One-Loop Electroweak Analysis for Third Family Scalar Quarks Production at LHC

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DOTTORANDO: Luca Panizzi

COORDINATORE DEL COLLEGIO DEI DOCENTI: Chiar.mo prof. Gaetano Senatore (Univ. Trieste)

FIRMA: 

TUTORE: Chiar.mo prof. Claudio Verzegnassi (Univ. Trieste)

FIRMA: 

RELATORE: Chiar.mo prof. Claudio Verzegnassi (Univ. Trieste)

FIRMA:

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Introduction

The LHC is starting and there are great expectations about what kind of new physics will be discovered in the next years. Despite the success of past experiments in identifying all the building blocks of the Standard Model and in fixing with great accuracy the parameters of the theory, still something has escaped detection. Noticeably, the particle which is responsible for the electroweak symmetry breaking through the Higgs mechanism has not yet been discovered, just to mention one of the most significant issues the LHC is expected to answer. Due to the lack of experimental data, many theories of physics beyond the SM have been constructed to solve the open issues in the description of high energy physics.

Among these theories, Supersymmetry (SUSY) is one of the most promising and phenomenologically acceptable (not to mention its elegance). Besides providing solutions to issues related to the Higgs hierarchy problem or gauge coupling unification at GUT scales, SUSY predicts the existence of a plethora of new particles which have not yet been discovered. One of these new particles, the Lightest Supersymmetric Particle, is (under certain assumptions about the properties of the supersymmetric lagrangian) one of the most promising candidates to be the cosmological Dark Matter. Therefore, one of the key issues for studying SUSY is to find observables which might be used to identify unambiguously these new particles at accelerators such as the LHC or future linear colliders (ILC, CLIC). Unfortunately, most of the new particles predicted by supersymmetric theories are extremely heavy and difficult to produce at currently available energies, except for a handful of them. In the context of the widely studied framework of the Minimal Supersymmetric Standard Model (MSSM), the scalar superpartners of Top and Bottom quarks, the Stop and the Sbottom respectively, are among the particles which are supposed to have a sufficiently low mass to be produced and detected at the LHC. Third family squarks are lighter than squarks belonging to the first two families, contrarily to
their fermionic partners: this is a direct consequence of the structure of renormalisation group equations and of the fact that Yukawa couplings of superpartners must be equal by construction in a SUSY theory. Due to their predicted lightness, studying third family squark properties and production channels is mandatory: stop and sbottom, if they are ever detected at LHC, would open a wide window on the determination of the parameters of SUSY theories.

This work is meant to provide a detailed description of two of the most significant production processes for stop and sbottom at the LHC, namely $PP \xrightarrow{gg} \tilde{t}_a \tilde{t}_b^* (\tilde{b}_a \tilde{b}_b^*)$ and $PP \xrightarrow{bg} \tilde{t}_a \chi^-_1$, through the calculation of the one-loop electroweak corrections to meaningful observables and through an analysis of the connection between these observables and the parameters of MSSM and mSUGRA. The analysis of EW NLO corrections is relevant for two reasons:

- involving the electroweak sector of the theory, these corrections are sensitive to supersymmetric parameters which are expected to be among the first signals of new physics to be measured at the LHC; one of these parameters is the angle which parametrises the ratio between Higgs VEVs, $\tan \beta$.

- EW NLO effects could give sizable contributions to the determination of observables of the processes making their detection at the LHC possible. Searching for possible direct signals of EW contributions is therefore mandatory.

A full knowledge of EW NLO corrections is also fundamental to obtain the complete one-loop description of the processes when joined to the QCD NLO sector.

The work is organised as follows: the first chapter serves as an introduction to Supersymmetry and the mSUGRA scenario, and its main purpose is to fix the notation that will be used throughout the thesis, the second chapter describes the methods used to perform the computations: Passarino Veltman decomposition, Monte Carlo technique, regularisation of divergences and various conventions; the subsequent chapters contain the EW NLO description of the processes $PP \xrightarrow{gg} \tilde{t}_a \tilde{t}_b^* (\tilde{b}_a \tilde{b}_b^*)$ and $PP \xrightarrow{bg} \tilde{t}_a \chi^-_1$. Some cosmological motivations for the possibility that a very light stop, $m_{\tilde{t}} \lesssim m_t$, may be necessary for electroweak baryogenesis are discussed in appendix together with an analysis of the results obtained in the context of scenarios with such a very light stop.
Chapter 1

Supersymmetric Models

In spite of the great number of successes in describing three of the four fundamental interactions of Nature, the Standard Model appears to be incomplete in many respects. There are both phenomenological and conceptual issues that still remain unsolved. Starting from the phenomenological side, the assumption that neutrinos are massless is clearly in contrast with the evidence of neutrino oscillations, the great gaps in the masses of fermions belonging to different families is completely unexplained, not to mention the process of electroweak symmetry breaking and the generation of particle masses through the Higgs mechanism, which is undoubtedly the key problem of current experimental research in high energy physics. On the conceptual side, the clearer hint that some new physics should occur beyond the TeV scale is the so called “hierarchy problem”, that is the (quadratic) dependence of the Higgs boson mass on the ultraviolet cutoff of loop integrals, or the scale at which new physics comes into play; moreover, as it is possible to see in Fig. [1.1] running the gauge coupling constants to high energies, they come very close to each other near to $10^{15}$ GeV, but do not reach the same value at the same point: it would seem more natural if all the coupling unify at some very high energy (Grand Unification Theory).

The fact that the SM should be completed in some way in order to be a successful description of Nature has also cosmological grounds: the growing evidence for the existence of dark matter, which contribute for nearly a quarter of the energy of the Universe, is usually related to the hypothesis of the existence of yet undiscovered weakly interacting particles and the fact that the cosmological constant has a very tiny value cannot be easily explained.
Supersymmetric models are among the most appealing theories which try to predict what the physics beyond the Standard Model might be. Albeit they do not provide answers to all SM problems, SUSY models (or at least the minimal versions of them) solve in an elegant way the hierarchy problem, predict the unification of gauge couplings and introduce a huge number of new particles which could be candidates for dark matter.

Roughly speaking, supersymmetry means that the number of bosonic and fermionic degrees of freedom of the Lagrangian are equal, so that for every fermion and boson of the SM there are respectively a supersymmetric boson and fermion partner with exactly the same mass. Moreover, the new SUSY particles interact with the same coupling constants of SM particles, therefore, in principle, supersymmetry does not introduce many new parameters. There is a problem, however: the hypothetical supersymmetric partners of SM particles have never been observed so far and this sounds odd since the superpartner of the electron, the selectron, having the same mass as the electron, should have been detected long ago. To explain this problem supersymmetry is supposed to be a broken symmetry. The way supersymmetry is broken determines the features of the model, and in particular its spectrum. In this thesis the analysis will be performed within the framework
of the Minimal Supersymmetric Standard Model (MSSM) assuming minimal Supergravity (mSUGRA) scenarios. The next sections are devoted to illustrate these frameworks. It must be stressed, however, that this chapter is not supposed to be a detailed and exhaustive analysis of the MSSM and mSUGRA, but only a general overview of the most relevant relations which will be useful to understand the analysis of the processes of production of third family squarks illustrated in the next chapters. The main purpose of this chapter is therefore to fix the notation and conventions that will be used throughout the whole thesis and to focus on why third family squarks have a significant role in SUSY phenomenology. A thorough and detailed analysis of these models can be found, e.g., in [1], [2] and references therein, while a complete set of Feynman rules is given in [3].

1.1 The Minimal Supersymmetric Standard Model

The Minimal Supersymmetric Standard Model is, as its name suggests, the simplest phenomenologically consistent extension of the SM. In the unbroken state it can be characterised by fixing the gauge transformation properties and an analytic function of the scalar fields, called superpotential, whose most general gauge invariant form is:

$$W = \mu \epsilon_{ij}(H_d)_i(H_u)_j + \epsilon_{ij}Y^I_{ij}(H_d)_iL^j_iE^j_I + \epsilon_{ij}Y^J_{ij}(H_d)_iQ^J_iD^j_J + \epsilon_{ij}Y^U_{ij}(H_u)_iQ^U_jU^j$$

The fields which enter the superpotential, with the exception of the Higgs bosons, are the scalar superpartners of the fermions of the SM. The 3×3 matrices $Y_l$, $Y_d$ and $Y_u$ are the Yukawa couplings of leptons and quarks ($I, J$ are family indexes), while $\mu$ is a parameter with dimension of [mass]. From the superpotential it is possible to construct the whole lagrangian of the MSSM. Without entering in the details of such calculation, it is useful to illustrate the particle content of the MSSM:

- multiplets belonging to the gauge group $SU(3)_C \times SU(2)_W \times U(1)_Y$ have fermionic partners called *gauginos*:

<table>
<thead>
<tr>
<th>SM</th>
<th>SUSY</th>
</tr>
</thead>
<tbody>
<tr>
<td>hypercharge: $B_\mu$</td>
<td>$\lambda_B$</td>
</tr>
<tr>
<td>weak isospin: $A^i_\mu$</td>
<td>$\lambda^i_A$</td>
</tr>
<tr>
<td>QCD: $G^a_\mu$</td>
<td>$\lambda^a_G$</td>
</tr>
</tbody>
</table>
• matter multiplets have scalar partners called sfermions (squarks and sleptons):

\[
\begin{align*}
\Psi^I_Q &= \begin{pmatrix} u^I \\ d^I \end{pmatrix}_L \\
Q^I &= \begin{pmatrix} \tilde{u}^I \\ \tilde{d}^I \end{pmatrix}_L \\
\Psi^I_L &= (u^I_L)^c \\
U^I &= \tilde{u}^I_R \\
\Psi^I_D &= (d^I_L)^c \\
D^I &= \tilde{d}^I_R \\
\Psi^I_L &= (\nu^I) \\
L^I &= \begin{pmatrix} \tilde{\nu}^I \\ \tilde{e}^I \end{pmatrix}_L \\
\Psi^I_E &= (e^I_L)^c \\
E^I &= \tilde{e}^I_R
\end{align*}
\]

where the index \( I \) runs on the number of families (QCD indexes are not shown).

• Higgs multiplets deserve more attention. The superpotential is an analytic function of the superfields, that is, it cannot contain the conjugates of the fields; therefore it is not possible to have one Higgs doublet as in the SM since it would be necessary to use the conjugate field to give mass to either the up or down members of matter multiplets. Within the MSSM, the solution is to consider two Higgs doublets which then acquire their fermionic superpartners, the higgsinos:

\[
\begin{align*}
H_d &= \begin{pmatrix} H^0_d \\ \tilde{H}^-_d \end{pmatrix} \\
\Psi_{H_d} &= \begin{pmatrix} \Psi^0_{H_d} \\ \tilde{\Psi}^-_{H_d} \end{pmatrix} \\
H_u &= \begin{pmatrix} H^+_u \\ H^0_u \end{pmatrix} \\
\Psi_{H_u} &= \begin{pmatrix} \Psi^+_u \\ \tilde{\Psi}^0_{H_u} \end{pmatrix}
\end{align*}
\]

In principle it would be possible to add to the superpotential gauge invariant terms such as \( \varepsilon^I_B \varepsilon_{ij} (H_u)_i Q^I_j \) which, however, break baryon or lepton number conservation at tree level. It is necessary to get rid of these dangerous terms and this is possible imposing a global symmetry, called R-parity:

\[
R = (-1)^{3(B-L)+2S} \tag{1.2}
\]

where \( B \) and \( L \) are, respectively, the baryon and lepton numbers and \( S \) is the spin of the particle. Under R-parity all SM particles and the Higgs bosons are even, while all
SUSY particles are odd. This symmetry has a number of phenomenologically interesting consequences: there cannot be mixing between SM and SUSY particles; each interaction vertex must contain an even number of supersymmetric particles, which therefore can be created only in pairs; the lightest supersymmetric particle (LSP) must be stable, since it cannot decay into SM particles.

### 1.1.1 Supersymmetry breaking terms

Without any supersymmetry breaking interactions every SUSY particle would share the same mass and couplings with its SM partner, but in Nature supersymmetry must be broken, otherwise the superpartners of the lightest particles (electron, muon, light quarks) would have been discovered long ago. It is thus necessary to introduce SUSY breaking terms. At high energies the vacuum state of the model is invariant under supersymmetric transformations, but at low energy it is not, so it seems sensible that SUSY is an exact symmetry that is spontaneously broken at low energies. It is quite difficult, however, to construct a model of spontaneous supersymmetry breaking because it involves the description of new physics at very high energy scales, which is totally unknown. Hence, to perform calculations at low energy it is useful to introduce explicit supersymmetry breaking terms by hand. These terms must not introduce quadratically divergent corrections to the Higgs boson mass which might spoil the solution to the hierarchy problem: to avoid this problem it is enough to require them to be “soft”, or of positive mass dimension. The lagrangian of the MSSM thus becomes:

\[
\mathcal{L}_{\text{MSSM}} = \mathcal{L}_{\text{SUSY}} + \mathcal{L}_{\text{soft}}
\]  

The most general soft supersymmetry breaking lagrangian contains:

1. mass terms for the scalar fields, of the type \(-m_{ij}^2 \phi_i^* \phi_j^*\):

\[
\begin{align*}
- m_{H_d}^2 (H_d)_i (H_d)_i &- m_{H_u}^2 (H_u)_i (H_u)_i \\
- (m_{L_i}^2)^{I\bar{J}} L_i^{I*} L_i^{J} &- (m_{E_i}^2)^{I\bar{J}} E_i^{I*} E_i^{J} \\
- (m_{Q_i}^2)^{I\bar{J}} Q_i^{I*} Q_i^{J} &- (m_{U_i}^2)^{I\bar{J}} U_i^{I*} U_i^{J} \\
- (m_{L_i}^2)^{I\bar{J}} L_i^{I*} L_i^{J} &- (m_{E_i}^2)^{I\bar{J}} E_i^{I*} E_i^{J} \\
- (m_{Q_i}^2)^{I\bar{J}} Q_i^{I*} Q_i^{J} &- (m_{U_i}^2)^{I\bar{J}} U_i^{I*} U_i^{J} \\

\end{align*}
\]  

2. mass terms for gauginos, of the type \(-\frac{1}{2} M_\alpha \lambda_\alpha\lambda_\alpha\):

\[
- \frac{1}{2} M_1 \lambda_B \lambda_B - \frac{1}{2} M_2 \lambda_A \lambda_A - \frac{1}{2} M_3 \lambda_G \lambda_G + h.c.
\]
3. bilinear and trilinear couplings of scalar fields, of the type $b_{ij}\phi_i\phi_j$ and $a_{ijk}\phi_i\phi_j\phi_k$, corresponding to the Yukawa terms in the superpotential:

$$m_{12}^2\epsilon_{ij}(H_d)_i(H_u)_j + \epsilon_{ij}A^I_i(H_d)_iL^I_jE^J + \epsilon_{ij}A^I_d(H_d)_iQ^I_jD^J + \epsilon_{ij}A^I_u(H_u)_iQ^I_jU^J$$

(1.6)

where $m_{12}^2$ is a squared mass term for the Higgs bosons, while $A_i, A_d$ and $A_u$ are $3\times3$ matrices in family space with dimension of [mass] which correspond to the Yukawa terms in the superpotential.

There could be also non analytic terms of the type $c_{ijk}\phi_i^*\phi_j\phi_k$, but in general they are neglected because in order to build any phenomenologically acceptable model with spontaneous symmetry breaking, the constants $c_{ijk}$ should usually be very small [1].

1.1.2 The mass eigenstates

To perform phenomenological computations it is necessary to work with physical particles, which in general do not correspond to gauge eigenstates. They are obtained through the usual mechanism of spontaneous symmetry breaking in the Higgs sector:

$$\langle H_d \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_d \\ 0 \end{pmatrix} \quad \langle H_u \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_u \end{pmatrix}$$

(1.7)

Through the VEVs of the Higgs bosons it is possible to reconstruct the masses of MSSM particles (except gluinos, whose mass terms are already present in $L_{soft}$):

1. Gauge bosons

As in the SM, the photon and the gluons are massless, while the Z and W bosons acquire masses:

$$M_Z = \frac{e}{2s_Wc_W} (v_d^2 + v_u^2)^{1/2}$$

$$M_W = \frac{e}{2s_W} (v_d^2 + v_u^2)^{1/2}$$

(1.8)

(1.9)

where $s_W$ and $c_W$ are the sine and cosine of the Weinberg angle $\theta_W$. These relations show that the parameters $v_d$ and $v_u$ are not independent: they must reproduce the measured values of gauge bosons masses. In fact the VEVs of the 2 Higgs bosons are related by the relation:

$$v_u^2 + v_d^2 \equiv v_{SM}^2 \implies \frac{v_u}{v_d} = \frac{v_{SM} \sin \beta}{v_{SM} \cos \beta} = \tan \beta$$

(1.10)

which introduces the phenomenological parameter $\tan \beta$. 
2. Higgs particles

The gauge eigenstates can be expressed in terms of mass eigenstates as:

$$
\begin{pmatrix}
H^0_u \\
H^0_d
\end{pmatrix}
= \begin{pmatrix} v_u \\ v_d \end{pmatrix}
+ \frac{1}{\sqrt{2}} Z_R \begin{pmatrix} h_0 \\ H_0 \end{pmatrix}
+ \frac{1}{\sqrt{2}} Z_H \begin{pmatrix} G_0 \\ A_0 \end{pmatrix}
$$  \hfill (1.11)

$$
\begin{pmatrix}
H^+_u \\
H^{-}_d
\end{pmatrix}
= Z_H \begin{pmatrix} G^+ \\ H^+ \end{pmatrix}
$$  \hfill (1.12)

a) Neutral scalars $H_0$ and $h_0$ masses are diagonalised through the matrix $Z_R$, defined such as:

$$
Z_R \begin{pmatrix}
-m^2_{12} v_u \\
2 v_u^2 + 4 s^2_W c^2_W
\end{pmatrix}
+ \begin{pmatrix}
m^2_{12} v_d \\
2 v_d^2 + 4 s^2_W c^2_W
\end{pmatrix}
= \begin{pmatrix} M^2_{H_0} & 0 \\ 0 & M^2_{h_0} \end{pmatrix}
$$  \hfill (1.13)

The matrix $Z_R$ can be parametrised as:

$$
Z_R = \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix}
$$  \hfill (1.14)

b) Neutral pseudoscalars $A_0$ and $G_0$ masses can be diagonalised through the matrix $Z_H$:

$$
Z_H = \begin{pmatrix} \sin \beta & \cos \beta \\ -\cos \beta & \sin \beta \end{pmatrix}
$$  \hfill (1.15)

where $\beta$ has been defined in (1.10). The longitudinal degree of freedom of $G_0$ is eaten by the $Z$ boson, so $G_0$ becomes massless, while $A_0$ acquires mass:

$$
m_{A_0}^2 = m_{H_u}^2 + m_{H_d}^2 + 2 |\mu|^2
$$  \hfill (1.16)

c) Charged scalars $H^+$ and $G^+$ masses can be diagonalised by the same matrix $Z_H$; $G^+$ is the massless Goldstone boson, whose longitudinal degree of freedom is eaten by the W boson, while $H^+$ acquires mass:

$$
m_{H^+} = m_{A_0}^2 + m_W^2
$$  \hfill (1.17)

3. Matter fermions

As in the SM, quark and lepton masses are determined by their Yukawa couplings with the Higgs:

$$
m_e^I = -\frac{v_d Y^l}{\sqrt{2}} \\
m_d^I = -\frac{v_u Y^l}{\sqrt{2}} \\
m_u^I = \frac{v_u Y^l}{\sqrt{2}}
$$  \hfill (1.18)
4. Charginos
The charged parts of $\lambda_A^i$ combine with the charged higgsinos $\Psi_{H_d}^-$ and $\Psi_{H_u}^+$ to form two 4-component Dirac fermions $\chi_1$ and $\chi_2$. The mass matrix for gauge eigenstates is:

$$X = \begin{pmatrix} M_2 & \sqrt{2}M_W \sin \beta \\ \sqrt{2}M_W \cos \beta & \mu \end{pmatrix}$$

(1.19)

Two unitary mixing matrices $Z^\pm$ are necessary to diagonalise $X$: they are not uniquely defined and can be chosen such as $m_{\chi_1} < m_{\chi_2}$. Their defining properties are:

$$\sum_k Z^\pm_{ik} Z^\pm_{jk} = \delta_{ij}$$

(1.20)

$$\sum_k m_{\chi_k} Z^-_{ik} Z^+_{jk} = X_{ij}$$

(1.21)

5. Neutralinos
The neutral gauginos, $\lambda_3^A$ and $\lambda_B$, and the neutral higgsinos, $\Psi_{H_d}^0$ and $\Psi_{H_u}^0$, combine to form four Majorana fermions $\chi^0_i$, $i = 1 \ldots 4$. The mass matrix for gauge eigenstates is:

$$Y = \begin{pmatrix} M_1 & 0 & -\frac{\epsilon v_d}{2s_W} & \frac{\epsilon v_u}{2c_W} \\ 0 & M_2 & \frac{\epsilon v_d}{2s_W} & -\frac{\epsilon v_u}{2c_W} \\ -\frac{\epsilon v_d}{2s_W} & \frac{\epsilon v_d}{2s_W} & 0 & -\mu \\ \frac{\epsilon v_u}{2c_W} & -\frac{\epsilon v_u}{2c_W} & -\mu & 0 \end{pmatrix}$$

(1.22)

The defining properties of the mixing matrix $Z^N$ needed to diagonalise $Y$ and obtain the masses of the neutralinos are:

$$\sum_k Z^N_{ik} Z^N_{jk} = \delta_{ij}$$

(1.23)

$$\sum_k m_{\chi_k} Z^N_{ik} Z^N_{jk} = Y_{ij}$$

(1.24)

6. Gluinos
$SU(3)_c$ gauginos $\lambda_G^a$ do not mix, therefore the gluinos are, in four component notation:

$$\Lambda_G^a = \begin{pmatrix} -i\lambda_G^a \\ i\tilde{\lambda}_G^a \end{pmatrix}$$

