Semi-inclusive associated
bottom-Higgs production at LHC:
The complete one-loop
electroweak effect in the MSSM

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DOTTORANDO: Giacomo Oliviero Dovier

COORDINATORE DEL COLLEGIO DEI DOCENTI: Chiar.mo prof. Camerini (Univ. di Trieste)

FIRMA: __________________________

TUTORE: Chiar.mo prof. Claudio Verzegnassi (Univ. di Trieste)

FIRMA: __________________________

RELATORE: Chiar.mo prof. Claudio Verzegnassi (Univ. di Trieste)

FIRMA: __________________________
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Chapter 1

Introduction

Since the end of 2009, the Large Hadron Collider started producing collisions. The machine will provide access to physics at the highest scale ever, with a center of mass energy for the colliding proton-proton system of 7 TeV in the preliminary stage of running, that will be increased further increased in the second stage, starting in 2012.

One of the main goal of the different experiments (in particular ATLAS and CMS) will be to find evidence for the Higgs boson, the last missing piece of the Standard Model picture. The particle has yet eluded all precedent searches, but the LHC should give the final answer to the question of its existence. The machine will also search for new exotic physics, seeking possible extensions to the Standard Model. In particular, a big effort has been devoted in the hunt for the supersymmetric partners of the Standard Model particles. Since the Higgs sector is heavily modified by the presence of SUSY, studying Higgs phenomenology has a twofold importance.

1.1 Semi Inclusive Bottom Higgs Production

The associated bottom-Higgs production could be a very relevant channel for the discovery of the elusive boson. In the Minimal Supersymmetric Standard Model (MSSM), the nature of the Yukawa couplings should en-
1.1. Semi Inclusive Bottom Higgs Production

hance the Higgs production in association with a bottom quark, in particular in a high \( \tan \beta \) scenario. In addition, the single \( b \) quark in the final state can be tagged in the data analysis, making it easier for the experimentalists to identify the process.

The possible relevance of the associated bottom-Higgs production in the experimental search of the Higgs at the LHC gives a strong motivation to this study. The process has already been studied in the literature (proper references will be given later), but a computation of the full MSSM one-loop electroweak effect has never been performed. This is what we calculated, including the overall QED contribution and the SUSY QCD effect.

The calculation has been performed in two different renormalization schemes, the so-called \( \text{DR} \) and DCPR. The final result should be scheme-independent, barring higher order corrections: should this property hold to be true, this will provide also an important check of the validity of our calculation. We will check if this is indeed the case, and provide some suggestions about the best scheme to use once the data begin to arrive from the LHC.

Having computed the full result, we will then compare it with two possible approximation schemes (one of common use and one of our creation), and discuss their accuracy in reproducing the exact one loop result.

Finally, we will study the case of a Two Higgs Doublet Standard model, computing the one loop corrections in different scenarios, and comparing them to the corresponding SUSY results.
1.2 This Thesis

In this work the process of the calculation will be illustrated in detail, including all the obtained results and their discussion. Our main goal will be to see if the electroweak one loop effects are negligible, or if instead they are relevant, and have to be taken into account in a dedicated experimental analysis.

This thesis will be organized as follows:

- Chapter 2 gives a brief introduction to the Standard Model and the MSSM
- Chapter 3 describes some of the methods employed in the calculation
- Chapter 4 illustrates in full details the process of the calculation and discusses the results we obtained
1.2. This Thesis

- Chapter 5 contains a brief summary about the most important conclusions.

- Appendix A and Appendix B contain some technical details about the renormalization of the MSSM Higgs sector, the contribution of the counterterms and the cancellation of the UV divergences arising in different sectors.
Chapter 2

The Higgs in the Standard Model and in the MSSM

This chapter presents a brief review of the Higgs mechanism and the Minimal Supersymmetric Standard Model, to provide a reference for the notation to be used in the following chapters and motivations for the main work.

2.1 The Standard Model

The Standard Model is a gauge theory that provides a precise description of the interaction of all the known particles. The gauge group is $SU(3)_{\text{STRONG}} \times SU(2)_{L} \times U(1)_{Y}$, and the electroweak gauge lagrangian can be written as:

$$L_{EW} = -\frac{1}{4} W_{\mu\nu}^{i} W^{i\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \bar{\psi}_{L} i \gamma^\mu D_{\mu} \psi_{L} + \bar{\psi}_{R} i \gamma^\mu D_{\mu} \psi_{R},$$

(2.1)

where:

$$W_{\mu\nu}^{i} = \partial_{\mu} W_{\nu}^{i} - \partial_{\nu} W_{\mu}^{i} + g e_{ijk} W_{\mu}^{j} W_{\nu}^{k}$$

$$B_{\mu\nu}^{i} = \partial_{\mu} B_{\nu}^{i} - \partial_{\nu} B_{\mu}^{i}.$$  

(2.2)
2.1. The Standard Model

$W^i, B^i$ are the $SU(2)_L \times U(1)_Y$ gauge bosons, $\psi_{L/R}$ are the left and right handed fermion fields, defined as:

$$\psi_R = P_R \psi, \quad \psi_L = P_L \psi,$$

(2.3)

where $g$ and $g'$ are the $SU(2)$ and $U(1)$ coupling constants, $\tau_i$ are the generators of $SU(2)$ and $y_L$ is the $U(1)$ quantum number.

The QCD piece of the lagrangian can be written instead as:

$$L_{QCD} = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} - \sum_q \bar{\psi}_q (i \gamma^\mu D_\mu) \psi_q,$$

(2.6)

where $\psi_q$ are the quark fields and

$$G^a_{\mu\nu} = \partial_\mu G^a_{\nu} - \partial_\nu G^a_{\mu} + i \alpha_s [G^a_{\mu}, G^a_{\nu}],$$

(2.7)

$$D_\mu q = (\partial_\mu - i \alpha_s G^a_{\mu} T_a) q.$$  

(2.8)

The index $a$ is the $SU(3)$ index and goes from 1 to 8, $G^a_{\mu}$ are the gluon fields, $T_a$ are the generators of the gauge group and $\alpha_s$ is the strong coupling constant. Under the gauge symmetries of the group, the fermionic content of the model has the following structure:
2.2 The EWSB Mechanism in the SM

In the SM Lagrangian written before, all the particles involved are massless. It’s in fact impossible to naturally include mass terms (of the type $m^2 \psi_L \psi_R$) in the Standard Model Lagrangian, since those terms would break the gauge symmetries of the model. If one wishes to preserve those symmetries, explicitly breaking terms are not allowed to appear. The mechanism for the generation of the fermions’ and bosons’ masses is called spontaneous Electro-Weak Symmetry Breaking (EWSB), and involves the introduction of a scalar doublet in the model, the so called Higgs Boson:

$$\phi = \begin{pmatrix} \phi^- \\ \phi^0 \end{pmatrix}$$

(2.9)

With a potential (only the pure Higgs part):

$$V(\phi) = -\mu^2 \phi^* \phi + \lambda \phi^* \phi \phi^* \phi$$

(2.10)

This boson will couple with the fermions in the Standard Model through a Yukawa-type interaction:

$$\mathcal{L}_{Yuk} = -\lambda \psi_L \psi_R \phi + H.C.,$$

(2.11)
2.2. The EWSB Mechanism in the SM

Figure 2.1: The particle content of the Standard Model\textsuperscript{[2]}

were $\lambda_\psi$ is the coupling constant associated to the fermion species. The covariant derivative of the Higgs doublet will be

$$D_\mu \phi = (\partial_\mu - igW_\mu^i \frac{\tau^i}{2} - ig'^y_L B_\mu)^\phi.$$ (2.12)

Symmetry breaking occurs when the Higgs acquires a vacuum expectation value different from zero, and we have, in the Unitary Gauge:

$$\phi = \phi_0 + \phi'$$ (2.13)

with

$$\phi_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$$ (2.14)

and

$$\phi' = \begin{pmatrix} 0 \\ \rho \end{pmatrix}, \quad <0|\rho|0> = 0.$$ (2.15)

We have then for the fermion masses:

$$\mathcal{L}_{\text{Mass}} = -\lambda_v \frac{v}{\sqrt{2}} (\bar{e}_L e_R + \bar{e}_R e_L) - \lambda_{ij}^d \overline{Q_L}^i \phi^j R_j - \lambda_{ij}^u \overline{Q_L}^i \hat{\phi} u_R^j + H.C.,$$ (2.16)
2.3. The MSSM

The quark mass matrices are not diagonal, giving rise to the phenomenon of quark mixing. After the symmetry breaking, three degrees of freedom of the Higgs doublet are absorbed to become the longitudinal component of the gauge bosons. For those bosons one has that the physical states are:

\[
W_\mu^+ = W^1_\mu + iW^2_\mu \quad W^-_\mu = W^1_\mu - iW^2_\mu, \quad \text{(2.17)}
\]

with mass \( M_W = g^2 v \), while \( B_\mu \) and \( W^3_\mu \) mix to give \( Z_\mu \) and \( A_\mu \):

\[
\begin{pmatrix} Z \\ A \end{pmatrix} = \begin{pmatrix} \cos \theta_w & -\sin \theta_w \\ \sin \theta_w & \cos \theta_w \end{pmatrix} \begin{pmatrix} W^3 \\ B \end{pmatrix}, \quad M_Z = g \cos \theta_w \frac{v}{2} \quad \text{(2.18)}
\]

The fourth field is a mass eigenstate, the Higgs boson.

2.3 The MSSM

While the Standard Model did an excellent job in explaining the phenomenology of high energy physics up to now, there is more than one hint that points to the necessity of an extension of the model. The two most compelling reasons to extend the model are the Dark Matter puzzle and the so-called hierarchy problem. Cosmological observations point to one or more stable, relatively heavy and weak interacting particle(s) contributing to more than 20% to the value of \( \Omega \), for which there is no account in the Standard Model. On the other hand, the main theoretical motivation for an extension of the SM is the Higgs’ hierarchy problem. In the Standard Model, the Higgs mass is very sensitive to loop corrections. From indirect observations, we expect the Higgs mass to be roughly of the order of 100 GeV. This value, however, receives huge corrections from loops containing Dirac fermions which couple to the Higgs, of the type:

\[
\Delta m_H^2 = -\frac{|\lambda|^2}{8\pi^2} \Lambda_{UV}^2 + O(\log \Lambda_{UV}), \quad \text{(2.19)}
\]
where $f$ is a fermion with a Yukawa type coupling $-\lambda_f H \bar{f} f$, and $\Lambda_{UV}$ is a cutoff energy scale. For the nature of the Yukawa couplings in the Standard Model, the biggest contribution should arise from diagrams involving top quarks, since $|\lambda_t| \sim 1$. If we take the value of $\Lambda_{UV}$ to be the one of the Planck scale ($10^{18}$ GeV), then the one loop correction to the value of the Higgs mass turns out to be $30$ orders of magnitudes bigger than the mass itself.

It looks then reasonable to pick a smaller cutoff, meaning that some new theory could arise to keep the corrections in check at an energy scale well below the Planck one.

The most compelling extension to the Standard Model is obtained adding a new space-time symmetry to the theory, called Supersymmetry (SUSY). The effect of this symmetry is to introduce a superpartner for each of the particles appearing in the Standard Model. These superpartners have the same quantum numbers of their corresponding standard particles, but the spin is shifted by $\frac{1}{2}$, making the superparticle a boson if the “original” particle was a fermion and vice versa. Since none of these superparticles can be identified with an object already observed, SuperSymmetry implies the existence of a whole new spectrum of particles to be discovered.

SUSY does indeed solve the hierarchy problem: since particles and sparticles have the same couplings and quantum numbers, the divergence arising in the top loop would be canceled by an identical divergence coming from a loop involving the superpartner of the top, that will come with an opposite sign since the stop is a scalar, while the top is a fermion.

The simplest supersymmetric extension of the Standard Model is the Minimal Supersymmetric Standard Model (MSSM). The particle content of the model includes the Standard Model particles and their partners, and no other particle is added to the model. One other condition is usually required: that barion and lepton numbers are conserved. This requirement is driven by the absence of observations of proton decay, and can be achieved imposing the so-called R-parity conservation. R-parity is a new discrete $Z_2$
2.3. The MSSM

symmetry, and is defined by a conserved multiplicative quantum number as follows:

\[ R = (-1)^{L \cdot 3B + 2S}, \]  

(2.20)

where \( L \) is the lepton quantum number, \( B \) is the barion quantum number, and \( S \) is the spin quantum number. The conservation of lepton and barion numbers is then obtained imposing that each term appearing in the supersymmetric lagrangian has a total value of \( R = +1 \), thus prohibiting renormalizable interactions that violate those numbers.

It is important to notice that, according to the definition above, all the old Standard Model particles have \( R = 1 \), while the superpartners have \( R = -1 \). This implies that if R-parity is conserved, superparticles can be produced only in pairs in collider experiments, and cannot decay into Standard Model particles. This means that each decay chain of a supersymmetric particle should end with an odd number of the so-called LSPs (Lightest Supersymmetric Particle) and that this particle should be stable. The LSP turns out to be, then, a natural candidate for the solution of the Dark Matter puzzle, provided that it is neutral and weakly interacting.

2.3.1 SUSY spectrum

If SUSY was an exact and conserved symmetry of nature, the masses of the particles and their superpartners should have been the same. Unfortunately, this appears not to be true in nature, since we have yet to observe a superpartner candidate for any of the SM particles.

We expect, then, that SUSY is broken at some high energy scale. The description of the SUSY breaking is still an open question. In the MSSM the problem is solved with a phenomenological approach, introducing by hand in the lagrangian the SUSY breaking terms. These terms are often called "soft", since they should not not reintroduce quadratic divergences in the theory. The allowed terms, then, are all the gauge invariant operators of positive (2 or 3) mass dimensions. From a
2.3. The MSSM

Theoretical point of view, these new terms of the lagrangian represent an effective theory, parameterizing at a lower scale the unknown process of the SUSY breaking. These terms are:

1. Mass terms for the scalar fields

\[ -m_{H_1}^2 H_1^1 H_1^1 - m_{H_2}^2 H_2^2 H_2^2 - (m_{L}^2)^{IJ} L_i^I L_i^J - (m_{R}^2)^{IJ} R_i^I R_i^J - (m_{Q}^2)^{IJ} Q_i^I Q_i^J - (m_{D}^2)^{IJ} D_i^I D_i^J - (m_{U}^2)^{IJ} U_i^I U_i^J \]  \hspace{1cm} (2.21)

2. Mass terms for the gauginos

\[ \frac{1}{2} M_1 \lambda_B \lambda_B + \frac{1}{2} M_2 \lambda_A^i \lambda_A^i + \frac{1}{2} M_3 \lambda_G^a \lambda_G^a + \text{H.c.} \]  \hspace{1cm} (2.22)

3. Bi- and trilinear couplings of the scalar fields corresponding to the Yukawa terms in the superpotential

\[ m_{12}^2 \epsilon_{ij} H_1^1 H_2^2 + \epsilon_{ij} A_i^{IJ} H_1^1 L_i^I R_j^J + \epsilon_{ij} A_d^{IJ} H_1^1 Q_i^I D_j^J + \epsilon_{ij} A_u^{IJ} H_2^2 Q_j^I U_i^J + \text{H.c.} \]  \hspace{1cm} (2.23)

**Higgs sector**  Differently from the Standard Model, in the MSSM two different Higgs doublets are needed. Since the superpotential is holomorphic, one can’t use the doublet’s conjugate to account for the down type (s)quarks masses, since terms like $\bar{d}QH_1^*$ break holomorphy, and are therefore prohibited. A second doublet is then needed, and the Yukawa part of the lagrangian has this form:

\[ \mathcal{L}_{\text{yukawa}} = -\lambda_d^{ij} \bar{Q}^*_L H_1^i d^j R_j + \lambda_u^{ij} \bar{Q}^*_L u^j H_2^i u^j \]  \hspace{1cm} (2.24)

The two MSSM Higgs doublets $H_1$ and $H_2$ can be decomposed as:

\[ H_1 = \begin{pmatrix} v_1 + \frac{1}{\sqrt{2}} (\phi_1^0 - i \chi_1^0) \\ -\phi_1^- \end{pmatrix}, \quad H_2 = \begin{pmatrix} \frac{\phi_2^+}{\sqrt{2}} \\ v_2 + \frac{1}{\sqrt{2}} (\phi_2^0 + i \chi_2^0) \end{pmatrix} \]  \hspace{1cm} (2.25)

Electroweak symmetry breaking happens like in the SM, but now both Higgses can acquire a v.e.v.:
2.3. The MSSM

\[
<H^1> = \frac{1}{\sqrt{2}} \begin{pmatrix} v_1 \\ 0 \end{pmatrix} \quad \quad <H^2> = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}
\]

(2.26)

With

\[v_1^2 + v_2^2 = v_{SM}^2,\]

(2.27)

and

\[
M_Z = \frac{e}{2 s_W c_W} \left(v_1^2 + v_2^2\right)^{\frac{1}{2}}
\]

(2.28)

\[
M_W = \frac{e}{2 s_W} \left(v_1^2 + v_2^2\right)^{\frac{1}{2}}
\]

(2.29)

The Higgs mass eigenstates can be obtained rotating the scalar fields appearing in equation 2.25:

\[
\begin{pmatrix} H^0 \\ h^0 \end{pmatrix} = \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} \phi_1^0 \\ \phi_2^0 \end{pmatrix}
\]

\[
\begin{pmatrix} G^0 \\ A^0 \end{pmatrix} = \begin{pmatrix} \cos \beta_1 & \sin \beta_1 \\ -\sin \beta_1 & \cos \beta_1 \end{pmatrix} \begin{pmatrix} \chi_1^0 \\ \chi_2^0 \end{pmatrix}
\]

\[
\begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} = \begin{pmatrix} \cos \beta_2 & \sin \beta_2 \\ -\sin \beta_2 & \cos \beta_2 \end{pmatrix} \begin{pmatrix} \phi_1^\pm \\ \phi_2^\pm \end{pmatrix}
\]

(2.30)

