes the value true for at least one argument. With Crispin Wright’s words: «The first consideration presupposes Frege’s insight that statements of number, like statements of existence, are in a sense second level. That is, they are most fruitfully and least confusingly seen as ascribing a property not to an object but to a concept. To affirm that lions exist is to say something not about those animals which happen to be lions but about the concept of a lion; namely, that it has the property of having instances» (Wright 1983: 10). The real form of our ‘There are dogs’ then is as follows: ‘Being instantiated by at least one object (Being a dog())’. This, on the linguistic level, means that there is at least an object sign that when used to fill the gap in ‘Being a dog()’, makes it a name of the true (see Dummett 1973: 17). It is very important to notice that even quantified statements obey to the principle of compositionality, since their meaning is obtained from the meanings of the parts involved even though the parts are all function signs.

The second level function we have just seen is called by logicians particular or existential quantifier and it is used to express what in the traditional logic are called particular judgments that share the form ‘Some A is B’. In Frege’s system though, all such statements are expressed by means of what is called universal quantifier and the negation function (Frege 1979: 19). The universal quantifier is used to express general statements like ‘All dogs are mammals’ and in the Fregean system it is a second order function like all objects satisfy the function(). Our statement about dogs, for instance, becomes ‘All objects satisfy the function(if Dog() then Mammal())’. As I said, particular statements like ‘Some A is B’, are expressed by the universal quantifier and the negation and that’s done in the following way: not-all objects satisfy the function: not-(A() and B()).

As said before, Frege’s aim in formulating a formal language was that of building an instrument capable of rendering on the linguistic level the conceptual links at the semantic level. In order to do so, he introduced a new couple of notions in logic, namely that of function and argument by which he was aiming at replacing the old distinction between subject and predicate. Among functions, Frege introduced a multilevel distinction: there are first order functions that take objects as arguments and there are second level functions that take first level functions as arguments. In this way, he was able to give a precise account of quantification by taking quantifiers as second level functions. What I find interesting for the sake of my argument is that all statements in Frege’s systems are complex terms standing for truth-values. And all complex terms have a meaning that is composed from the meanings of the constituent parts. Hence, all

statements have a meaning that is composed from the meanings of the parts. Let us consider a simple example: 'Frege is a logician' is true because the term 'Frege' denotes an object that satisfies the function being a logician. What happens though when a term denotes nothing?

5. *Void terms*

5.1. *Sense and denotation*

Before answering this question, it is necessary now to present another distinction introduced by Frege at the semantic level, namely the distinction between sense and reference⁴. The reference of a term is what it stands for, the object it is a term of, the sense is the way in which the term refers to its object. Let us consider '1 + 1': this complex term refers to the number 2 by describing it as the sum of 1 with 1. The sense of this expression is the way in which it connotes the object that is its denotation. Let us consider now '5 - 3', this term has the same referent as '1 + 1', but a different sense since it connotes the number two as the result of subtracting the number three from the number five. Different senses can have the same denotation, but different denotations cannot have the same sense. For instance, as we have seen, all statements are complex terms denoting one of the two truth-values, all the true statements denote the true and hence they are terms denoting the same object. What differentiates them is their sense, namely the way in which they refer to the true. '1 + 1 = 2' and 'Plato was a great philosopher' denote the same thing, but they express different propositions or, in Frege's terminology, thoughts. The sense of a statement is then a proposition, and a proposition is a way of "describing" a truth value.

Given Frege's double layer semantics, it is necessary to reformulate the principle of compositionality in a double way: one for senses and one for denotations. The principle of compositionality for senses:

- The sense of a complex expression is a function of its structure and of the senses of its parts.

The principle of compositionality of denotation:

- The denotation of an expression is a function of its structure and the denotations of its parts.

An important aspect to mention is that of substitutions. If we replace in a complex term, like a statement, a constituent term with a different one having the same

⁴ For an in-depth discussion of Frege's doctrine of sense and reference see Textor (2010).

sense, what we get is a different statement expressing the same sense and denoting the same thing. If we substitute a term with another one with a different sense, but the same denotation, what we obtain is a different statement expressing a different proposition but denoting the same object. Let us consider ' $1+1 = 3 - 1$ ', if we substitute ' $1 + 1$ ' with ' $5 - 3$ ', what we get is ' $5 - 3 = 3 - 1$ ' that denotes the true since ' $1+1 = 3 - 1$ ' denotes the true and ' $1+1 = 5 - 3$ ' is true.