(1.25)
7. Sfermions

a) There are three sneutrinos $\tilde{\nu}^I$ which in general mix as:

$$ L_1^I = Z_{\nu}^{IJ} \tilde{\nu}^J $$

(1.26)

$$ Z_{\nu}^I M_{\tilde{\nu}}^2 Z_{\nu} = \begin{pmatrix} M_{\tilde{\nu}_1}^2 & M_{\tilde{\nu}_2}^2 & M_{\tilde{\nu}_3}^2 \\ M_{\tilde{\nu}_1}^2 & M_{\tilde{\nu}_2}^2 & M_{\tilde{\nu}_3}^2 \end{pmatrix} $$

(1.27)

$$ M_{\tilde{\nu}}^2 = \frac{e^2 (v_u^2 - v_d^2)}{8 s_W c_W} \hat{1} + m_L^2 $$

(1.28)

and six charged sleptons $L_i$, $i=1\ldots6$:

$$ L_2^I = Z_{L}^{Ii} L_i^- $$

(1.29)

$$ E^I = Z_{E}^{(i+e)i} L_i^+ $$

(1.30)

$$ Z_L^I \begin{pmatrix} (M_L^2)_{LL} & (M_L^2)_{LR} \\ (M_L^2)_{LR}^T & (M_L^2)_{RR} \end{pmatrix} Z_L = \begin{pmatrix} M_{L_1}^2 & 0 \\ 0 & M_{L_6}^2 \end{pmatrix} $$

(1.31)

$$ (M_L^2)_{LL} = \frac{e^2 (v_u^2 - v_d^2)(1 - 2 c_W^2)}{8 s_W^2 c_W^2} \hat{1} + \frac{v_d^2 Y_l^2}{2} + (m_L^2)^T $$

(1.32)

$$ (M_L^2)_{RR} = -\frac{e^2 (v_u^2 - v_d^2)}{4 c_W^2} \hat{1} + \frac{v_d^2 Y_l^2}{2} + m_E^2 $$

(1.33)

$$ (M_L^2)_{LR} = \frac{1}{\sqrt{2}} (v_u Y_l \mu^* + v_d A_l) $$

(1.34)

b) Six up squarks $U_i$, $i=1\ldots6$ are generated by the mixing of $Q_1^I$ and $U^I$:

$$ Q_1^I = Z_U^{Ii} U_i^+ $$

(1.35)

$$ U_I = Z_U^{(I+3)i} U_i^- $$

(1.36)

$$ Z_U^T \begin{pmatrix} (M_U^2)_{LL} & (M_U^2)_{LR} \\ (M_U^2)_{LR}^T & (M_U^2)_{RR} \end{pmatrix} Z_U = \begin{pmatrix} M_{U_1}^2 & 0 \\ 0 & M_{U_6}^2 \end{pmatrix} $$

(1.37)
\( (\mathcal{M}_U^2)_{LL} = -\frac{e^2(v_d^2 - v_u^2)(1 - 4e^2_W)}{24s_W^2c_W^2} - \frac{v_u^2Y_u^2}{2} + (Km_QK^\dagger)^T \) (1.38)

\( (\mathcal{M}_U^2)_{RR} = \frac{e^2(v_d^2 - v_u^2)}{6c_W^2} + \frac{v_u^2Y_u^2}{2} + m_U^2 \) (1.39)

\( (\mathcal{M}_U^2)_{LR} = -\frac{1}{\sqrt{2}}(v_uY_uh^* + v_dA_d) \) (1.40)

where \( K \) is the CKM matrix.

The mixing of \( Q^I_2 \) and \( D^I \) generates six down squarks \( D_i, i=1...6: \)

\[ Q^I_2 = Z^I_D D^+_i \] (1.41)

\[ D^I = Z^I_D (I + e^{i\theta}) D^+_i \] (1.42)

\[ Z^I_D \begin{pmatrix} (\mathcal{M}_D^2)_{LL} & (\mathcal{M}_D^2)_{LR} \\ (\mathcal{M}_D^2)_{LR}^\dagger & (\mathcal{M}_D^2)_{RR} \end{pmatrix} \] \[ Z_D = \begin{pmatrix} M^2_{D_1} & 0 \\ \vdots & \vdots \\ 0 & M^2_{D_6} \end{pmatrix} \] (1.43)

\( (\mathcal{M}_D^2)_{LL} = -\frac{e^2(v_d^2 - v_u^2)(1 - 2c_W^2)}{24s_W^2c_W^2} - \frac{v_u^2Y_d^2}{2} + (m_Q^2)^T \) (1.44)

\( (\mathcal{M}_D^2)_{RR} = -\frac{e^2(v_d^2 - v_u^2)}{12c_W^2} + \frac{v_u^2Y_d^2}{2} + m_D^2 \) (1.45)

\( (\mathcal{M}_D^2)_{LR} = \frac{1}{\sqrt{2}}(v_uY_dh^* + v_dA_d) \) (1.46)

1.2 mSUGRA

The SUSY breaking part of the Lagrangian introduces many new parameters, so that the total number of degrees of freedom of the MSSM is 110: 69 real parameters (9 fermion masses, 21 scalar masses and 39 mixing angles) and 41 phases [4]. The MSSM adds 96 degrees of freedom to the SM. Whereas the supersymmetric lagrangian elegantly solves some problems of the SM, the soft lagrangian on the other hand seems to be poorly predictive and, moreover, it contains dangerous terms which imply flavour mixing or CP violations that are severely constrained by experiment. Nevertheless, simple assumptions about the values of soft parameters at some very high energy (not accessible by experiment) allow to get rid of all these terms. These assumptions essentially consist in assigning universal values to soft parameters:

\[ (m_Q^2) = m_Q^2 \hat{1} \quad (m_U^2) = m_U^2 \hat{1} \quad (m_D^2) = m_D^2 \hat{1} \quad (m_L^2) = m_L^2 \hat{1} \quad (m_E^2) = m_E^2 \hat{1} \] (1.47)

\[ A_u \propto -Y_u \quad A_d \propto -Y_d \quad A_l \propto -Y_l \] (1.48)
and assuming that these parameters do not introduce new complex phases such that CP violation can only arise from the usual CKM phase of the SM. Then, the supersymmetric spectrum can be derived running the soft parameters from high to low energy, keeping all the dangerous interactions negligible.

Even with the assumption of universality between the values of soft parameters, however, supersymmetry has been broken \textit{explicitly} in the soft lagrangian, while some mechanism of \textit{spontaneous} breaking at some high energy scale would be more natural. The spontaneous breaking is supposed to happen in a “hidden sector” which communicates to the “visible sector” through some suppressed interactions generating the soft terms. One of the most studied scenarios which can lead to spontaneous supersymmetry breaking is supergravity. The supergravity lagrangian contains non-renormalizable terms suppressed by powers of the Planck mass $M_P$ and proportional to some auxiliary field $F$ which acquires a VEV generating in this way the soft terms of the lagrangian. By dimensional analysis, the soft terms are of order:

\begin{equation}
 m_{\text{soft}} \sim \frac{\langle F \rangle}{M_P}
\end{equation}

The parameters of the supergravity lagrangian are of course unknown and their determination is quite difficult, but a great simplification occurs within the mSUGRA scenario, a minimal version of supergravity in which the parameters which generate the soft terms are assumed to be proportional to universal dimensionless constants at some energy scale. This assumption becomes clear when looking at the running equations of the gauge coupling constants: since they merge at the GUT scale $M_X$, it seems sensible to assume that the mSUGRA parameters become universal at the same scale. In the mSUGRA framework the parameters which determine the soft terms of the lagrangian are:

- the universal gaugino mass

\begin{equation}
 m_{1/2} = f \frac{\langle F \rangle}{M_P} \quad M_1(M_X) = M_2(M_X) = M_3(M_X) \equiv m_{1/2}
\end{equation}

- the universal scalar mass

\begin{equation}
 m^2_0 = k \frac{|\langle F \rangle|^2}{M_P^2}
\end{equation}

\begin{align*}
 m^2_Q(M_X) = m^2_U(M_X) = m^2_D(M_X) = m^2_L(M_X) = m^2_E(M_X) \equiv m^2_0 \\
 m^2_H_u(M_X) = m^2_H_d(M_X) \equiv m^2_0
\end{align*}
• the universal trilinear coupling

\[ A_0 = \alpha \frac{\langle F \rangle}{M_P} \]  
\[ A_u(M_X) = A_0 Y_u \quad A_d(M_X) = A_0 Y_d \quad A_l(M_X) = A_0 Y_l \]  

• the bilinear coupling of the Higgs

\[ B_0 = \beta \frac{\langle F \rangle}{M_P} \quad m_{12}^2 = B_0 \mu \]  

The “fundamental” set of parameters at \( M_X \) is therefore: \( m_{1/2}, m_0, A_0, B_0, \mu \). When imposing the electroweak symmetry breaking equations through the minimisation of Higgs potential at a scale \( Q \sim O(m_Z) \), however, some relations between these parameters arise allowing the possibility to use a different fundamental set: \( \mu^2(Q) \) can be traded for \( m_Z^2 \) (but its sign remains free and is scale independent) and \( m_{12}^2(Q) \) for \( \tan \beta(Q) \) \([5]\). The independent set of parameters that will be used throughout this thesis is:

\[ m_{1/2}(M_X) \quad m_0(M_X) \quad A_0(M_X) \quad \tan \beta(Q) \quad \text{sign}(\mu) \]  

The spectrum of the MSSM at low energy has been calculated from the various mSUGRA benchmark points using the publicly available codes SUSPECT \([6]\) and FeynHiggs \([7]\).

Nevertheless, the C++ codes used to perform the one-loop computations do not necessarily require mSUGRA-inspired spectra as input: they have been used mainly because of their physical consistency and simplicity.

### 1.3 Why stop and sbottom squarks are so interesting

The squark mass matrices Eqs. (1.37-1.40) and Eqs. (1.43-1.46) can be obtained running the mSUGRA parameters down to the scale of electroweak breaking. It will be assumed that mixing between squarks belonging to different families is negligible, so that the mixing matrices can be block-diagonalized separately for each family (Constrained Minimal Flavour Violation \([8,9,10]\)). It is now possible to inspect the various elements of the mass matrices for the different families.
Squarks of the first and second family

The non-diagonal components contain the Yukawa and trilinear coupling: the first is negligible due to the smallness of light quark masses, the second, being proportional to the Yukawa coupling itself, becomes negligible too. Therefore the mass matrices for the first two squark families can be approximated as diagonal matrices with elements:

\[ M_{\tilde{u}}^2 \simeq \begin{pmatrix} \left( \frac{1}{2} - \frac{2}{3} s_W^2 \right) \cos(2\beta) M_Z^2 + m_{Q_1}^2 & 0 \\ 0 & \frac{2}{3} s_W^2 M_Z^2 \cos(2\beta) + m_{U_1}^2 \end{pmatrix} \]

\[ M_{\tilde{d}}^2 \simeq \begin{pmatrix} \left( -\frac{1}{2} + \frac{1}{3} s_W^2 \right) \cos(2\beta) M_Z^2 + m_{Q_1}^2 & 0 \\ 0 & -\frac{1}{3} s_W^2 M_Z^2 \cos(2\beta) + m_{D_1}^2 \end{pmatrix} \]

where Eq. (1.8) has been used to express the matrix elements in terms of \( M_Z \). The terms \( m_{Q_1}, m_{U_1}, \) and \( m_{D_1} \) are common for squarks of the first two families. It is possible to see that for the first two families, mass eigenstates coincide with gauge eigenstates.

Squarks of the third family

The non-diagonal terms are no more negligible since Yukawa couplings of top and bottom quarks are much bigger than those of the first two families. The mass matrices can then be written as:

\[ M_{\tilde{t}}^2 \simeq \begin{pmatrix} \left( \frac{1}{2} - \frac{2}{3} s_W^2 \right) \cos(2\beta) M_Z^2 + m_t^2 + m^2_{Q_3} & v (Y_t \mu \cos \beta + A_t \sin \beta) \\ v (Y_t \mu \cos \beta + A_t \sin \beta) & 2 \frac{s_W^2 M_Z^2 \cos(2\beta)}{3} + m_t^2 + m_{U_3}^2 \end{pmatrix} \]

\[ M_{\tilde{b}}^2 \simeq \begin{pmatrix} \left( -\frac{1}{2} + \frac{1}{3} s_W^2 \right) \cos(2\beta) M_Z^2 + m_b^2 + m^2_{Q_3} & v (Y_b \mu \cos \beta + A_b \sin \beta) \\ v (Y_b \mu \cos \beta + A_b \sin \beta) & -\frac{1}{3} s_W^2 M_Z^2 \cos(2\beta) + m_b^2 + m_{D_3}^2 \end{pmatrix} \]

The non-diagonal terms induce a strong mixing between squarks of the third family, which can be parametrized through the angles \( \theta_t \) and \( \theta_b \):

\[ \begin{pmatrix} \tilde{t}_1 \\ \tilde{t}_2 \end{pmatrix} = \begin{pmatrix} \cos \theta_t & \sin \theta_t \\ -\sin \theta_t & \cos \theta_t \end{pmatrix} \begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \end{pmatrix} \]

\[ \begin{pmatrix} \tilde{b}_1 \\ \tilde{b}_2 \end{pmatrix} = \begin{pmatrix} \cos \theta_b & \sin \theta_b \\ -\sin \theta_b & \cos \theta_b \end{pmatrix} \begin{pmatrix} \tilde{b}_L \\ \tilde{b}_R \end{pmatrix} \]
Renormalization group equations are also strongly affected by Yukawa couplings: the numerical determination of the soft terms $m_{Q_3}$, $m_{U_3}$ and $m_{D_3}$ reveals that they are much smaller than the analogous terms for the first two families. Even if $m_t$ and $m_b$ in the diagonal terms tend to compensate this effect, the non-diagonal mixing terms contribute to push the stop and sbottom masses to low values, making them the lightest squarks. In general, then, stop and sbottom are found to be mainly right handed. As will be shown in the next chapters, the stop mass can even be pushed down to values close to the top mass. The lightness of squarks of the third family makes them very interesting: since they might be among the first supersymmetric particles to be discovered at the LHC, it is worth to analyse their properties.
Chapter 2

One-Loop Techniques

One-loop analysis requires the computation of a great number of Feynman diagrams, therefore it is quite important to simplify the work as much as possible using numerical libraries. The standard library that has been used to perform the integrations is the package LoopTools [11], which exploits the reduction of Feynman diagrams to linear combinations of Passarino-Veltman functions [12].

2.1 Passarino-Veltman reduction

The general one-loop tensor integral for a loop with Q propagators in D-dimensions is (for the direction of momenta see Fig. 2.1):

$$T^{\mu_1...\mu_p}_Q(p_1,\ldots,p_{Q-1},m_1,\ldots,m_Q) = \frac{(2\pi \mu)^2}{i \pi^2} \int d^D k \frac{k^\mu \cdots k^p}{N_1 \cdots N_Q}$$  \hspace{1cm} (2.1)

where $D = 4 - 2\epsilon$, the parameter $\mu$ is necessary to ensure that the integral has the correct dimension in a D-dimensional space and the denominators are:

$$N_1 = k^2 - m_1^2 + i\epsilon$$
$$N_2 = (k + p_1)^2 - m_2^2 + i\epsilon$$
$$\vdots$$
$$N_Q = (k + \cdots + p_{Q-1})^2 - m_Q^2 + i\epsilon$$  \hspace{1cm} (2.2)

Conventionally $T_1 \equiv A$, $T_2 \equiv B,\ldots$, thus to compute $2 \to 2$ processes, no more than D tensors will be necessary. Scalar integrals have an index 0. The scalar one-point function
Figure 2.1: General one-loop Feynman diagram

is:

\[ A_0(m_1) = -m_1^2 \left( \frac{m_1^2}{4\pi\mu^2} \right)^\epsilon \Gamma \left( 1 - \frac{D}{2} \right) = m_1^2 \left( \Delta - \log \frac{m_1^2}{\mu^2} + 1 \right) + O(\epsilon) \quad (2.3) \]

where:

\[ \Delta = \frac{1}{\epsilon} - \gamma_E + \log 4\pi \quad (2.4) \]

It is possible to expand the tensorial functions of B, C and D in terms of scalar functions \( B_j, C_j \) and \( D_j \):

\[ B^\mu(12) = p_1^\mu B_1(12) \]
\[ B^{\mu\nu}(12) = p_1^\mu p_1^\nu B_{21}(12) + g^{\mu\nu} B_{22}(12) \]
\[ B_j(12) = B_j(p_1^2; m_1, m_2) = B_j(p_2^2; m_1, m_2) \quad (2.5) \]

\[ C^\mu(123) = p_1^\mu C_{11}(123) + p_2^\mu C_{12}(123) \]
\[ C^{\mu\nu}(123) = p_1^\mu p_1^\nu C_{21}(123) + p_2^\mu p_2^\nu C_{22}(123) + (p_1^\mu p_2^\nu + p_2^\mu p_1^\nu) C_{23}(123) + g^{\mu\nu} C_{24}(123) \]
\[ C^{\mu\nu\rho}(123) = \sum_{i=1,2} C_{00i}(123)(g^{\mu\nu} p_i^\rho + g^{\nu\rho} p_i^\mu + g^{\mu\rho} p_i^\nu) + \sum_{i,j,k=1,2} C_{ijk}(123) p_i^\mu p_j^\nu p_k^\rho \]
\[ C_j(123) = C_j(p_1^2, p_2^2, p_3^2; m_1, m_2, m_3) \quad (2.6) \]
\[ D^\mu(1234) = p_1^\mu D_{11}(1234) + p_2^\mu D_{12}(1234) + p_3^\mu D_{13}(1234) \]
\[ D^{\mu\nu}(1234) = p_1^\mu p_1^\nu D_{21}(1234) + p_2^\mu p_2^\nu D_{22}(1234) + p_3^\mu p_3^\nu D_{23}(1234) \]
\[ + (p_1^\mu p_2^\nu + p_2^\mu p_1^\nu) D_{24}(1234) + (p_1^\nu p_3^\mu + p_3^\nu p_1^\mu) D_{25}(1234) \]
\[ + (p_2^\mu p_3^\nu + p_3^\mu p_2^\nu) D_{26}(1234) + g^{\mu\nu} D_{27}(1234) \]
\[ D^{\mu\nu\rho}(1234) = \sum_{i=1,2,3} D_{00i}(1234)(g^{\mu\nu} p_1^\rho + g^{\nu\rho} p_1^\mu + g^{\mu\rho} p_1^\nu) + \sum_{i,j,k=1,2,3} D_{ijk}(1234)p_i^\mu p_j^\nu p_k^\rho \]
\[ D_j(1234) = D_j(p_1^2, p_2^2, p_3^2, p_4^2, (p_1 + p_2)^2, (p_2 + p_3)^2; m_1, m_2, m_3, m_4) \] (2.7)

In the library \textit{LoopTools} the PV functions are expressed with a different notation. Internal momenta are defined by:
\[ k_1 = p_1 \]
\[ k_2 = p_1 + p_2 \]
\[ \vdots \]
\[ k_Q = p_1 + \ldots + p_Q \] (2.8)

and the tensorial coefficients are denoted in the following way:
\[ B^\mu = k_1^\mu B_1^L \]
\[ B^{\mu\nu} = k_1^\mu k_1^\nu B_{11}^L + g^{\mu\nu} B_{00}^L \] (2.9)
\[ C^\mu = k_1^\mu C_1^L + k_2^\mu C_2^L \]
\[ C^{\mu\nu} = \sum_{i,j=1}^2 k_i^\mu k_j^\nu C_{ij}^L + g^{\mu\nu} C_{00}^L \]
\[ C^{\mu\nu\rho} = \sum_{i,j,l=1}^2 k_i^\mu k_j^\nu k_l^\rho C_{ijl}^L + \sum_{i=1}^2 (g^{\mu\nu} k_i^\rho + g^{\nu\rho} k_i^\mu + g^{\mu\rho} k_i^\nu) C_{i00}^L \] (2.10)
\[ D^\mu = k_1^\mu D_1^L + k_2^\mu D_2^L + k_3^\mu D_3^L \]
\[ D^{\mu\nu} = \sum_{i,j=1}^3 k_i^\mu k_j^\nu D_{ij}^L + g^{\mu\nu} D_{00}^L \] (11.1)

where \( C_{ij...}^L \) and \( D_{ij...}^L \) are completely symmetric in \( \{i, j, \ldots\} \). Expanding and going to the conventional basis, it is possible to obtain relations between the standard definitions of PV functions and the functions computed by \textit{LoopTools}. These relations (which are not very illuminating and therefore will not be written) have been implemented in the
C++ codes used to compute the Feynman diagrams.

The PV functions exhibit a number of interesting properties that have been exploited to perform checks about the correctness of the computations.

- **Divergences**
  Some of the PV functions diverge, and their divergences must cancel in the total amplitude of the process. The divergent PV functions are:

\[
\begin{align*}
A_0(m_1^2) & \sim m_1^2\Delta \\
B_0(q^2, m_1^2, m_2^2) & \sim \Delta \\
B_1(q^2, m_1^2, m_2^2) & \sim -\frac{\Delta}{2} \\
B_{21}(q^2, m_1^2, m_2^2) & \sim \frac{\Delta}{3} \\
B_{22}(q^2, m_1^2, m_2^2) & \sim \frac{\Delta}{4}(m_1^2 + m_2^2 - \frac{q^2}{3}) \\
C_{24}(q^2, m_1^2, m_2^2, m_3^2) & \sim \frac{\Delta}{4} \\
C_{001}(q^2, m_1^2, m_2^2, m_3^2) & \sim -\frac{\Delta}{6} \\
C_{002}(q^2, m_1^2, m_2^2, m_3^2) & \sim -\frac{\Delta}{12}
\end{align*}
\]  

(2.12)

Since only a limited number of PV function is divergent, tests based on their divergence properties do not provide a complete check of the correctness of the computations. A more powerful type of check involves the asymptotic features of PV functions.