Written in terms of the mass eigenstates the linear and bilinear terms of the Higgs potential reads:

\[
V = -T_{h^0} h^0 - T_{H^0} H^0 + \frac{1}{2} \left( h^0 H^0 \right) \begin{pmatrix} M_{h^0}^2 & M_{h^0 H^0}^2 \\ M_{h^0 H^0}^2 & M_{H^0}^2 \end{pmatrix} \begin{pmatrix} h^0 \\ H^0 \end{pmatrix}
\]

\[
+ \frac{1}{2} \begin{pmatrix} A^0 & G^0 \end{pmatrix} \begin{pmatrix} M_{A^0}^2 & M_{A^0 G^0}^2 \\ M_{A^0 G^0}^2 & M_{G^0}^2 \end{pmatrix} \begin{pmatrix} A^0 \\ G^0 \end{pmatrix}
\]

\[
+ \begin{pmatrix} H^- G^- \end{pmatrix} \begin{pmatrix} M_{H^-}^2 & M_{H^- G^+}^2 \\ M_{G^- H^+}^2 & M_{G^-}^2 \end{pmatrix} \begin{pmatrix} H^+ \\ G^+ \end{pmatrix}
\]

(2.31)

This potential depends on seven parameters. Beside the Standard model parameters, \(e, M_W, M_Z\), we have four new parameters, namely:

\[
T_{h^0}, \quad T_{H^0}, \quad M_{A^0}, \quad \tan \beta,
\]

(2.32)
where \( T_{h^0} \) and \( T_{H^0} \) are the tadpole terms for \( h_0, H_0 \) and

\[
\tan \beta = \frac{v_2}{v_1}.
\]

(2.33)

In terms of the new parameters, the non-diagonal entries of the mass matrices describing the mixing between Higgs bosons are given by

\[
M^2_{h^0 H^0} = -\frac{1}{2} M_Z^2 \sin(\alpha + \beta) + \frac{1}{2} M_A^2 \sin(\alpha - \beta)/ \cos^2(\beta - \beta_1)
\]
\[
+ \frac{e}{2M_Z s_W c_W} T_{h^0} \sin(\alpha - \beta) \sin^2(\alpha - \beta_1)/ \cos^2(\beta - \beta_1)
\]
\[
- \frac{e}{2M_Z s_W c_W} T_{h^0} \cos(\alpha - \beta) \cos^2(\alpha - \beta_1)/ \cos^2(\beta - \beta_1),
\]

\[
M^2_{A^0 G^0} = -M_A^2 \tan(\beta - \beta_1) - \frac{e}{2M_Z s_W c_W} T_{H^0} \sin(\alpha - \beta_1)/ \cos(\beta - \beta_1)
\]
\[
- \frac{e}{2M_Z s_W c_W} T_{h^0} \cos(\alpha - \beta_1)/ \cos(\beta - \beta_1),
\]

\[
M^2_{H^+ - G^+} = -M_{H^+}^2 \tan(\beta - \beta_2) - \frac{e}{2M_Z s_W c_W} T_{H^0} \sin(\alpha - \beta_2)/ \cos(\beta - \beta_2)
\]
\[
- \frac{e}{2M_Z s_W c_W} T_{h^0} \cos(\alpha - \beta_2)/ \cos(\beta - \beta_2),
\]

(2.34)

where \( s_W \) and \( c_W \) denote the sine and cosine of the weak mixing angle, respectively. The condition that \( M^2_{A^0 G^0}, M^2_{H^+ - G^+} \) and the tadpoles vanish (to minimize the three level potential) yields \( \beta_1 = \beta_2 = \beta \), while \( M^2_{h^0 H^0} = 0 \) fixes the mixing angle \( \alpha \). \( \alpha \) is given by:

\[
\tan 2\alpha = \tan 2\beta \frac{M_{A^0}^2 + M_Z^2}{M_{A^0}^2 - M_Z^2}; \quad \frac{\pi}{2} < \alpha < 0.
\]

(2.35)

From the initial two complex doublet, we obtain a total of 8 degrees of freedom. Three of them give origin to the Goldstone bosons \( G^0, H^+_2 (\equiv G^\pm) \) (like in the SM, in the Physical gauge they are “eaten” and become the longitudinal components of the \( Z \) and \( W \) bosons) and five to the physical Higgs bosons.

The relations between the gauge and physical Higgs states are given below.

1. Charged scalars

\[
M^2_{H^+} = M_W^2 + m_{H_1}^2 + m_{H_2}^2 + 2|\mu|^2
\]

(2.36)
2.3. The MSSM

\[
\begin{pmatrix}
H_2^{1*} \\
H_1^2
\end{pmatrix}
= Z_H
\begin{pmatrix}
H_1^+ \\
H_2^+
\end{pmatrix} \quad (2.37)
\]

\[
Z_H = (v_1^2 + v_2^2)^{-\frac{1}{2}}
\begin{pmatrix}
v_2 & -v_1 \\
v_1 & v_2
\end{pmatrix} \quad (2.38)
\]

2. Neutral scalars

"Scalar" particles (CP even Higgses) \( H_i^0 \), \( i = 1, 2 \), defined as:

\[
\sqrt{2} \Re H_i^0 = Z_R^{ij} H_j^0 + v_i \text{ (no sum over } i) \]

The matrix \( Z_R \) and the masses of \( H_i^0 \) can be obtained by diagonalizing the \( M_R^2 \) matrix

\[
Z_R^T \begin{pmatrix}
-m_1^2 v_1^2 + \frac{\epsilon_1^2 v_1^2}{4 s_W c_W} & m_2^2 - \frac{\epsilon_1^2 v_2}{4 s_W c_W} \\
m_1^2 - \frac{\epsilon_1^2 v_2}{4 s_W c_W} & -m_1^2 v_1^2 + \frac{\epsilon_1^2 v_2}{4 s_W c_W}
\end{pmatrix} Z_R =
\begin{pmatrix}
M_{H_1^0}^2 & 0 \\
0 & M_{H_2^0}^2
\end{pmatrix} \quad (2.39)
\]

"Pseudoscalar" particles (CP odd Higgses) \( A_i^0 \), \( i = 1, 2 \):

\[
\sqrt{2} \Im H_i^0 = Z_H^{ij} A_j^0 \text{ (no sum over } i) \]

\( A_1^0 (\equiv A^0) \) has mass

\[
M_{A_1}^2 = m_{H_1}^2 + m_{H_2}^2 + 2|\mu|^2, \quad (2.40)
\]

while \( A_2^0 (\equiv G^0) \) is the massless Goldstone boson which disappears in the unitary gauge. The \( Z_H \) matrix is the same as in the case of the charged Higgs bosons.
Neutralinos and Charginos  The fermionic superpartners of the Higgses and gauge bosons are not mass eigenstates. They undergo a mixing procedure to obtain the physical particles, called Neutralinos and Charginos. The Neutralinos originate from the neutral higgsinos and gauginos. The mixing of the four spinors is governed by the $Z_N$ matrices:

$$
Z_N^T \begin{pmatrix}
M_1 & 0 & -eW_1 & eW_2 \\
0 & M_2 & eW_1 & -eW_2 \\
-eW_1 & eW_1 & 0 & -\mu \\
eW_2 & -eW_2 & -\mu & 0
\end{pmatrix} Z_N = 
\begin{pmatrix}
M_{\chi_1^0} & 0 & \\
& \ddots & \\
0 & & M_{\chi_4^0}
\end{pmatrix},
$$

(2.41)

where $\chi_1^0, \chi_2^0, \chi_3^0,$ and $\chi_4^0$ are the four Neutralinos.

The Charginos are originated instead from the charged component of the Higgsinos and the charged gauge bosons (called Winos). Their mixing generates two mass eigenstates, the Charginos $\chi_1, \chi_2$. This mixing is governed by two unitary matrices called $Z_+$ and $Z_-$$

$$(Z_-)^T \begin{pmatrix}
M_2 & -eW_2 \\
eW_1 & \mu \\
\end{pmatrix} Z_+ = 
\begin{pmatrix}
M_{\chi_1} & 0 & \\
& \ddots & \\
0 & & M_{\chi_2}
\end{pmatrix}
$$

(2.42)

Gluinos  The superpartners of the gluons are called Gluinos. They compose an octet, like their counterparts, and are only subjected to strong interactions. Therefore, there is no mixing in this sector.

Squarks and Sleptons  The SUSY particles corresponding to quarks and leptons are the scalar sfermions, Squarks and Sleptons. The superpartners of the left-handed and right-handed components of the standard model fermions mix to give rise to the physical eigenstates. In the most general situation of full mixing, one should diagonalize a single
3 × 3 matrix for the sneutrinos, and three 6 × 6 mass matrices for charged leptons, up and down squarks instead.

Fields $L_2^I$ and $R^I$ mix to give six charged sleptons $L_i, i = 1 \ldots 6$:

$$L_2^I = Z_{L}^{Ii} L_i^- \quad R^I = Z_{L}^{I(I+3)i} + L_i^+$$  (2.43)

$$Z_{L}^{I} \left( \begin{pmatrix} (M^{2}_{L})_{ii} & (M^{2}_{L})_{iR} \\ (M^{2}_{L})_{iR} & (M^{2}_{L})_{R} \end{pmatrix} \right) Z_{L} = \begin{pmatrix} M^{2}_{L_i} & 0 \\ \vdots & \ddots \\ 0 & M^{2}_{L_6} \end{pmatrix}$$  (2.44)

$$M^{2}_{L} = \frac{e^2(v_1^2 - v_2^2)(1 - 2c^2_W)}{8s_W c_W} + \frac{v_1^2 Y_1^2}{2} + (m_{L_i}^2)^T$$  (2.45)

$$M^{2}_{R} = -\frac{e^2(v_1^2 - v_2^2)}{4c_W^2} + \frac{v_1^2 Y_1^2}{2} + m_{R}^2$$  (2.46)

$$M^{2}_{LR} = \frac{1}{\sqrt{2}} \left( v_2 (Y_l u - A^T_{1}) + v_1 A_{l} \right)$$  (2.47)

Fields $Q^I_1$ and $U^I$ turn into six up squarks $U_i$.

$$Q^I_1 = Z_{U}^{Ii} U_i^+ \quad U^I = Z_{U}^{I(I+3)i} U_i^-$$  (2.48)

$$Z_{U}^{I} \left( \begin{pmatrix} (M^{2}_{U})_{ii} & (M^{2}_{U})_{iR} \\ (M^{2}_{U})_{iR} & (M^{2}_{U})_{R} \end{pmatrix} \right) Z_{U}^* = \begin{pmatrix} M^{2}_{U_i} & 0 \\ \vdots & \ddots \\ 0 & M^{2}_{U_6} \end{pmatrix}$$  (2.49)

$$M^{2}_{U} = -\frac{e^2(v_1^2 - v_2^2)(1 - 4c^2_W)}{24s_W c_W} + \frac{v_2^2 Y_u^2}{2} + (Km_{Q}^2 K^T)^T$$  (2.50)

$$M^{2}_{R} = \frac{e^2(v_1^2 - v_2^2)}{6c_W^2} + \frac{v_2^2 Y_u^2}{2} + m_{U}^2$$  (2.51)

$$M^{2}_{LR} = -\frac{1}{\sqrt{2}} \left( v_1 (A_u Y_{u} + A_{u}^T) + v_2 A_{u} \right)$$  (2.52)

Six down-squarks $D_i$ composed from fields $Q^I_2$ and $D^I$:

$$Q^I_2 = Z_{D}^{Ii} D_i^- \quad D^I = Z_{D}^{I(I+3)i} D_i^+$$  (2.53)
Three complex scalar fields $L_1^I$ form three sneutrino mass eigenstates $\tilde{\nu}^I$ with masses given by diagonalization of a matrix $\mathcal{M}_\nu^2$:

\begin{align}
L_1^I &= Z^{I J}_\nu \tilde{\nu}^J \\
Z^{I J}_\nu \mathcal{M}_\nu^2 Z_\nu &= \begin{pmatrix} M_{\nu_1}^2 & 0 \\ 0 & \ddots \\ 0 & M_{\nu_3}^2 \end{pmatrix} \\
\mathcal{M}_\nu^2 &= \frac{e^2 (v_1^2 - v_2^2)}{8 s_w^2 c_w^2} \hat{1} + m_L^2
\end{align}
### 2.3. The MSSM

<table>
<thead>
<tr>
<th>Particle Type</th>
<th>Particles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Photon</td>
<td>$F_\mu$</td>
</tr>
<tr>
<td>Gauge bosons</td>
<td>$Z^\mu_\mu, W^\pm_\mu$</td>
</tr>
<tr>
<td>Charginos</td>
<td>$\chi_i, i = 1, 2$</td>
</tr>
<tr>
<td>Neutralinos</td>
<td>$\chi^0_i, i = 1, \ldots, 4$</td>
</tr>
<tr>
<td>Gluons</td>
<td>$g^a_\mu, a = 1, \ldots, 8$</td>
</tr>
<tr>
<td>Gluinos</td>
<td>$\Lambda^a_\mu, a = 1, \ldots, 8$</td>
</tr>
<tr>
<td>Neutrinos</td>
<td>$\nu^i, i = 1, \ldots, 3$</td>
</tr>
<tr>
<td>Electrons</td>
<td>$e^i, i = 1, \ldots, 3$</td>
</tr>
<tr>
<td>Charged Sleptons</td>
<td>$L^\pm_i, i = 1, \ldots, 6$</td>
</tr>
<tr>
<td>Sneutrinos</td>
<td>$\tilde{\nu}^i, i = 1, \ldots, 3$</td>
</tr>
<tr>
<td>Up Quarks</td>
<td>$u^i, i = 1, \ldots, 3$</td>
</tr>
<tr>
<td>Down Quarks</td>
<td>$d^i, i = 1, \ldots, 3$</td>
</tr>
<tr>
<td>Squarks</td>
<td>$U^{\pm}_i, D^{\pm}_i, i = 1, \ldots, 6$</td>
</tr>
<tr>
<td>Higgs particles</td>
<td>$H^0, h^0, A^0, H^\pm$</td>
</tr>
</tbody>
</table>

Table 2.1: The particle spectrum of the MSSM
Chapter 3

Analytical and Numerical Methods

This brief section will be dedicated to illustrating some of the tools used in the computation, to introduce some notation and discuss some choices.

3.1 Analytical Methods for One Loop Calculations

To compute the different one loop integrals appearing in a next to leading order calculation, it is convenient to decompose the results of the different Feynman diagrams in terms of the so called Passarino-Veltmann functions. This will allow us to use some standard precomputed libraries when we proceed to the numerical evaluation of the result. The general form of the 1,2,3,4 point function in $D$ dimension can be written:

\[
A_0(a) = \int \frac{d^D k}{i \pi^2} \frac{1}{N_a}
\]

\[
[B_0, B^\mu, B^{\nu\rho}(ab)] = \int \frac{d^D k}{i \pi^2} \frac{[1, k_\mu, k_\mu k_\nu]}{N_a N_b}
\]

\[
[C^\mu_0, C^\mu, C^{\mu\rho}] (abc) = \int \frac{d^D k}{i \pi^2} \frac{[1, k_\mu, k_\mu k_\nu]}{N_a N_b N_c}
\]
3.1. Analytical Methods for One Loop Calculations

Figure 3.1: Convention for the masses and momenta in the PV functions

\[
[D_0, D^\mu, D^{\mu\nu}, D^{\mu\nu\rho}]^{(abcd)} = \int \frac{d^Dk}{i\pi^2} \frac{[1, k_\mu, k_\nu, k_\rho]}{N_a N_b N_c N_d} \ 
\]

Where the denominators \( N_i \) are of the form:

\[
\begin{align*}
N_1 &= k^2 - m_1^2 + i\epsilon \\
N_2 &= (k + p_1)^2 - m_2^2 + i\epsilon \\
N_3 &= (k + p_1 + p_2)^2 - m_3^2 + i\epsilon \\
N_4 &= (k + p_1 + p_2 + p_3)^2 - m_4^2 + i\epsilon,
\end{align*}
\]  

(3.2)

the \( p_i \) indicate the external momenta (all incoming in this convention), the \( m_i \) are the masses of the particle running in the loop (see figure 3.1 for the labeling convention), and \( k \) is the free loop momentum.

The tensorial integrals can be re-expressed as a linear combination of tensorial objects built with the momenta \( k_\mu \), the metric \( g_{\mu\nu} \) and the scalar Passarino-Veltman functions \[3\]. Following the standard decomposition we obtain

\[
\begin{align*}
B^{\mu}(12) &= p_1^\mu B_1(12) \\
B^{\mu\nu}(12) &= p_1^\mu p_2^\nu B_{21}(12) + g^{\mu\nu} B_{22}(12)
\end{align*}
\]
3.1. Analytical Methods for One Loop Calculations

\[ B_j(12) = B_j(p_1^2; m_1, m_2) = B_j(p_2^2; m_1, m_2) \]  

(3.3)

\[ C^\mu(123) = p_1^\mu C_{11}(123) + p_2^\mu C_{12}(123) \]

\[ C^{\mu\nu}(123) = p_1^\mu p_2^\nu C_{21}(123) + p_2^\mu p_3^\nu C_{22}(123) \]

\[ + (p_1^\mu p_2^\nu + p_2^\mu p_3^\nu) C_{23}(123) + g^{\mu\nu} C_{24}(123) \]

\[ C^{\mu\nu\rho}(123) = \sum_{i=1,2} C_{00i}(123) (g^{\mu\nu} p_i^\rho + g^{\mu\rho} p_i^\nu + g^{\nu\rho} p_i^\mu) + \]

\[ \sum_{i,j,k=1,2} C_{ijk}(123) p_i^\mu p_j^\nu p_k^\rho \]

\[ C_j(123) = C_j(p_1^2, p_2^2, p_3^2; m_1, m_2, m_3) \]  

(3.4)

\[ D^\mu(1234) = p_1^\mu D_{11}(1234) + p_2^\mu D_{12}(1234) + p_3^\mu D_{13}(1234) \]

\[ D^{\mu\nu}(1234) = p_1^\mu p_2^\nu D_{21}(1234) + p_2^\mu p_3^\nu D_{22}(1234) + p_3^\mu p_4^\nu D_{23}(1234) \]

\[ + (p_1^\mu p_2^\nu + p_2^\mu p_3^\nu) D_{24}(1234) + (p_1^\mu p_3^\nu + p_3^\mu p_4^\nu) D_{25}(1234) \]

\[ + (p_2^\mu p_3^\nu + p_3^\mu p_4^\nu) D_{26}(1234) + g^{\mu\nu} D_{27}(1234) \]

\[ D^{\mu\nu\rho}(1234) = \sum_{i=1,2,3} D_{00i}(1234) (g^{\mu\nu} p_i^\rho + g^{\mu\rho} p_i^\nu + g^{\nu\rho} p_i^\mu) + \]

\[ \sum_{i,j,k=1,2,3} D_{ijk}(1234) p_i^\mu p_j^\nu p_k^\rho \]

\[ D_j(1234) = D_j(p_1^2, p_2^2, p_3^2, p_4^2, (p_1 + p_2)^2, (p_2 + p_3)^2; m_1, m_2, m_3, m_4) \]  

(3.5)

Many of this functions have divergent parts, one should be very careful in the calculation to cancel them in the end.