5.2. *Non-denoting terms*

A central principle of Frege's semantics is that by which if a complex term contains as constituent a non denoting term, it lacks denotation and since all terms with no sense lack denotation (at least according to Frege) no term containing a constituent term lacking sense can have a denotation. This can be seen as a corollary of compositionality and the principle of context, even though the fact that the meaning of the whole is a function of the meanings of the parts, doesn't imply that all the parts must have a meaning. On the other hand the principle of context says that it is in the context of a sentence that a word gets its meaning, and, as we have seen, this means that we use words to form sentences in order to say something about the world. A term without reference is useless from this point of view. Other evidences in favor of Frege's idea that a void term spreads its lack of reference to all the complex terms in which it takes part, can be taken from everyday life. If we consider our experience as language users it is clear that, most of the times, we are not able to decide whether a statement is true or false if we don't know the meaning of one of its terms. Let us consider 'John is a thief', this statement could be either true or false, but if we don't know the referent of 'John' we cannot decide whether it is a name of the true or of the false.

What I described is an epistemic condition, but, if we recall the way in which statements are formed in Frege's language, it is quite obvious that, if a term lacks denotation, the entire expression lacks denotation and the same holds for senses. Let us consider 'Being a dog(Fgfgfgfg)', the argument sign is a random sequence of letters and it is just a sign without meaning. Since it lacks any denotation it cannot stand for an object that taken as the argument of the function being a dog can possibly make it true or false and this makes the entire statement devoid of reference. The same can be said for 'Batman is strong', even though 'Batman' is a term of our language and Frege would have conceded that it has a sense, it, apparently, lacks denotation. This means that, even though we grasp the sense of the entire statement, we cannot decide whether it is true or false, since 'Batman' is not giving any contribution to the determination of the denotation of the sentence in which it appears and that, for Frege, holds for all the terms lacking denotation. On the converse, if a complex term denotes, it means that its parts denote too and this because of the way it is constructed. This means that a true statement only contains denoting terms. And that no term containing a non-denoting sub-term can be a true statement. This conclusion is essentially correct,

but, as we will see, one has to be careful when deciding whether a term is denoting or not.

6. *Existential Problems*

Frege's treatment of quantification has been particularly appreciated among analytic philosophers also because it allows one to deal with some problematic statements in an apparently satisfactory way. The kind of statements I'm referring to are existential statements. Despite the great success reached among philosophers, Frege's solution is not devoid of undesirable consequences⁵.

As I said, the particular quantifier is also called existential quantifier. This is so since it is assumed that particular quantification is a notion capable of expressing existence. Let us consider 'Aliens exist', the real form of this statement, according to Frege, is 'There is an x such that (Alien (x))' or equivalently 'There exist an x such that (Alien(x))'. The existence of the aliens is expressed by affirming that the function being an alien is satisfied by at least one object. As we have seen, quantifiers are second order functions and this means that only first order functions can be taken as arguments. How can one express the fact that an individual exist then? If the notion of existence is correctly expressed by the quantifier it seems like it is a property of concepts and not of individuals. How then to explain the fact that existence in natural language is treated like a property of objects? Well, knowing Frege's diffidence towards natural language it is not surprising that he is inclined to blame the deceiving nature of language. Misguided by natural language grammar we have thought existence to be a first order predicate, but it is not so. Should we then accept the idea that our way of talking about individuals and their existence is illusory? Not necessarily. According to Frege, existence may not be directly predicable of individuals, but could be predicable of functions satisfied only by one object, namely the object of which we want to assert the existence. Such function is self-identity. Let us consider 'Eva Green exists'. This statement affirms that a certain individual exists, but as we have seen Frege does not allow in his language for existence to be predicated directly of individuals. The way out of this situation is to paraphrase our true statement as 'There is an x such that: ($x = \text{Eva Green}$)'. In this way we apply the quantifier to the function being identical with Eva Green and since only one object can possibly satisfy it and being that object Eva Green herself, it is equivalent to 'Eva Green exists'. It is worthwhile to note that the function sign ' $x = \text{Eva Green}$ ' is obtained by substituting the first occurrence of 'Eva Green' in ' $\text{Eva Green} = \text{Eva Green}$ ' that is a complex term denoting the true.

⁵ For a defense of the quantificational account of existence see van Inwagen (1998), especially thesis 4.