- **Sudakov Expansion**
  The cross section at high CM energies has a specific behaviour which is described by the Sudakov expansion in terms of logarithms of the CM energy itself. This expansion reads:

\[
\sigma_{\text{tot}} \simeq \sigma_{\text{tot}}^{\text{Born}} \left(1 + \sum_i a_i \log^2 \frac{q^2}{M_{ai}^2} + \sum_i b_i \log \frac{q^2}{M_{bi}^2} + \text{constant}\right)
\]  

(2.13)

where \(M_{ai}\) and \(M_{bi}\) are the masses of the particles appearing in the loops and the coefficient of the double and single logarithmic terms can be calculated following
the methods of Ref. [13]. The PV functions, however, can be Sudakov expanded as well [14], therefore, the coefficient calculated following the two methods should match and this is a very precise test of the correctness of the amplitude.

2.2 Montecarlo Procedure

In the C++ codes a Montecarlo procedure has been used to perform the integration of the partonic differential cross sections convoluted with the parton distribution functions. To illustrate this method the specific example of \( gg \rightarrow \tilde{t}_a \tilde{t}_b^* \) will be described. The case of \( bg \rightarrow \tilde{t}_i \chi_j \) is analogous and does not add further knowledge.

The invariant mass distribution is obtained from the partonic differential cross section through the relation (see also Sect. 3.4):

\[
\frac{d\sigma}{dM_{\text{inv}}} = \int dx_1 dx_2 d\cos \theta g(x_1, \mu) g(x_2, \mu) \times \frac{d\sigma_{gg-\tilde{t}_a \tilde{t}_b^*}}{d\cos \theta} \delta(\sqrt{x_1 x_2 S} - M_{\text{inv}}) \tag{2.14}
\]

where \( \sqrt{S} \) is the proton-proton c.m. energy (14 TeV at LHC), \( M_{\text{inv}} \) is the \( \tilde{t}_a + \tilde{t}_b^* \) invariant mass, \( \theta \) is the stop squark scattering angle in the partonic CM frame, and \( g(x_i, \mu) \) are the distributions of the gluon inside the proton with a momentum fraction \( x_i \) at the scale \( \mu \). In this section only the treatment of the integrand function is described, while the implementation of the procedure in the C++ code will be illustrated in Appendix A since it is rather technical. The random numbers generator spans the interval \([0, 1]\) with uniform distribution, therefore, integrating through the Montecarlo procedure requires to change the variables of integration (the parton momentum fractions, \( x_1 \) and \( x_2 \), and the cosine of the scattering angle \( \theta \)) into functions of these uniformly distributed random variables.

- Integration over the scattering angle

  The integration over \( \cos \theta \) is performed on the whole range of the \( \theta \) angle, thus the simplest function is:

  \[
  \cos \theta = 2\hat{x} - 1 \quad \Longrightarrow \quad d\cos \theta = 2d\hat{x} \tag{2.15}
  \]

  where \( \hat{x} \) is the random variable uniformly distributed between 0 and 1.
• Integration over the momentum fractions

A different approach must be considered in the case of \( x_1 \) and \( x_2 \). Partons with very high momentum fractions are quite unlikely, therefore a function which spans mainly over low values of these integration variables must be privileged. One possibility is to consider integration variables with a distribution of the type:

\[
P(x_i) \sim \frac{1}{x_i^{1+\beta}} \quad \text{with} \quad \alpha < x_i < 1 \tag{2.16}
\]

It is then possible to obtain an identity which relates a uniformly distributed random variable \( \hat{x}' \) to the integration variable distributed with a (normalised) probability density of the type (2.16):

\[
\int_{-\infty}^{\infty} d\hat{x}' P(\hat{x}') = \int_{\alpha}^{1} dx_i \frac{\beta}{\alpha^{-\beta} - 1} \frac{1}{x_i^{1+\beta}} \tag{2.17}
\]

where \( P(\hat{x}') = 1 \) in the interval \([0, 1]\) and zero elsewhere. From this identity it is possible to obtain a relation between \( x_i \) and \( \hat{x}' \):

\[
\int_{0}^{x} d\hat{x}' = \int_{\alpha}^{x} dx_i \frac{\beta}{\alpha^{-\beta} - 1} \frac{1}{x_i^{1+\beta}} \implies x_i = \left[ \alpha^{-\beta} + \hat{x}'(1 - \alpha^{-\beta}) \right]^{-\frac{1}{\beta}} \tag{2.18}
\]

The independent variables chosen for the computation of the integral are the momentum fraction \( x_1 \) and the product of momentum fractions \( y = x_1 x_2 \) (in this case a Jacobian \( J = \frac{dx_2}{dy} = \frac{1}{x_1} \) arises). \( x_1 \) is supposed to be distributed with a probability distribution which goes as \( \sim \frac{1}{x} \); this is the \( \beta \to 0 \) limit of the general case (2.16), which gives:

\[
P(x_1) = -\frac{1}{x_1 \log \alpha_1} \implies x_1 = \alpha_1^{-\hat{x}'} = e^{(1-\hat{x}')(\log \alpha_1)} \tag{2.19}
\]

\[
dx_1 = \log \alpha_1 e^{(1-\hat{x}')(\log \alpha_1)} d\hat{x}' \tag{2.20}
\]

Since \( y \) is the product of two quantities with the same probability distribution, the distribution for \( y \) is of the form \( \sim \frac{1}{x^2} \). This is the \( \beta \to 1 \) limit of the general case (2.16), which gives:

\[
P(y) = \frac{1}{\left(\frac{1}{\alpha_2} - 1\right) y^2} \implies y = \frac{1}{\alpha_2} + (1 - \frac{1}{\alpha_2}) \hat{x}'' \tag{2.21}
\]

\[
dy = -\frac{(1 - \frac{1}{\alpha_2})}{\left[\frac{1}{\alpha_2} + (1 - \frac{1}{\alpha_2}) \hat{x}''\right]^2} d\hat{x}'' \tag{2.22}
\]
What are the ranges of $\alpha_1$ and $\alpha_2$?

The minimum value for the product of momentum fraction is the threshold ratio $y_{\text{min}} = \frac{(m_a + m_b)^2}{S}$, therefore:

$$\hat{x}'' = [0, 1] \Rightarrow y = [\alpha_2, 1] \implies \alpha_2 = y_{\text{min}} \quad (2.23)$$

When $x_1$ reaches its maximum value ($x_1 = 1$), $x_2$ reaches its minimum ($x_2 = y$) and vice versa, therefore:

$$\hat{x}' = [0, 1] \Rightarrow x_1 = [\alpha_1, 1] \implies \alpha_1 = y \quad (2.24)$$

Thus $\alpha_1 = \alpha_1(\hat{x}'', \alpha_2)$.

Summing up, the function to integrate depends only on the variables $\hat{x}$, $\hat{x}'$ and $\hat{x}''$, which are uniformly distributed between 0 and 1:

$$\frac{d\sigma(pp \rightarrow \tilde{t}_a \tilde{t}_b^* + X)}{dM_{\text{inv}}} = \int d\hat{x} \, d\hat{x}' \, d\hat{x}'' \frac{1}{x_1(\hat{x}')} \left( -2 \log \alpha_1 \, e^{(1-\hat{x}') \log \alpha_1} \frac{1 - \frac{1}{\alpha_2}}{\left[ \frac{1}{\alpha_2} + (1 - \frac{1}{\alpha_2}) \hat{x}'' \right]^2} \right)$$

$$\times g(x_1(\hat{x}'), \mu) \, g(x_2(\hat{x}''), \mu) \times \frac{d\sigma_{gg-\tilde{t}_a \tilde{t}_b^*}}{d\hat{x}} \, \delta(\sqrt{x_1(\hat{x}') x_2(\hat{x}'') S - M_{\text{inv}}})$$

(2.25)

where, to avoid clutter, $\alpha_2$ and $\alpha_1$ have not been substituted with their values (2.23) and (2.24).

### 2.3 Regularisation of divergences

The library LoopTools automatically regularises the ultraviolet divergences through two parameters: $\mu$ and $\Delta$, already defined in (2.1) and (2.4). They are redundant since it is always possible to redefine $\mu$ to have the same effect as $\Delta$ as: $\mu_{\text{new}}^2 = e^\Delta \mu_{\text{old}}^2$. The C++ codes must not produce divergences: to test this requirement the parameter $\Delta$ has been varied while keeping $\mu$ fixed and the results have been verified to be independent of $\Delta$ (up to the numerical precision of the code).
The infrared divergences arise from terms involving virtual photons connected to charged external particles. To regularise these divergences, in the package LoopTools a photon mass $\lambda$ is introduced; $\lambda$ is treated as an infinitesimal quantity, so that only terms proportional to $\log \lambda$ are kept. In the C++ codes the IR divergences have been cancelled including the soft photon emission in the final state, and the cancellation of IR divergences has been tested by verifying that the results do not depend on the parameter $\lambda$ up to a limiting energy $E_{\gamma}^{\text{max}}$.

Further details on the regularisation of divergences will be given in dedicated sections in the following chapters.

### 2.4 Assumptions on mixing

The computation of the one-loop diagrams has been performed under some simplifying assumptions:

- **Mixing angles of sfermion**
  As already described in Sect. [1.3], only the mixing angles of the third family of squarks and sleptons have been kept, assuming a standard Minimal Flavour Violation scenario. Therefore, the physical states $(\alpha = 1, 2)$ of third family sfermions are obtained from chiral states $(i = L, R)$ through the relation:

  $$(f_3)_\alpha = R_{\alpha i}(f_3)_i$$  \hspace{1cm} (2.26)

  while sfermions belonging to the first two families have physical eigenstates that coincide with gauge eigenstates.

- **Mixing matrices of charginos and neutralinos**
  The mixing matrices for charginos, $Z^\pm$ defined in (1.20) and (1.21), and neutralinos, $Z^N$ defined in (1.23) and (1.24), are assumed to have real elements, and therefore to be orthogonal matrices.
Chapter 3

EW NLO description of the processes

\[ PP \xrightarrow{gg} \tilde{t}_a \tilde{t}^*_b \text{ and } PP \xrightarrow{gg} \tilde{b}_a \tilde{b}^*_b \]

3.1 Introduction

In this chapter the computation of the one loop electroweak corrections for the processes of stop-antistop and sbottom-antisbottom production, and a detailed analysis of the dependence of the corrections on mSUGRA parameters are presented.

The expected values of the cross sections for squark-antisquark pair production at LHC should allow their relatively quick identification and this might be particularly true for the case of final stops, that, as already mentioned in Sect. 1.3, are supposed to be the lightest squarks in the MSSM. Not surprisingly, three calculations of stop-antistop production already exist, two at the electroweak Born level (including NLO QCD effects) for diagonal [15] and non diagonal [16] production and a recent and very exhaustive one [17] for diagonal production at electroweak NLO. For what concerns the dependence on the MSSM involved parameters, at the Born level for diagonal production this is limited to the masses of the two produced stops, \( \tilde{t}_1 \) (conventionally the lighter one) and \( \tilde{t}_2 \). The production of sbottom-antisbottom, that might be an interesting source e.g. of very light stop subsequent decays, has also been studied in [16], together with the stop-antistop one, for both diagonal and non diagonal cases, also considering the mixed stop-sbottom production, at the Born level and for the two benchmark points SPS1a and SPS5.

Given these premises, the purpose of this analysis is twofold:

1. to perform in an independent way and for a large set of benchmark points the calcu-
3. EW NLO description of the processes $PP \to \tilde{t}_a \tilde{t}_a^* \tilde{b}_a \tilde{b}_a^*$ and $PP \to \tilde{t}_a \tilde{t}_a^* \tilde{b}_a \tilde{b}_a^*$

lation of the EW NLO effects on the diagonal and non-diagonal stop-antistop and sbottom-antisbottom productions.

2. to perform a search of extra SUSY parameter dependence (different from stop and sbottom masses) for all the processes considered at EW NLO.

This chapter is organised as follows: Sect. 3.2 describes the initial state, focusing on why only the gluon initiated process has been considered for the analysis; Sect. 3.3 contains the necessary details of the NLO electroweak calculation, including the treatment of the soft photon contribution; Sects. 3.4 and 3.5 contain the definition of the proposed observables and the various computed NLO effects for different choices of SUSY parameters together with the analysis of the results.

Part of the results described in this chapter are based on the paper [18].

### 3.2 The Initial State

The process of diagonal pair production of top and bottom squarks can be initiated by the same partonic initial states:

$$PP \to qq \to \tilde{t}_a \tilde{t}_a^* \tilde{b}_a \tilde{b}_a^*$$

$$PP \to gg \to \tilde{t}_a \tilde{t}_a^* \tilde{b}_a \tilde{b}_a^*$$

as depicted in Fig. 3.1.

![Fig. 3.1: The process of diagonal production of stop-antistop and sbottom-antisbottom.](image-url)
The non-diagonal pair production cannot be achieved at Born level for the $gg$-initiated process since the couplings $g\tilde{q}\tilde{q}^*$ and $gg\tilde{q}\tilde{q}^*$ are diagonal. Therefore, the EW contributions to non-diagonal stop or sbottom production can only arise from tree-level Z exchange for the $q\bar{q}$-initiated process and from the NLO computation for the $gg$-initiated process.

Since the PdF’s of the quarks are suppressed with respect to those of the gluons at the energy scale of the LHC, the EW NLO effects have been computed only for the gluon initiated process, while for the $q\bar{q}$-initiated process only a tree-level computation has been performed for the non-diagonal production, where LO results from Z exchange could be comparable to NLO results from the $gg$-initiated process.

3.3 Kinematics and Amplitudes

Only the case of stop pair production initiated by 2 gluons $gg \rightarrow \tilde{t}_a \tilde{t}_b^*$ is discussed in detail: the kinematics of the process $gg \rightarrow \tilde{b}_a \tilde{b}_b^*$ is completely analogous to the stop case, the only differences being the substitution of stop masses and mixing angles with sbottom ones.

Physical (mixed) stops and antistops are denoted as $\tilde{t}_a$ and $\tilde{t}_b^*$, with $a, b$ running over 1 and 2. They are obtained from the chirality states $\tilde{t}_i$, $i = 1, 2$ standing for L,R as in (2.26):

$$\tilde{t}_a = R_{a1} \tilde{t}_i$$  \hspace{1cm} (3.3)

or explicitly:

$$\tilde{t}_1 = \cos \theta_t \tilde{t}_L + \sin \theta_t \tilde{t}_R \hspace{1cm} \tilde{t}_2 = -\sin \theta_t \tilde{t}_L + \cos \theta_t \tilde{t}_R$$  \hspace{1cm} (3.4)

The momenta, polarisation vectors and helicities are defined by:

$$g(p_g, \epsilon(\lambda_g)) + g(p'_g, \epsilon'(\lambda'_g)) \rightarrow \tilde{t}_a(p_a) + \tilde{t}_b(p_b)$$  \hspace{1cm} (3.5)

The gluon polarisation vectors depend on the helicities as:

$$\epsilon(g) = \left(0; -\frac{\lambda_g}{\sqrt{2}}, -\frac{i}{\sqrt{2}}, 0\right) \hspace{1cm} \epsilon'(g) = \left(0; \frac{\lambda'_g}{\sqrt{2}}, -\frac{i}{\sqrt{2}}, 0\right)$$  \hspace{1cm} (3.6)

The Mandelstam variables are defined as:

$$s = (p_g + p'_g)^2 = (p_a + p_b)^2 \hspace{1cm} u = (p_g - p_b)^2 = (p'_g - p_a)^2 \hspace{1cm} t = (p_g - p_a)^2 = (p'_g - p_b)^2$$  \hspace{1cm} (3.7)
with:

\[ p_g = \frac{\sqrt{s}}{2} (1; 0, 0, 1) \quad p_g' = \frac{\sqrt{s}}{2} (1; 0, 0, -1) \quad (3.8) \]

\[ p_a = (E_a; p \sin \theta, 0, p \cos \theta) \quad p_b = (E_b; -p \sin \theta, 0, -p \cos \theta) \quad (3.9) \]

\[ E_a = \frac{s + m_a^2 - m_b^2}{2\sqrt{s}} \quad E_b = \frac{s + m_b^2 - m_a^2}{2\sqrt{s}} \quad p = \sqrt{E_a^2 - m_a^2} \quad \beta = \frac{2p}{\sqrt{s}} \quad (3.10) \]

The helicity amplitudes \( F_{\lambda_g, \lambda'_g} \), computed from the Feynman diagrams listed in the next Section using the polarisation vectors of Eq. (3.6), will appear with various combinations of colours of the external particles. Firstly, it is possible to write the colour structure in the form:

\[
F_{\lambda_g, \lambda'_g} = \{ F_{\lambda_g, \lambda'_g}^{ij} \left[ i f_{ijl} \left( \frac{\lambda_j}{2} \right) \right] + F_{\lambda_g, \lambda'_g}^{ij} \left[ \frac{1}{3} \delta_{ij} + d_{ij} \left( \frac{\lambda_j}{2} \right) \right] \\
+ F_{\lambda_g, \lambda'_g}^{ij} \left[ \left( \frac{\lambda_j^2}{4} \right) \right] + F_{\lambda_g, \lambda'_g}^{ij} \left( \frac{\lambda_j \lambda_i}{4} \right) + F_{\lambda_g, \lambda'_g}^{ij} \left( I \right) \} \alpha \beta \quad (3.11)
\]

where \( i, j \) running from 1 to 8 refer to the gluon colours and \( \alpha, \beta \) running from 1 to 3 refer to stop and antistop colours.

The polarised cross section of the process \( gg \rightarrow \bar{t}_a \bar{t}_b \) (averaged over initial and summed over final colours) reads:

\[
\frac{d\sigma(\lambda_g, \lambda'_g)}{d\cos \theta} = \frac{\beta}{2048\pi s} \sum_{col} |F_{\lambda_g, \lambda'_g}|^2 \quad (3.12)
\]

and the unpolarised cross section is:

\[
\frac{d\sigma}{d\cos \theta} = \frac{1}{4} \sum_{\lambda_g, \lambda'_g} \frac{d\sigma(\lambda_g, \lambda'_g)}{d\cos \theta} \quad (3.13)
\]

The colour summation can be explicitly written as:

\[
\sum_{col(ija)} |F_{\lambda_g, \lambda'_g}|^2 = 12|F_{\lambda_g, \lambda'_g}^{ij}|^2 + \frac{28}{3}|F_{\lambda_g, \lambda'_g}^{ij}|^2 + \frac{16}{3}(|F_{\lambda_g, \lambda'_g}^{ij}|^2 + |F_{\lambda_g, \lambda'_g}^{ij}|^2) \\
+ 12(F_{\lambda_g, \lambda'_g}^{ij} F_{\lambda_g, \lambda'_g}^{ij} - F_{\lambda_g, \lambda'_g}^{ij} F_{\lambda_g, \lambda'_g}^{ij}) + 28(F_{\lambda_g, \lambda'_g}^{ij} F_{\lambda_g, \lambda'_g}^{ij} + F_{\lambda_g, \lambda'_g}^{ij} F_{\lambda_g, \lambda'_g}^{ij}) \\
- \frac{4}{3} F_{\lambda_g, \lambda'_g}^{ij} F_{\lambda_g, \lambda'_g}^{ij} + 16 F_{\lambda_g, \lambda'_g}^{ij} F_{\lambda_g, \lambda'_g}^{ij} + 8(F_{\lambda_g, \lambda'_g}^{ij} F_{\lambda_g, \lambda'_g}^{ij} + F_{\lambda_g, \lambda'_g}^{ij} F_{\lambda_g, \lambda'_g}^{ij}) \\
+ 24|F_{\lambda_g, \lambda'_g}^{ij}|^2 \quad (3.14)
\]
3.3.1 The Born Terms

The Born terms exist only for diagonal stop-antistop pairs \((a \equiv b)\). They are given by 4 diagrams and the amplitude reads:

\[
A^{\text{Born}} = A^{\text{Born } A} + A^{\text{Born } A'} + A^{\text{Born } B} + A^{\text{Born } C} \tag{3.15}
\]

where the four parts are:

(A) s-channel gluon exchange:

\[
A^{\text{Born } A}_{ab} = \left[ if^{ijkl} \frac{\lambda^l}{2} \right] (4\pi\alpha_s)(\epsilon.\epsilon') \frac{t - u}{s} \delta_{ab} \tag{3.16}
\]

(A’) 4-leg \(g^i g^j \tilde{t}_a \tilde{t}_a\) diagram:

\[
A^{\text{Born } A'}_{ab} = \left[ \frac{1}{3} \delta_{ij} + d^{ijkl} \frac{\lambda^l}{2} \right] (4\pi\alpha_s)(\epsilon.\epsilon') \delta_{ab} \tag{3.17}
\]

(B) stop exchange in the t-channel:

\[
A^{\text{Born } B}_{ab} = -\frac{16\pi\alpha_s}{t - m_t^2} \frac{\lambda^i \lambda^j}{2} \left( \epsilon.p \right) \left( \epsilon'.p' \right) \delta_{ab} \tag{3.18}
\]


(C) stop exchange in the $u$-channel:

$$A_{ab}^{\text{Born}} C = -\frac{16\pi\alpha_s}{u - m_t^2} \left[ \frac{\lambda^j \lambda^i}{2} \right] (\epsilon_p)(\epsilon'_p)\delta_{ab}$$  \hspace{1cm} (3.19)

(where $\epsilon'_p = -\epsilon_p$ and $\epsilon_p' = -\epsilon_p$).