The analytical calculation of the PV decomposition of the various diagrams has been performed by hand first, and then cross-checked with a Mathematica notebook written for this purpose. The same has been done to check the analytical cancelation of the divergences.

3.1.1 Numerical methods

For the numerical evaluation of the cross sections of the process, a C++ code called lepre has been written. The results from the code have been
cross-checked at the parton level with the output of the Mathematica notebook. The code calculates the results for the one-loop cross sections in the MSSM, with some simplifying assumptions for the MSSM parameters. First of all, a minimal flavour violation scenario is chosen for the sfermion sector. This means that the mixing between squarks and sfermions is only possible between members of the third generation, first and second generation members are not allowed to mix. The physical sfermion states come from a mixing of $\tilde{q}_L$ and $\tilde{q}_R$:

$$A_i^{L,R}(\tilde{q}_a) = R_{an} A_i^{L,R}(\tilde{q}_n)$$  \hspace{1cm} (3.6)

with

$$R_{1L} = R_{2R} = \cos \theta_q \hspace{0.5cm} R_{1R} = -R_{2L} = \sin \theta_q$$  \hspace{1cm} (3.7)

where $q$ are the third family squark $\tilde{t}, \tilde{b}$. An analogous equation holds for the stau sleptons, while for the first two generations we assume:

$$\theta_{\tilde{f}} = 0,$$  \hspace{1cm} (3.8)

which means

$$\tilde{f}_{1,2} = \tilde{f}_{L,R}$$  \hspace{1cm} (3.9)

with $\tilde{f}$ is a squark or a slepton of the first two families. This choice is motivated by the expectation of a very small mixing in the sfermion sector corresponding to the light fermions, due to the small Yukawa couplings.

In addition, the mixing matrices for the charginos and the neutralinos are taken to have real elements, and we don’t allow mixing between CP even and CP odd Higgses (i.e. our calculation is performed in the so-called rMSSM).

For the evaluation of the one-loop integrals (both in the code and in the Mathematica notebook), the LoopTools [4] package has been employed. This package can be used to evaluate the tensorial and 1-, 2-, 3- and 4-point scalar integrals once they have been decomposed to scalar PV functions.
3.1. Analytical Methods for One Loop Calculations

In LoopTools the ultraviolet divergences are treated with dimensional regularizations, using two parameters $\Delta$ and $\mu$. $\Delta$ is the proper divergent part, $\Delta = 2/(4 - n) + \gamma_E + \log 4\pi$, while $\mu$ is a dimensionful parameter used to keep the integral mass dimension identical for all values of the dimension $n$. In the full calculation, the Dimensional Reduction (DRED) scheme has been employed. In this scheme, only the $g^{\mu\nu}$ coming from the PV decomposition have dimension $n = 4 - 2\epsilon$; vector propagators and Dirac matrices have dimension $n = 4$.

It is then straightforward to check the presence of Ultra Violet (UV) divergences in the code, in any expression. Since UV-finite expressions should be independent of $\Delta$ and $\mu$, comparing two numerical values where the parameters have been changed, one can see the presence of UV divergences if those values are different.

There is another source of possible divergences, that can arise in diagrams involving a virtual photon running in the loop. These divergences are regularized by the package assigning an infinitesimal $\lambda$ mass to the photon, and then keeping the only relevant terms, which are the ones proportional to the logarithm of the parameter.

This divergences should be canceled with the ones arising from the soft photon emission, and the final result should not depend on the fictitious photon mass. The full details of this procedure are given in the following chapter.

---

As opposed to Dimensional Regularization (DREG), where all $g^{\mu\nu}$ and Dirac matrices have dimension $n = 4 - 2\epsilon$. This scheme, however, doesn’t preserve Supersymmetry, as the bosonic and fermionic degrees of freedom don’t match anymore.
Chapter 4

$bH^0$ production at the LHC

In this chapter we will focus on the description of the associated $bH^0$ production at the LHC, where a neutral Higgs is produced through Yukawa interaction with a bottom quark in the final state.

We will provide a full description of the electroweak corrections at the one loop level in the MSSM, discuss the behaviour of the cross section in the two different renormalization schemes $\overline{DR}$ and DCPR, provide two possible approximations to the full calculation, and compare the results obtained in the MSSM with the ones for a Two Higgs Doublet Standard Model. Finally, we will have a brief look at the Standard Model case.

4.1 Introduction and Previous Literature on the Process

In the last few years the associated bottom-Higgs production has been extensively studied in the literature. Depending on the choice of the flavour-scheme in the partonic description of the initial state and on the identified final state, one can consider a number of different partonic sub-processes for $H^0 + b_{jets}$ production: while the choice of the 4 versus 5 flavour scheme is mainly theoretically motivated, resulting in a reordering of the pertur-
4.1. Introduction and Previous Literature on the Process

Bative expansion [5], the requirement of a minimum number of tagged $b$ in the final state is physically relevant in the signal extraction. In a four-flavor-number scheme, there are no $b$ quarks in the initial state, and they’re originated from a gluon splitting into $b\bar{b}$ pairs. Thus, the lowest order process in the scheme is $gg \rightarrow b\bar{b}H^0$. In the five-flavor scheme, instead, one resums the collinear logarithms through the use of a $b$ quark parton distribution. The two schemes are equivalent to all orders in perturbation theory, but the results do not match exactly at finite order. It has however been shown in [6] that the two schemes should produce compatible results at NLO for $b\bar{b}H$ production at the LHC. A detailed description and comparison of the two schemes can also be found in [7], [8], and [9]. Five-flavor scheme has also been employed in [10].

Assuming the 5-flavour scheme (which ensures a better convergence of the perturbative series resumming large logarithms in the bottom PDF), one can consider three different types of production processes, depending on the required final states: the exclusive one where both bottom jets are tagged ($b\bar{b}H^0$ final state), the semi-inclusive one where only one bottom quark is tagged ($bH^0$), and the inclusive one where no bottom quark jets are tagged. While the inclusive process has a larger cross section [8], [10], the semi-inclusive with a high $p_{b,T}$ bottom in the final state is experimentally more appealing [11].

The relative weights of the partonic processes ($b\bar{b} \rightarrow H^0$, $bg \rightarrow bH^0$, $gg \rightarrow b\bar{b}H^0$) are analyzed in [8], where also the $\alpha_s$ corrections (NLO) to the leading sub-process $b\bar{b} \rightarrow H^0$ are computed. The NNLO order in QCD ($\alpha_s^2$) for the same sub-process is calculated in [8], while the electroweak (SM and MSSM) and SUSY-QCD NLO corrections have been computed in [12], showing that the size of electroweak corrections can be comparable, for large $\tan \beta$, with that of the strong ones.

The associated semi-inclusive production process ($bH^0$ final state) is
4.2 Kinematics

analyzed at the NLO in QCD in [11] and [9], while the effect of the SUSY QCD is given in [13]. Very recently, Dawson and Jaiswal have also computed, for the Standard Model process $bg \rightarrow bh_{SM}$, the one-loop weak corrections [14].

Finally, the exclusive process, where two bottom jets are tagged in the final state, is considered at the NLO in QCD in [5], [7], [15] and [6]. The leading Yukawa corrections for this partonic process are considered in [16] and SUSY QCD effects have also been computed in [17]. A more detailed summary of the results of the previously cited calculations can be found in [18].

4.2 Kinematics

Focusing then on the semi-inclusive process, and having chosen the five-flavour-number scheme, at lowest order there is only one partonic channel to be considered for bottom-Higgs production:

$$b(p_b) g(p_g) \rightarrow b(p'_b) H^0(p_{H^0})$$  \hspace{1cm} (4.1)

where $H^0$ is one of the three MSSM neutral Higgs bosons ($h^0, H^0, A^0$). One could furthermore distinguish two subprocesses, depending if the virtual bottom quark is exchanged in the u-channel or in the s-channel (see figure 4.1).

Choosing the partonic center of mass frame, the momenta of the particles

\footnote{One should in principle also consider the photon induced process $b\gamma \rightarrow bH^0$. The photon-induced process needed to subtract the collinear photon emission and absorb the divergence into the pdf. For this reason, the size of these processes depends on the factorization scheme, and it is in general small for MS or DIS schemes. Further discussion on the QED contributions can be found in sections 4.6 and 4.8; for the moment it is only important to acknowledge that, since the main goal of this work is the calculation of the NLO electroweak effects for $bh^0$ production, and the $b\gamma \rightarrow bH^0$ can be safely computed at the LO, we do not take into account the photon induced production in the following.}
4.2. Kinematics

Figure 4.1: Tree level processes for semi-inclusive bottom-Higgs associated production at the LHC: on the left the s-channel subprocess, on the right the u-channel subprocess.

can be written as:

\[
\begin{align*}
p_b &= (E_b; 0, 0, p), \\
p_g &= (p; 0, 0, -p), \\
p'_b &= (E'_b; p' \sin \theta, 0, p' \cos \theta), \\
p_{H^0} &= (E_{H^0}; -p' \sin \theta, 0, -p' \cos \theta).
\end{align*}
\]

In the following, it will be often useful to employ the Mandelstam variables, that are as usual defined as:

\[
\begin{align*}
s &= (p_g + p_b)^2 \\
t &= (p_b - p'_b)^2 \\
u &= (p_g - p'_b)^2
\end{align*}
\]

In addition, it will also be convenient to define two auxiliary momenta \( q \) and \( q' \) as follows

\[q = p_b + p_g, \quad q' = p'_b - p_g\]
4.3 The Amplitude

4.3.1 Born and one-loop amplitudes

We denote the $O(\alpha_s^a \alpha^b)$ contribution to the amplitude (differential cross section) of the process $X$ as $\mathcal{M}_X^{a,b} (d\sigma_X^{ab})$. The Born terms result from the $s$- and $u$-channel $b$ quark exchange of Figure 4.1. The color stripped tree-level amplitude reads as follows

\[
\mathcal{M}_{1/2}^{bg \rightarrow bH^0} = -\frac{g_s}{s-m_b^2} \bar{u}_b(\lambda_b') [c^L(bbH^0) P_L + c^R(bbH^0) P_R] (\hat{q} + m_b) \gamma_\mu u_b(\lambda_b) \\
- \frac{g_s}{u-m_b^2} \bar{u}_b(\lambda_b') \gamma_\mu (\hat{q}' + m_b) [c^L(bbH^0) P_L + c^R(bbH^0) P_R] u_b(\lambda_b),
\]  

(4.4)

where $\lambda_b$, ($\lambda_b'$) is the helicity of the initial (final) bottom quark while $\mu$ is the polarization of the gluon. $u_b(\lambda_b)$ [$u_b'(\lambda_b')$] is the spinor of the initial [final] bottom quark, $\epsilon_g(\mu) = (0; \mu/\sqrt{2}, -i/\sqrt{2}, 0)$ is the gluon polarization vector and $P_{R,L} = (1 \pm \gamma^5)/2$ are the chirality projectors. The first piece comes from the $s$-channel subprocess, while the second piece is due to the $u$-channel subprocess.

The relevant couplings $c^\eta(bbH^0)$ ($\eta = L, R$) are defined as

\[
c^\eta(bbH^0) = -\left(\frac{e m_b}{2 S_W M_W}\right) \cos \alpha \cos \beta, \quad c^\eta(bbA^0) = \left(\frac{e m_b}{2 S_W M_W}\right) \sin \alpha \cos \beta, \\
c^L(bbA^0) = -i \left(\frac{e m_b}{2 S_W M_W}\right) \tan \beta, \quad c^R(bbA^0) = c^L*(bbA^0).
\]  

(4.5)

We factorize out of the gluon couplings the colour matrix element $\lambda^a/2$. The sum over colors leads to a factor

\[
\sum_{a=1}^{8} \text{tr} \left\{ \frac{\lambda^a}{2} \frac{\lambda^a}{2} \right\} = 4
\]  

(4.6)

that multiplies the squared amplitude.

The generic helicity amplitude can be decomposed on a set of eight forms factors $J^{k\eta}$ ($\eta = L, R$) as follows

\[
\mathcal{M}_{1/2}^{bg \rightarrow bH^0} = \bar{u}_b(\lambda_b') \left( \sum_{k=1}^{4} \sum_{\eta=L,R} J^{k\eta} N_{bg \rightarrow bH^0}^{k\eta} \right) u_b(\lambda_b),
\]  

(4.7)
4.3. The Amplitude

where

\[ J_{1\eta} = \psi_d(\mu)P_\eta, \]
\[ J_{2\eta} = (\epsilon_\eta(\mu)\cdot p_\eta') P_\eta, \]
\[ J_{3\eta} = \phi_d(\mu) P_\eta, \]
\[ J_{4\eta} = (\epsilon_\eta(\mu)\cdot p_\eta')\psi_\eta P_\eta. \]  \hspace{1cm} (4.8)

The Helicity amplitudes are \( F_{\lambda,\mu,\lambda'}, \lambda, \mu, \lambda' \) and one obtains them from Dirac decompositions of invariants.

- From \( J_{1\eta} \):

\[ F_{+++} = -\left(\frac{pR}{\sqrt{2}}\right)(1 + r_b)(1 + \eta)(1 - r'_b) \sin \frac{\theta}{2} \] \hspace{1cm} (4.9)
\[ F_{++-} = -\left(\frac{pR}{\sqrt{2}}\right)(1 + r_b)(1 + \eta)(1 + r'_b) \cos \frac{\theta}{2} \] \hspace{1cm} (4.10)
\[ F_{-++} = -\left(\frac{pR}{\sqrt{2}}\right)(1 - \eta)(1 + r'_b) \cos \frac{\theta}{2} \] \hspace{1cm} (4.11)
\[ F_{--} = \left(\frac{pR}{\sqrt{2}}\right)(1 + r_b)(1 - \eta)(1 - r'_b) \sin \frac{\theta}{2} \] \hspace{1cm} (4.12)

- From \( J_{2\eta} \):

\[ F_{+++} = -\left(\frac{p'R}{\sqrt{2}}\right)(1 + \eta r_b - r'_b(\eta + r_b)) \cos \frac{\theta}{2} \] \hspace{1cm} (4.13)
\[ F_{++-} = \left(\frac{p'R}{\sqrt{2}}\right)(1 + \eta r_b - r'_b(\eta + r_b)) \cos \frac{\theta}{2} \] \hspace{1cm} (4.14)
\[ F_{-++} = \left(\frac{p'R}{\sqrt{2}}\right)(1 + \eta r_b + r'_b(\eta + r_b)) \sin \frac{\theta}{2} \] \hspace{1cm} (4.15)
\[ F_{--} = -\left(\frac{p'R}{\sqrt{2}}\right)(1 + \eta r_b + r'_b(\eta + r_b)) \sin \frac{\theta}{2} \] \hspace{1cm} (4.16)
\[ F_{+++} = -\left(\frac{p'R}{\sqrt{2}}\right)(1 - \eta r_b - r'_b(\eta - r_b)) \sin \frac{\theta}{2} \] \hspace{1cm} (4.17)
\[ F_{++-} = \left(\frac{p'R}{\sqrt{2}}\right)(1 - \eta r_b - r'_b(\eta - r_b)) \sin \frac{\theta}{2} \] \hspace{1cm} (4.18)
\[ F_{-++} = -\left(\frac{p'R}{\sqrt{2}}\right)(1 - \eta r_b + r'_b(\eta - r_b)) \cos \frac{\theta}{2} \] \hspace{1cm} (4.19)
\[ F_{--} = \left(\frac{p'R}{\sqrt{2}}\right)(1 - \eta r_b + r'_b(\eta - r_b)) \cos \frac{\theta}{2} \] \hspace{1cm} (4.20)
4.3. The Amplitude

- From $J_{3\eta}$:

\[
F_{+++} = -\left(\frac{R}{\sqrt{2}}\right)(\eta + r_b + r'_b(1 + \eta r_b)) \sin \frac{\theta}{2} \quad (4.21)
\]
\[
F_{++-} = -\left(\frac{R}{\sqrt{2}}\right)(\eta + r_b - r'_b(1 + \eta r_b)) \cos \frac{\theta}{2} \quad (4.22)
\]
\[
F_{-++} = \left(\frac{R}{\sqrt{2}}\right)(\eta - r_b + r'_b(1 - \eta r_b)) \cos \frac{\theta}{2} \quad (4.23)
\]
\[
F_{-+-} = -\left(\frac{R}{\sqrt{2}}\right)(\eta - r_b - r'_b(1 - \eta r_b)) \sin \frac{\theta}{2} \quad (4.24)
\]

- From $J_{4\eta}$:

\[
F_{+++} = -\left(\frac{y'pR \sin \theta}{2\sqrt{2}}\right)(1 + r_b)(1 + \eta)(1 + r'_b) \cos \frac{\theta}{2} \quad (4.25)
\]
\[
F_{++-} = \left(\frac{y'pR \sin \theta}{2\sqrt{2}}\right)(1 + r_b)(1 + \eta)(1 + r'_b) \cos \frac{\theta}{2} \quad (4.26)
\]
\[
F_{-++} = -\left(\frac{y'pR \sin \theta}{2\sqrt{2}}\right)(1 + r_b)(1 + \eta)(1 - r'_b) \sin \frac{\theta}{2} \quad (4.27)
\]
\[
F_{-+-} = \left(\frac{y'pR \sin \theta}{2\sqrt{2}}\right)(1 + r_b)(1 + \eta)(1 - r'_b) \sin \frac{\theta}{2} \quad (4.28)
\]
\[
F_{++-} = -\left(\frac{y'pR \sin \theta}{2\sqrt{2}}\right)(1 + r_b)(1 - \eta)(1 + r'_b) \sin \frac{\theta}{2} \quad (4.29)
\]
\[
F_{-++} = \left(\frac{y'pR \sin \theta}{2\sqrt{2}}\right)(1 + r_b)(1 - \eta)(1 + r'_b) \sin \frac{\theta}{2} \quad (4.30)
\]
\[
F_{+-+} = -\left(\frac{y'pR \sin \theta}{2\sqrt{2}}\right)(1 + r_b)(1 - \eta)(1 - r'_b) \cos \frac{\theta}{2} \quad (4.31)
\]
\[
F_{-+} = \left(\frac{y'pR \sin \theta}{2\sqrt{2}}\right)(1 + r_b)(1 - \eta)(1 - r'_b) \cos \frac{\theta}{2} \quad (4.32)
\]

In the previous formulae $R$, $r_b$ and $r'_b$ stand for:

\[
R = \sqrt{(E_b + m_b)(E'_b - m_b)}
\]
\[
r_b = \frac{p}{E_b + m_b} \quad r'_b = \frac{p'}{E'_b + m_b} \quad (4.33)
\]

The only non-zero scalar functions at the tree level are $N^{1\eta}_{bg\to b\eta^0}$ and $N^{2\eta}_{bg\to b\eta^0}$. They read as follows

\[
N^{1\eta}_{bg\to b\eta^0} = -g_s \frac{c^\eta (bb\eta^0)}{s - m_b^2} - g_s \frac{c^\eta (bb\eta^0)}{u - m_b^2}, \quad N^{2\eta}_{bg\to b\eta^0} = -2g_s \frac{c^\eta (bb\eta^0)}{u - m_b^2} \quad (4.34)
\]
4.3. The Amplitude

The one-loop electroweak virtual contributions arise from many different types of diagrams. We divide the large number of diagrams appearing in the following classes:

- Vertex corrections or “Triangular” diagrams
- Four legs one-particle irreducible diagrams, or “Boxes”
- Self energies
- Counterterms (for the various bottom quarks lines, for the $\mathcal{H}^0$ line, and for the $bb\mathcal{H}^0$ coupling constants)
- Real photon radiation

The corresponding generic diagrams, where virtual particles running in the loops are left unspecified except for their spin, have been produced with \texttt{FeynArts}\cite{19}, and can be read off from Fig.\texttt{4.2, 4.3, 4.4}.