6.1. Non-existence

The solution of applying the existential quantifier to identity functions works well and it is respectful of the principle of compositionality. In fact, every term in ‘There is an x such that: $x = \text{Eva Green}$ ’ has a sense and a meaning and the meaning of the complex is a function of the meanings of the parts. What happens though when we want to deny existence of something? In this case, it seems that compositionality clashes against the analysis of existence in terms of quantification. Let us consider ‘Aliens do not exist’, here we are saying that no object satisfies the function being an alien, or equivalently, that the number of aliens is zero. Also this translation respects the principle of compositionality and works well. The problems start when we try to deny the existence of an individual. Let us consider the statement ‘Superman does not exist’, in this case the strategy to apply is the same as the positive existential statements, but some problems arise. Intuitively, it seems plausible to say that Superman does not exist is to deny that some object satisfies the function being identical with Superman, but this seems to be incompatible with the principle of compositionality since the expression ‘ $x = \text{Superman}$ ’, is non-denoting, since it contains an apparently non denoting-term. If the expression ‘ $x = \text{Superman}$ ’ is not denoting, any complex sentence that contains it is non-denoting too and this is true also for ‘There is no x such that: $(x = \text{Superman})$ ’. Since all singular negative existential statements, interpreted *à la* Frege, express the idea that no object satisfy the identity function, it follows that they all contain a non-denoting expression, and hence they are all not true. The problem in general seems to be that in order for a sentence to be true, its terms must denote, but this conflicts with compositionality when considering singular negative existential statements.

Frege’s account of existence seems to fail when dealing with singular negative existential statements and it seems not to avoid what Quine (1953: 2) called Plato’s beard, namely the apparent impossibility of denying existence of anything. When we deny meaningfully that an individual does not exist, we are affirming something inconsistent since in a meaningful sentence all terms are meaningful and hence the term referring to the thing we are saying that it doesn’t exist must refer to something. The contradiction seems to arise from the fact that ‘ a does not exist’ implies ‘ a denotes’, which in turn apparently implies that ‘ a exists’ is true⁶.

⁶ Frege proposed also a different solution, namely to consider ‘Superman does not exist’ as tantamount to “‘Superman’ does not refer”. This solution is problematic though since it seems to imply that, for instance, ‘If Philip II hadn’t existed, Alexander the Great wouldn’t have either’ is tantamount to ‘If ‘Philip II’ hadn’t referred, ‘Alexander the Great’ wouldn’t have either’, but the former is certainly true, whereas the latter is not.

7. Non-existent objects

Frege's account of how language works compositionally seems very plausible⁷, but his analysis of existence in terms of quantification appears to be in conflict with compositionality and with the meaningfulness of some, apparently true, statements of natural language. I think that neither compositionality nor true singular negative existential statements should be dropped and this means that existence has to be conceived differently⁸. A solution may come from the disregarded and deprecated Meinongian doctrine.

The problem of negative existential statements exposed above seems to presuppose the premise that if 'a' refers to something, then 'a' refers to something existent. This assumption is based upon the idea, explicitly expressed by Quine (1953), that everything exists. But if it is so, how can we meaningfully say that something does not exist? An attempt to resolve this puzzle is proposed by the Meinongian philosophers who reject what Berto (2012) calls received view, namely the set of assumptions inherited by analytic philosophers from the works of Frege and Russell in logic. One of this assumption is exactly that everything exists and that the existential quantifier has to be conceived as ontologically loaded (see Priest 2007). In other words, that existence can be paraphrased in quantificational terms, since the quantifiers range over a domain constituted only of existent objects. To use Parson's words: «Are there objects that don't exist? The orthodox, mainstream answer (in Anglo-American philosophy, anyway) is a resounding "No! There's no such thing as a thing that doesn't exist. Though there may be kinds of things that are nowhere exemplified (*e.g.* being a winged horse) there is no particular thing that fails to exist." Or, to put in positive terms: "Everything exists"» (Parsons 1980: 1).

The Meinongian strategy contemplates the possibility that not everything exists and this is equivalent to say that something does not exist. On this ground, the fact that a term denotes does not necessarily imply that it denotes something that exists, since the domain of quantification contains also non-existent objects. In this sense to say that 'Superman does not exist' does not generate any paradox since, if true, it just asserts that Superman lacks existence, but this does not imply that the term 'Supermen' is void and so it can contribute to the formation of the denotation of the expression in which it appears. As Kit Fine points out: «Is 'Hamlet' a proper name and, if so, to what does it refer? The correct answers are: yes, it refers to Hamlet. But because of their distrust of non-existence, many philosophers have hunted around for another reference for 'Hamlet' or, failing

⁷ Among others, two very influential authors that share Frege's view about the compositional nature of language are Davidson (1965; 1967) and Montague (1974; 1974b).