The Born terms involve only 2 invariant forms:

$$I_1 = (\epsilon_p)(\epsilon'_p) \quad I_2 = (\epsilon\epsilon')$$  \hspace{1cm} (3.20)

(and 4 colour components, $C = 1, 4$), so that writing the invariant amplitude as

$$A = N_1(s, t, u)I_1 + N_2(s, t, u)I_2$$  \hspace{1cm} (3.21)

the helicity amplitudes are given by:

$$F_{\lambda_g \lambda_g'} = -\frac{1}{2} \lambda_g \lambda_g' p^2 \sin^2 \theta N_1(s, t, u) + \frac{1}{2} (1 + \lambda_g \lambda_g') N_2(s, t, u)$$  \hspace{1cm} (3.22)

From Eqs. (3.12), (3.14) and (3.22) it is possible to obtain the polarised Born cross sections:

$$\frac{d\sigma^{\text{Born}}(\lambda_g, \lambda_g)}{d \cos \theta} = \frac{\pi\alpha_s^2\beta}{24s} \left( \frac{m_t^4}{s^2} \right) \left[ \frac{28 + 36\beta^2 \cos^2 \theta}{(1 - \beta^2 \cos^2 \theta)^2} \right]$$  \hspace{1cm} (3.23)

$$\frac{d\sigma^{\text{Born}}(\lambda_g, -\lambda_g)}{d \cos \theta} = \frac{\pi\alpha_s^2\beta^3}{384s} \left[ \frac{28 + 36\beta^2 \cos^2 \theta}{(1 - \beta^2 \cos^2 \theta)^2} \right] \sin^4 \theta$$  \hspace{1cm} (3.24)

in agreement with the results of Refs. [16, 19]. Note that, at this Born level, $\sigma^{\text{Born}}(++) = \sigma^{\text{Born}}(--)$ and $\sigma^{\text{Born}}(+-) = \sigma^{\text{Born}}(-+)$. 

It is useful, for later discussions of one loop effects, to emphasise the energy and angular dependences of the two types of polarised cross sections which are illustrated in Figs. 3.2(a) and 3.2(b). At low energy the dominant cross sections are the so-called [20, 21] Gauge Boson Helicity Violating (GBHV) ones $\sigma(++, --)$ of Eq. (3.23). This arises because the invariant form $I_2$ has no threshold suppression factor, contrarily to $I_1$ which
Figure 3.2: Differential cross section at parton level. Energy (a) and angular (b) dependence of the helicity violating (+++) and conserving (+-) components. The dashed lines include the one-loop corrections.
vanishes like $\beta^2$ near threshold. However at high energy the GBHV cross sections become mass suppressed like $m_{\tilde{t}_a}^4/s^2$, as one can check from Eq. (3.23), in agreement with the general HC rule of Refs. [20, 21]. Consequently, as it is possible to see in Fig. 3.2(a) between threshold ($2m_{\tilde{t}_a}$) and about $3m_{\tilde{t}_a}$, the stop pair is essentially produced through $\sigma(++,--)$, whereas for higher energies ($\sqrt{s} > 3m_{\tilde{t}_a}$) it is dominated by the GBHC cross sections ($\sigma(+-,--)$) of Eq. (3.24). Fig. 3.2(b) shows the corresponding angular distributions which appear to be also totally different in the two cases, larger for central angles in $\sigma(+-,--)$, see Eq. (3.24), as opposed to forward and backward peaks in $\sigma(++,--)$, Eq. (3.23). These various features will be essential for understanding the sensitivity to one loop effects in this process at LHC.

As shown in Ref. [17], the stop pair can also be produced through the $q\bar{q}$ channel, Fig. 3.3 with a cross section:

$$\frac{d\sigma}{d\cos \theta} = \frac{\pi \alpha_s^2 \beta^3}{18s} \sin^2 \theta$$

(3.25)

and through photon-induced mechanisms present only at NLO.

Figure 3.3: Born diagram for non-diagonal squark production $q\bar{q} \rightarrow \tilde{q}_a \tilde{q}_b^*$, $(a \neq b)$, via Z boson exchange.

Even if at LHC these processes are depressed as compared to the gluon-gluon one because of the smaller (quark) or non-existent (photon) PdF’s, the authors of Ref. [17] have shown that the one-loop corrections can be numerically significant or even bigger than those of the gluon fusion initiated process, but essentially independent of extra (i.e. different from the stop mass) SUSY parameters, and for this reason they are not included in this analysis.
3.3.2 EW NLO Corrections for Stop Pair Production

The one loop electroweak contributions come from counter terms (c.t.) and self-energy (s.e.) corrections to the Born terms, and from triangle and box diagrams, Fig. 3.4.

![Diagrams](image)

Figure 3.4: Self-energy, triangle and box (generic) diagrams for diagonal production $gg \rightarrow \tilde{t}_1 \tilde{t}_1$. 

In the on-shell scheme the c.t.+s.e. corrections are, for $a = b$:

$$A_{aa}^{Born+c.t.+s.e.} = A_{aa}^{Born} A [1 + \delta Z_{aa}]$$  \hspace{1cm} (3.26)

$$A_{aa}^{Born+c.t.+s.e.} = A_{aa}^{Born} A' [1 + \delta Z_{aa}]$$  \hspace{1cm} (3.27)
\[ A_{aa}^{\text{Born} + \text{c.t.} + \text{s.e.}} = A_{aa}^{\text{Born}} B [1 + 2\delta Z_{aa} - \frac{\hat{\Sigma}_{aa}(t)}{t - m^{2}_{t_a}}] \] (3.28)

\[ A_{aa}^{\text{Born} + \text{c.t.} + \text{s.e.}} = A_{aa}^{\text{Born}} C [1 + 2\delta Z_{aa} - \frac{\hat{\Sigma}_{aa}(u)}{u - m^{2}_{t_a}}] \] (3.29)

and for \( a \neq b \), using the relation \( A_{aa}^{\text{Born} A,A'} = A_{bb}^{\text{Born} A,A'} \):

\[ A_{ab}^{\text{Born} + \text{c.t.} + \text{s.e.}} = A_{aa}^{\text{Born}} \delta Z_{ba} \] (3.30)

\[ A_{ab}^{\text{Born} + \text{c.t.} + \text{s.e.}} = A_{aa}^{\text{Born}} A_{bb}^{\text{Born}} \delta Z_{ba} \] (3.31)

\[ A_{ab}^{\text{Born} + \text{c.t.} + \text{s.e.}} = A_{aa}^{\text{Born}} B [\delta Z_{ba} - \frac{\hat{\Sigma}_{ab}(t)}{2(t - m^{2}_{t_b})}] + A_{bb}^{\text{Born}} B [\delta Z_{ba} - \frac{\hat{\Sigma}_{ab}(t)}{2(t - m^{2}_{t_b})}] \] (3.32)

\[ A_{ab}^{\text{Born} + \text{c.t.} + \text{s.e.}} = A_{aa}^{\text{Born}} C [\delta Z_{ba} - \frac{\hat{\Sigma}_{ab}(u)}{2(u - m^{2}_{t_b})}] + A_{bb}^{\text{Born}} C [\delta Z_{ba} - \frac{\hat{\Sigma}_{ab}(u)}{2(u - m^{2}_{t_b})}] \] (3.33)

with the c.t. terms expressed in terms of stops self-energies:

\[ \delta Z_{aa} = \frac{d\Sigma_{aa}(p^2)}{dp^2} \bigg|_{p^2 = m^{2}_{t_a}} \] (3.34)

and for \( a \neq b \):

\[ \delta Z_{ba} = \frac{2\Sigma_{ba}(m^{2}_{t_a})}{m^{2}_{t_b} - m^{2}_{t_a}} \] (3.35)

\[ \delta Z_{ba} = \frac{1}{2} [\delta Z_{ba}^{*} + \delta Z_{ab}] \] (3.36)

the renormalised s.e. functions being given by:

\[ \hat{\Sigma}_{aa}(p^2) = \Sigma_{aa}(p^2) - \Sigma_{aa}(m^{2}_{t_a}) - (p^2 - m^{2}_{t_a}) \frac{d\Sigma_{aa}(p^2)}{dp^2} \bigg|_{p^2 = m^{2}_{t_a}} \] (3.37)

and for \( a \neq b \):

\[ \hat{\Sigma}_{ba}(p^2) = \Sigma_{ba}(p^2) + \frac{p^2 - m^{2}_{t_b}}{m^{2}_{t_b} - m^{2}_{t_a}} \Sigma_{ba}(m^{2}_{t_a}) + \frac{p^2 - m^{2}_{t_a}}{m^{2}_{t_a} - m^{2}_{t_b}} \Sigma_{ab}(m^{2}_{t_b}) \] (3.38)
The needed $\Sigma(p^2)$ functions are obtained from the various $(qV)$, $(\tilde{q}H)$, $(q\chi)$ bubbles and from the gauge boson (V) and the 4-leg $(SS\tilde{t}t)$ tadpoles depicted in Fig. 3.4.

Triangle and boxes corrections affect respectively each sector (A), (A’), (B) and (C) appearing in the Born case. In the s-channel one finds "left" and "right" triangles and in the t- and u-channels one has "up" and "down" ones. Contributions of sector (C) are obtained from those of sector (B) by symmetrisation rules for the 2 gluons: interchange of momenta, polarisation vectors and colours ($p_g, \epsilon(\lambda_g), i$) and ($p'_g, \epsilon'(\lambda'_g), j$). The 3 types of boxes can be identified through their (clockwise) internal contents $(SSV S)$, $(qq\chi q)$ and $(SSHS)$ for sector (B), the above symmetrisation rules giving the crossed sector (C); $S$ refer to all possible scalar states.

These electroweak corrections can also be classified into:

- **gauge** terms due to internal exchanges of gauge bosons ($V = \gamma, Z, W$) and of charginos, neutralinos (through their gaugino components),

- **Yukawa** terms due to exchanges of Higgs bosons ($H$), and also charginos, neutralinos (now through their higgsino components).

The contributions of these various diagrams to the helicity amplitudes are obtained after colour decomposition according to Eq. (3.11) and are expressed in terms of the Passarino-Veltman (PV) functions described in Sect. 2.1. The numerical computation is then done with dedicated C++ codes, stst and sbsb, exploiting the LoopTools library [11] and the Montecarlo techniques described in Sect. 2.2.

**Analytic Checks**

The first check of the computation is obtained by observing the cancellation of the divergences appearing in counter terms, self-energies, triangles and boxes. For some parts these cancellations occur separately in each sector, but for other parts they involve contributions from several sectors as required by gauge invariance.

Another type of check is provided by the high energy behaviour of the helicity amplitudes which has to satisfy a number of "asymptotic" rules (Sudakov expansion). As
already noticed in Sect. 3.3.1 at high energy, neglecting masses the only surviving Born helicity amplitudes obtained from the addition of (A+A’+B+C) terms are the GBHC ones:

$$F_{\lambda g, -\lambda g}^{\text{Born}} = (4\pi\alpha_s)\left[\frac{\sin^2 \theta}{2}c_{ij} + \frac{c'_{ij}}{1 + \cos \theta}\right]$$ \hspace{1cm} (3.39)

with:

$$c_{ij} = \frac{1}{3}\delta_{ij} + d_{ijl}(\lambda_l^2) + if_{ijl}(\lambda_l^2)$$

$$c'_{ij} = \frac{1}{3}\delta_{ij} + d_{ijl}(\lambda_l^2) - if_{ijl}(\lambda_l^2)$$ \hspace{1cm} (3.40)

in agreement with the theorem given in [20, 21], whereas the GBHV ones (with $\lambda_g = \lambda'_g$) are mass suppressed (vanish like $m^2/s$).

From the general logarithmic rules established in [13, 22], one expects the one loop virtual electroweak contributions to give, for final unmixed $L, R$ states (before applying the mixing matrices $R_{ai}$), the following corrections to the GBHC Born amplitudes:

$$F_{\lambda g, -\lambda g} = F_{\lambda g, -\lambda g}^{\text{Born}}[1 + C_{ii}]$$ \hspace{1cm} (3.41)

$$e_{L}^{L} = \frac{\alpha(1 + 26c^2_W)}{144\pi c^2_W s^2_W}[2\ln \frac{s}{M^2} - \ln^2 \frac{s}{M^2}] - \frac{\alpha(\tilde{m}_t^2 + \tilde{m}_b^2)}{8\pi s^2_W M^4_W}$$ \hspace{1cm} (3.42)

$$e_{R}^{L} = \frac{\alpha}{9\pi c^2_W}[2\ln \frac{s}{M^2} - \ln^2 \frac{s}{M^2}] - \frac{\alpha\tilde{m}_t^2}{4\pi s^2_W M^4_W}$$ \hspace{1cm} (3.43)

in which one identifies the "gauge" and the "Yukawa" parts. $M$ is a typical mass scale whose precise value does not matter at Log accuracy.

By taking the leading logarithmic expressions of the PV functions it has been checked analytically that the various self-energy, triangle and box contributions reproduce the above expressions in both gauge and Yukawa sectors.

### Infrared singularities

As usual, QED radiation effects can be split into a soft part which is infrared (IR) singular and a hard part including the emission of photons with an energy which is not small compared to the process energy scale. In this analysis, only the soft part which is necessary to cancel the IR singularities associated with the photonic virtual corrections has been included. The hard part of QED effects has not been included since the main focus of this analysis is to search for extra SUSY parameter dependence.
3.3. Kinematics and Amplitudes

\( A^{\text{Born}} \) and \( A^{1 \text{ loop}} \) denote any invariant helicity scattering amplitude evaluated at Born or one loop level. IR divergences are regulated by a small photon mass \( \lambda \); IR cancellation holds for every helicity channel separately and it has been checked numerically by taking the \( \lambda \to 0 \) limit of the calculation. The real radiation factorises on the Born amplitude leading to:

\[
(A^{\text{Born}})^2 \left(1 + \frac{\alpha}{2\pi} \delta_s \right) + 2 A^{\text{Born}} A^{1 \text{ loop}} = \text{IR finite.} \quad (3.45)
\]

The universal correction factor \( \delta_S \) takes into account the emission of soft real photons with energy from \( \lambda \) up to \( \Delta E^\gamma_{\text{max}} \ll \sqrt{s} \) \cite{23}. In this analysis, \( \Delta E^\gamma_{\text{max}} \) has been fixed at 0.1 GeV.

3.3.3 EW NLO Corrections for Sbottom Pair Production

The treatment of the one loop corrections for the sbottom case is again analogous to that of the stop, but the particles involved in the loops are different, so the numerical results for the one loop contributions obtained in the stop production process cannot be trivially transposed to the sbottom case. In practice, all the expression given in the above section have to be “mirrored” substituting every top-tagged quantity with its bottom-tagged counterpart. For this reason this section contains only a brief overview of the main differences that arise between the two processes.

Since the main parameters that control the processes are the masses of the final state squarks, the starting point to consider is how such masses affect the physical observables. As it is possible to see in Tab.\[3.1\] the masses of stop and sbottom squarks change within a wide range of values depending on the scenario considered, and the thresholds for the production of \( \tilde{q}, \tilde{q}^* \) vary accordingly affecting the values of the cross section. Thus, since at tree-level the only difference is the sbottom masses instead of the stop masses in \( t - m^2 \) and \( u - m^2 \), considering scenarios with not too different masses, the tree-level cross sections should be comparable. At high energy, however, all the masses can be neglected, so the cross sections are identical to a great approximation.

At one loop level, two types of differences appear: a) the different masses in the various propagators, b) the different couplings in gauge, SUSY gauge and Yukawa couplings. This second type can be very simply pointed out by comparing the Sudakov coefficients \( \delta_S \) for the two processes.
controlling the high energy behaviour:

\[ c_{\tilde{b}L\tilde{b}L} = c_{\tilde{t}L\tilde{t}L} = \frac{\alpha(1 + 26c_W^2)}{144\pi c_W^2 s_W^2} \left[ 2\ln \frac{s}{M^2} - \ln^2 \frac{s}{M_W^2} \right] - \frac{\alpha(\tilde{m}_t^2 + \tilde{m}_b^2)}{8\pi s_W^2 M_W^2} \left[ \ln \frac{s}{M^2} \right] \]

(3.46)

\[ c_{\tilde{t}R\tilde{t}L} = \frac{\alpha}{9\pi c_W^2} \left[ 2\ln \frac{s}{M^2} - \ln^2 \frac{s}{M_W^2} \right] - \frac{\alpha(\tilde{m}_t^2)}{4\pi s_W^2 M_W^2} \left[ \ln \frac{s}{M^2} \right] \]

(3.47)

\[ c_{\tilde{b}R\tilde{b}R} = \frac{\alpha}{36\pi c_W^2} \left[ 2\ln \frac{s}{M^2} - \ln^2 \frac{s}{M_W^2} \right] - \frac{\alpha(\tilde{m}_b^2)}{4\pi s_W^2 M_W^2} \left[ \ln \frac{s}{M^2} \right] \]

(3.48)

the only difference coming from the R part.

Again this should give only a slight difference at high energy when mass effects are negligible.

### 3.4 One-Loop Results

The starting observable for this process is the invariant mass distribution defined as:

\[
\frac{d\sigma(pp \rightarrow \tilde{t}_a \tilde{t}_b^* + X)}{dM_{\text{inv}}} = \int dx_1 \, dx_2 \, d\cos \theta \, g(x_1, \mu) \, g(x_2, \mu) \times \frac{d\sigma_{gg \rightarrow \tilde{t}_a \tilde{t}_b^*}}{d\cos \theta} \delta(\sqrt{s} x_1 x_2 S - M_{\text{inv}}),
\]

(3.49)

where \(\sqrt{s}\) is the proton-proton c.m. energy, \(M_{\text{inv}}\) is the \(\tilde{t}_a + \tilde{t}_b^*\) invariant mass, \(\theta\) is the stop squark scattering angle in the partonic c.m. frame, and \(g(x_1, \mu)\) are the distributions of the gluon inside the proton with a momentum fraction \(x_i\) at the scale \(\mu\). The parton distribution functions used in this analysis are the LO PDF set CTEQ6L [24] with \(\mu = m_{\tilde{t}_a} + m_{\tilde{t}_b}\).

As already mentioned, soft QED real radiation is included to cancel IR singularities. For the \(2 \rightarrow 2 + \gamma(\text{soft})\) process it is possible to identify \(M_{\text{inv}}\) with the partonic c.m. energy \(\sqrt{s}\).

The first observable that has been considered is the total rate \(\sigma_{\text{tot}}\) of the process defined by integrating the distribution \(d\sigma/dM_{\text{inv}}\) over the full range of invariant mass values, from the threshold \(m_{\tilde{t}_a} + m_{\tilde{t}_b}\), for the diagonal light squark production (\(\tilde{t}_1 \tilde{t}_1^*\)) and for the non-diagonal case (\(\tilde{t}_1 \tilde{t}_2^* + \tilde{t}_2 \tilde{t}_1^*\)).

The analysis has been performed for a choice of 12 mSUGRA inspired points: the eight SPS points (SPS1a, SPS1a’, SPS1a slope, SPS2-6) [25] which allow, as far as SPS1a, SPS1a’, SPS1a slope, SPS2 and SPS5 are concerned, a direct comparison with the results.
of [16], [17], the two SU1, SU6 ATLAS points [26] and two light SUSY scenarios LS1 and LS2 discussed in [27]. The values of the chosen set of parameters ($m_0$, $m_{1/2}$, $A_0$, $\tan \beta$ and sign $\mu$) are listed in Tab. 3.1 together with the values of stop and sbottom masses obtained through RG running equations from the mSUGRA input parameters using the codes SUSPECT [6] and FeynHiggs [28].

Table 3.1: mSUGRA benchmark points and masses of stops and sbottoms (all the values are in GeV).

<table>
<thead>
<tr>
<th></th>
<th>$m_0$</th>
<th>$m_{1/2}$</th>
<th>$A_0$</th>
<th>$\tan \beta$</th>
<th>$\mu$</th>
<th>$m_{\tilde{t}_1}$</th>
<th>$m_{\tilde{t}_2}$</th>
<th>$m_{\tilde{b}_1}$</th>
<th>$m_{\tilde{b}_2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LS1</td>
<td>300</td>
<td>150</td>
<td>-500</td>
<td>10</td>
<td>+</td>
<td>214.6</td>
<td>460.5</td>
<td>377.1</td>
<td>444.7</td>
</tr>
<tr>
<td>LS2</td>
<td>300</td>
<td>150</td>
<td>-500</td>
<td>50</td>
<td>+</td>
<td>224.7</td>
<td>430.4</td>
<td>301.6</td>
<td>399.3</td>
</tr>
<tr>
<td>SPS1a</td>
<td>100</td>
<td>250</td>
<td>-100</td>
<td>10</td>
<td>+</td>
<td>399.7</td>
<td>585.5</td>
<td>515.7</td>
<td>546.6</td>
</tr>
<tr>
<td>SPS1a'</td>
<td>0.4$m_{1/2}$</td>
<td>$m_{1/2}$</td>
<td>-0.4$m_{1/2}$</td>
<td>10</td>
<td>+</td>
<td>399.7</td>
<td>585.5</td>
<td>515.7</td>
<td>546.6</td>
</tr>
<tr>
<td>SPS2</td>
<td>1450</td>
<td>300</td>
<td>0</td>
<td>10</td>
<td>+</td>
<td>921.4</td>
<td>1289</td>
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<td>1540</td>
</tr>
<tr>
<td>SPS3</td>
<td>90</td>
<td>400</td>
<td>0</td>
<td>10</td>
<td>+</td>
<td>645.2</td>
<td>840.3</td>
<td>790.1</td>
<td>823.7</td>
</tr>
<tr>
<td>SPS4</td>
<td>400</td>
<td>300</td>
<td>0</td>
<td>50</td>
<td>+</td>
<td>540.1</td>
<td>692.5</td>
<td>614.9</td>
<td>687.2</td>
</tr>
<tr>
<td>SPS5</td>
<td>150</td>
<td>300</td>
<td>-1000</td>
<td>5</td>
<td>+</td>
<td>279.0</td>
<td>651.2</td>
<td>566.3</td>
<td>651.1</td>
</tr>
<tr>
<td>SPS6</td>
<td>150</td>
<td>300</td>
<td>0</td>
<td>10</td>
<td>+</td>
<td>494.6</td>
<td>675.6</td>
<td>617.0</td>
<td>649.4</td>
</tr>
<tr>
<td>SU1</td>
<td>70</td>
<td>350</td>
<td>0</td>
<td>10</td>
<td>+</td>
<td>566.4</td>
<td>754.0</td>
<td>698.6</td>
<td>729.8</td>
</tr>
<tr>
<td>SU6</td>
<td>320</td>
<td>375</td>
<td>0</td>
<td>50</td>
<td>+</td>
<td>634.1</td>
<td>794.7</td>
<td>712.1</td>
<td>785.8</td>
</tr>
</tbody>
</table>

The results are shown in the next Figures: only a limited number of curves that contain all the information that seems more relevant have been drawn. With this purpose, Fig. 3.5 shows the shape of the differential distribution $d\sigma/dM_{\text{inv}}$ with the related relative effect for two representative points, chosen as LS1 and SPS5, both for stop and sbottom production. It is possible to see that both for the stop and sbottom cases the relative effect is positive near the threshold, but drops to negative values in the high invariant mass region. The same feature persists in all the remaining considered points. This can be understood from the discussion of the various helicity amplitudes in Sec. (II). At large $M_{\text{inv}}$, the helicity conserving amplitude dominates with its Sudakov negative correction, while at small $M_{\text{inv}}$ the helicity violating amplitude is the larger one and receives a positive correction in a narrow region near the production threshold.