All these contributions have been computed using the usual decomposition in terms of Passarino-Veltman functions and the complete amplitude has been implemented in a C++ numerical code called \texttt{Lepre}. 

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Triangle diagrams (part I)

Triangle diagrams (part II)

Figure 4.2: Diagrams for the vertex corrections, of triangular shape.
Figure 4.3: Four legs one-particle irreducible diagrams, or “Boxes”
4.3. The Amplitude

b quark self energy

\[
\begin{array}{c}
\text{S} & \text{b} & \text{F} \\
\text{b} & \text{F} & \text{b}
\end{array}
\]

h\(0\) self energy

\[
\begin{array}{c}
\text{S} & \text{V} & \text{F} \\
\text{F} & \text{S} & \text{V}
\end{array}
\]

internal self–energy diagrams

\[
\begin{array}{c}
\text{U} & \text{V} & \text{S} \\
\text{V} & \text{S} & \text{V}
\end{array}
\]

Figure 4.4: Bottom quark self energies, higgs self energies (only the diagonal case) and internal self energies.
4.4 Self Energies and Counterterms

To obtain the full one loop electroweak corrections, as well as for canceling the divergences arising from some triangular diagrams, the counterterms for the various bottom quarks lines, for the $H^0$ line, and for the $bbH^0$ coupling constants should be computed. To obtain them, one should calculate the self energy function for the various particles appearing, taking all the contributions in the loops.

With reference to the amplitude decompositions into different form factors detailed in the previous section (4.7), one has that the counterterms contributes to the different scalar functions can be written in the following way:

$$\bar{u}_b(\lambda)_b \left( \sum_{k=1}^{4} \sum_{\eta=L,R} J^{k\eta} N^{k\eta}_{gg \to bH^0} \right) u_b(\lambda)_b, \quad (4.35)$$

The various $N^{k\eta}$ functions are explicitly written below. More details on the origin of the counterterms will be given in Appendix B.

**Total s-channel c.t. terms**

$$N_{1L}^{c.t. s} = -\frac{g_s}{s - m_b^2} \left\{ \frac{3}{2} \delta Z^b_L c^L(bbH^0_i) \right. \right. \left\} \right\}$$

(4.36)

$$N_{1R}^{c.t. s} = -\frac{g_s}{s - m_b^2} \left\{ \frac{1}{2} \delta Z^b_R c^R(bbH^0_i) \right. \right. \left\} \right\}$$

(4.37)

$$N_{3L}^{c.t. s} = -\frac{m_b g_s}{s - m_b^2} \left\{ \delta Z^b_L + \frac{1}{2} \delta Z^b_R c^R(bbH^0_i) + \frac{1}{2} \delta Z_L^c c^R(bbH^0_i) \right\}$$

(4.38)
4.4. Self Energies and Counterterms

\[ + \delta c^R (bbH^0_i) + \frac{1}{2} \sum_j (\delta Z_{Rj}^b c^R_j) \] - m_b N_{1R}^{c.t.} s

\[ = \frac{m_b g_s}{s - m_b^2} (\delta Z_{R}^b - \delta Z_{L}^b) c^R \] (4.38)

\[ N_{3R}^{c.t.} s = - \frac{m_b g_s}{s - m_b^2} \left\{ \left( \delta Z_{R}^b + \frac{1}{2} \delta Z_{L}^b \right) c^L (bbH^0_i) + \frac{1}{2} (\delta Z_{R}^b c^L) (bbH^0_i) \right\} + \delta c^L (bbH^0_i) + \frac{1}{2} \sum_j \delta Z_{j}^b c^L_j - m_b N_{1L}^{c.t.} s

\[ = \frac{m_b g_s}{s - m_b^2} (\delta Z_{L}^b - \delta Z_{R}^b) c^L \] (4.39)

From the b self energies one gets (\( \eta = +1, -1 \) means R, L):

\[ N_{1_R}^{b \text{ s.e.}} = g_s \frac{c^R (bbH^0_i)}{(s - m_b^2)^2} \left\{ \frac{1}{2} \sum_j (\delta Z_{Rj}^b + \delta Z_{Rj}^b) \right\} + \frac{1}{2} \sum_j \delta Z_{j}^b c^L_j \]

\[ = g_s \frac{c^R (bbH^0_i)m_b}{(s - m_b^2)^2} \left[ \Sigma_{\eta}^b(s) + \frac{1}{2} \left( \delta Z_{R}^b + \delta Z_{L}^b \right) \right] \] (4.40)

\[ N_{3_R}^{b \text{ s.e.}} = g_s \frac{c^{-\eta} (bbH^0_i)m_b}{(s - m_b^2)^2} \left[ \Sigma_{\eta}^b(s) + \frac{1}{2} \left( \delta Z_{R}^b + \delta Z_{L}^b \right) \right] \] (4.41)

Total u-channel c.t. terms

\[ N_{1_L}^{c.t.} u = - g_s \frac{1}{(u - m_b^2)} \left\{ \left( \frac{3}{2} \delta Z_{R}^b + \frac{1}{2} \delta Z_{L}^b \right) c^L \right\} + \delta c^L + \frac{1}{2} \sum_j \delta Z_{j}^b c^L_j \] (4.42)

\[ N_{1_R}^{c.t.} u = - g_s \frac{1}{(u - m_b^2)} \left\{ \left( \frac{3}{2} \delta Z_{L}^b + \frac{1}{2} \delta Z_{R}^b \right) c^R \right\} + \delta c^R + \frac{1}{2} \sum_j \delta Z_{j}^b c^R_j \] (4.43)

\[ N_{2_L}^{c.t.} u = - 2 g_s \frac{1}{(u - m_b^2)} \left\{ \left( \frac{3}{2} \delta Z_{R}^b + \frac{1}{2} \delta Z_{L}^b \right) c^L \right\} + \delta c^L + \frac{1}{2} \sum_j \delta Z_{j}^b c^L_j \] (4.44)

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4.4. Self Energies and Counterterms

\[ N_{2R}^{ct} u = -2g_s \frac{1}{(u - m_b^2)} \left\{ \left( \frac{3}{2} \delta Z^b_L + \frac{1}{2} \delta Z^b_R \right) c_R \right\} + \delta c_R + \frac{1}{2} \sum_j \delta Z^s_{ji} c^R_j \]  \hspace{1cm} (4.45)

\[ N_{3L}^{ct} u = g_s \frac{m_b c_L}{(u - m_b^2)} \left\{ \delta Z^b_R - \delta Z^b_L \right\} \]  \hspace{1cm} (4.46)

\[ N_{3R}^{ct} u = g_s \frac{m_b c_R}{(u - m_b^2)} \left\{ \delta Z^b_L - \delta Z^b_R \right\} \]  \hspace{1cm} (4.47)

From the self energies one gets (\( \eta = +1, -1 \) means \( R, L \)):

\[ N_{1\eta}^{s.e.} = g_s \frac{c_\eta}{(u - m_b^2)^2} \left[ u \left( \Sigma^b_{-\eta}(u) + \delta Z^b_{-\eta} \right) + m_b^2 \left( \Sigma^b_\eta(u) + \delta Z^b_\eta \right) \right. \]
\[ \left. + 2m_b^2 \left( \Sigma^b_S(u) - \frac{1}{2} \left( \delta Z^b_\eta + \delta Z^b_{-\eta} \right) - \frac{\delta m_b}{m_b} \right) \right] \]  \hspace{1cm} (4.48)

\[ N_{2\eta}^{s.e.} = 2N_{1\eta}^{s.e.} \]  \hspace{1cm} (4.49)

\[ N_{3\eta}^{s.e.} = g_s \frac{c_\eta m_b}{(u - m_b^2)} \left[ \Sigma^b_\eta(u) + \delta Z^b_\eta + \Sigma^b_S(u) - \frac{1}{2} \left( \delta Z^b_\eta + \delta Z^b_{-\eta} \right) - \frac{\delta m_b}{m_b} \right] \]  \hspace{1cm} (4.50)

The explicit expressions for the various counterterms in term of the self energies are, for the gauge boson part:

\[ \delta Z^W_1 - \delta Z^W_2 = \frac{\Sigma^\gamma Z(0)}{s_W c_W M_Z^2} \]  \hspace{1cm} (4.51)

\[ \delta Z^W_2 = -\Sigma'^\gamma(0) + 2 \frac{c_W}{s_W M_Z^2} \Sigma^\gamma Z(0) + \frac{c_W^2}{s_W^2} \left[ \frac{\delta M_W^2}{M_W^2} - \frac{\delta M_W^2}{M_W^2} \right] \]  \hspace{1cm} (4.52)

\[ \delta M_W^2 = Re \Sigma^WW(M_W^2) \quad \delta M_Z^2 = Re \Sigma^{ZZ}(M_Z^2) \]  \hspace{1cm} (4.53)
4.4. Self Energies and Counterterms

We will use

$$\delta g \frac{g}{g} = \delta Z_1^W - \frac{3}{2} \delta Z_2^W$$ \hspace{1cm} (4.54)

The counterterms for the bottom quark will be detailed and discussed in a following section.

4.4.1 Higgs sector

As anticipated we performed the calculation using two different renormalization schemes: the DR scheme \cite{20} is defined by the following renormalization conditions

$$\delta Z_{H_1}^{\text{DR}} = - \left[ \text{Re} \left( \frac{\partial \Sigma_{H_0}}{\partial k^2} \right) \bigg|_{k^2 = M_{H_0}^2, \alpha = 0} \right]_{\text{div}}$$

$$\delta Z_{H_2}^{\text{DR}} = - \left[ \text{Re} \left( \frac{\partial \Sigma_{h_0}}{\partial k^2} \right) \bigg|_{k^2 = M_{h_0}^2, \alpha = 0} \right]_{\text{div}}$$

$$\delta T_{h_0} = - T_{h_0}$$

$$\delta T_{H_0} = - T_{H_0}$$

$$\delta M_A^2 = \text{Re} \Sigma_{A^0}(M_{A^0}^2) - M_{A^0}^2 \Sigma'_{A^0}(M_{A^0}^2)$$

$$\delta \tan \beta^{\text{DR}} = \frac{1}{2} \left( \delta Z_{H_2}^{\text{DR}} - \delta Z_{H_1}^{\text{DR}} \right) \tan \beta.$$ \hspace{1cm} (4.55)

$\delta Z_{H_i}^{\text{DR}}$ define the wave function renormalization constant of the Higgs field $H_i$, the third and fourth line fix the tadpole renormalization and the last one the $\tan \beta$ renormalization constant. $[A]_{\text{div}}$ means keeping the UV divergent part of $A$, discarding the finite contribution. In this scheme, as one can see from the previous formulae, there is no finite contribution to $\delta \tan \beta^{\text{DR}}$: only divergent part are kept, and one has explicitly:

$$\left( \frac{\delta \tan \beta}{\tan \beta} \right) \equiv - \frac{\alpha}{4\pi} \left( \frac{\Delta}{4 s_W^2 m_t^2} \right) \sum_{\text{families}} N_{\text{col}} \left( \frac{m_t^2}{\sin^2 \beta} - \frac{m_b^2}{\cos^2 \beta} \right)$$ \hspace{1cm} (4.56)

with

$$\Delta = \frac{2}{4 - D} - \gamma_E + \log 4\pi$$ \hspace{1cm} (4.57)
4.4. Self Energies and Counterterms

In the DCPR scheme \cite{21, 22} the independent parameters are the same, and the renormalization conditions of the Higgs wavefunctions change as follows

\[
\delta Z_{H_1}^{DCPR} = -\text{Re} \left( \frac{\partial \Sigma_{A^0}(k^2)}{\partial k^2} \right)_{k^2=M_{A^0}^2} - \frac{1}{\tan \beta M_Z} \text{Re} \Sigma_{A^0 Z}(M_{A^0}^2) \\
\delta Z_{H_2}^{DCPR} = -\text{Re} \left( \frac{\partial \Sigma_{A^0}(k^2)}{\partial k^2} \right)_{k^2=M_{A^0}^2} + \frac{\tan \beta}{M_Z} \text{Re} \Sigma_{A^0 Z}(M_{A^0}^2) \\
\delta T_{h^0} = -T_{h^0} \\
\delta T_{H^0} = -T_{H^0} \\
\delta M_{A^0}^2 = \text{Re} \Sigma_{A^0}(M_{A^0}^2) - M_{A^0}^2 \Sigma'_{A^0}(M_{A^0}^2) \\
\delta \tan \beta^{DCPR} = \frac{1}{2} \left( \delta Z_{H_2}^{DCPR} - \delta Z_{H_1}^{DCPR} \right) \tan \beta
\]

(4.58)

Differently from \(\overline{DR}\), in this scheme \(\delta \tan \beta^{DCPR}\) receives also finite corrections. It is important to notice that because of this, one should then expect to have scheme independence violated at born level, and then restored at Next to Leading Order. The conversion procedure for translating \(\tan \beta\) from one scheme to another is \cite{12}:

\[
\{\tan \beta\}_{\overline{DR}} = \{\tan \beta\}_{DCPR} + \frac{1}{2 M_Z C_\beta} \Sigma'_{A^0 Z}(m_A^2)
\]

(4.59)

We choose to impose on-shell (OS) condition for the mass of CP-odd \(A^0\) Higgs in both schemes. This means imposing the following conditions:

\[
\hat{\Sigma}_{A^0}(M_{A^0}^2) = 0 \\
\hat{\Sigma}'_{A^0}(M_{A^0}^2) = 0
\]

(4.60)  (4.61)

\[
\delta M_{A^0}^2 = \Sigma_{A^0}(M_{A^0}^2) - M_{A^0}^2 \Sigma'_{A^0}(M_{A^0}^2)
\]

(4.62)

The renormalization constants of the Higgs bosons wavefunctions and of the \(c^0(bb)H^0\) couplings can be written in terms of the of the renormalization constants defined above. Their explicit expression is given in Appendix A.
4.4. Self Energies and Counterterms

4.4.2 Bottom sector

The mass of the bottom and its wavefunction renormalization function is fixed in the on-shell scheme:

\[
\delta m_b^{\text{OS}} = \frac{1}{2} m_b \left[ \text{Re} \Sigma_{b_L}(m_b^2) + \text{Re} \Sigma_{b_R}(m_b^2) + 2 \text{Re} \Sigma_{b_S}(m_b^2) \right],
\]

\[
\delta Z_L^b = - \text{Re} \Sigma_{b_L}(m_b^2) - m_b^2 \frac{\partial}{\partial k^2} \text{Re} \left[ \Sigma_{b_L}(k^2) + \Sigma_{b_R}(k^2) + 2 \Sigma_{b_S}(k^2) \right] \Big|_{k^2 = m_b^2},
\]

\[
\delta Z_R^b = - \text{Re} \Sigma_{b_R}(m_b^2) - m_b^2 \frac{\partial}{\partial k^2} \text{Re} \left[ \Sigma_{b_L}(k^2) + \Sigma_{b_R}(k^2) + 2 \Sigma_{b_S}(k^2) \right] \Big|_{k^2 = m_b^2},
\]

where the bottom self energies are defined according to following Lorentz decomposition:

\[
\Sigma_b(p) = \not p \not P L \Sigma_{b_L}(p^2) + \not p \not P R \Sigma_{b_R}(p^2) + m_b \Sigma_{b_S}(p^2).
\]

Resummation of large logarithms from the running of the bottom mass suggests to trade the on-shell bottom mass appearing the couplings with the DR bottom mass, \(m_b^{\text{DR}}\) [23]. The resummation of the \((\alpha_s \tan \beta)^n\) contributions can be achieved modifying the tree level relation between the bottom Yukawa coupling and the bottom mass. This kind of approach should be accurate to the few percent level in taking into account SQCD corrections [13]. The (DR) bottom mass of the couplings - which is related to the bottom Yukawa coupling - is replaced by an effective mass

\[
m_b^{\text{DR}} \to m_b^{\text{eff}} = \frac{m_b^{\text{DR}}}{1 + \Delta_b}
\]

where \(\Delta_b\) is given by

\[
\Delta_b = \frac{2}{3} \frac{\alpha_s}{\pi} M_{\tilde{g}} \mu \tan \beta I(M_{b_1}, M_{b_2}, M_{\tilde{g}})
\]

\[
I(a, b, c) = \frac{-1}{(a^2 - b^2)(b^2 - c^2)(c^2 - a^2)} \left( a^2 b^2 \ln \frac{a^2}{b^2} + b^2 c^2 \ln \frac{b^2}{c^2} + c^2 a^2 \ln \frac{c^2}{a^2} \right).
\]