⁸ I am not taking into account here free logics since the basic idea of those systems seems to consist in giving up the principle of compositionality.

that, have tried to give some other account of how it functions in ordinary discourse» (Fine 1982: 100).

Even though the appeal to non-existent objects resolves the problem of singular negative existential statements, another paradox seems to be around the corner. Meinongian philosophers take quantifiers to be ontologically neutral, that means that to say that some aliens are blue does not commit us to the existence of aliens, but existence is explicitly expressed by a first order predicate like in natural languages. This could be a problem, since from the statement ‘not-Exist(Superman)’ one can draw the conclusion ‘There exist an x such that: (not-exist(x))’. The problem here arises only for those that identify existence with quantification and conceive quantification as ontologically loaded. For the Meinongian the particular quantifier is not to be read as ‘There exist’, but rather as ‘There is’ or ‘For some’ without presupposing that it ranges only on existents. The identification of existence with the notion of quantification seems to incorporate some metaphysical assumptions as Parsons explains:

It is relatively common to teach elementary logic in a manner that presupposes the orthodox view. In particular, when students are asked to symbolize these sentences:

(a) Tables exist.

(b) There are tables

the instructor expects the same symbolization, namely:

(c) $(\exists x) Tx$.

And this expectation is not presented as embodying a metaphysical view that might be wrong; instead it is treated as a matter of pure logic. Symbolizing both (a) and (b) in the same way amounts to equating the quantifier ‘there is’ with the quantifier ‘there exists’, an equation which makes sense only if what exists exhaust what there is[...]. (Parsons 1980: 6)

So “something does not exist” is not an inconsistent statement for the Meinongian but it is so for the philosophers that accepts the view that the domain over which the quantifiers range is composed entirely of things that exist.

I believe that, besides the ontological interpretation of the quantifiers, Frege’s logic is perfectly compatible with the idea that there are non-existent objects. The predicates can still be interpreted as functions and statements as complex terms of truth-values. Compositionality is save since every term, also in singular negative existential statements, is provided with a denotation that gives its contribution to the determination of the denotation of the whole. In this way Meinong’s intuitions can be used to resolve Frege’s problem with singular negative existential statements. I perfectly agree with Edward Zalta when he writes: «While a posteriori investigation is required to discover the facts about Reagan, a priori and a posteriori investigation is required to discover the facts about Holmes and Zeus. The metaphysical theory of objects proves to be an

important foundation for constructing compositional semantic analysis of sentences expressing such facts» (Zalta 2003: 2).

What is needed in order to express existential statements, positive and negative, after the definition of the domain of quantification as containing also non-existent objects, is a first order predicate for existence that gives the value true for every argument that exist. On this conditions statements of the form 'There is not an x such that: $(\text{not}(x = a))$ ' are always false, but meaningful, this doesn't imply that everything exists, but rather that everything is self identical. Existence is then to be taken as a first order predicate whose extension is not the entire domain, but rather a subset of it. From this point of view, I believe that Frege's diffidence towards natural languages was unfair and perhaps due to the influence of the prejudice according to which everything exists. Taking existence to be a first order predicate seems to be the easiest way to conciliate compositionality with the possibility of meaningfully expressing singular negative existential statements. The price to pay for this simple semantic theory is that of accepting a wider ontology that Quineans and all that, like them, «have a taste for desert landscapes» (Quine 1953: 4) are not going to like.

8. *Conclusions*

Frege's revolution in logic brought the discipline to a higher level of rigor by applying some mathematical notions. This gave rise to what it's nowadays known as classical mathematical logic. The main innovation was to treat predication in a functional way introducing the notions of function and argument in logic in order substitute the old couple subject/predicate. The functional approach led Frege to give an account of quantification involving second order functions like quantifiers. Quantifiers were conceived as ontologically loaded, namely as quantifying on a domain of existent objects. This assumption generates some problems when dealing with singular negative existential statements because it seems somehow to clash against our common sense and the principle of compositionality. The appeal to non-existent objects seems to allow us to save both compositionality and common sense even though it might seem ontologically extravagant.

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