Nevertheless, due to the different masses of stops and sbottoms and to the different
Figure 3.5: Born and one-loop distribution $d\sigma/dM_{\text{inv}}$ for diagonal stop and sbottom production in LS1 and SPS5. The bottom panels show the relative effect.
particles involved in the loops, there are substantial differences between the two processes: in the stop case the positive relative effects in the very low mass region soon vanishes, approaching typically a -10% limit, while in the sbottom case it is possible to note that threshold effects (the peaks and troughs in the low \( M_{\text{inv}} \) region) are more pronounced and produce a typically bigger positive contribution which drops slowly to different limits in the high \( M_{\text{inv}} \) region. As a consequence, in the stop case one may expect to find a rather small effect in the total rate due to the cancellation between the corrections in these two regimes; in the sbottom case, by contrast, it is not possible, a priori, to predict whether the total one-loop effect will be positive or negative and to what extent, therefore to analyse the corrections to the Born results the numerical evaluation is necessary.

The numerical values of the total rates for the different benchmark points are shown in Tab. 3.2 together with the one-loop effects.

<table>
<thead>
<tr>
<th></th>
<th>( \sigma_{\text{Born}}^{gg \rightarrow \tilde{t}_1 \tilde{t}_1^*} )</th>
<th>( \sigma_{\text{1-loop}}^{gg \rightarrow \tilde{t}_1 \tilde{t}_1^*} )</th>
<th>1-loop effect</th>
<th>( \sigma_{\text{Born}}^{gg \rightarrow \tilde{b}_1 \tilde{b}_1^*} )</th>
<th>( \sigma_{\text{1-loop}}^{gg \rightarrow \tilde{b}_1 \tilde{b}_1^*} )</th>
<th>1-loop effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>LS1</td>
<td>27.47</td>
<td>27.00</td>
<td>-1.71</td>
<td>1.5769</td>
<td>1.5427</td>
<td>-2.17</td>
</tr>
<tr>
<td>LS2</td>
<td>22.03</td>
<td>21.51</td>
<td>-2.36</td>
<td>5.116</td>
<td>4.846</td>
<td>-5.28</td>
</tr>
<tr>
<td>SPS5</td>
<td>7.590</td>
<td>7.457</td>
<td>-1.75</td>
<td>0.1542</td>
<td>0.1561</td>
<td>1.23</td>
</tr>
<tr>
<td>SPS1a’</td>
<td>1.822</td>
<td>1.763</td>
<td>-3.24</td>
<td>0.307</td>
<td>0.300</td>
<td>-2.28</td>
</tr>
<tr>
<td>SPS1a</td>
<td>1.144</td>
<td>1.096</td>
<td>-4.19</td>
<td>0.2699</td>
<td>0.2615</td>
<td>-3.11</td>
</tr>
<tr>
<td>SPS6</td>
<td>0.3438</td>
<td>0.3288</td>
<td>-4.36</td>
<td>0.0912</td>
<td>0.0908</td>
<td>-0.44</td>
</tr>
<tr>
<td>SPS4</td>
<td>0.2047</td>
<td>0.1936</td>
<td>-5.42</td>
<td>0.0931</td>
<td>0.0900</td>
<td>-3.33</td>
</tr>
<tr>
<td>SU1</td>
<td>0.1538</td>
<td>0.1475</td>
<td>-4.10</td>
<td>0.04143</td>
<td>0.04157</td>
<td>0.34</td>
</tr>
<tr>
<td>SU6</td>
<td>0.0766</td>
<td>0.0731</td>
<td>-4.57</td>
<td>0.03658</td>
<td>0.03585</td>
<td>-2.00</td>
</tr>
<tr>
<td>SPS3</td>
<td>0.0687</td>
<td>0.0661</td>
<td>-3.78</td>
<td>0.01833</td>
<td>0.01852</td>
<td>1.04</td>
</tr>
<tr>
<td>SPS2</td>
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<td>0.00617</td>
<td>-1.28</td>
<td>0.000498</td>
<td>0.000520</td>
<td>4.42</td>
</tr>
</tbody>
</table>

Table 3.2: Total cross sections and one-loop corrections (in pb) for diagonal stop and sbottom production and for all mSUGRA benchmark points. The point SPS1a slope has not been included since it coincides with SPS1a at \( m_{1/2} = 250 \) GeV.

It is possible to get rid of the compensations of positive and negative correction for the case of stop production and of the threshold effects for the case of sbottom production.
defining the partial rate as:

\[ \sigma(M_{\text{inv}}) = \int_{M_{\text{inv}}}^{\infty} \frac{d\sigma}{dM'_{\text{inv}}} \, dM'_{\text{inv}}. \]  

(3.50)

This definition must not be confused with that of the conventional partial decay rates. In this case partial refers to the constrained integration range. In Fig. 3.6 the partial rates for stop and sbottom production in the LS1 and LS2 cases are shown. As one can see, raising the integration limit, the value of the rate decreases, but the relative one-loop electroweak effect increases. In the stop case, considering 1 pb as a realistic observable limit for the rates, one concludes that for a value of \( M_{\text{inv}} \) of approximately 1.2 TeV a relative effect of 10% on the partial rate would be present for both benchmark points. The behaviour of the one-loop effect for sbottom production is qualitatively similar to the stop case, but since the cross sections are quite small (1.5 pb and 5 pb for LS1 and LS2 respectively), it is possible to see that considering again 1 pb as a lower observable limit threshold effects are still present, so that to get rid of them one should integrate from higher values of \( M_{\text{inv}} \): with a partial rate of the order of 0.1 pb one obtains a one-loop effect of -6% in the LS1 case and of -10% in the LS2 case.

Fig. 3.7 show, for all points, the relative electroweak one-loop effects that one would find by considering first the various total rates, and secondly the partial rates that would correspond to an integration from \( 3 m_{\tilde{t}_1} \) (\( 3 m_{\tilde{b}_1} \)) to infinity (this choice is purely indicative).

The non diagonal total rate derived from one-loop gg electroweak diagrams has also been computed, for all benchmark points. Note that a calculation of the non diagonal rate, derived from \( q\bar{q} \) annihilation at Born level via \( Z \) exchange, already exists [16] for SPS5. In the stop case, the value that is obtained is larger than that coming from the kinematically depressed NLO QCD diagrams, and is equal to \( \sim 6 \cdot 10^{-4} \) pb. In the sbottom case, the value that is obtained is equal to \( \sim 1.5 \cdot 10^{-5} \) pb. Tab. 3.3 shows the values of the total cross section derived by the codes used in this analysis for the different benchmark points both for the one-loop gg diagrams and from the \( Z \) exchange calculations. The one-loop electroweak values are of the same size as those due to \( Z \) exchange and in some cases larger. This could have some relevance for the “meaningful” benchmark points. For example, in the LS2 stop production case, summing the one-loop with the \( Z \)-exchange contribution, it is possible to get a total rate of approximately \( 10^{-2} \) pb. This is a factor 15 larger than the SPS5 point of [16], but realistically hard for experimental detection.
Figure 3.6: LS1, LS2, partial rate $\sigma(M_{\text{inv}})$ and one-loop effect for stop and sbottom production.
3. EW NLO description of the processes $PP \frac{g}{g} \tilde{t}_a \tilde{t}_b^*$ and $PP \frac{g}{g} \tilde{b}_a \tilde{b}_b^*$

Figure 3.7: Summary plots showing the electroweak one-loop effect on the total cross section (circles) and on the partial rate obtained integrating $d\sigma/dM_{\text{inv}}$ from $3 m_{\tilde{t}_1}$ ($3 m_{\tilde{b}_1}$) to infinity (squares). Since in the SPS2 case the sbottom is very heavy, it has been excluded.
A similar conclusion might be drawn for the rates of the remaining meaningful points if one sums the one-loop with the \(Z\)-exchange contributions. The results obtained for the sbottoms are similar, but because of the tiny cross sections involved, the experimental confirmation of the predictions will again be problematic.

Nevertheless, it is possible to identify a sort of pattern in the differences between \(gg\)- and \(q\bar{q}\)-initiated processes. Both for stop and sbottom production, the cross section is larger in the \(gg\) case only for the scenarios LS2, SPS4 and SU6: these points share the same value of \(\tan \beta = 50\), so it seems that in the non-diagonal production process this parameter might be able to affect the observables.

<table>
<thead>
<tr>
<th></th>
<th>(\sigma_{q\bar{q}\to \tilde{t}_1 \tilde{t}_2^* + \tilde{t}_2 \tilde{t}_1^*})</th>
<th>(\sigma_{gg\to \tilde{t}_1 \tilde{t}_2^* + \tilde{t}_2 \tilde{t}_1^*})</th>
<th>(\sigma_{q\bar{q}\to \tilde{b}_1 \tilde{b}_2^* + \tilde{b}_2 \tilde{b}_1^*})</th>
<th>(\sigma_{gg\to \tilde{b}_1 \tilde{b}_2^* + \tilde{b}_2 \tilde{b}_1^*})</th>
</tr>
</thead>
<tbody>
<tr>
<td>LS2</td>
<td>0.0034</td>
<td>0.0058</td>
<td>0.0027</td>
<td>0.011</td>
</tr>
<tr>
<td>LS1</td>
<td>0.0026</td>
<td>0.0012</td>
<td>0.00020</td>
<td>0.000024</td>
</tr>
<tr>
<td>SPS5</td>
<td>0.00057</td>
<td>0.00049</td>
<td>0.000013</td>
<td>0.00000087</td>
</tr>
<tr>
<td>SPS1a</td>
<td>0.00054</td>
<td>0.00038</td>
<td>0.00020</td>
<td>0.000049</td>
</tr>
<tr>
<td>SPS6</td>
<td>0.00022</td>
<td>0.00013</td>
<td>0.000068</td>
<td>0.000067</td>
</tr>
<tr>
<td>SPS4</td>
<td>0.00017</td>
<td>0.00045</td>
<td>0.00016</td>
<td>0.0006</td>
</tr>
<tr>
<td>SU1</td>
<td>0.00011</td>
<td>0.00057</td>
<td>0.000040</td>
<td>0.000032</td>
</tr>
<tr>
<td>SU6</td>
<td>0.000080</td>
<td>0.00016</td>
<td>0.000081</td>
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</tr>
<tr>
<td>SPS3</td>
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<td>0.000017</td>
<td>0.000012</td>
</tr>
<tr>
<td>SPS2</td>
<td>0.00000044</td>
<td>0.00000023</td>
<td>(2.49 \times 10^{-9})</td>
<td>(2.1 \times 10^{-10})</td>
</tr>
</tbody>
</table>

Table 3.3: Total cross-sections (in pb) for non-diagonal stop and sbottom production starting from \(q\bar{q}\) and \(gg\).

### 3.5 Dependence on Supersymmetric Parameters

The search for extra SUSY parameter dependence has been performed in the following way. For each benchmark point, the relative one loop effect and rate have been calculated varying in turn one of the four conventional parameters (\(\tan \beta\), \(m_{1/2}\), \(m_0\) and \(A_0\)) in a reasonable range. For practical reasons, only the largely dominant \(\tilde{t}_1 \tilde{t}_1^*\) component of diagonal stop production has been considered; the value of the \(\tilde{t}_1\) mass which is generated by the variation of the chosen parameter is indicated in the plots. Only the complete numerical results for those cases that seem to be reasonably meaningful, in particular that
correspond to a total rate not below the 1 pb (extreme?) limit are shown in Figs. 3.8-3.10. This choice selects the set of LS1, LS2, SPS1a, SPS1a’, SPS1a slope and SPS5 benchmark points, but to perform a comparison with Ref. [17] the (perhaps academical) case of SPS2 has also been included. For the remaining benchmark points, given the negligible value of their rates, only the dominant relative one loop effects have been shown in Fig. 3.12.

Fig. 3.8 shows the results obtained for the point SPS5, which is perhaps the most relevant one. As a general feature, common to all the considered cases, the SUSY one loop effects are almost systematically negative and small, of the few percent size. For what concerns the dependence on the chosen parameter, for SPS5 the variation of $m_{1/2}$ can produce a maximal variation of the relative effect of approximately three percent. A smaller variation, of approximately 1.5 percent, is generated in the considered range of $\tan \beta$. Varying $m_0$ and $A_0$ has essentially no practical effect ($\sim$ below one percent) on the one loop contribution. The latter remains, in all cases, of the few percent at most. In Figs. 3.9-3.10 only the maximal relative variation and the corresponding parameter have been shown. This choice selects $\tan \beta$ for SPS1a, $m_{1/2}$ for SPS1a’, $\tan \beta$ for SPS2, $m_{1/2}$ for LS1 and LS2. To perform a comparison with Ref. [17], in the SPS1a’ case also the $\tan \beta$ dependence has been plotted. In all cases, the relative one loop effect is negative and small, typically of the one-two percent size.

A special case is that of the benchmark point SPS1a slope, where the parameters $m_{1/2}$, $m_0$ and $A_0$ are related. In this case, in Fig. 3.11 the variations with $m_{1/2}$ and $\tan \beta$ are shown. In the first case the relative negative effect can vary between one and four percent, remaining often in the three-four percent range. This represents the most relevant extra parameter dependence of this stop analysis. Varying $\tan \beta$ can produce a smaller ($\sim 1.5$) effect, with an overall negative relative contribution in the five percent region which a priori might be visible with a dedicated experimental search.

An important step is now the comparison with previous results. Concerning the total rates, the values obtained for the points SPS1a, SPS1a’, SPS2 and SPS5 essentially reproduce, taking the corresponding stop mass values, the gluon-gluon component of Ref. [17] Table 1. For the parameter dependence, Ref. [17] shows the SPS1a’ case but uses, apart from $\tan \beta$, a different set of parameters. A comparison of the $\tan \beta$ dependences for this point shows a qualitative agreement, i.e. a small and negative effect that increases with $\tan \beta$, although the values obtained in this analysis are slightly larger, in the three percent
Figure 3.8: SPS5: scans over $\tan \beta$, $m_{1/2}$, $m_0$ and $A_0$ for diagonal stop production. Top panels show the 1-loop effect on the total cross section, bottom panels show the variation in the value of the total cross section; the numbers above the curves in the bottom panels represent the value of the stop mass $m_{\tilde{t}_1}$ (in GeV).
Figure 3.9: SPS1a, SPS1a’ and SPS2: scans over \( \tan \beta \) for diagonal stop production.
Figure 3.10: LS1, LS2 and SPS1a’: scans over $m_{1/2}$ for diagonal stop production.
Figure 3.11: SPS1a slope: scans over $\tan \beta$ and $m_{1/2}$ for diagonal stop production.
range. It is possible to conclude in this case, in full agreement with Ref. [17], that the
dependence on the extra SUSY parameters is for SPS1a’ extremely small.

Another comparison can be performed for the SPS1a slope and SPS5 cases with the
plots of Ref. [16]. Again it is possible to notice an essential agreement between the one-
loop results of this analysis and the Born results of Ref. [16], as one would expect given
the smallness of the obtained one loop effects.

The conclusion is that supersymmetric contributions due to extra SUSY parameters
exist, but are generally apparently too small, at the few percent level, to produce an ap-
preciable effect under realistic LHC experimental conditions, at least in a first luminosity
phase.
Figure 3.12: Dominant parameter dependence on one loop effects for the benchmark points with small cross section for diagonal stop production.
The next step has been that of repeating the analysis for the diagonal sbottom-antisbottom production. Here only the LS1 and LS2 points, that would have a rate of the pb size, have been considered. As it is possible to see, the dependence on $\tan\beta$, Fig. 3.13, exhibits a possibly appreciable feature. The negative effect regularly increases with $\tan\beta$, like in the stop cases, but changing more, i.e. from $\sim$2 percent to $\sim$6 percent in the explored range. In particular, a relative effect of approximately six percent, in correspondence to a rate of the 5 pb size, might be, in principle, proposed for a highly dedicated experimental search.

![Figure 3.13: LS1 and LS2: scan over the mSUGRA parameter $\tan\beta$ for diagonal sbottom production.](image)

Figs. 3.14-3.15 show the scans on the remaining mSUGRA parameters. The dependence of the effects on $m_0$ and $A_0$ is essentially negligible. For $m_{1/2}$ there is also no dependence on LS1, and a larger but irregular dependence (same values for different $m_{1/2}$) on LS2.

In conclusion, a reasonable picture that seems to emerge from this combined analysis of the stop-antistop and sbottom-antisbottom diagonal production processes is that of a possible, although mild, dependence of the one-loop electroweak effect in the mSUGRA scenario on extra parameters.
Figure 3.14: LS1: scan over $m_{1/2}$, $m_0$ and $A_0$ for diagonal sbottom production.
Figure 3.15: LS2: scan over $m_{1/2}$, $m_0$ and $A_0$ for diagonal sbottom production.
3. EW NLO description of the processes $PP \overset{gg}{\rightarrow} t_t \bar{t}_t$ and $PP \overset{gg}{\rightarrow} b_b \bar{b}_b$
Chapter 4

EW NLO description of the processes

\[ PP^{bg} \rightarrow \tilde{t}_a \tilde{\chi}^-_i \]

4.1 Introduction

In this chapter the analysis of the one-loop electroweak effects on the process of stop-chargino production is presented, together with a detailed analysis of the dependence on supersymmetric parameters at Born level.

For associated stop-chargino production the available calculations have been performed at NLO for the QCD component [29, 30]. The latter effect has been found to be large, and the very positive feature has been the sensible reduction of the theoretical QCD uncertainty.

This chapter is organised as follows: Sect. 4.2 describes the initial state of the process; Sect. 4.3 will be devoted to a description of the kinematics and of the shape and basic properties of parton level amplitudes at Born and at one-loop level. Illustrations will be given for the angular distributions and for the invariant mass dependence of the helicity amplitudes near threshold and in the high energy range, where a check of the agreement with the logarithmic terms of the Sudakov expansion will be discussed; Sect. 4.4 will exhibit the numerical one-loop effects on the production rates for a large number of typical SUSY benchmark points. Given the relatively small value of the one-loop effects, a search at the apparently safe Born level of possible dependences of the predictions on a couple of dimensionless parameters, chosen as \(\tan \beta\) and \(\theta_i\) and on the MSSM parameter \(\mu\) will be presented in Sect. 4.5; this will be done assuming a previous determination of the relevant
masses (stop and chargino) of the process. For sufficiently small values of the latter masses, an interesting strong dependence on the dimensionless parameters will appear and will be shown in some detail for realistic cases of “potentially LHC visible” rates. The results described in this chapter are based on Refs. [31, 32] and on a contribution for Ref. [33].

4.2 The Initial State

The process of production of stop and chargino is initiated by the configuration depicted in Fig. 4.1 in which there are three particles in the final state. As it is possible to see, the bottom quark is not a real parton as light quarks or gluons, but is generated from the splitting of a gluon parton. This process, however, produces large logarithms which arise from collinearities of the bottom-antibottom pair with the gluon itself, and this makes the process non perturbative [34]. It is possible to restore the perturbative nature of the calculation introducing an effective bottom PdF (see Fig. 4.2) and considering a 2→2 process where the collinearities arising from the $\bar{b}$ are included into the NLO QCD corrections of the bottom PdF.
4.3 Kinematics and Amplitudes

The kinematics of the process $b \, g \rightarrow \tilde{t}_a \, \chi_{-i}$ is expressed in terms of the Dirac spinors $u(p_b, \lambda_b)$ and $\bar{v}_c(p_{\chi_{-i}}, \lambda_{\chi_{-i}})$ with the momenta:

\[
\begin{align*}
  p_b &= (E_b; 0, 0, p) \\
  p_{\tilde{t}_a} &= (E_{\tilde{t}_a}; p' \sin \theta, 0, p' \cos \theta) \\
  p_g &= (p; 0, 0, -p) \\
  p_{\chi_{-i}} &= (E_{\chi_{-i}}; -p' \sin \theta, 0, -p' \cos \theta)
\end{align*}
\]

and the gluon polarisation vector:

\[
  e_g(\lambda_g) = (0; \frac{\lambda_g}{\sqrt{2}}, -\frac{i}{\sqrt{2}}, 0)
\]

referring to the helicity labels $\lambda_b = \pm \frac{1}{2}$, $\lambda_g = \pm 1$, $\lambda_{\chi_{-i}} = \pm \frac{1}{2}$. $E_b$ and $E_{\chi}$ are the b quark and chargino c.m. energies and the scattering angle $\theta$ refers to $p_{\tilde{t}_a}$ and $p_b$.

The Mandelstam variables for the process are:

\[
  s = (p_b + p_g)^2 \quad t = (p_b - p_{\tilde{t}_a})^2 \quad u = (p_b - p_{\chi_{-i}})^2
\]

The process will be described by 8 helicity amplitudes $F(\lambda_b, \lambda_g, \lambda_{\chi_{-i}})$ related to the 8 invariant amplitudes ($k = 1, 4$ and $\eta = R, L$):

\[
  A = \left(\frac{\lambda}{2}\right)_{b,\tilde{f}_a} \sum_{k,\eta} \left[ \bar{v}_c(p_{\chi_{-i}}) J_{k\eta} u(p_b, \lambda_b) \right] N_{k\eta}(s, t, u)
\]
\[ J_{1\eta} = \bar{\psi}_g P_\eta \]
\[ J_{2\eta} = (e \cdot p_\chi) P_\eta \]
\[ J_{3\eta} = \bar{\psi} P_\eta \]
\[ J_{4\eta} = (e \cdot p_\chi) \bar{\psi}_g P_\eta \]  

(4.5)

with \( P_\eta = P_{R,L} = (1 \pm \gamma^5)/2 \). A colour matrix element \((\lambda/2)_{b,\tilde{t}_a}\) relating the initial \( b \) quark and the final \( \tilde{t}_a \) squark has been systematically factorised out.