Moreover, the \(\tilde{b}\tilde{b}H_1\) coupling is dynamically generated at \(O(\alpha_s)\) and can be enhanced if \(\tan \beta\) is large. This effect can be included modifying the \(c^n(\tilde{b}\tilde{b}\mathcal{H}^0)\) couplings. The actual effect of this modification and of the bottom
4.5. Vertex and Box Corrections

mass resummation, Eq. (4.65), is to substitute the $c^3(bbH^0)$ couplings in Eq. (4.5) as follows

\[
\begin{align*}
  c^3(bbH^0) & \rightarrow \frac{c^3(bbH^0)}{m_b} \times \frac{m^\text{pr}}{m_b} \left(1 - \frac{\Delta_b}{\tan \beta \tan \alpha}\right) \\
  c^3(bbA^0) & \rightarrow \frac{c^3(bbA^0)}{m_b} \times \frac{m^\text{pr}}{m_b} \left(1 - \frac{\Delta_b}{\tan^2 \beta}\right) \\
  c^3(bbH^0) & \rightarrow \frac{c^3(bbH^0)}{m_b} \times \frac{m^\text{pr}}{m_b} \left(1 + \frac{\Delta_b \tan \alpha}{\tan \beta}\right)
\end{align*}
\]

(4.67)

4.5 Vertex and Box Corrections

This class of diagrams involves the triangle-like vertex corrections and the four leg box-like loops. Schematically we can further subdivide them as follows ($q$ stands for quarks, $f$ for a generic fermion particle $V$ for $\gamma, Z, W$, $S$ for any scalar particle, $\chi$ for chargino or neutralino):

- S-channel “Left” Triangles: ($Vqq), (Sqq), (\chi\tilde{q}\tilde{q})$
- S-channel “Right” Triangles: ($qSV), (Vq'q), (qVS), (Sf'f), (fSS')$
- ($VVq$)
- U-channel “Up” Triangles: ($ffV), (ffS), (SSf)$
- U-channel “Down” Triangles: ($VSq), (q'qV), (SVq), (f'fS), (S'Sf), (VVq)$
- “Direct” Boxes: ($qqqV), (qqqS), (\tilde{q}\tilde{q}\tilde{q}\chi)$
- “Crossed” Boxes ($qqVS), (qqSV), (\tilde{q}\tilde{q}\chi, \chi)$, ($qqSS'), (qqVV)$
- “Twisted” Boxes: ($qqqV), (qqqS), (\tilde{q}\tilde{q}\tilde{q}\chi)$

The notation corresponds to the clockwise ordering of the internal particles inside the diagrams, and each of the subclasses contains many different diagrams depending on the exact particles running in the loops.
All the one-loop corrections have been computed using the Passarino-Veltman functions formalism: the obtained analytical results have been tested checking analytically and numerically the cancellation of the UV divergent terms. Divergent terms appear in the self energy functions $\Sigma(k^2)$, (and so in the various counterterms) and in the various triangles. Box corrections are convergent (as one can simply see by power counting). We checked the cancellation when summing all of these terms, both analytically and numerically in the program. This cancellation occurs in several independent sectors (gauge, Higgs, SM, SUSY), and further details about it can be found in Appendix B.

4.6 QED Radiation

As previously anticipated, diagrams involving an internal photon in the loops can give rise to divergences.

The infrared (IR) singularities affecting the virtual contributions are cancelled by those arising in the bremsstrahlung of real photons at $O(\alpha_s \alpha^2)$,

\[
b(p_b) \ g(p_y) \rightarrow b(p'_b) \ H^0(p_{4\gamma}) \ \gamma(p_\gamma),
\]

arising from the diagrams in Figure 4.5. This contribution has been computed using FeynArts [19] and FormCalc [24]. The integral over the photon phase space is IR divergent in the soft-photon region, i.e. for $p_\gamma^0 \rightarrow 0$. The IR divergences are regularized within mass regularization, giving a small mass $m_\gamma$ to the photon. The phase space integration has been performed using the phase space slicing method. This method introduces a fictitious separator $\Delta E$ and restricts the numerical phase space integration in the region characterized by $p_\gamma > \Delta E$. The integral over the region $p_\gamma < \Delta E$ is performed analytically in the eikonal approximation [25].

Large collinear logarithms containing the bottom mass can be re-absorbed into the redefinition of the parton distribution function (PDF) of the bot-
4.6. QED Radiation

\[\gamma\text{ emission}\]

\[
f_b(x, \mu) \rightarrow f_b(x, \mu) \left\{ 1 - \frac{\alpha e_b^2}{\pi} \left[ 1 - \ln \delta_s - \ln \delta_s^2 + \left( \ln \delta_s + \frac{3}{4} \right) \ln \left( \frac{\mu^2}{m_b^2} \right) - \frac{1}{4} \lambda_{\text{FC}} \kappa_1 \right] \right\} - \frac{\alpha}{2\pi e_b^2} \int_\delta^1 \frac{dz}{z} f_b \left( \frac{z}{\chi}, \mu \right) \left[ 1 + \frac{z^2}{1 - z} \ln \left( \frac{\mu^2}{m_b^2} \right) \frac{1}{(1 - z)^2} - \frac{1}{1 - z} + \lambda_{\text{FC}} \kappa_2 \right], \] (4.69)

and setting \( \lambda_{\text{FC}} = 0 \) (\( \lambda_{\text{FC}} = 1 \)). \( \mu \) is the factorization scale, \( \delta_s = 2\Delta E/\sqrt{s} \), while \( e_b \) is the bottom charge. \( \kappa_1 \) and \( \kappa_2 \) are defined as follows,

\[
\kappa_1 = 9 + \frac{2}{3} \pi^2 + 3 \ln \delta_s - 2 \ln^2 \delta_s, \\
\kappa_2 = \frac{1 + z^2}{1 - z} \ln \left( \frac{1 - z}{z} \right) - \frac{3}{2} \frac{1}{1 - z} + 2z + 3. \] (4.70)

The cancellation of IR divergences, the independence of our results on \( m_\gamma \) (in the sum of the soft and virtual part) and on the fictitious separator \( \Delta E \) have been numerically tested. The results of those checks are illustrated in the following Figures 4.6, 4.7, 4.8.
Figure 4.6: $h^0$ production: dependence of the $\mathcal{O}(\alpha)$ soft+virtual, hard, and total sum corrections on the separator $\Delta E \ (s^{1/2})$
Figure 4.7: $H^0$ production: dependence of the $\mathcal{O}(\alpha)$ soft+virtual, hard, and total sum corrections on the separator $\Delta E (s^{1/2})$.
4.6. QED Radiation

Figure 4.8: $A^0$ production: dependence of the $\mathcal{O}(\alpha)$ soft+virtual, hard, and total sum corrections on the separator $\Delta E$ ($s^{1/2}$)
4.7 Total cross sections

Including the finite wave function renormalization for the Higgs field we obtain the following expressions for the tree-level differential partonic cross section of the processes we are considering,

\[ d\hat{\sigma}^{1,1}_{bg\rightarrow bH^0} = \frac{\beta'}{768 \pi s \beta} Z_{H^0} |\mathcal{M}_{bg\rightarrow bH^0}|^2 \]  

(4.71)

where \( \beta = 2p/\sqrt{s} \), \( \beta' = 2p'/\sqrt{s} \), and \( s \) is the Mandelstam variable defined in Eq. (4.3); the NLO-EW contribution to the differential cross section reads as follows

\[ d\hat{\sigma}^{1,2}_{bg\rightarrow bh^0} = \frac{\beta'}{768 \pi s \beta} Z_{h^0} \left\{ |1 - Z_{h^0 H^0} \frac{\cos\alpha}{\sin\alpha}|^2 |\mathcal{M}_{bg\rightarrow bh^0}|^2 \right. 
+ 2 \text{Re} \mathcal{M}_{bg\rightarrow bh^0} \left( \mathcal{M}_{bg\rightarrow bh^0}^\ast \right) 
\]  

\[ d\hat{\sigma}^{1,2}_{bg\rightarrow bH^0} = \frac{\beta'}{768 \pi s \beta} Z_{H^0} \left\{ |1 - Z_{h^0 H^0} \frac{\sin\alpha}{\cos\alpha}|^2 |\mathcal{M}_{bg\rightarrow bH^0}|^2 \right. 
+ 2 \text{Re} \mathcal{M}_{bg\rightarrow bH^0} \left( \mathcal{M}_{bg\rightarrow bH^0}^\ast \right) 
\]  

\[ d\hat{\sigma}^{1,2}_{bg\rightarrow bA^0} = \frac{\beta'}{768 \pi s \beta} Z_{A^0} \left\{ 2 \text{Re} \mathcal{M}_{bg\rightarrow bA^0} \left( \mathcal{M}_{bg\rightarrow bA^0}^\ast \right) \right\}, \]  

(4.72)

where the Z factors \( Z_{h^0}, Z_{H^0}, Z_{A^0}, Z_{h^0 H^0}, \) and \( Z_{H^0 h^0} \) in the two renormalization schemes we are considering read as follows \[20], \[21]:

\[ Z_{h^0} = \frac{1}{1 + \text{Re} \left\{ \Sigma_{h^0}^\prime(k^2) - \left( \frac{\Sigma_{h^0 A^0}^2(k^2)}{k^2 - M_{h^0}^2 + \Sigma_{h^0}(k^2)} \right) \right\}} \]  

\[ Z_{H^0} = \frac{1}{1 + \text{Re} \left\{ \Sigma_{H^0}^\prime(k^2) - \left( \frac{\Sigma_{h^0 A^0}^2(k^2)}{k^2 - M_{H^0}^2 + \Sigma_{h^0}(k^2)} \right) \right\}} \]  

\[ Z_{A^0} = \frac{1}{1 + \Sigma_{A^0}^\prime(k^2)} \]

\[ ^2 \text{Note that this formulae hold for the case of the rMSSM where no mixing is allowed between } A^0 \text{ and } h^0, H^0. \]

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4.8. Results

There are only two independent input parameters in the MSSM Higgs sector: the $A^0$ mass and $\tan\beta$. Regarding the behaviour of these parameters in the two renormalization schemes, $M_{A^0}$ remains the same, since $A^0$ is taken to be on shell in both cases, while this is not true for $\tan\beta$. At the one loop level, the following relation should be used to relate the values of $\tan\beta$ in the two schemes:

$$\tan\beta_{\text{DCPR}} = \tan\beta_{\text{DR}} + \delta \tan\beta_{\text{DR}} - \delta \tan\beta_{\text{DCPR}},$$

(4.74)

The bottom masses $m_b^{\text{CS}}$ and $m_b^{\text{DR}}(\mu)$ are calculated starting from the value of $m_b^{\text{MS}}(m_b) = 4.2$ GeV, and can be expressed following the procedure fully detailed in from Section 3.2.2 of [23]. One has:

$$m_b^{\text{CS}} = m_b^{\text{MS}}(m_b)b^{\text{shift}},$$

(4.75)

where

$$b^{\text{shift}} = 1 + \frac{\alpha_s}{\pi} \left( \frac{4}{3} - \ln \left( \frac{(m_b^{\text{MS}})^2}{m_b^2} \right) \right),$$

(4.76)

and, for $m_b^{\text{DR}}$:

$$m_b^{\text{DR}}(\mu) = m_b^{\text{MS}}(m_b)b^{\text{shift}} + \frac{1}{2}m_b \left( \Sigma_{bL}^{\text{fin}}(m_b^2) + \Sigma_{bR}^{\text{fin}}(m_b^2) \right) + m_b \Sigma_{bS}^{\text{fin}}(m_b^2).$$

(4.77)

For the numerical evaluations we used the supersymmetric scenario SPP$_1$ and a class of points of the parameter space SPP$_2$, with variable $\tan\beta = 10, 20, 30, 40$. The input parameters characterizing these scenarios are summarized in Table 4.10. The sparticle masses and mixing angles have been obtained with the code FeynHiggs [28]. The one-loop Higgs
Table 4.1: Inputs parameters for the SUSY scenarios considered in our numerical discussion. $M_{\tilde{q},j}$ is the common value of the breaking parameters in the sector of the squarks belonging to the $j^{th}$ generation. The dimensionful parameters are given in GeV.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$\tan \beta$</th>
<th>$M_{A_0}$</th>
<th>$M_{\tilde{q},1}$</th>
<th>$M_{\tilde{q},2}$</th>
<th>$M_{\tilde{q},3}$</th>
<th>$M_1$</th>
<th>$M_2$</th>
<th>$M_{\tilde{g}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPP$_1$</td>
<td>15</td>
<td>350</td>
<td>350</td>
<td>350</td>
<td>250</td>
<td>90</td>
<td>150</td>
<td>800</td>
</tr>
<tr>
<td>SPP$_2$</td>
<td>variable</td>
<td>250</td>
<td>500</td>
<td>500</td>
<td>400</td>
<td>90</td>
<td>200</td>
<td>800</td>
</tr>
</tbody>
</table>

Table 4.2: $A^0$ production, SPP$_2$ spectra: total cross sections [pb], $K$-factors and NLO $\text{DR}/\text{DCPR}$ ratio

<table>
<thead>
<tr>
<th>$\tan \beta$</th>
<th>$\sigma^{\text{DR,NLO}}$</th>
<th>$\sigma^{\text{DR,LO}}$</th>
<th>$\sigma^{\text{DCPR,NLO}}$</th>
<th>$\sigma^{\text{DCPR,LO}}$</th>
<th>$K^{\text{DR}}$</th>
<th>$K^{\text{DCPR}}$</th>
<th>NLO ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1.367</td>
<td>1.281</td>
<td>1.371</td>
<td>1.253</td>
<td>1.067</td>
<td>1.093</td>
<td>0.997</td>
</tr>
<tr>
<td>20</td>
<td>5.040</td>
<td>4.784</td>
<td>5.060</td>
<td>4.278</td>
<td>1.053</td>
<td>1.182</td>
<td>0.995</td>
</tr>
<tr>
<td>30</td>
<td>10.601</td>
<td>10.295</td>
<td>10.785</td>
<td>8.505</td>
<td>1.029</td>
<td>1.268</td>
<td>0.98</td>
</tr>
<tr>
<td>40</td>
<td>17.118</td>
<td>17.125</td>
<td>17.615</td>
<td>13.038</td>
<td>0.999</td>
<td>1.350</td>
<td>0.97</td>
</tr>
</tbody>
</table>

Table 4.3: $H^0$ production SPP$_2$ spectra: total cross sections [pb], $K$-factors and NLO $\text{DR}/\text{DCPR}$ ratio

<table>
<thead>
<tr>
<th>$\tan \beta$</th>
<th>$\sigma^{\text{DR,NLO}}$</th>
<th>$\sigma^{\text{DR,LO}}$</th>
<th>$\sigma^{\text{DCPR,NLO}}$</th>
<th>$\sigma^{\text{DCPR,LO}}$</th>
<th>$K^{\text{DR}}$</th>
<th>$K^{\text{DCPR}}$</th>
<th>NLO ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.282</td>
<td>0.248</td>
<td>0.282</td>
<td>0.243</td>
<td>1.135</td>
<td>1.156</td>
<td>1.002</td>
</tr>
<tr>
<td>20</td>
<td>0.255</td>
<td>0.254</td>
<td>0.254</td>
<td>0.230</td>
<td>1.005</td>
<td>1.107</td>
<td>1.003</td>
</tr>
<tr>
<td>30</td>
<td>0.228</td>
<td>0.258</td>
<td>0.230</td>
<td>0.217</td>
<td>0.882</td>
<td>1.059</td>
<td>0.988</td>
</tr>
<tr>
<td>40</td>
<td>0.204</td>
<td>0.267</td>
<td>0.213</td>
<td>0.211</td>
<td>0.764</td>
<td>1.012</td>
<td>0.955</td>
</tr>
</tbody>
</table>

Table 4.4: $h^0$ production SPP$_2$ spectra: total cross sections [pb], $K$-factors and NLO $\text{DR}/\text{DCPR}$ ratio

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masses are numerically computed by finding the zero of inverse one-loop propagator matrix determinant

\[ \left[ k^2 - M_{H^0}^2 + \hat{\Sigma}_{H^0}(k^2) \right] \left[ k^2 - M_{h^0}^2 + \hat{\Sigma}_{h^0}(k^2) \right] - \hat{\Sigma}_{H^0 h^0}(k^2) = 0. \] (4.78)

Since we require semi-inclusive production (i.e. the bottom quark must be tagged) we impose the following kinematical cuts on the bottom in the final state, limiting the transferred momentum \( p_{b,T} > 20 \text{ GeV} \) (due to resolution limitations of the hadronic calorimeter) and the rapidity \( |y_b| < 2 \) (in order to be able to perform inner tracking). The process we are considering is leading order in QCD. Therefore, analogously to \[29, 30, 31\], we use a LO QCD PDF set, namely the LO CTEQ6L \[32\]. Our choice is justified since the QED effects in the DGLAP evolution equations are known to be small \[33\]. The factorization of the bottom PDF has been performed in the DIS scheme, with factorization scale \( \mu = M_{H^0} + m_b^{\text{OS}} \). Finally, the center of mass energy for the proton-proton collision, has been set to 14 TeV.

In Figures 4.10, 4.11, 4.12 we show the total cross section for \( h^0, H^0 \) and \( A^0 \) production in the class of supersymmetric scenarios SPP\textsubscript{2}, as functions of \( \tan \beta \). We present both the results in the \( \overline{\text{DR}} \) and in the DCPR schemes. The numerical values and the \( K \)-factors in the two schemes (defined as usual as the ratios \( \sigma^{NLO}/\sigma^{LO} \); note that the LO is computed with the resummed/modified SUSY QCD coupling, so our \( K \)-factors account of the pure electroweak NLO effect), as well as the ratios of the NLO cross sections in the two scheme are reported in Table 4.2, 4.3, 4.4.

As one sees, the values of the total cross sections do coincide in the overall range, apart from small differences of the few percent size for very large \( \tan \beta \) values. This result is twofold important, confirming the equivalence of the calculations done in the two different schemes at Next to Leading Order, and providing also a reassuring check of the reliability of our results.
4.8. Results

**Figure 4.9:** K-factors for the production of the three neutral Higgses in the \( \overline{\text{DR}} \) scheme

Having verified the realistic one-loop equivalence of the two schemes, we have decided to perform our analysis in the \( \overline{\text{DR}} \) scheme. The main theoretical reasons of our choice have been fully illustrated in [34]. In particular this scheme is known to be generally more stable numerically: our results confirm mainly this expectation but it is worth to note that for \( h^0 \) production both schemes can produce (in different \( \tan \beta \) regions) relatively large effects; nevertheless the good agreement between the two schemes leads to suppose that the perturbative expansion is well behaved, and NNLO effects are well under control.

Figure 4.8 shows the \( K \)-factors for the three Higgs bosons in \( \overline{\text{DR}} \) as function of \( \tan \beta \) while Figures 4.15, 4.14, 4.13 show, for the scenario SPP\(_2\) \( \tan \beta = 30 \), the invariant mass distribution and the relative NLO effect. In the next Figures 4.16, 4.17, 4.18 we again plot the differential distributions...
Figure 4.10: Values of the Born and one loop cross sections in the two renormalization schemes for $h^0$

Figure 4.11: Values of the Born and one loop cross sections in the two renormalization schemes for $H^0$
Figure 4.12: Values of the Born and one loop cross sections in the two renormalization schemes for $A^0$ for the SPP$_1$ scenario; the total cross sections for this scenario are reported in Table 4.5.

Table 4.5: SPP$_1$ spectrum: total cross sections [pb] for the three Higgs and DR $K$-factors

<table>
<thead>
<tr>
<th>$H^0$</th>
<th>$\sigma_{\text{DR, NLO}}^{\text{DR}}$</th>
<th>$\sigma_{\text{DR, LO}}^{\text{DR}}$</th>
<th>$K_{\text{DR}}^{\text{DR}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A^0$</td>
<td>0.768</td>
<td>0.724</td>
<td>1.060</td>
</tr>
<tr>
<td>$H^0$</td>
<td>0.769</td>
<td>0.727</td>
<td>1.056</td>
</tr>
<tr>
<td>$h^0$</td>
<td>0.213</td>
<td>0.222</td>
<td>0.961</td>
</tr>
</tbody>
</table>

From inspection of the figures, one can draw the following main conclusions:

1. For all the three Higgses the one loop correction to the cross section
Figure 4.13: Invariant mass distribution, $h^0$ production, DR scheme. $M_{A^0} = 250$ GeV, $\tan\beta = 30$, $p_{b,T} > 20$ GeV, $|y_b| < 2$.

is roughly independent of the final state invariant mass

2. The $K$-factors for $H^0, A^0$ are systematically small for large $\tan\beta$, and would reach a larger size (roughly, 8 %) for small $\tan\beta$ values around 10.

3. The $K$-factor for $h^0$ varies drastically with $\tan\beta$, changing from positive values of about 15 % for $\tan\beta$ around 10 to negative values of about 25 % for $\tan\beta$ around 40. These extreme negative and positive values are of a size that cannot be ignored in a dedicated experimental analysis.

This last feature follows from the Two Higgs Doublet structure of the
model, resulting in an $h^0 - H^0$ mixing, similar to the $B^0 - W^3$ mixing in the Standard Model. Recalling 4.73, one can see how the mixed self energy appears in the $Z$-factors, leading to corrections to the one loop cross section, as shown in 4.72. In the decoupling limit, in which our choice for the mass of $A^0$ falls, $\alpha$ is close to $\beta - \pi/2$; this leads to a $\tan \beta$ enhancement in the $h^0$ case but to a $1/\tan \beta$ suppression in the $H^0$ cases, since $\frac{\cos \alpha}{\sin \alpha} \sim \tan \beta$ and $\frac{\sin \alpha}{\cos \alpha} \sim \frac{1}{\tan \beta}$ in this limit. Table 4.6 illustrates the contribution of the relevant factor in the $h^0$ case.

In conclusion, what we believe to be the main message of our calculation is the fact that, as it was to be expected from the analysis of Dittmaier
4.8. Results

Figure 4.15: Invariant mass distribution, $A^0$ production, DR scheme. $M_{A^0} = 250$ GeV, $\tan\beta = 30$, $p_{b,T} > 20$ GeV, $|y_b| < 2$.

et al. [12], the one-loop electroweak contribution in the semi-inclusive bottom-Higgs production processes can’t be a priori discarded as negli-

| $\tan\beta$ | $|1 - Z_{h^0 H^0} \cos\alpha|^2$ |
|------------|-------------------------------|
| 10         | 1.10                          |
| 20         | 1.01                          |
| 30         | 0.93                          |
| 40         | 0.84                          |

Table 4.6: SPP$_2$ spectrum: value of the $Z$ depending factor in the $h^0$ production
4.8. Results

Figure 4.16: Invariant mass distribution, $h^0$ production, DR scheme. $M_{A^0} = 350$ GeV, $\tan \beta = 15$, $p_{T,b} > 20$ GeV, $|y_b| < 2$.

gible, and a careful evaluation of such contribution is needed.
Figure 4.17: Invariant mass distribution, $H^0$ production, DR scheme. $M_{A^0} = 350$ GeV, $\tan \beta = 15$, $p_{T,b} > 20$ GeV, $|y_b| < 2$. 

4.8. Results
4.8. Results

Figure 4.18: Invariant mass distribution, $A^0$ production, DR scheme. $M_{A^0} = 350$ GeV, $\tan \beta = 15$, $p_{T,b} > 20$ GeV, $|y_b| < 2$. 
4.9 Possible Numerical Approximations

After having performed the full calculation for the one loop effect, we considered testing the validity of some possible approximations involving a lesser amount of computations. A simpler approximation that reproduces the full result well enough could be useful for Monte Carlo Generators or in case of a wide parameter space scan, to reduce the otherwise huge machine-time required to compute the full one loop result.

Following these considerations, we tried at first the so-called “improved Born Approximation” (IBA in the following), which has been already successfully applied in similar cases in the past, e.g. in [12].

In this case the IBA approximation is obtained following these prescriptions:

- \( \Delta_b \) (see eq. 4.65) should be computed including also the electroweak contributions

- The mixing angle \( \alpha \) should be replaced with an effective angle \( \alpha_{eff} \), whose value is derived by the diagonalization of the one loop Higgs mass matrix appearing in eq. 2.52 of [12]), which we reproduce here:

\[
\begin{pmatrix}
  m_{H^0}^2 - \hat{\Sigma}_{H^0}(m_{H^0}^2) & \hat{\Sigma}_{H^0 A^0}(\frac{1}{2}(m_{H^0}^2 + m_{A^0}^2)) \\
  \hat{\Sigma}_{H^0 A^0}(\frac{1}{2}(m_{H^0}^2 + m_{A^0}^2)) & m_{A^0}^2 - \hat{\Sigma}_{A^0}(m_{A^0}^2)
\end{pmatrix}
\]

(4.79)

The effect of this redefinition of \( \alpha \) turns out to be negligible for \( H^0 \) and \( A^0 \), but is instead significant for \( h^0 \).

As one can see from the plots (Figures 4.19, 4.20, 4.21) this version of IBA gives a reliable approximation of the total cross section for \( H^0, A^0 \) only for sufficiently small \( \tan \beta \), while it fails to reproduce the cross section for \( h^0 \), that turns out to be underestimated in the whole range.
4.9. Possible Numerical Approximations

Having noticed the unsatisfactory behaviour of the IBA, we decided to try a second kind of approximation, that we would call RVA ("Reduced Vertex Approximation").

In this second attempt, we approximate the complete NLO by keeping only the one loop corrections to the "final" Yukawa $b\bar{b}H^0$ vertex (all of them), plus the subset of counterterms needed to cancel the divergences arising in those diagrams and get a UV-finite result. We don’t include any QED radiation, and regulate (somewhat arbitrarily) the photon mass as $M_\gamma = M_Z/\sqrt{2}$. As for the Higgs sector, we keep the one loop values for the Higgs masses in the kinematics as well as the $Z$-factors (4.73) in the definition of the cross section. All the other diagrams appearing in the full computation (Boxes, Initial and Up Triangles, Self Energies) are neglected. As a check we computed the cross section in this approximation in both schemes (the subset of diagrams, with the right choice of counterterms, should be scheme independent). As one can see from the updated figures our RVA turns out to provide very efficient description of the total NLO cross sections; the difference between the NLO and the RVA is of order of the 1%, 3.4% in the worst case. This is numerically summarized in Tables 4.7,4.8,4.9 and Figure 4.19.

---

1No thorough tests have yet been made on the dependence of the results on the arbitrary regulator. This investigation will be interesting to perform if one wants to properly check the validity of this approximation.
Figure 4.19: Values of the cross section in the two approximations and at full one loop for $h^0$.

<table>
<thead>
<tr>
<th>$\tan \beta$</th>
<th>$\sigma_{\text{DR}}^{\text{NLO}}$</th>
<th>$\sigma_{\text{RVA}_{bbH}}$</th>
<th>$\sigma_{\text{RVA}<em>{bbH}} / \sigma</em>{\text{IBA}}$</th>
<th>$\sigma_{\text{IBA}}$</th>
<th>$\sigma_{\text{IBA}} / \sigma_{\text{IBA}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1.338</td>
<td>1.32623</td>
<td>1.00888</td>
<td>1.34087</td>
<td>0.997861</td>
</tr>
<tr>
<td>20</td>
<td>5.133</td>
<td>5.08324</td>
<td>1.00979</td>
<td>5.48397</td>
<td>0.936</td>
</tr>
<tr>
<td>30</td>
<td>10.975</td>
<td>10.8433</td>
<td>1.01215</td>
<td>12.6044</td>
<td>0.87073</td>
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<tr>
<td>40</td>
<td>18.613</td>
<td>18.3461</td>
<td>1.01455</td>
<td>22.6229</td>
<td>0.822749</td>
</tr>
</tbody>
</table>

Table 4.7: $H^0$ production: comparison between the complete NLO prediction and the two approximations: total cross sections and ratios $\sigma_{\text{DR}}^{\text{NLO}} / \sigma_{\text{APP}}$. 
4.9. Possible Numerical Approximations

Figure 4.20: Values of the cross section in the two approximations and at full one loop for $H^0$

Table 4.8: $h^0$ production: comparison between the complete NLO prediction and the two approximations: total cross sections and ratios $\sigma_{\text{NLO}}^{\text{DR}}/\sigma_{\text{APP}}$.
4.9. Possible Numerical Approximations

Figure 4.21: Values of the cross section in the two approximations and at full one loop for $A^0$

<table>
<thead>
<tr>
<th>$\tan \beta$</th>
<th>$\sigma_{NLO}^{DR}$</th>
<th>RVA_{bbH}</th>
<th>$\sigma$/RVA_{bbH}</th>
<th>IBA</th>
<th>$\sigma$/IBA</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1.367</td>
<td>1.35328</td>
<td>1.01014</td>
<td>1.36737</td>
<td>0.999729</td>
</tr>
<tr>
<td>20</td>
<td>5.04</td>
<td>4.98026</td>
<td>1.01199</td>
<td>5.4543</td>
<td>0.924042</td>
</tr>
<tr>
<td>30</td>
<td>10.601</td>
<td>10.4581</td>
<td>1.01366</td>
<td>12.2948</td>
<td>0.862232</td>
</tr>
<tr>
<td>40</td>
<td>17.118</td>
<td>16.8292</td>
<td>1.01716</td>
<td>21.7326</td>
<td>0.787663</td>
</tr>
</tbody>
</table>

Table 4.9: $A^0$ production: comparison between the complete NLO prediction and the two approximations: total cross sections and ratios $\sigma_{NLO}^{DR}/\sigma_{APP}$. 
4.10 \( bh_0 \) production in a Two Higgs Doublet Model

The Two Higgs Doublet Model (2HDM in the following) is a minimal extension of the Standard Model, where a second Higgs doublet is added to the field sector.

The Higgs particle spectrum is totally analogous to the MSSM one (in fact the MSSM is a Two Higgs Doublet Model theory), and thus consist of the two newtral CP even states \( H^0 \) and \( h^0 \), the neutral CP odd state \( A^0 \), and the two charged states \( H^\pm \). However, no other SUSY contribution is present, the rest of the model is identical to the usual Standard Model except for the Higgs sector.

It is rather straightforward to tweak the calculation made for the MSSM case to obtain the results for 2HDM; it is sufficient to kill all the diagrams including supersymmetric particles. The divergences arising from the calculation cancel separately in the Standard Model and Susy sector, with the exception of the Higgs renormalization sector (because of the Hierarchy Problem, see section 2.3).

The treatment of \( h^0 \) and \( H^0 \) in the renormalization sector should then be different, and they should now also be treated in the on-shell scheme, like \( A^0 \). This means one should write \( \hat{\Sigma}_{hh,HH} \) in terms of \( \Sigma_{hh,HH} \) (without the squark and inos), of \( \delta Z_{ij} \) and of \( \delta m^2_{ij} \) (\( i, j \) stay for \( h^0 \) or \( H^0 \)):

\[
\hat{\Sigma}_{ij}(k^2) = \Sigma_{ij}(k^2) + k^2\delta Z_{ij} - \delta m^2_{ij}
\] (4.80)

Then impose the on-shell constraints like it’s been done for the MSSM case for \( A^0 \) (REF):

\[
\hat{\Sigma}_{ii}(M_i^2) = 0 \tag{4.81}
\]
\[
\hat{\Sigma}'_{ii}(M_i^2) = 0 \tag{4.82}
\]

which give \( \delta Z_{ii} \) and \( \delta m^2_{ii} \) for \( i = h, H \):

\[
\delta Z_{ii} = -\Sigma'_{ii}(M_i^2) \tag{4.83}
\]
4.10. $bh_0$ production in a Two Higgs Doublet Model

\[ \delta m^2_{\tilde{t}_i} = \Sigma_{ii}(M_i^2) - M_i^2 \Sigma'_{ii}(M_i^2). \]  

(4.84)

For the mixed self energy one has:

\[ \hat{\Sigma}_{hh}(k^2) = \hat{\Sigma}_{hh}(k^2) = \Sigma_{hh}(k^2) + k^2 \delta Z_{hh} - \delta m^2_{\tilde{h}_H}. \]  

(4.85)

We can then impose the two demixing conditions:

\[ \hat{\Sigma}_{hh}(M_h^2) = 0 \]  

(4.86)

\[ \hat{\Sigma}_{hh}(M_H^2) = 0 \]  

(4.87)

giving

\[ \delta Z_{hh} = \frac{\Sigma_{hh}(M_h^2) - \Sigma_{hh}(M_H^2)}{M_H^2 - M_h^2} \]  

(4.88)

\[ \delta m^2_{\tilde{h}_H} = \frac{M_H^2 \Sigma_{hh}(M_h^2) - M_h^2 \Sigma_{hh}(M_H^2)}{M_H^2 - M_h^2} \]  

(4.89)

Finally, the SUSYQCD corrections to the bottom mass and to the couplings have of course been set to zero.

The input for the code have then been generated with FeynHiggs for twelve different scenarios (three different values of $M_{A^0}$ and four different values of $\tan \beta$, see table). For the purpose of comparison with the MSSM results (i.e. of determining the effect of the pure SUSY virtual corrections), the six free parameters in the 2HDM potential (corresponding to $M_{h^0}$, $M_{H^0}$, $M_{A^0}$, $M_{H^\pm}$, $\beta$ and $\alpha$) are set to the values of the corresponding MSSM scenarios. The procedure has been therefore to run first the code for the MSSM scenarios, and then take parameters calculated in this way as an input for the 2HDM runs.

The code has been run for three different values of $M_{A^0}$ (150, 250 and 350 GeV) and four values of $\tan \beta$ ( 10, 20, 30 and 40 GeV). The values of the other parameters can be found in tables 4.11, 4.12, 4.13, all the other relevant settings are the same as of the previous sections.
4.10. $bh_0$ production in a Two Higgs Doublet Model

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$\tan \beta$</th>
<th>$M_{A_0}$</th>
<th>$M_{\tilde{q},1}$</th>
<th>$M_{\tilde{q},2}$</th>
<th>$M_{\tilde{q},3}$</th>
<th>$M_1$</th>
<th>$M_2$</th>
<th>$M_{\tilde{g}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPP$_2$</td>
<td>variable</td>
<td>250</td>
<td>500</td>
<td>500</td>
<td>400</td>
<td>90</td>
<td>200</td>
<td>800</td>
</tr>
<tr>
<td>SPP'$_2$</td>
<td>variable</td>
<td>150</td>
<td>500</td>
<td>500</td>
<td>400</td>
<td>90</td>
<td>200</td>
<td>800</td>
</tr>
<tr>
<td>SPP''$_2$</td>
<td>variable</td>
<td>350</td>
<td>500</td>
<td>500</td>
<td>400</td>
<td>90</td>
<td>200</td>
<td>800</td>
</tr>
</tbody>
</table>

Table 4.10: Inputs parameters for the SUSY scenarios considered in our numerical discussion. $M_{\tilde{q},j}$ is the common value of the breaking parameters in the sector of the squarks belonging to the $j^{th}$ generation. The dimensionful parameters are given in GeV.

<table>
<thead>
<tr>
<th>$\tan \beta$</th>
<th>$\alpha$</th>
<th>$M_{H^0}$</th>
<th>$M_{h^0}$</th>
<th>$M_{H^\pm}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>-0.2068</td>
<td>152.20</td>
<td>114.4</td>
<td>170.2</td>
</tr>
<tr>
<td>20</td>
<td>-0.1072</td>
<td>149.8</td>
<td>115.5</td>
<td>170.2</td>
</tr>
<tr>
<td>30</td>
<td>-0.07199</td>
<td>148.3</td>
<td>115.8</td>
<td>170.2</td>
</tr>
<tr>
<td>40</td>
<td>-0.05413</td>
<td>146.8</td>
<td>115.9</td>
<td>170.2</td>
</tr>
</tbody>
</table>

Table 4.11: 2HDM inputs parameters for the corresponding SUSY scenario SPP'$_2$. The dimensionful parameters are given in GeV.