Averaging over initial spins and colours and summing over final spins and colours with:

\[ \sum_{\text{col}} \frac{\lambda^I}{2} \otimes \frac{\lambda^I}{2} = 4 \]  

(4.6)

leads to the partonic differential cross section:

\[ \frac{d\sigma_{bg \rightarrow \tilde{t}_a \chi^{-i}}}{d \cos \theta} = \frac{\beta'}{768 \pi s \beta} \sum_{\text{spins}} |F_{\lambda_b,\lambda_g,\lambda_\chi}|^2 \]  

(4.7)

where \( \beta = 2p/\sqrt{s}, \beta' = 2p'/\sqrt{s} \).

The 8 scalar functions \( N_{k\eta}(s,t,u) \) are obtained in terms of Born and one-loop diagrams.

### 4.3.1 The Born Terms

The Born terms, depicted in Fig. 4.3, result from the s-channel \( b \) exchange and the u-channel \( \tilde{t}_a \) exchange:

\[ N_{1\eta}^{\text{Born } s} = -g_s A^q_{i}(\tilde{t}_a) \]
\[ N_{2\eta}^{\text{Born } u} = 2g_s A^q_{i}(\tilde{t}_a) \]

(4.8)

(4.9)

with the couplings:

\[ A^L_i(\tilde{t}_L) = -\frac{e}{s_W} Z^+_{2i} \]
\[ A^L_i(\tilde{t}_R) = \frac{e m_t}{\sqrt{2} M_W s_W \sin \beta} Z^+_{2i} \]
\[ A^R_i(\tilde{t}_L) = \frac{e m_b}{\sqrt{2} M_W s_W \cos \beta} Z^-_{2i} \]

(4.10)

The Born level helicity amplitude is therefore:

\[ F_{\lambda_b,\lambda_g,\lambda_\chi} = \sum_{k=L,R} \sum_{k=1,2} N_k^q \mathcal{H}_{k,\lambda_\chi,\lambda_g,\lambda_\chi} \]

(4.11)
4.3. Kinematics and Amplitudes

(a) s-channel bottom exchange

(b) u-channel stop exchange

Figure 4.3: Born diagrams for stop-chargino production

where

\[
\mathcal{H}_{1,+++}^\eta = -\frac{pR}{\sqrt{2}}(1 + r_b)(1 + \eta)(1 - r_\chi) \cos \frac{\theta}{2}
\]  
\[\text{(4.12)}\]

\[
\mathcal{H}_{1,++-}^\eta = -\frac{pR}{\sqrt{2}}(1 + r_b)(1 + \eta)(1 + r_\chi) \sin \frac{\theta}{2}
\]  
\[\text{(4.13)}\]

\[
\mathcal{H}_{1,-+-}^\eta = \frac{pR}{\sqrt{2}}(1 + r_b)(1 - \eta)(1 + r_\chi) \sin \frac{\theta}{2}
\]  
\[\text{(4.14)}\]

\[
\mathcal{H}_{1,---}^\eta = -\frac{pR}{\sqrt{2}}(1 + r_b)(1 - \eta)(1 - r_\chi) \cos \frac{\theta}{2}
\]  
\[\text{(4.15)}\]

\[
\mathcal{H}_{2,+++}^\eta = -\frac{p'R}{\sqrt{2}} \sin \theta (1 + \eta r_b - r_\chi (\eta + r_b)) \sin \frac{\theta}{2}
\]  
\[\text{(4.16)}\]

\[
\mathcal{H}_{2,++-}^\eta = \frac{p'R}{\sqrt{2}} \sin \theta (1 + \eta r_b - r_\chi (\eta + r_b)) \sin \frac{\theta}{2}
\]  
\[\text{(4.17)}\]

\[
\mathcal{H}_{2,+++}^\eta = \frac{p'R}{\sqrt{2}} \sin \theta (1 + \eta r_b + r_\chi (\eta + r_b)) \cos \frac{\theta}{2}
\]  
\[\text{(4.18)}\]

\[
\mathcal{H}_{2,+-+}^\eta = -\frac{p'R}{\sqrt{2}} \sin \theta (1 + \eta r_b + r_\chi (\eta + r_b)) \cos \frac{\theta}{2}
\]  
\[\text{(4.19)}\]

\[
\mathcal{H}_{2,--+}^\eta = \frac{p'R}{\sqrt{2}} \sin \theta (1 - \eta r_b - r_\chi (\eta - r_b)) \cos \frac{\theta}{2}
\]  
\[\text{(4.20)}\]

\[
\mathcal{H}_{2,-++}^\eta = -\frac{p'R}{\sqrt{2}} \sin \theta (1 - \eta r_b - r_\chi (\eta - r_b)) \cos \frac{\theta}{2}
\]  
\[\text{(4.21)}\]

\[
\mathcal{H}_{2,-+-}^\eta = \frac{p'R}{\sqrt{2}} \sin \theta (1 - \eta r_b + r_\chi (\eta - r_b)) \sin \frac{\theta}{2}
\]  
\[\text{(4.22)}\]

\[
\mathcal{H}_{2,--+}^\eta = -\frac{p'R}{\sqrt{2}} \sin \theta (1 - \eta r_b + r_\chi (\eta - r_b)) \sin \frac{\theta}{2}
\]  
\[\text{(4.23)}\]

The kinematical parameters \(R, r_b,\) and \(r_\chi\) appearing in the above expressions are defined...
as

\[ R = \sqrt{(E_b + m_b)(E_\chi + m_\chi)}, \quad (4.24) \]

\[ r_b = \frac{p}{E_b + m_b}, \quad (4.25) \]

\[ r_\chi = \frac{p'}{E_\chi + m_\chi}, \quad (4.26) \]

Their basic properties will be essential to understand the final results. First, because of the small value of \( m_b \), the \( b \) helicity corresponds to the chirality \( \eta \) (L for \( \lambda_b = -1/2 \) and R for \( \lambda_b = +1/2 \)). In the case of the production of the lightest chargino (\( i = 1 \)) this means that the \( \lambda_b = -1/2 \) amplitudes will generally dominate because the R chirality couplings (see Eq. (4.10)) are depressed by the \( m_b \) factor and by the non-diagonal chargino mixing element \( Z_{21}^* \). It is now possible to predict the main features of the angular and of the energy dependences. At low energy (near above threshold) the u-channel contribution is suppressed by the final momentum \( p' \). Only the s-channel contribution survives and the leading amplitudes should be \( F_{-++} \) and \( F_{----} \). They respectively produce an angular distribution \((1 - \cos \theta)\) and \((1 + \cos \theta)\). Having the same magnitude (at Born level) the unpolarised cross section should then be flat.

At High energy \((\sqrt{s} \gg m)\) one observes a cancellation between the s-channel and the u-channel Born contributions to \( F_{+++, F_{++-}, F_{--}, F_{--}}, \) as well as the mass suppression of the u-channel contribution to \( F_{++-}, F_{--} \) (because \( r_b \) and \( r_\chi \) tend to 1). The only surviving amplitudes at high energy are then \( F_{----} \) and \( F_{+++} \):

\[
F_{++-}^{\text{Born}} u \rightarrow g_s \sqrt{2} \bar{A}_L^R (\tilde{t}_a) \sin \frac{\theta}{2} \quad F_{----}^{\text{Born}} u \rightarrow -g_s \sqrt{2} \bar{A}_L^L (\tilde{t}_a) \sin \frac{\theta}{2} \quad (4.27)
\]

In this high energy limit the quantities of Eq. (4.27) can be expressed in terms of 3 basic amplitudes, one of gaugino type \( F_{+++}(\tilde{t}_L) \) and two of higgsino type \( F_{++-}(\tilde{t}_R), F_{--}(\tilde{t}_L) \).

In all cases the high energy distribution should tend to a \((1 - \cos \theta)\) shape. For \( \chi_1 \) production and for the reasons already given above, \( F_{+++} \) should dominate. For a light stop \( \tilde{t}_1 \), mixture of \( \tilde{t}_L \) and \( \tilde{t}_R \), this amplitude will be:

\[
\cos \theta \tilde{t} F_{+++}(\tilde{t}_L) + \sin \theta \tilde{t} F_{++-}(\tilde{t}_R) \quad (4.28)
\]

### 4.3.2 EW NLO Corrections

For the calculation of the one-loop amplitude the on-shell scheme has been used. The one-loop electroweak terms can be classified in:
— counter terms for \( b, \tilde{t}_a, \chi_i^- \) lines, coupling constants and mixing elements, all of them being expressed in terms of self-energy diagrams;

— self-energy corrections for \( b \) and \( \tilde{t}_a \) propagators;

— s-channel left and right triangles;

— u-channels bubbles with 4-leg couplings and up, down triangles;

— direct boxes, crossed boxes, twisted boxes;

The loop diagrams are represented in Fig. 4.4.
The contributions to the s-channel of the counter terms terms are, symbolically:

\[ N_{1L}^{c.t.} = - \frac{g_s \langle \xi \rangle}{s - m_b^2} \left\{ \frac{3}{2} \delta Z_R^b A_i^L (\tilde{t}_a) + \frac{1}{2} \sum_{a'} \delta Z_{a'a}^s A_i^L (\tilde{t}_{a'}) \right\} + \frac{1}{2} \sum_{j} \delta \chi_{jL}^L A_j^L (\tilde{t}_a) \]  

\[ N_{1R}^{c.t.} = - \frac{g_s \langle \xi \rangle}{s - m_b^2} \left\{ \frac{3}{2} \delta Z_R^b A_i^R (\tilde{t}_a) + \frac{1}{2} \sum_{a'} \delta Z_{a'a}^s A_i^R (\tilde{t}_{a'}) \right\} + \frac{1}{2} \sum_{j} \delta \chi_{jR}^R A_j^R (\tilde{t}_a) \]  

\[ N_{3L}^{c.t.} = - \frac{m_b g_s \langle \xi \rangle}{s - m_b^2} \left\{ \left( \delta Z_L + \frac{1}{2} \delta Z_R^b \right) A_i^L (\tilde{t}_a) + \frac{1}{2} \sum_{a'} \delta Z_{a'a}^s A_i^L (\tilde{t}_{a'}) \right\} + \frac{1}{2} \sum_{j} \delta \chi_{jL}^L A_j^L (\tilde{t}_a) \]  

\[ N_{3R}^{c.t.} = - \frac{m_b g_s \langle \xi \rangle}{s - m_b^2} \left\{ \left( \delta Z_L + \frac{1}{2} \delta Z_R^b \right) A_i^R (\tilde{t}_a) + \frac{1}{2} \sum_{a'} \delta Z_{a'a}^s A_i^R (\tilde{t}_{a'}) \right\} + \frac{1}{2} \sum_{j} \delta \chi_{jR}^R A_j^R (\tilde{t}_a) \]  

and from b s.e. one gets \((\eta = +1, -1)\ means \(R, L)\):

\[ N_{1\eta}^{s.e.} = g_s \langle \xi \rangle \left\{ \frac{1}{2} \left( \Sigma_b^b - \frac{1}{2} \delta Z_L^b \right) \right\} A_i^L (\tilde{t}_a) (s(\Sigma_b^b - \frac{1}{2} \delta Z_L^b) + m_b^2 (\Sigma_{-\eta}^b - \frac{1}{2} \delta Z_{-\eta}^b) \right\} \]  

\[ N_{3\eta}^{s.e.} = g_s \langle \xi \rangle \left\{ \frac{1}{2} \left( \Sigma_b^b - \frac{1}{2} \delta Z_L^b \right) \right\} A_i^L (\tilde{t}_a) (s(\Sigma_b^b - \frac{1}{2} \delta Z_L^b) + m_b^2 (\Sigma_{-\eta}^b - \frac{1}{2} \delta Z_{-\eta}^b) \right\} \]  

For the u-channel c.t. one obtains:

\[ N_{2L}^{c.t.} = 2 g_s \langle \xi \rangle \left\{ \frac{1}{2} \delta Z_L^b A_i^L (\tilde{t}_a) \right\} \left( \frac{1}{u - m_{t_a}^2} \right) + \frac{1}{2} \sum_{a'} \delta Z_{a'a}^s A_i^L (\tilde{t}_{a'}) \left( \frac{1}{u - m_{t_{a'}}^2} \right) \]  

\[ + \frac{1}{2} \sum_{j} \delta \chi_{jL}^L A_j^L (\tilde{t}_a) \left( \frac{1}{u - m_{t_a}^2} \right) \]  

\[ + \frac{1}{2} \sum_{a'} \delta Z_{a'a}^s A_i^L (\tilde{t}_{a'}) \left( \frac{1}{u - m_{t_{a'}}^2} \right) \]  

\[ + \frac{1}{2} \sum_{j} \delta \chi_{jL}^L A_j^L (\tilde{t}_a) \left( \frac{1}{u - m_{t_a}^2} \right) \]
N_{2R}^{a.e.} = -2g_s(\frac{\lambda}{2}) \sum_{a'} \tilde{v}_c(\chi_i^-) [A^L_t(\tilde{\chi}_i^-) P_L + A^R_t(\tilde{\chi}_i^-) P_R] \frac{\hat{\Sigma}_{a'a}(u)}{u - m_{t_a}^2} \tag{4.37}

and from \tilde{\ell}_a s.e.:

N_{2s}^{a,e.} = -2g_s(\frac{\lambda}{2}) \sum_{a'} \tilde{v}_c(\chi_i^-) [A^L_t(\tilde{\chi}_i^-) P_L + A^R_t(\tilde{\chi}_i^-) P_R] \frac{\hat{\Sigma}_{a'a}(u)}{u - m_{t_a}^2} \tag{4.38}

The renormalised self-energy \( \hat{\Sigma}_{a'a}(u) \) is defined below. Following [35, 36, 37, 38] it is possible to write:

\[
\delta Z_{ba} = \frac{2\Sigma_{ba}(m_a^2)}{m_b^2 - m_a^2}, \quad \delta Z_{aa} = -\left[ \frac{d\Sigma_{aa}(p^2)}{dp^2} \right]_{p^2 = m_a^2} \tag{4.39}
\]

These results allow to write the renormalised stop self-energies as:

\[
\hat{\Sigma}_{aa}(p^2) = \Sigma_{aa}(p^2) - \Sigma_{aa}(m_a^2) - (p^2 - m_a^2) \left[ \frac{d\Sigma_{aa}(p^2)}{dp^2} \right]_{p^2 = m_a^2} \tag{4.40}
\]

and for \( a \neq b \)

\[
\hat{\Sigma}_{ba}(p^2) = \Sigma_{ba}(p^2) + \frac{p^2 - m_a^2}{m_b^2 - m_a^2} \Sigma_{ba}(m_a^2) + \frac{p^2 - m_a^2}{m_a^2 - m_b^2} \Sigma_{ab}(m_b^2) \tag{4.41}
\]

The renormalization condition on the mixing angle is defined [37] in order to ensure the finiteness of the squark vertices.

\[
\delta \theta_t = \frac{\Sigma_{12}(m_t^2) + \Sigma_{21}(m_t^2)}{2(m_t^2 - m_\tau^2)} = \frac{1}{4} [\delta Z_{12} - \delta Z_{21}] \tag{4.42}
\]

which gives the needed:

\[
\delta R_{1L} = \delta R_{2R} = \delta \cos \theta_t = -\sin \theta_t \delta \theta_t \quad \delta R_{1R} = -\delta R_{2L} = \delta \sin \theta_t = \cos \theta_t \delta \theta_t \tag{4.43}
\]

The various counter terms for the quarks and gauge bosons have the following explicit form in terms of self-energies; for b, t quark and gauge part:

\[
\delta Z_L^b = \delta Z_L^t \equiv \delta Z_L = -\Sigma_L^b(m_b^2) - m_b^2[\Sigma_L^b(m_b^2) + \Sigma_L^b(m_b^2) + 2\Sigma_L^b(m_b^2)] \tag{4.44}
\]

\[
\delta Z_R^b = -\Sigma_R^b(m_b^2) - m_b^2[\Sigma_R^b(m_b^2) + \Sigma_R^b(m_b^2) + 2\Sigma_R^b(m_b^2)] \tag{4.45}
\]
\[
\delta m_b = \frac{m_b}{2} Re[\Sigma_L^b(m_b^2) + \Sigma_R^b(m_b^2) + 2\Sigma_S^b(m_b^2)]
\] (4.45)

\[
\delta Z_1^W - \delta Z_2^W = \frac{\Sigma^{\gamma Z}(0)}{s_W c_W M_Z^2}
\] (4.46)

\[
\delta Z_2^W = -\Sigma^{\gamma \gamma}(0) + \frac{c_W^2}{s_W^2} \left[ \frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2} \right]
\] (4.47)

\[
\delta M_W^2 = Re \Sigma^{WW}(M_W^2)
\]

while the counterterms for the gauge coupling lead to:

\[
\frac{\delta g}{g} = \delta Z_1^W - \frac{3}{2} \delta Z_2^W
\] (4.49)

From [39] one can write for \(\frac{\delta \tan \beta}{\tan \beta}\):

\[
\frac{\delta \tan \beta}{\tan \beta} = \frac{Re \Sigma_{H^+W^+}(m_{H^+}^2)}{M_W \sin 2\beta}
\] (4.55)

Also the counterterms for the chargino mixing matrices are necessary. Applying the method of [37, 39] requiring the cancellation of the antihermitian part of the wave function renormalization, one obtains:

\[
\delta Z_{1i}^+ = \frac{1}{4} \sum_k Z_{ik}^+ (\delta \chi_{ik}^L - \delta \chi_{ki}^L)
\] (4.57)
\[ \delta Z^+_{2i} = \frac{1}{4} \sum_k Z^+_{2k} (\delta \chi^L_{ik} - \delta \chi^R_{ki}) \] (4.58)

\[ \delta Z^-_{2i} = \frac{1}{4} \sum_k Z^-_{2k} (\delta \chi^R_{ik} - \delta \chi^L_{ki}) \] (4.59)

Using the \( \chi^+ \) chargino c.t. and s.e. \( \delta \chi^L,R \) listed below the counter terms for \( \chi^+ \) are obtained from the field transformation:

\[ \chi^+_i \rightarrow (1 + \frac{1}{2} [\delta \chi^L_{ij} P_L + \delta \chi^R_{ij} P_R]) \chi^+_j \] (4.60)

They are obtained by applying the method proposed in [39, 40, 41] and in terms of the \( j \rightarrow i \) bubble of momentum \( p \) they read:

\[ \Sigma_{ij} = \hat{p} P_L \Sigma^L_{ij} + \hat{p} P_R \Sigma^R_{ij} + P_L \Sigma^S_{ij} + P_R \Sigma^\bar{S}_{ij} \] (4.61)

with \( \Sigma^S_{ij} = \Sigma_{ji}^\bar{S} \) and:

\[ \delta \chi^L_{ii} = -\{ \Sigma^L_{ii}(M^2_i) + M^2_i [\Sigma^L_{ii}(M^2_j) + \Sigma^R_{ii}(M^2_j)] + M_i \Sigma^S_{ii}(M^2_j) + \Sigma^\bar{S}_{ii}(M^2_j) \} \] (4.62)

\[ \delta \chi^R_{ii} = -\{ \Sigma^R_{ii}(M^2_i) + M^2_i [\Sigma^L_{ii}(M^2_j) + \Sigma^R_{ii}(M^2_j)] + M_i \Sigma^S_{ii}(M^2_j) + \Sigma^\bar{S}_{ii}(M^2_j) \} \] (4.63)

and for \( i \neq j \)

\[ \delta \chi^L_{ij} = \frac{2}{M^2_i - M^2_j} \{ M^2_j \Sigma^L_{ij}(M^2_j) + M_i M_j \Sigma^R_{ij}(M^2_j) + M_i \Sigma^S_{ij}(M^2_j) + M_j \Sigma^\bar{S}_{ij}(M^2_j) \} \] (4.64)

\[ \delta \chi^R_{ij} = \frac{2}{M^2_i - M^2_j} \{ M^2_j \Sigma^R_{ij}(M^2_j) + M_i M_j \Sigma^L_{ij}(M^2_j) + M_j \Sigma^S_{ij}(M^2_j) + M_i \Sigma^\bar{S}_{ij}(M^2_j) \} \] (4.65)

All these contributions together with the virtual vertex and box diagrams (whose expressions are rather technical and not illuminating and therefore will not be shown) have been computed using the decomposition in terms of Passarino-Veltman functions described in Sect. 2.1 and the complete amplitude has been implemented in the numerical code TigrMC, which exploits the LoopTools library [11] and Montecarlo Techniques described in Sect. 2.2.

**Analytic Checks**

The first check of the correctness of the computation is the cancellation of the UV divergences among counter terms, self-energies and triangles, this cancellation occurring...
separately for s-channel and for u-channel, as well as for gauge-left, gauge-right, Yukawa-left and Yukawa-right sectors separately.

For the treatment of infrared singularities arising from the exchange of virtual photons see Sect.3.3.2. The maximum energy for the soft photon has been fixed at $\Delta E_{\gamma}^{\text{max}} = 0.1 \text{ GeV}$.

Another useful check can be done using the high energy behaviour of the amplitudes. High energy rules [13] predict the logarithmic behaviour of these amplitudes at one-loop level. They use splitting functions for external particles $b$, $\tilde{t}_{L,R}$, $\chi^{- i}$ and Renormalization Group effects on the parameters appearing in the Born terms.

They read:

$$F_{-++}(\tilde{t}_L) = -g_s\sqrt{2}A^L_i(\tilde{t}_L) \sin \frac{\theta}{2} \left\{ 1 + \frac{\alpha}{4\pi} \right\}$$

$$\frac{1 + 2\alpha^2}{18s^2_W c^2_W} \log \frac{s}{M^2_W} - \left[ \frac{m^2_t}{2s^2_WM^2_W} (1 + \cot^2 \beta) + \frac{m^2_b}{2s^2_WM^2_W} (1 + \tan^2 \beta) \right] \log \frac{s}{M^2_W}$$

$$-\left\{ \frac{1}{2s^2_W} \left[ \ln^2 \frac{1}{m^2_t} + \frac{1 - 10c^2_W}{36s^2_W c^2_W} \ln^2 \frac{1}{m^2_Z} \right] \right\}$$

(4.66)

$$F_{-++}(\tilde{t}_R) = -g_s\sqrt{2}A^R_i(\tilde{t}_R) \sin \frac{\theta}{2} \left\{ 1 + \frac{\alpha}{4\pi} \right\}$$

$$\left\{ -\left[ \frac{1}{3c_W^2} \right] \log^2 \frac{s}{m^2_Z} - \left[ \frac{1}{9c_W^2} \right] \log^2 \frac{-u}{m^2_W} \right\}$$

$$+ \frac{1 - 4c^2_W}{12s^2_W c^2_W} \left[ \log^2 \frac{-u}{M^2_Z} - \frac{1}{2s^2_W} \left[ \log^2 \frac{-u}{M^2_W} \right] \right\}$$

(4.67)

$$F_{+-+}(\tilde{t}_L) = g_s\sqrt{2}A^L_i(\tilde{t}_L) \sin \frac{\theta}{2} \left\{ 1 + \frac{\alpha}{4\pi} \right\}$$

$$\left\{ -\left[ \frac{1 + 2c^2_W}{12s^2_W c^2_W} \right] \log^2 \frac{s}{m^2_Z} - \left[ \frac{1}{2s^2_W} \right] \log^2 \frac{s}{m^2_W} \right\}$$

$$+ \frac{1}{18c^2_W} \left[ \log^2 \frac{-t}{M^2_W} - \frac{1}{6s^2_W} \left[ \log^2 \frac{-u}{M^2_W} \right] \right\}$$

(4.68)

The logarithmic part of the gaugino amplitude (4.66) is similar to the one obtained for the process $bg \rightarrow tW^-$ with transverse $W$ [42]. Higgsino amplitudes $F_{-++}(\tilde{t}_R)$, $F_{+-+}(\tilde{t}_L)$ in (4.67,4.68) get logarithmic terms similar to the ones in both $bg \rightarrow tW^-$ for longitudinal $W$ and $bg \rightarrow tH^-$ [43]. One notices that there is no linear logarithmic
contribution, but only quadratic logarithmic terms, in these Higgs or Higgsino type of amplitudes. The coefficients of these quadratic logarithms are of pure gauge origin and do not involve any free parameter.

Considering the complete one-loop computation and retaining only the logarithmic parts of the B,C,D Passarino-Veltman functions appearing in the various diagrams, the above expressions for the 3 types of leading amplitudes have been obtained.

### 4.4 One-Loop Results

Starting from the results for the partonic process described in the previous section, it is now possible to define the invariant mass distribution for the hadronic process $PP \rightarrow \tilde{t}_a \tilde{\chi}_i^-$ as:

$$
\frac{d\sigma}{dM_{\tilde{t}_a \tilde{\chi}_i^-}} = \int dx_1 dx_2 d\cos \theta \left[ b(x_1, \mu)g(x_2, \mu) + g(x_1, \mu)b(x_2, \mu) \right]
\times \frac{d\sigma_{bg \rightarrow \tilde{t}_a \tilde{\chi}_i^-}}{d\cos \theta} \delta(\sqrt{x_1 x_2 S} - M_{\tilde{t}_a \tilde{\chi}_i^-}),
$$

(4.69)

where $\sqrt{S}$ is the proton-proton c.m. energy (at LHC $\sqrt{S} = 14$ TeV), $M_{\tilde{t}_a \tilde{\chi}_i^-}$ is the $\tilde{t}_a \tilde{\chi}_i^-$ invariant mass, $\theta$ is the scattering angle in the partonic c.m. frame, and $b(x_i, \mu), g(x_i, \mu)$ are the parton distributions of the bottom and gluon partons inside the proton with a momentum fraction $x_i$ at the scale $\mu$.

The invariant mass distribution $d\sigma/dM_{\tilde{t}_a \tilde{\chi}_i^-}$ has been evaluated at the one-loop electroweak level for a large set of representative SUSY benchmark points as in the case of stop and sbottom pair production. The properties of these points are shown in Tab. 3.1. The obtained cross sections at the Born and one-loop level are collected in Tab. 4.1 where the values of stop and chargino masses are also shown: for the realistic case of production of the lightest stop and chargino states $\tilde{t}_1$ and $\tilde{\chi}_1^-$, only the couple LS1 - LS2 give a cross section of order of the $pb$ (considering a global factor 2, arising from the conjugate process), considered as a reasonable limit for realistic detections at the LHC. All the other input sets, including the SPS5 "Light Stop", give smaller rates, and will not be further considered in what follows.

For what concerns the one-loop electroweak corrections, they are generally small, of the order of a relative few percent for all the considered scenarios. As an example of this behaviour, in Fig. 4.5 the differential distributions for the LS1 and LS2 benchmark points,
Figure 4.5: Born and one-loop distribution $d\sigma/dM_{\text{inv}}$ in LS1 and LS2. The bottom panels show the relative effect.
Table 4.1: Total cross sections and one-loop corrections (in pb) for all mSUGRA benchmark points. The masses (in GeV) of the lightest stop and chargino states are also indicated.

<table>
<thead>
<tr>
<th>mSUGRA scenario</th>
<th>$m_{\tilde{t}_1}$</th>
<th>$m_{\chi_1}$</th>
<th>$\sigma_{tot}^{\text{Born}}$</th>
<th>$\sigma_{tot}^{1\text{-loop}}$</th>
<th>1-loop effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>LS2</td>
<td>224.7</td>
<td>106.9</td>
<td>0.5403</td>
<td>0.5264</td>
<td>-2.5</td>
</tr>
<tr>
<td>LS1</td>
<td>214.6</td>
<td>103.6</td>
<td>0.4278</td>
<td>0.4310</td>
<td>0.7</td>
</tr>
<tr>
<td>SPS5</td>
<td>279.0</td>
<td>226.2</td>
<td>0.0573</td>
<td>0.0567</td>
<td>-1.0</td>
</tr>
<tr>
<td>SPS1a</td>
<td>367.3</td>
<td>178.8</td>
<td>0.0396</td>
<td>0.0385</td>
<td>-2.8</td>
</tr>
<tr>
<td>SPS1a</td>
<td>399.7</td>
<td>175.9</td>
<td>0.0311</td>
<td>0.0302</td>
<td>-2.9</td>
</tr>
<tr>
<td>SPS6</td>
<td>494.6</td>
<td>215.3</td>
<td>0.00962</td>
<td>0.00937</td>
<td>-2.6</td>
</tr>
<tr>
<td>SPS4</td>
<td>540.1</td>
<td>219.6</td>
<td>0.00741</td>
<td>0.00725</td>
<td>-2.2</td>
</tr>
<tr>
<td>SU1</td>
<td>566.4</td>
<td>255.7</td>
<td>0.00402</td>
<td>0.00388</td>
<td>-3.5</td>
</tr>
<tr>
<td>SU6</td>
<td>634.1</td>
<td>279.7</td>
<td>0.00251</td>
<td>0.00246</td>
<td>-2.0</td>
</tr>
<tr>
<td>SPS3</td>
<td>645.2</td>
<td>296.2</td>
<td>0.001703</td>
<td>0.001641</td>
<td>-3.6</td>
</tr>
<tr>
<td>SPS2</td>
<td>921.4</td>
<td>224.9</td>
<td>$5.91 \times 10^{-5}$</td>
<td>$5.88 \times 10^{-5}$</td>
<td>-0.51</td>
</tr>
</tbody>
</table>

The points with highest cross sections are shown: as one can notice, the one-loop effect is positive in the low energy region (near the production threshold) and drops to negative values when increasing the final invariant mass. The global effect on the totally integrated cross section, being the result of the sum of two opposite contributions, is positive and below 1% in the LS1 case, slightly greater and negative (−2.5%) in the LS2 case.

The compensation of the opposite effects which reduces the global one-loop contribution can be understood by looking at the properties of the helicity amplitudes. Fig. 4.6 shows the energy dependence of each helicity amplitude from threshold to high energy for a given (central) angle $\theta = \pi/4$. The Born and full one-loop results are shown for each amplitude. They confirm the expectations described in the previous subsection, namely the nature of the dominant amplitudes at low energy and the dominant amplitudes at high energy and it is possible to see that the high energy behaviour is quickly reached as soon as the threshold is crossed. The difference between the LS1 and LS2 cases is due to the increase in the final masses and in the change in stop and chargino mixings. In particular for LS2 the R chirality amplitudes are less depressed because of the difference in $\tan \beta$ and in the mixing element $Z_{2i1}$, which increase the value of $A_i^R(\tilde{t}_L)$ (see Eq.4.10), and
Figure 4.6: Energy dependence for leading helicity amplitudes in LS1 and LS2.
this can be clearly seen both at low and high energies.

Fig. 4.7 gives the angular distributions at low energy ($s/s_{\text{thresh}} = 1.001$) and at high energy ($s/s_{\text{thresh}} = 30$). At low energy the leading amplitudes give indeed the expected $(1 + \cos \theta)$ and $(1 - \cos \theta)$ distributions, whereas at high energy one tends to a limiting $(1 - \cos \theta)$ distribution, at least away from purely backward scattering. The one-loop corrections make only little changes in the shape of the angular distributions as expected from the Sudakov rules.

Again, it is possible to get rid of the cancellation between positive and negative effects at different energies through the partial rate, defined in (3.50). The results, obtained for the points LS1 and LS2 are shown in Fig. 4.8, where it is possible to see that to obtain relative effects of the order of 10% the LHC should be able to detect a rate of 0.1pb or even 0.01pb, admittedly a quite difficult achievement.
Figure 4.8: LS1, LS2, partial rate $\sigma(M_{\text{inv}})$ and one-loop effect
4.5 Dependence on Supersymmetric Parameters

From the discovery that the one-loop electroweak effect is small, one concludes that the main relevant EW information can be essentially obtained remaining at the Born level, where a limited set of parameters affects the determination of physical observables. Besides the values of the stop and chargino masses, which are obviously crucial for the definition of the production threshold, other SUSY parameters contribute to the coupling $b\tilde{t}_a\tilde{\chi}_i^-$, Eq.(4.10). While $\tan \beta$ explicitly appears in the various terms, the chargino mixing matrices $Z_{ij}^\pm$ depend in a non-trivial way on the chargino mass matrix parameters:

$$X = \begin{pmatrix} M_2 & \sqrt{2}M_W \sin \beta \\ \sqrt{2}M_W \cos \beta & \mu \end{pmatrix}$$

Moreover, for production of physical stops, the mixing angle $\theta_{\tilde{t}}$ mixes the various terms of Eq.(4.10). In conclusion, it is possible to identify a set of independent parameters which determine the amplitude at Born level:

$$\tan \beta \quad M_2 \quad \mu \quad m_{\tilde{t}} \quad \theta_{\tilde{t}}$$

The chargino masses are determined by a combination of $M_2$, $\mu$ and $\tan \beta$. Since to perform a parameter analysis of the process it seems reasonable to fix all the masses, it is possible to trade, e.g., $M_2$ for $m_{\chi_1}$ and $\mu$ for $m_{\chi_2}$, depending on the chargino of the final state.

Given these premises, the process of production of the lightest stop and chargino $bg \rightarrow \tilde{t}_1\chi_1$ will now be analyzed at Born level to investigate possible dependences on supersymmetric parameters. Rather than looking for a dependence of the cross section on the stop or chargino masses that can be assumed to be experimentally known from previous discovery, the analysis will focus on the dependences on $\tan \beta$, $\mu$ and $\theta_{\tilde{t}}$. The results are shown in Fig. 4.9. All panels show the dependence of the total cross section on the mixing angle $\theta_{\tilde{t}}$ for different values of $\tan \beta$, at different values of $\mu$. Figs. 4.9(a) and 4.9(b) show the results obtained for low values of $\mu$, close to the chargino mass, while in Figs. 4.9(c) and 4.9(d) $\mu$ has been pushed to a value ($800\text{ GeV}$), which is high compared to $m_{\chi_1}$. It is possible to notice that the cross section depends very strongly on the value of $\theta_{\tilde{t}}$ and that there is always a range for which the cross section drops near to zero. In Fig. 4.9(a), where the final state particles have been assumed to be light, and therefore the low threshold allows a cross section of the order of some pb, it is possible to see that $\sigma$
Figure 4.9: Parameter dependence of the total cross section at Born level.
4.5. Dependence on Supersymmetric Parameters

changes from $\sim 6$ pb for $\theta_i \simeq \pi/8$ to less than 0.5 pb for $\theta_i \simeq 5\pi/8$ (when $\tan \beta = 40$). Hence there are regions of the parameter space where, even if the masses of final state particles are very low, the stop mixing angle can acquire values for which the cross section is driven to nearly undetectable levels. The dependence on $\theta_i$ can indeed be understood looking at the amplitude, which is a sum of terms of the form:

$$A_1 \cos \theta_i + A_2 \sin \theta_i$$  \hspace{1cm} (4.72)

Depending on the values of $A_1$ and $A_2$, the squared amplitude generates the curves shown in Fig. 4.9.

The results also depend in a weaker and less trivial way on $\tan \beta$. The cross section tends to somewhat higher values as $\tan \beta$ increases, but to different extents in the various cases considered; it is also possible to see that in all cases this is not true within a small range of $\theta_i$. The variation of the total cross sections with different values of $\tan \beta$ is strongly affected by the choice of $\mu$ resulting in a very mild dependence on $\tan \beta$ for high values of $\mu$ and viceversa, so that the determination of this supersymmetric parameter from this process could be ambiguous, unless the rate turns out to be larger than a certain “threshold” value. Further constraints coming from other processes could however limit the range of $\tan \beta$, and the determination of the two remaining parameters through the analysis of this process would then be more relevant.
4. EW NLO description of the processes $PP \xrightarrow{bg} \tilde{t}_a \chi_i$
Conclusions

This work has been devoted to the analysis of one-loop electroweak effects on processes of production of third family squarks in the context of the Minimal Supersymmetric Standard Model. Due to their light masses, which might lie in a range detectable by LHC, the stop and sbottom squarks play a relevant role in the study of Supersymmetry phenomenology. An accurate analysis of their properties and of the observables related to their production channels is therefore mandatory.

The results of the EW NLO analysis of the processes $PP \rightarrow \tilde{t}_a \tilde{t}_b^{*} (\tilde{b}_a \tilde{b}_b^{*})$ and $PP \rightarrow \tilde{t}_a \chi_i$ have been illustrated in the preceding chapters. The analysis has been divided into two parts: firstly, the numerical outcomes of the EW NLO calculations have been presented for both processes; afterwards, the dependence on supersymmetric parameters of the total cross sections has been analysed. The discussion of the results is now summarised.

General considerations

As a general outcome, it has been found that one-loop electroweak effects are quite mild, see Tabs. 3.2-4.1 of difficult experimental detection and in some cases even too small to be realistically testable even in a future high luminosity stage at LHC. For each process the suppression of NLO effects is due to a compensation of different behaviours at low and high $M_{inv}$ regions: in the low $M_{inv}$ range a limited number of helicity amplitudes are dominant, with generally positive one-loop corrections and eventually affected by threshold effects, while at high energies the dominant helicity amplitudes (different from the low energy ones) receive negative one-loop corrections. This behaviour, though quite annoying, can in principle be eliminated through a sensible definition of the observables related to the processes. In this respect, the partial rate, defined in (3.50), is an
observable through which it is possible to get rid of the positive low energy effects: it is
defined as the integral of the differential cross section where the lower extreme of inte-
gration is not the threshold of the process but a specific value of $M_{\text{inv}}$. For some “light”
supersymmetric scenarios the value of $M_{\text{inv}}$ can be chosen in such a way that the partial
rate can still acquire detectable values and receive (sizable) corrections only by negative
high energy effects. The dependence on supersymmetric parameters is quite specific to
the process under investigation and, due to the different detectability of the predicted sig-
 nal, the strategies used to analyse such dependence have been different as well: the results
will be discussed in the next Section.

**Specific results**

The outcomes of the analysis which are specific to the process under investigation are
now illustrated:

**Stop-antistop and sbottom-antisbottom production**

The results coming from the partial rate, shown in Fig. 3.7, show that, while for stop-
antistop production the one-loop effects are strongly enhanced for all benchmark points,
the picture is not so clear for the sbottom-antisbottom case. However, in the former case
the enhancement might have some relevance only for a limited number of points, those
with suitably light spectra, for which the cross section does not drop to undetectable
levels. For the sbottom case the partial rate analysis is not worth the effort due to the
small partial rates involved.

For what concerns the non diagonal rates, Tab. 3.3, the one-loop values obtained both
for stop-antistop and sbottom-antisbottom, summed with those coming from the single
Z-exchange, might be in principle observable in a hopefully high luminosity stage. A fur-
ther interesting feature that emerges from the non-diagonal results is that for benchmarks
which share the same value of $\tan \beta = 50$, the total cross section for the gluon initiated
NLO process is bigger than that coming from Z exchange at tree level and that, despite
the final state particles are heavier for sbottom production, the total cross section is bigger
than the stop case.

The search for extra (i.e. different from the final squark masses) parameters depen-
dence has been performed for a choice of twelve representative mSUGRA benchmark
Specific results

points: the mSUGRA parameters specific to each benchmark point have been varied in reasonable ranges and the spectrum at low energy has been obtained running the parameters through the codes SUSPECT and FeynHiggs. In all cases the results reveal the presence of a small (at the few percent level) relative difference of the effects with a more important role apparently played by different parameters for different points, in particular by $\tan \beta$ in a case of sbottom production. Certainly, the possibility of experimental verification of such conclusions would require very high luminosity scenarios and accuracies, representing a real challenge for the LHC experimental groups. This might, though, become interesting in case of a previous production of supersymmetric particles, which might justify the idea of the dedicated experimental effort.

Stop-chargino production

The results of the analysis show that the complete one-loop electroweak effect is of the relative few percent size, that would make it hardly visible in a realistic LHC situation. The analysis of the partial rate reveals an enhancement of the corrections when considering sufficiently high ranges for the integration, see Fig. 4.8. This enhancement, however, is spoiled by the low detectability of the processes in all the considered mSUGRA scenarios, since the total cross section is usually below the reasonable limit for detection (supposed to be around 1pb). Due to this result, the analysis on parametric dependences has been performed on the total rate at Born level, which will embody the total electroweak content of the process. The supersymmetric parameters which enter the rate and that cannot be directly measured from direct production are $\mu$, $\tan \beta$ and $\cos \theta_{\tilde{t}}$. Assuming a previous measurement of the stop and chargino masses, it has been verified that the dependence of the rates on $\cos \theta_{\tilde{t}}$ and $\mu$ might be rather strong in the case of light final state masses, and would influence the milder dependence on $\tan \beta$, see Fig. 4.9. This would indicate that, given a picture with light stop and chargino masses, a measurement of the light stop-chargino process might provide an original and interesting type of constraints on the size of these MSSM parameters.
Appendix A

Implementation of the Montecarlo procedure in C++

In this appendix the implementation in the C++ codes of the Montecarlo procedure described in Sect. 2.2 will be described. The excerpts are taken from the code \texttt{stst}, used for the computation of the process $PP \rightarrow \tilde{\nu}_a \tilde{\tau}_b$. Only the information necessary to follow the logic of the computation is described in the following.

The partonic differential cross section is the result of the function:

$$\text{DifferentialSigma}(\text{double MandelS, double costheta, Kinematics & Kin, FormFactors & FF\_Full\_Born, FormFactors & FF\_Full\_OneLoop})$$

This function depends on 2 variables which are relevant for this description: MandelS and costheta. The other variables are necessary for the loop computation and will not be described. The code defines the Jacobian of the change of variables, the part in parenthesis of (2.25), the (cosine of the) scattering angle and inserts everything into the integrand function weight:

$$\text{Jac} = 1.0/x1;$$
$$\text{densities} = 1.0/(\text{InverseDistribution1}(x1, y) \ast \text{InverseDistribution2}(y, y\_min));$$
$$\text{cos}_\text{theta}_\text{star} = 2.0 \ast \text{RandomUniform()} - 1.0;$$
$$\text{weight} = \text{densities} \ast \text{Jac};$$
The functions \texttt{InverseDistribution1} and \texttt{InverseDistribution2} define the distributions of $x_1, (2.19)$, and $y, (2.21)$. The squared invariant mass $M_{\text{inv}}^2$ is then defined:

$$\text{MandelS} = x_1 \times x_2 \times \text{HadronS};$$

where \texttt{HadronS} for LHC is $(14 \text{ TeV})^2$. The code then computes the parton distribution functions and the partonic differential cross section:

\begin{verbatim}
DSigma = DifferentialSigma(MandelS, cos_theta_star,
Kin, FF_Full_Born, FF_Full_OneLoop);
pdf.ComputePDF(x1, Q2); f_gluon = pdf.f_g;
pdf.ComputePDF(x2, Q2); f_gluon_prime = pdf.f_g;
\end{verbatim}

and inserts them into \texttt{weight} (the factor 2 comes from the Jacobian of the change $\cos \theta \rightarrow \hat{x}$):

$$\text{weight} \times= 2.0 \times f_{\text{gluon}} \times f_{\text{gluon}\_\text{prime}} \times \text{DSigma};$$

The invariant mass distribution is now computed. First of all the code generates a random value for $M_{\text{inv}}$, which is called $x$:

$$x = \sqrt{\text{Ev.MandelS}};$$

The extreme values of $M_{\text{inv}}$ and the density of its partition are taken as external inputs:

$$\text{X\_min} = \text{xmin}; \quad \text{X\_max} = \text{xmax}; \quad \text{n\_bin} = \text{nbin};$$

The code then defines the coordinates of the small intervals by which the range is divided:

$$\text{X\_Left[i]} = \text{X\_min} + i \times ((\text{X\_max}-\text{X\_min})/\text{n\_bin});$$
$$\text{X\_Right[i]} = \text{X\_min} + (i+1) \times ((\text{X\_max}-\text{X\_min})/\text{n\_bin});$$

where $i$ is defined as:

$$\text{int } i = \text{(int)} \times ((x-\text{X\_min})/((\text{X\_max}-\text{X\_min})/\text{n\_bin}));$$

Then the code defines \texttt{value\_normalized} which represents the area below the rectangle with height \texttt{weight} and depth 1 in the $\hat{x}$ (i.e. $\cos \theta$) dimension by dividing the related volume by the width $\text{X\_Right[i]}-\text{X\_Left[i]}$:

$$\text{value\_normalized} = \text{Ev.weight}/(\text{X\_Right[i]}-\text{X\_Left[i]});$$
After summing all these areas (within the range $X_{\text{Right}[i]}-X_{\text{Left}[i]}$ and labelled by the integer $i$):

$$\text{hist}[i] += \text{value}_\text{normalized};$$

the code calculates the average of these values dividing by the number of events and then plots the result, which is the invariant mass distribution:

```cpp
for(int i=0; i<n\_bin; i++)
  if(ndata[i]>0)
    value = hist[i]/nev;
  hist\_file << X\_Left[i] << " \" << value << endl;
```

The errors associated to these quantities are calculated as well.