<table>
<thead>
<tr>
<th>$\tan \beta$</th>
<th>$\alpha$</th>
<th>$M_{H^0}$</th>
<th>$M_{h^0}$</th>
<th>$M_{H^\pm}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>-0.1291</td>
<td>250.8</td>
<td>114.8</td>
<td>262.6</td>
</tr>
<tr>
<td>20</td>
<td>-0.06513</td>
<td>249.1</td>
<td>115.7</td>
<td>262.6</td>
</tr>
<tr>
<td>30</td>
<td>-0.04350</td>
<td>247.1</td>
<td>115.9</td>
<td>262.6</td>
</tr>
<tr>
<td>40</td>
<td>-0.03264</td>
<td>244.4</td>
<td>116.</td>
<td>262.6</td>
</tr>
</tbody>
</table>

Table 4.12: 2HDM inputs parameters for the corresponding SUSY scenario SPP2. The dimensionful parameters are given in GeV.
4.10. $bh_0$ production in a Two Higgs Doublet Model

Table 4.13: 2HDM inputs parameters for the corresponding SUSY scenario SPP$''_2$. The dimensionful parameters are given in GeV.

<table>
<thead>
<tr>
<th>$\tan \beta$</th>
<th>$\alpha$</th>
<th>$M_{H^0}$</th>
<th>$M_{h^0}$</th>
<th>$M_{H^\pm}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>−0.1137</td>
<td>351.1</td>
<td>115.6</td>
<td>359.1</td>
</tr>
<tr>
<td>20</td>
<td>−0.05717</td>
<td>349.3</td>
<td>116.4</td>
<td>359.1</td>
</tr>
<tr>
<td>30</td>
<td>−0.03815</td>
<td>347.0</td>
<td>116.6</td>
<td>359.1</td>
</tr>
<tr>
<td>40</td>
<td>−0.02863</td>
<td>500</td>
<td>344.0</td>
<td>116.7</td>
</tr>
</tbody>
</table>

The results are shown in Tables 4.14 to 4.22 and Figures 4.22 to 4.30. It is evident that in the 2HDM the pure electroweak effect is very small, negligible for all practical purposes. The ratio between the cross sections in the two models $R_{\sigma} := \frac{\sigma_{\text{2HDM}}}{\sigma_{\text{MSSM}}}$, however, is very sensitive to the variation of the two parameters $M_{A^0}$ and $\tan \beta$, and can reach values very far from 1.

The first thing one can notice is that the ratio $R_{\sigma}$ is much more sensitive to $\tan \beta$, and in general can vary in a broader range for smaller values of $M_{A^0}$. In particular, one can see from the figures for $M_{A^0} = 150$ GeV, that for $A^0$ and $H^0$ the values of the cross sections in the two models are rather close for low $\tan \beta$, while, for $\tan \beta = 30 - 40$, $R_{\sigma}$ grows significantly, reaching values higher of 1.5. The case is even more interesting for $h^0$, were the huge correction coming from the $Z$ factors in the MSSM case plays a big role in enhancing the sensitivity of $R_{\sigma}$ to the value of $\tan \beta$. Figure 4.8 shows clearly the behaviour of the one loop effect for $h^0$ for $M_{A^0}$ (as already remarked previously, the correction is large and negative for low $\tan \beta$ and then becomes positive for high $\tan \beta$). For $M_{A^0} = 150$ GeV this effect is even bigger, and combines to the tendency already appearing in the case of the other two Higgses making $R_{\sigma}$ vary roughly from 0.65 to 1.5. This huge sensitivity to the parameters could make this process a very good candidate not only for discriminating the case for a 2HDM from the case for MSSM, but also for constraining the values for the parameters...
themselves.
For higher values of $M_{A^0}$, the described behaviour is still present, although its magnitude is partially suppressed.
The origin of this big difference between 2HDM and MSSM is to be located mostly in the corrections due to SUSY QCD, not present in the 2HDM case. The pure electroweak corrections to the 2HDM cross sections can be, for all practical purposes, safely neglected (but they should still be taken into account for the MSSM results, if one wants to discriminate between the two scenarios).
Figure 4.22: Comparison between the MSSM and 2HDM cross sections for $h^0$ in the scenario SPP'$_2$.

<table>
<thead>
<tr>
<th>tan $\beta$</th>
<th>$R_\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.66</td>
</tr>
<tr>
<td>20</td>
<td>0.90</td>
</tr>
<tr>
<td>30</td>
<td>1.16</td>
</tr>
<tr>
<td>40</td>
<td>1.45</td>
</tr>
</tbody>
</table>

Table 4.14: Results for $h^0$ in the scenario SPP'$_2$. The dimensionful parameters are given in GeV.
Figure 4.23: Comparison between the MSSM and 2HDM cross sections for $H^0$ in the scenario SPP'.

Table 4.15: Results for $H^0$ in the scenario SPP'. The dimensionful parameters are given in GeV.

<table>
<thead>
<tr>
<th>$\tan{\beta}$</th>
<th>$R_\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1.06</td>
</tr>
<tr>
<td>20</td>
<td>1.19</td>
</tr>
<tr>
<td>30</td>
<td>1.39</td>
</tr>
<tr>
<td>40</td>
<td>1.58</td>
</tr>
</tbody>
</table>
4.10. \( bh_0 \) production in a Two Higgs Doublet Model

![Graphical Representation]

Figure 4.24: Comparison between the MSSM and 2HDM cross sections for \( A^0 \) in the scenario SPP_{2}'.

<table>
<thead>
<tr>
<th>( \tan \beta )</th>
<th>( R_\sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1.03</td>
</tr>
<tr>
<td>20</td>
<td>1.20</td>
</tr>
<tr>
<td>30</td>
<td>1.31</td>
</tr>
<tr>
<td>40</td>
<td>1.59</td>
</tr>
</tbody>
</table>

Table 4.16: Results for \( A^0 \) in the scenario SPP_{2}'. The dimensionful parameters are given in GeV.
Figure 4.25: Comparison between the MSSM and 2HDM cross sections for $h^0$ in the scenario SPP$_2$.

<table>
<thead>
<tr>
<th>$\tan \beta$</th>
<th>$R_\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.97</td>
</tr>
<tr>
<td>20</td>
<td>1.14</td>
</tr>
<tr>
<td>30</td>
<td>1.35</td>
</tr>
<tr>
<td>40</td>
<td>1.55</td>
</tr>
</tbody>
</table>

Table 4.17: Results for $h^0$ in the scenario SPP2. The dimensionful parameters are given in GeV.
4.10. $bh_0$ production in a Two Higgs Doublet Model

Figure 4.26: Comparison between the MSSM and 2HDM cross sections for $H^0$ in the scenario SPP2.

<table>
<thead>
<tr>
<th>$\tan \beta$</th>
<th>$R_\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1.03</td>
</tr>
<tr>
<td>20</td>
<td>1.19</td>
</tr>
<tr>
<td>30</td>
<td>1.32</td>
</tr>
<tr>
<td>40</td>
<td>1.50</td>
</tr>
</tbody>
</table>

Table 4.18: Results for $H^0$ in the scenario SPP2. The dimensionful parameters are given in GeV.
Figure 4.27: Comparison between the MSSM and 2HDM cross sections for $A_0$ in the scenario SPP$_2$.

Table 4.19: Results for $A_0$ in the scenario SPP2. The dimensionful parameters are given in GeV.

<table>
<thead>
<tr>
<th>$\tan \beta$</th>
<th>$R_\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1.06</td>
</tr>
<tr>
<td>20</td>
<td>1.17</td>
</tr>
<tr>
<td>30</td>
<td>1.38</td>
</tr>
<tr>
<td>40</td>
<td>1.59</td>
</tr>
</tbody>
</table>
4.10. $b h_0$ production in a Two Higgs Doublet Model

![Graph showing the comparison between the MSSM and 2HDM cross sections for $h_0$ in the scenario SPP$_2''$.]

Figure 4.28: Comparison between the MSSM and 2HDM cross sections for $h_0$ in the scenario SPP$_2''$.  

<table>
<thead>
<tr>
<th>$\tan \beta$</th>
<th>$R_\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.96</td>
</tr>
<tr>
<td>20</td>
<td>1.02</td>
</tr>
<tr>
<td>30</td>
<td>1.10</td>
</tr>
<tr>
<td>40</td>
<td>1.19</td>
</tr>
</tbody>
</table>

Table 4.20: Results for $h_0$ in the scenario SPP$_2''$. The dimensionful parameters are given in GeV.
Figure 4.29: Comparison between the MSSM and 2HDM cross sections for $H^0$ in the scenario SPP$^\prime\prime_2$.

<table>
<thead>
<tr>
<th>$\tan\beta$</th>
<th>$R_\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.99</td>
</tr>
<tr>
<td>20</td>
<td>1.10</td>
</tr>
<tr>
<td>30</td>
<td>1.25</td>
</tr>
<tr>
<td>40</td>
<td>1.33</td>
</tr>
</tbody>
</table>

Table 4.21: Results for $H^0$ in the scenario SPP$^\prime\prime_2$. The dimensionful parameters are given in GeV.
Figure 4.30: Comparison between the MSSM and 2HDM cross sections for $A^0$ in the scenario SPP$^\prime\prime_2$. 

Table 4.22: Results for $A^0$ in the scenario SPP$^\prime\prime_2$. The dimensionful parameters are given in GeV.
4.11 Standard Model Higgs

For the sake of completeness we quickly review the pure SM case, that has been extensively studied in [14]. Like we did for the 2HDM case, the SM Higgs is taken to be on shell, such that there is no external self energy, and:

$$\delta Z_{H_{SM}} = -\Sigma'_H(M^2_{H_{SM}})$$  \hspace{1cm} (4.90)

All supersymmetric contributions are now neglected, as well as those coming from the different Higgses (except, of course, the SM one itself), and the Higgs-bottom coupling is now:

$$c^b(bb^0) = \left(\frac{e m_b}{2 s_W M_W}\right)$$  \hspace{1cm} (4.91)

Our results appear to be consistent with those of [14]. The SM one loop corrections are rather small, as it was for the 2HDM case. It is interesting to notice, looking at the bottom-Higgs coupling, that at the born level the SM Higgs behaviour should approach the one of $h^0$ in the case were $\beta - \alpha = \frac{\pi}{2}$, but there is a big difference at the one loop level. Even if the small cross section makes the SM process uninteresting to study per se (the inclusion of the QED effects doesn’t change the picture of [14]), it could still be interesting to look at this channel as a way to discriminate between a SM Higgs and a supersymmetric one.
Figure 4.31: Integrated cross sections and K-factors for SM Higgs production, for different values of the Higgs' mass
Chapter 5

Conclusions

In this thesis we report our results for the first complete calculation of the one-loop electroweak corrections for the semi-inclusive \( b\Bar{b}H' \) production process at the Large Hadron Collider. Our calculation includes all SM and MSSM electroweak contributions, QED radiation and SUSY QCD corrections to the bottom mass and the bottom-Higgs coupling for the two CP-even particles \( h^0 \) and \( H^0 \) and the CP-odd one \( A^0 \). As our computation shows, one loop electroweak corrections to the process cross section can’t be always considered negligible, and can reach sizable dimensions for some regions of the parameter’s space, in particular for \( h^0 \). A scan of the pMSSM [?] parameter’s space could be needed to further investigate this property, and it’s already under work.

Moreover, the prediction that the one loop results for the two different renormalization schemes, On-Shell and \( \overline{\text{DR}} \) are in good agreement is confirmed, even if the bigger one loop correction emerging from the calculation with the On-Shell scheme suggest that \( \overline{\text{DR}} \) is employed when the first input for the parameters will come from the LHC.

We have also compared our complete results with two possible approximations, and shown that while the IBA fails to reproduce correctly the results, the RVA is in good agreement with the full calculation.

Finally, we have provided a comparison between the results for some
SUSY scenarios and for the corresponding 2HDM set of parameters, and shown that the difference in the cross section can be big, and can be used to discriminate between the two cases.
Appendix A

Renormalization in the Higgs sector

The independent parameters are split into renormalization constants and renormalized parameters as follows,

\[ T_{h^0} \rightarrow T_{h^0} + \delta T_{h^0}, \quad T_{H^0} \rightarrow T_{H^0} + \delta T_{H^0}, \quad \tan \beta \rightarrow \tan \beta + \delta \tan \beta, \quad (A.1) \]

The actual effect of the renormalization is to substitute every mass matrix \( M \) appearing Eq. (2.34) by \( M + \delta M \), i.e.,

\[
M = \begin{pmatrix}
M_{1,1} & M_{1,2} \\
M_{2,1} & M_{2,2}
\end{pmatrix} \rightarrow M + \delta M = \begin{pmatrix}
M_{1,1} & M_{1,2} \\
M_{2,1} & M_{2,2}
\end{pmatrix} + \begin{pmatrix}
\delta M_{1,1} & \delta M_{1,2} \\
\delta M_{2,1} & \delta M_{2,2}
\end{pmatrix}
\]

\[(A.2)\]

Notice that the mixing angle \( \alpha \) do not need to be renormalized, and the tree-level relations, Eq. (2.35), still hold. \( \delta M_{i,j} \) can be obtained by differentiation

\[
\delta M_{K^0 H^0} = -\frac{1}{2} \delta M_Z^2 \sin 2(\alpha + \beta) + \frac{1}{2} \delta M_{A^0}^2 \sin 2(\alpha - \beta)
+ \frac{e}{2 M_Z s_W c_W} \left[ \delta T_{H^0} \sin^3 (\alpha - \beta) - \delta T_{Z^0} \cos^3 (\alpha - \beta) \right]
- \delta \tan \beta \cos^2 \beta [M_Z^2 \cos 2(\alpha + \beta) + M_{A^0}^2 \cos 2(\alpha - \beta)]
\]

\[
\delta M_{K^0} = \delta M_Z^2 \sin^2 (\alpha + \beta) + \delta M_{A^0}^2 \cos^2 (\alpha - \beta)
+ \frac{e}{2 M_Z s_W c_W} \left[ \delta T_{H^0} \cos (\alpha - \beta) \sin^2 (\alpha - \beta) \right]
\]
\[ 4.11. \text{Standard Model Higgs} \]

\[
\delta M_{H^0}^2 = \delta M_Z^2 \cos^2(\alpha + \beta) + \delta M_A^2 \sin^2(\alpha - \beta) \\
- \frac{e}{2M_Z s_W c_W} [\delta T_{H^0} \cos(\alpha - \beta)(1 + \sin^2(\alpha - \beta)) \\
+ \delta T_{h^0} \sin(\alpha - \beta) \cos^2(\alpha - \beta)] \\
- \delta \tan \beta \cos^2 \beta \left[ M_Z^2 \sin^2(\alpha + \beta) + M_A^2 \sin^2(\alpha - \beta) \right] \\
\delta M_{A^0 G^0}^2 = -\frac{e}{2M_Z s_W c_W} [\delta T_{H^0} \sin(\alpha - \beta) + \delta T_{h^0} \cos(\alpha - \beta)] \\
- \delta \tan \beta \cos^2 \beta M_A^2, \\
\delta M_{G^0}^2 = -\frac{e}{2M_Z s_W c_W} [\delta T_{H^0} \cos(\alpha - \beta) - \delta T_{h^0} \sin(\alpha - \beta)], \\
\delta M_{H^0 - G^+}^2 = -\frac{e}{2M_Z s_W c_W} [\delta T_{h^0} \cos(\alpha - \beta) + \delta T_{h^0} \cos(\alpha - \beta)] \\
- \delta \tan \beta \cos^2 \beta M_{H^\pm}^2 \\
\delta M_{G^\pm}^2 = -\frac{e}{2M_Z s_W c_W} [\delta T_{H^0} \cos(\alpha - \beta) - \delta T_{h^0} \sin(\alpha - \beta)]. \quad (A.3)
\]

The renormalization constant for \( M_{H^\pm}^2 \) is given by

\[ \delta M_{H^\pm}^2 = \delta M_{A^0}^2 + \delta M_W^2. \quad (A.4) \]

In order to have finite Green functions, the fields have to be renormalized as well. We renormalize the unrotated Higgs doublets \( H_1 \) and \( H_2 \).

\[ H_1 \rightarrow \left(1 + \frac{1}{2} \delta Z_{H_1}\right) H_1, \quad H_2 \rightarrow \left(1 + \frac{1}{2} \delta Z_{H_2}\right) H_2. \quad (A.5) \]

and their vacuum expectation value is renormalized as follows:

\[ v_1 \rightarrow \left(1 + \frac{1}{2} \delta Z_{H_1}\right) (v_1 - \delta v_1) = v_1 \left(1 + \frac{1}{2} \delta Z_{H_1} - \frac{\delta v_1}{v_1}\right), \]
\[ v_2 \rightarrow \left(1 + \frac{1}{2} \delta Z_{H_2}\right) (v_2 - \delta v_2) = v_2 \left(1 + \frac{1}{2} \delta Z_{H_2} - \frac{\delta v_2}{v_2}\right). \quad (A.6) \]

Note that the freedom of field renormalization allows to impose the condition \( \frac{\delta v_1}{v_1} = \frac{\delta v_2}{v_2} \).

The CP-even Higgs bosons fields are therefore split into renormalized fields.