It is now possible to calculate the total cross section: it is enough to sum over all the weights and divide by the number of events.

```cpp
for(int iter=0; iter < Nevents; iter++){
  Ev\_GenerateEvent();
  Ev\_ComputeWeight();
  Hist\_Fill(Ev);
  Sigma += Ev\_weight;
}
Sigma /= Nevents;
```

This is the volume subtended by the average value of $\text{Ev}_\text{weight}$. In this case too, the code calculates the associated error.
A. Implementation of the Montecarlo procedure in C++
Appendix B

Light Stop and Electroweak Baryogenesis

Producing a stop at the LHC would be fundamental also in the context of the cosmological model of electroweak baryogenesis: for such model to be viable, a very light stop, even lighter than the top quark, is required [44, 45, 46, 47, 48, 49].

A stop with $m_\tilde{t} \lesssim m_t$ is indeed quite interesting because it would be produced with high cross sections and therefore its detection would be easier. Since one-loop effects might provide visible corrections to the observables, especially when the cross section is high, it is important to quantify such effects in the various stop production channels.

This appendix is divided in two parts: the first part provides a brief description of the model of electroweak baryogenesis, focusing on why a light stop is necessary, while in the second part the one-loop electroweak descriptions of processes of production of light stop are presented.

B.1 Electroweak Baryogenesis

Common experience shows that the world is composed of matter, while antimatter is totally negligible. In Nature antimatter can only be produced by high energy reactions taking place in stars or by the interaction of cosmic rays with the atmosphere, but quickly disappears by annihilation with surrounding matter. The observable which quantifies the unbalance between matter and antimatter is the parameter $\eta$, defined as the difference of
number densities of baryons \( n_B \equiv N_B/V \) and antibaryons \( n_{\bar{B}} \equiv N_{\bar{B}}/V \) divided by
the number density of photons \( n_\gamma \equiv N_\gamma/V \). It is found to be \([50]\):

\[
\eta \equiv \frac{n_B - n_{\bar{B}}}{n_\gamma} = (6.14 \pm 0.25) \times 10^{-10}
\]  \hspace{1cm} (B.1)

Albeit being quite small, \( \eta \) is different from zero. This difference could in principle be
explained by assuming that the dominance of baryons over antibaryons is an initial con-
dition of the Universe. Nevertheless, such initial condition would be in contrast with the
now widely accepted theory of Cosmological Inflation, a phase of enormous expansion
which took place in the very early stages of the Universe and efficiently erased any kind
of initial asymmetry. A symmetry between baryons and antibaryons after inflation would
lead, if baryons were free to annihilate with antibaryons, to \( n_B/n_\gamma = n_{\bar{B}}/n_\gamma \sim 10^{-18} \) at
present times, but these values are far too small to be consistent with observational data.
Therefore, in modern cosmology a dynamical mechanism is supposed to be responsible
for the generation of baryon-antibaryon asymmetry after inflation. Finding the nature of
this mechanism is the key problem of baryogenesis.

One of the ways to explain how the baryon-antibaryon asymmetry was dynamically
generated is the mechanism of Electroweak baryogenesis (EWBG). In the context of
EWBG, the asymmetry was generated during the electroweak phase transition when the
temperature of the universe was \( T \sim 100 \, GeV \). The way EWBG works is beyond the
purposes of this thesis: some exhaustive reviews of the model are \([51, 52, 53, 54, 55]\). It
is more interesting to note that, regardless of the specific mechanism of baryogenesis, to
dynamically generate a baryon-antibaryon asymmetry starting from a symmetric phase,
three conditions (the so called Sakharov conditions \([56]\)) must be satisfied:

1. Baryon number violation;
2. C and CP violation;
3. Departure from thermal equilibrium.

It is possible to prove in many ways why the Sakharov conditions are necessary for baryo-
genesis \([51, 52, 53, 54, 55]\). Here it is enough to assume that the Sakharov conditions are
true and show how they can be satisfied by the mechanism of EWBG. In this way it is
possible to verify that EWBG is not possible in the SM, given its experimental constraints,
but is allowed within the parameter space of the MSSM, provided the stop is light.
EWBG in the SM

1. Baryon number violation

The baryon current, defined as:

\[ J_B^\mu = \frac{1}{3} \sum_{\text{families}} (\bar{q}_L \gamma_\mu q_L + \bar{u}_R \gamma_\mu u_R + \bar{d}_R \gamma_\mu d_R) \] (B.2)

is conserved at the classical level in the SM, but is violated by quantum anomalies. It is possible to compute the non-conservation of \( B \) as:

\[ \partial_\mu J_B^\mu = \frac{N_f}{32\pi^2} \left( -g^2 W_I^{\mu\nu} \tilde{W}^{I\mu\nu} + g'^2 B_{\mu\nu} \tilde{B}^{\mu\nu} \right) \] (B.3)

where \( N_f \) is the number of families, \( W_I^{\mu\nu} \) and \( B_{\mu\nu} \) are the field strengths of the gauge groups \( SU(2)_L \) (coupling constant \( g \)) and \( U(1)_Y \) (coupling constant \( g' \)) respectively, and duals are defined as \( \tilde{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma} \).

The variation of baryon number over the time interval \( \Delta t = t_f - t_i \) is defined as:

\[ B(t_f) - B(t_i) = \int_{t_i}^{t_f} dt \int d^3x J_0^B \] (B.4)

and this integral can be written as:

\[ B(t_f) - B(t_i) = N_f \left[ \left( N_{CS}^{SU(2)}(t_f) - N_{CS}^{SU(2)}(t_i) \right) - \left( n_{CS}^{U(1)}(t_f) - n_{CS}^{U(1)}(t_i) \right) \right] \] (B.5)

where \( N_{CS}^{SU(2)} \) and \( n_{CS}^{U(1)} \) are the Chern-Simons numbers, functions of the gauge fields of \( SU(2)_L \) and \( U(1)_Y \) respectively. They describe the topology of the gauge groups \( SU(2)_L \) and \( U(1)_Y \) in the following way. A gauge transformation of \( U(1)_Y \):

\[ B_i \rightarrow B_i + \frac{i}{g'} (\partial_i U_Y) U_Y^{-1} \] (B.6)

with \( U_Y(x) = e^{i\alpha_Y(x)} \), does not change \( n_{CS}^{U(1)} \), while a gauge transformation of \( SU(2)_L \):

\[ A_i \rightarrow U A_i U^{-1} + \frac{i}{g} (\partial_i U) U^{-1} \] (B.7)

generates a variation of \( N_{CS}^{SU(2)} \):

\[ \Delta N_{CS} = \frac{1}{24\pi^2} \int d^3x Tr \left[ (\partial_i U) U^{-1} (\partial_j U) U^{-1} (\partial_k U) U^{-1} \right] \delta^{ijk} \] (B.8)

It can be shown that pure gauge configurations have integer Chern-Simons number and that \( SU(2)_L \) transformations change the Chern-Simons number by integer units. Therefore, moving from a \( SU(2)_L \) vacuum to another vacuum configuration, the Chern-Simons...
number changes from $n$ to $n \pm 1$. The $SU(2)$ vacuum structure is depicted in Fig. B.1. The interesting fact is that the violation of baryon number depends on the possibility to have transitions between vacua: going back to (B.5), the variation of $B$ can be expressed as:

$$\Delta B = N_f \Delta N_{CS} \quad (B.9)$$

The tunnelling probability between the vacua is so small that it could never be possible throughout the lifetime of the Universe $[53]$. Nevertheless it is possible to have transitions between the vacua at finite temperature. Between the two vacua there are static configurations with non vanishing field strength and half-integer Chern-Simons number. They are the so called Sphaleron configurations and their energy can be estimated on dimensional grounds as:

$$E_{\text{sph}}(T) = \min (E(A_i) + E(H)) \simeq \min \left( \frac{4\pi}{g^2 l} + 4\pi v^2(T) l \right) \sim \frac{8\pi}{g} v(T) \quad (B.10)$$

where $v(T)$ is the temperature dependent Higgs VEV and $l$ is the characteristic dimension of the sphaleron. Sphalerons mediate transitions between two vacua at finite temperature and it is possible to show that in the absence of any other charge asymmetries, sphalerons tend to erase the difference between baryons and antibaryons. CP violating interactions, on the other hand, can generate such asymmetries which are then converted by sphalerons into a net baryon number.
2. C and CP violation

C and CP are clearly violated in the SM; the amount of CP violation, however, is not enough for the generation of the observed baryon asymmetry. It is possible to define a dimensionless parameter which estimates the amount of CP violation needed for baryogenesis. The sphaleron transition rate is found to be proportional to $\alpha^4_W \equiv (\alpha/s_W^2)^4$, hence, working on dimensional grounds, $\eta$ can be parametrised as:

$$\eta \approx \frac{\alpha^4_W T^3}{n_\gamma} \delta_{CP} \simeq 10^{-6} \delta_{CP}$$

(B.11)

where the photon number density is $n_\gamma \simeq T^3$. To be consistent with the observed value for $\eta$ the CP violation parameter must be:

$$\delta_{CP} \gtrsim 10^{-4}$$

(B.12)

Starting from the CKM matrix, however, it is possible to find a parameter which quantifies the amount of CP violation in the SM, the so called Jarlskog invariant \[57\]. The dimensionless CP violation at a temperature of the order of 100 GeV, when sphalerons were effective, can be calculated through the Jarlskog invariant and reads:

$$\delta_{CP} \sim 10^{-20}$$

(B.13)

far too small to generate the observed value for $\eta$. This is just a naive estimate: even if no theorems prove the validity of the Jarlskog invariant as a tool to estimate the amount of CP violation for baryogenesis, there is wide consensus that within the SM, CP violation is too small.

Therefore one must find new sources of CP violation outside the SM.

3. Departure from thermal equilibrium

Thermal equilibrium in an expanding universe is obtained when the rate of the interactions which modify particle distribution functions to adjust to the changing temperature is faster than the expansion rate. When such interactions are no more able to maintain thermal equilibrium, they are said to freeze out.

For electroweak baryogenesis to work, sphalerons must freeze out after the electroweak transition, so that they cannot erase the baryon asymmetry generated during the symmetric phase. Computing the sphaleron rate it is possible to show that sphalerons were
in thermal equilibrium up to very high temperatures, $T \lesssim 10^{12}$ GeV; after the electroweak transition sphalerons go out of equilibrium and freeze out provided that:

$$\frac{\langle \phi(T_c) \rangle}{T_c} \gtrsim 1$$

(B.14)

where $\langle \phi(T_c) \rangle$ is the temperature dependent Higgs VEV and $T_c$ is the critical temperature of electroweak phase transition. This means that the phase transition must be strong enough. Now, the electroweak phase transition can be first or second order: it is crucial to determine which order is the correct one for baryogenesis.

During a first order phase transition, when the system reaches the critical temperature $T_c$, two degenerate minima of the potential coexist, separated by an energy barrier. During a second order phase transition the system reaches in a smooth way the true minimum of the potential. The difference between the two types of phase transition are sketched in Fig.(B.2).

![Figure B.2](image)

(a) first order and (b) second order phase transition

A second order phase transition would produce an adiabatic EW breaking thus allowing sphalerons to erase any baryon asymmetry during the smooth transition. A first order phase transition, on the other hand, proceeds through the nucleation of bubbles of the broken phase in the symmetric background. Inside the bubbles, sphalerons are frozen, while outside the bubbles they are effective and mediate baryon violating interactions. In the symmetric phase CP violation is converted into baryon asymmetry and, since the bubble is expanding, part of this asymmetry flows into the broken phase where it cannot be erased by sphalerons anymore. Therefore electroweak baryogenesis can be attained provided the electroweak phase transition is strongly enough first order. From the expression
of the finite temperature potential of the Higgs boson [58], the condition (B.14) implies a constraint on the Higgs mass:

$$1 \lesssim \frac{\langle \phi(T_c) \rangle}{T_c} = \frac{2E}{\lambda(T_c)} \approx \frac{4v^2}{m_h^2} \frac{2M_W^3 + M_Z^3}{4\pi v^3} \Rightarrow m_h \lesssim 32 \text{ GeV} \quad (B.15)$$

where $\lambda(T_c)$ is the temperature dependent quartic coupling of the Higgs boson. The parameter $E$ is contained in the finite temperature potential for the Higgs boson and gets contributions only from bosonic fields, which in the SM case are only $Z$ and $W$. This bound has already been excluded by LEP measurements:

$$m_{h}^{\text{obs}} \gtrsim 115 \text{ GeV} \quad (B.16)$$

A first order electroweak phase transition in the SM is clearly not strong enough to allow a successful baryogenesis.

Summing up, electroweak baryogenesis cannot be accomplished within the SM framework because two out of three Sakharov conditions are not fulfilled given the present experimental bounds on CP violation and Higgs mass. A possible solution to the problem is to consider the MSSM and see what are the conditions for electroweak baryogenesis to be effective.

**EWBG in the MSSM**

1. **Baryon number violation**

   In the MSSM sphaleron transitions are allowed in the same way as in the SM since the gauge structure $SU(2)_L \times U(1)_Y$ is preserved. The rate of sphaleron transitions can also be computed in a similar way because the lightest Higgs of the MSSM has the same role as the SM Higgs in the electroweak symmetry breaking mechanism.

2. **C and CP violation**

   The lagrangian of the MSSM contains many new sources of CP violation. It has been shown that the chargino sector contains the dominant contribution to CP violating currents which are converted into baryon asymmetry outside the bubble wall [54]. To have significant CP violation, charginos must be abundant during electroweak breaking, therefore their mass must be comparable to the critical temperature $T_c$. Chargino masses
depend on parameters of the supersymmetric lagrangian (4.70). This translates into a not too severe bound on $M^2$ and $\mu$:

$$M^2, \mu \lesssim 500 \text{ GeV}$$

(B.17)

3. Departure from thermal equilibrium

As pointed out in (B.14), to have a sufficiently strong phase transition of the first order, the vacuum expectation value of the Higgs boson at the critical temperature must be bigger than the critical temperature itself: this ratio is inversely proportional to the (squared) Higgs mass and proportional to the parameter $E$, which gets contributions only from bosonic fields in the finite temperature Higgs potential. The Higgs mass in the MSSM gets radiative contributions mainly from stop particles, which have the same Yukawa coupling of the top quark:

$$m^2_{h^0} \simeq m^2_Z \cos^2 2\beta + \frac{3}{4\pi} \frac{m^4_t}{v^2} \log \left( \frac{m^2_{\tilde{t}}}{m^2_t} \right)$$

(B.18)

To satisfy the LEP bounds on the Higgs mass one of the two stops must be heavy, and to be in agreement with precision measurements, the left handed stop must be the heaviest one. Now, increasing the Higgs mass implies a weakening of the phase transition strength, thus the allowed range has been found to be:

$$115 \text{ GeV} \lesssim m_{h^0} \lesssim 120 \text{ GeV}$$

(B.19)

with the additional constraint $5 \lesssim \tan \beta \lesssim 10$ [59]. Moreover, since the Higgs mass increases, an enhancement of the parameter $E$ is needed to satisfy (B.14); the strength of the phase transition can be expressed as:

$$1 \lesssim \frac{\langle \phi(T_c) \rangle}{T_c} = \left( \frac{\langle \phi(T_c) \rangle}{T_c} \right)_{\text{SM}} + \left( \frac{\langle \phi(T_c) \rangle}{T_c} \right)_{\text{MSSM}}$$

(B.20)

The first term is the SM contribution (B.15) and the second term is mainly given by finite temperature contribution of stop loops; it can be shown that it is bounded from above by:

$$\left( \frac{\langle \phi(T_c) \rangle}{T_c} \right)_{\text{MSSM}} \lesssim \frac{2m^2_{\tilde{t}} \left( 1 - \tilde{A}_t^2/m^2_Q \right)^{3/2}}{\pi v m^2_{h^0}}$$

(B.21)

where $\tilde{A}_t = A_t - \mu^*/\tan \beta$ is the non diagonal term in the stop mixing matrix (1.40) with the stop mass factorised. The ratio (B.21) is maximum when $\tilde{A}_t \ll m^2_Q$. The lightest stop mass reads:

$$m^2_{\tilde{t}} \simeq m^2_{\tilde{t}_L} + m^2_t \left( 1 - \tilde{A}_t^2/m^2_Q \right)$$

(B.22)
and it is possible to show \cite{46} that to have the biggest contribution to the parameter $E$ in the MSSM, $m_U$ must be small and negative. All these constraints sum up to give a bound on the stop mass: $m_{\tilde{t}} \lesssim m_t$. To avoid colour breaking minima, however, the stop mass must be bounded from below and it is possible to show \cite{59} that the allowed range for successful baryogenesis is:

$$120 \text{ GeV} \lesssim m_{\tilde{t}} \lesssim m_t$$

(B.23)

To conclude this section, EWBG is possible in the MSSM, provided some constraints are satisfied. The allowed parameter space is admittedly quite small, however the model cannot be excluded yet. A discovery of a light stop at the LHC could easily confirm or discard EWBG in the MSSM, therefore, performing a detailed analysis of stop production processes is mandatory.

### B.2 Light Stop production

#### B.2.1 The process $gg \to \tilde{t}_1 \tilde{t}_1^*$

Only the diagonal production of light stops have been considered since the heavy state does not play a significant role in supersymmetric electroweak baryogenesis. The analysis has been performed using the benchmark point LST-1 defined in \cite{59}, characterised by (only the relevant parameter are listed):

$$m_{\tilde{t}_1} \simeq 150 \text{ GeV} \quad m_{\tilde{q} \neq \tilde{t}_1} \simeq 1 \text{ TeV} \quad m_{h_0} \simeq 118 \text{ GeV} \quad \tan \beta \simeq 7$$

(B.24)

The results are shown in Fig.(B.3), where it is possible to see that one-loop contributions heavily affect the invariant mass distribution $d\sigma/dM_{\text{inv}}$. Nevertheless, even if at low energy the effects are more than 10%, they suddenly drop to negative values at high values of $M_{\text{inv}}$. The compensation is evident when looking at the total rate, $\sigma_{\text{tot}}^{1\text{-loop}} \simeq 146.00 \text{ pb}$ and $\sigma_{\text{tot}}^{\text{Born}} = 144.62 \text{ pb}$: the overall effect is slightly less than 1%.

Considering the partial rate, Eq. (3.50), it is possible to get rid of the positive contributions at low energy and, due to the light masses of the final state stops, with a partial rate of the order of 1 pb, the one-loop corrections are found to be sizable and of the order of -8%, as it is possible to see in Fig. (B.4).
Figure B.3: Diagonal stop production in LST-1. The top panel shows the differential distribution $d\sigma/dM_{inv}$; the bottom panel shows the relative one-loop effect on the differential distribution.

Figure B.4: Diagonal stop production in LST-1. The top panel shows the partial rate (in pb); the bottom panel shows the relative effect on the partial rate.
B.2.2 The process $bg \rightarrow \tilde{t}_1 \chi_1^-$

Here, only the production of light states of stop and chargino have been considered: the heavy stop, as already mentioned, is not relevant for baryogenesis, while the heavy chargino has not been considered due to the small cross sections involved, as it will be evident in the following discussion. The analysis has been performed using the same benchmark point of the previous section: the lightest chargino mass in this scenario is $m_\chi = 188.5 \text{ GeV}$. Fig. B.5 shows the results: the one-loop effects are quite relevant and, contrarily to most of the scenarios considered in Chapter 4, the effect on the total rate, given by the compensation of positive corrections at low $M_{\text{inv}}$ and negative corrections at high $M_{\text{inv}}$, is found to be positive and equal to 2.3%. The total rate is by itself not very high ($\sigma_{\text{tot}}^{1-\text{loop}} = 0.620$ and $\sigma_{\text{tot}}^{\text{Born}} = 0.606$), thus making the analysis in terms of the partial rate rather meaningless.

LST-1 stop-chargino production

Figure B.5: Stop-chargino production in LST-1. The top panel shows the differential distribution $d\sigma/dM_{\text{inv}}$; the bottom panel shows the relative one-loop effect on the differential distribution.
B.2.3 Discussion of results

Producing a very light stop at the LHC would indeed be a smoking gun for the model of EW baryogenesis in the MSSM. The analysis of the EW corrections to the two processes of stop production considered in this thesis, however, shows that looking for NLO EW effects at LHC would indeed be very challenging and that a careful choice of the observables must be made to perform the analysis. In this respect, the partial rate seems to be a privileged observable for the analysis of NLO EW effects for the stop-antistop production process: due to the high cross sections involved, the partial rate does not suddenly drop to undetectable levels, thus providing a chance for the detection of sizable effects of the order of 10%. Moreover, the scenario considered for the analysis is one of the most visible for stop-antistop production, and for this reason it is worth performing an accurate search for light stops and for loop effects on their pair production channel at the LHC.

On the other hand, the detection of a very light stop through the process of stop-chargino production would unfortunately be quite difficult because of the small cross section involved and to the mild NLO EW effects, of the order of few percent; the analysis of the partial rate is thus not worth the effort in this case.
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