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end wavefunction renormalization constants,

\[
\begin{pmatrix}
\h^0 \\
H^0
\end{pmatrix} \to \begin{pmatrix}
1 + \frac{1}{2}\delta Z_{h^0 h^0} & \frac{1}{2}\delta Z_{H^0 h^0} \\
\frac{1}{2}\delta Z_{H^0 h^0} & 1 + \frac{1}{2}\delta Z_{H^0 H^0}
\end{pmatrix} \begin{pmatrix}
\h^0 \\
H^0
\end{pmatrix} = (A.7)
\]

\[
\begin{pmatrix}
-\sin\alpha & \cos\alpha \\
\cos\alpha & \sin\alpha
\end{pmatrix} \begin{pmatrix}
1 + \frac{1}{2}\delta Z_{H_1} & 0 \\
0 & 1 + \frac{1}{2}\delta Z_{H_2}
\end{pmatrix} \begin{pmatrix}
-\sin\alpha & \cos\alpha \\
\cos\alpha & \sin\alpha
\end{pmatrix} \begin{pmatrix}
\h^0 \\
H^0
\end{pmatrix},
\]
giving rise to

\[
\delta Z_{h^0 h^0} = \sin^2\alpha \delta Z_{H_1} + \cos^2\alpha \delta Z_{H_2},
\]
\[
\delta Z_{H_0 h^0} = \delta Z_{H_0 h^0} = \sin\alpha \cos\alpha (\delta Z_{H_2} - \delta Z_{H_1}),
\]
\[
\delta Z_{H_0 H_0} = \cos^2\alpha \delta Z_{H_1} + \sin^2\alpha \delta Z_{H_2}.
\]

One has:

\[
\delta Z_{H_1} = -\Sigma'_{A, A}(M_{A}^2) - \frac{\cot\beta}{M_Z} \Sigma_{A, Z}(M_{A}^2) (A.9)
\]
\[
\delta Z_{H_2} = -\Sigma'_{A, A}(M_{A}^2) + \frac{\tan\beta}{M_Z} \Sigma_{A, Z}(M_{A}^2) (A.10)
\]

where \(\Sigma_{ij}(k^2)\) is the self energy function of the particles \(i, j\) evaluated at a squared momentum \(k^2\), and

\[
\Sigma'_{ij} = \frac{d\Sigma_{ij}}{dk^2} (A.11)
\]

One should also defined the renormalized self energy functions. These functions are UV finite, and are, for the CP even sector:

\[
\hat{\Sigma}_{H_0 H_0}(k^2) = \Sigma_{H_0 H_0}(k^2) + k^2 (\cos^2\alpha \delta Z_{H_1} + \sin^2\alpha \delta Z_{H_2}) - \delta M_{H_0}^2
\]
\[
\hat{\Sigma}_{h_0 h_0}(k^2) = \Sigma_{h_0 h_0}(k^2) + k^2 (\sin^2\alpha \delta Z_{H_1} + \cos^2\alpha \delta Z_{H_2}) - \delta M_{h_0}^2
\]
\[
\hat{\Sigma}_{H_0 h_0}(k^2) = \Sigma_{H_0 h_0}(k^2) + k^2 \cos\alpha \sin\alpha (\delta Z_{H_1} + \delta Z_{H_2}) - \delta M_{H_0 h_0}^2
\]

(A.12)

Finally, one also has that:

\[
\delta Z_{h_0 h_0}^{se} = -\hat{\Sigma}_{h_0 h_0}(M_{h_0}^2) \quad \delta Z_{H_0 H_0}^{se} = -\hat{\Sigma}_{H_0 H_0}(M_{H_0}^2)
\]

(A.13)
\[ \delta Z_{H^0 H^0}^{\text{se}} = - \frac{2 \tilde{\Sigma}_{h^0 H^0}(M_{h^0}^2)}{M_{h^0}^2 - M_{H^0}^2} \delta Z_{h^0 H^0}^{\text{se}} = - \frac{2 \tilde{\Sigma}_{h^0 H^0}(M_{H^0}^2)}{M_{H^0}^2 - M_{h^0}^2} \] (A.14)

For the CP odd sector one has (remembering that \( A^0 \) is renormalized on-shell):

\[ \tilde{\Sigma}_{A^0}(M_{A^0}^2) = 0 \quad \tilde{\Sigma}'_{A^0}(M_{A^0}^2) = 0 \] (A.15)

\[ \delta M_{A^0}^2 = \Sigma_{A^0}(M_{A^0}^2) - M_{A^0}^2 \Sigma'_{A^0}(M_{A^0}^2), \] (A.16)

and:

\[ \delta Z_{A^0 A^0}^{\text{se}} = \sin^2 \beta \delta Z_{H^1} + \cos^2 \beta \delta Z_{H^2} \] (A.17)

\[ \delta Z_{G^0 A^0}^{\text{se}} = \cos \beta \sin \beta (\delta Z_{H^2} - \delta Z_{H^1}) \] (A.18)

The explicit expressions for the self energies functions can be found in [35].

The renormalization constants for the \( c^0(bb h^0) \) and for the \( c^0(bb H^0) \) couplings are obtained differentiating the tree-level expressions in Eq. (4.5),

\[
\begin{align*}
\delta c^0(bb h^0) &= \left( \frac{\delta g}{g} + \frac{\delta m_b}{m_b} - \frac{\delta M_W^2}{2M_W^2} - \frac{\delta \cos \beta}{\cos \beta} \right) c^0(bb h^0), \\
\delta c^0(bb H^0) &= \left( \frac{\delta g}{g} + \frac{\delta m_b}{m_b} - \frac{\delta M_W^2}{2M_W^2} - \frac{\delta \cos \beta}{\cos \beta} \right) c^0(bb H^0).
\end{align*}
\] (A.19)

\( \delta c^0, \delta M_W^2, \delta M_Z^2 \) and \( \delta g \), read as follows

\[
\begin{align*}
\delta \cos \beta &= - \sin^2 \beta \frac{\delta \tan \beta}{\tan \beta}, \\
\delta M_W^2 &= \text{Re} \Sigma_W(M_W^2), \\
\delta M_Z^2 &= \text{Re} \Sigma_Z(M_Z^2) \\
\frac{\delta g}{g} &= \frac{\Sigma_{\gamma Z}(0)}{s_W c_W M_Z^2} - \frac{1}{2} \left[ + \frac{c_W}{s_W M_Z^2} \Sigma_{\gamma Z}(0) \frac{c^2_W}{s^2_W} \left( \frac{\delta M_W^2}{M_W^2} \frac{\delta M_W^2}{M_W^2} - \frac{\delta M_Z^2}{M_Z^2} \right) \right]
\end{align*}
\] (A.20)

The \( c^0(bb A^0) \) couplings depends only on the angle \( \beta \). When computing the the renormalization constant \( \delta c^0(bb A^0) \), one has to distinguish between the \( \beta \)-dependent factors originated by the \( H_1, H_2 \) mixing and the \( \beta \)-dependent
factors from the $H_1, H_2$ couplings. Only the latter have to be renormalized. In particular the factor $\sin \beta \left[ 1 / \cos \beta \right]$ entering the $c^\eta (bbA^0)$ coupling is originated from the $H_1, H_2$ mixing couplings, and thus $\delta c^\eta (bbA^0)$ reads

\[
\delta c^\eta (bbA^0) = \left( \frac{\delta g}{g} + \frac{\delta m_b}{m_b} - \frac{\delta M_W^2}{2M_W^2} - \frac{\delta \cos \beta \cos \beta}{\cos \beta} \right) c^\eta (bbA^0). \tag{A.21}
\]
Appendix B

Contributions of the counterterms

In this appendix we will detail the origin of the counterterms contribution, as well as the cancelation of the divergences in the UV sector.

For the s-channel subprocess, the following contributions shall be considered:

1. $\frac{1}{2} \delta Z_{b}^{b} L,R$ from $(gbb)$ vertex, leading to

\[
\bar{u}(b') [c^{L}(bbH_{i}^{0}) P_{L} + c^{R}(bbH_{i}^{0}) P_{R}] (\bar{q} + m_{b})\psi_{4}(\mu) u(b) \rightarrow \\
\bar{u}(b') [c^{L}(bbH_{i}^{0}) P_{L} + c^{R}(bbH_{i}^{0}) P_{R}] (\bar{q} + m_{b})\psi_{4}(\mu) [\delta Z_{b}^{L} P_{L} + \delta Z_{b}^{R} P_{R}] u(b) \rightarrow \\
\bar{q}\psi_{4}(\mu) [\delta Z_{b}^{L} c^{L}(bbH_{i}^{0}) P_{L} + \delta Z_{b}^{R} c^{R}(bbH_{i}^{0}) P_{R}] + m_{b}\psi_{4}(\mu) [\delta Z_{b}^{b} L c^{R}(bbH_{i}^{0}) P_{L} + \delta Z_{b}^{b} R c^{L}(bbH_{i}^{0}) P_{R}]
\]

2. $\frac{1}{2} \delta Z_{b}^{b} L,R$ from $(bbH_{i}^{0})$ vertex, leading to

\[
\bar{u}(b') [c^{L}(bbH_{i}^{0}) P_{L} + c^{R}(bbH_{i}^{0}) P_{R}] (\bar{q} + m_{b})\psi_{4}(\mu) u(b) \rightarrow \\
\bar{u}(b') [c^{L}(bbH_{i}^{0}) P_{L} + c^{R}(bbH_{i}^{0}) P_{R}] [\frac{1}{2} \delta Z_{b}^{L} P_{L} + \frac{1}{2} \delta Z_{b}^{R} P_{R}] (\bar{q} + m_{b})\psi_{4}(\mu) u(b) \rightarrow \\
\bar{q}\psi_{4}(\mu) [\frac{1}{2} \delta Z_{b}^{L} c^{L}(bbH_{i}^{0}) P_{L} + \frac{1}{2} \delta Z_{b}^{R} c^{R}(bbH_{i}^{0}) P_{R}]
\]
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\[ + m_b \phi'_L(\mu) \left[ \frac{1}{2} \delta Z_R^b c^R(\bar{b}bH_i^0))P_L + \frac{1}{2} \delta Z_L^b c^L(\bar{b}bH_i^0))P_R \right] \]

(B.2)

3. \( \frac{1}{2} \delta Z_{L,R}^b \) from \((\bar{b}bH_i^0)\) vertex, leading to

\[ \bar{u}(b') \left[ c^L(\bar{b}bH_i^0)P_L + c^R(\bar{b}bH_i^0)P_R \right] (\bar{q} + m_b) \phi'_L(\mu) u(b) \rightarrow \]
\[ \bar{u}(b') \left[ \frac{1}{2} (\delta Z_R^b) c^L(\bar{b}bH_i^0)P_L + \frac{1}{2} (\delta Z_L^b) c^R(\bar{b}bH_i^0)P_R \right] (\bar{q} + m_b) \phi'_L(\mu) u(b) \]

(B.3)

4. \( \delta c^L,R \) from the Yukawa couplings in the \((\bar{b}bH_i^0)\) vertex,

\[ \bar{u}(b') \left[ \delta c_L(\bar{b}bH_i^0) \right] P_L \]
\[ + \left[ \delta c_R(\bar{b}bH_i^0) \right] P_R (\bar{q} + m_b) \phi'_L(\mu) u(b) \]

(B.4)

5. \( \frac{1}{2} \delta Z_{ji}^{\text{res}} \) for \(H_i^0\) line of \((\bar{b}bH_i^0)\) vertex, \((j = 1, 2\) either for \(H^0, h^0\) case or for \(A^0, G^0\)\), leading to

\[ \bar{u}(t) \left[ c^L(\bar{b}bH_i^0)P_L + c^R(\bar{b}bH_i^0)P_R \right] (\bar{q} + m_b) \phi'_L(\mu) u(b) \rightarrow \]
\[ \frac{1}{2} \sum_j \bar{u}(b') \left[ c^L_j \delta Z_{ji}^{\text{res}} P_L + c^R_j \delta Z_{ji}^{\text{res}} P_R \right] (\bar{q} + m_b) \phi'_L(\mu) u(b) \]

(B.5)

6. \( \frac{1}{2} \delta Z_{ji}^{\text{res}} \) due to external \(H_i^0\) line residue, only for type \(H^0, h^0\) case (none for \(A^0\) which is taken on shell), leading to

\[ \bar{u}(t) \left[ c^L(\bar{b}bH_i^0)P_L + c^R(\bar{b}bH_i^0)P_R \right] (\bar{q} + m_b) \phi'_L(\mu) u(b) \rightarrow \]
\[ \frac{1}{2} \sum_j \bar{u}(b') \left[ c^L_j \delta Z_{ji}^{\text{res}} P_L + c^R_j \delta Z_{ji}^{\text{res}} P_R \right] (\bar{q} + m_b) \phi'_L(\mu) u(b) \]

(B.6)
The counterterms arising in the u-channel subprocess are instead the followings:

1. $\delta Z_{b_{L,R}}^b$ from $(gbb)$ vertex, leading to

$$
\bar{u}(b')\bar{\psi}(\mu)(g' + m_b)[c^L(bbH_0^0)P_L + c^R(bbH_0^0)P_R]u(b) \frac{1}{u - m_b^2} \rightarrow
$$
$$
\bar{u}(b')\bar{\psi}(\mu)\bar{g}[(\delta Z_{b_R}^b)c^L(bbH_0^0)P_L + (\delta Z_{b_L}^b)c^R(bbH_0^0)P_R]
+ \frac{1}{2}m_b[\delta Z_{b_{L,R}}^b]c^L(bbH_0^0)P_L + \frac{1}{2}m_b[c^R(bbH_0^0)P_R]u(b) \frac{1}{u - m_b^2}
$$

(B.7)

2. $\frac{1}{2}\delta Z_{b_{L,R}}^b$ from $(bbH_0^0)$ vertex, leading to

$$
\bar{u}(b')\bar{\psi}(\mu)(g' + m_b)[c^L(bbH_0^0)P_L + c^R(bbH_0^0)P_R]u(b) \frac{1}{u - m_b^2} \rightarrow
$$
$$
\frac{1}{2}u(b')\bar{\psi}(\mu)(g' + m_b)[\delta Z_{b_{L}}^b c^L(bbH_0^0)P_L + \delta Z_{b_{R}}^b c^R(bbH_0^0)P_R]u(b) \frac{1}{u - m_b^2}
$$

(B.8)

3. $\frac{1}{2}\delta Z_{b_{L,R}}^b$ from $(bbH_0^0)$ vertex, leading to

$$
\bar{u}(b')\bar{\psi}(\mu)(g' + m_b)[c^L(bbH_0^0)P_L + c^R(bbH_0^0)P_R]u(b) \rightarrow
$$
$$
\bar{u}(b')\bar{\psi}(\mu)\bar{g}[\frac{1}{2}\delta Z_{b_{L}}^b c^L(bbH_0^0)P_L + \frac{1}{2}\delta Z_{b_{R}}^b c^R(bbH_0^0)P_R]
+ \frac{1}{2}m_b[\frac{1}{2}\delta Z_{b_{L}}^b c^L(bbH_0^0)P_L + \frac{1}{2}\delta Z_{b_{R}}^b c^R(bbH_0^0)P_R]u(b)
$$

(B.9)

4. $\delta c_{L,R}^{bb}$ from the Yukawa couplings in the $(bbH_0^0)$ vertex,

$$
\bar{u}(b')\bar{\psi}(\mu)(g' + m_b)\{\delta c^L(bbH_0^0)P_L
+ \delta c^R(bbH_0^0)P_R\}u(b)
$$

(B.10)
5. \( \frac{1}{2} \delta Z_{ji}^{ct} \) for \( H_i^0 \) line of \((bbH_i^0)\) vertex, \((j = 1, 2 \text{ either for } H^0, h^0 \text{ case or for } A^0, G^0)\), leading to
\[
\bar{u}(b')\gamma_{\mu}(q' + m_b)\frac{1}{2} \sum_j \delta Z_{ji}^0 \{ c_j^L P_L + c_j^R P_R \} u(b)
\]
(B.11)

6. \( \frac{1}{2} \delta Z_{ji}^{res} \) due to external \( H_i^0 \) line residue, only for type \( H^0, h^0 \) case \((\text{none for } A^0 \text{ which is taken on shell}), \text{leading to}
\[
\bar{u}(b')\gamma_{\mu}(q' + m_b)\frac{1}{2} \sum_j \delta Z_{ji}^{res} \{ c_j^L P_L + c_j^R P_R \} u(b)
\]
(B.12)

The ultra violet divergences arising in the counterterms sector should cancel with the similar ones arising in the vertex corrections. In fact, the cancelation should happen in four separate subsectors.

1. In the \( s \)-channel, terms coming with a factor \( \frac{\alpha_s}{\pi} N_{1L}^{Born} \Delta \):

   - Left-side diagrams: \( \delta Z_L^b + (V qq) + (S qq) + (\chi qq) \)

   \[
   - 2\left( \frac{1 + 26 c_W^2}{36 s_W^2 c_W} \right) + \left( \frac{\tilde{m}_t^2 + \tilde{m}_b^2}{4 s_W^2 M_W^2} \right) + \left( \frac{1 + 26 c_W^2}{36 s_W^2 c_W} \right)
   + \left( \frac{\tilde{m}_t^2 + \tilde{m}_b^2}{4 s_W^2 M_W^2} \right) + \left( \frac{1 + 26 c_W^2}{36 s_W^2 c_W} \right) + \left( \frac{\tilde{m}_t^2 + \tilde{m}_b^2}{4 s_W^2 M_W^2} \right) \rightarrow 0
   \]
   (B.13)

   - Right-side diagrams: \( \frac{1}{2} \delta Z_L^b + \frac{1}{2} \delta Z_R^b + \frac{\delta y}{g} \delta M_W + \frac{\delta m_b}{m_b} - \frac{\delta \cos \beta}{\cos \beta} \)

   \[
   + \frac{1}{2} \sum_j \frac{c_j^L}{c_j^R} \delta Z_{ji}^{CT} + (q SV) + (q qq) + (q VS) + (SF f)
   \]

   \[
   - \left( \frac{1 + 26 c_W^2}{36 s_W^2 c_W} \right) - \left( \frac{\tilde{m}_t^2 + \tilde{m}_b^2}{4 s_W^2 M_W^2} \right) - \left( \frac{1}{9 c_W^2} \right)
   \]

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3. In the $u$-channel, factorize $\frac{a}{4\pi}N_{1R}^{Born} \Delta$:

- Left-side diagrams: $\delta Z_R^h + (V qq) + (S qq) + (\chi qq)$

$$ - \left( \frac{2}{9c_W^2} \right) - \left( \frac{\tilde{m}_b^2}{s_W^2 M_W^2} \right) + \left( \frac{1}{9c_W^2} \right) + \left( \frac{\tilde{m}_b^2}{2s_W^2 M_W^2} \right) + \left( \frac{1}{9c_W^2} \right) + \left( \frac{\tilde{m}_b^2}{2s_W^2 M_W^2} \right) \rightarrow 0 $$

(B.15)

- Right-side diagrams: $\frac{1}{2} \delta Z_R^h + \frac{1}{2} \delta Z_L^h + \frac{\delta g}{g} \frac{\delta M_W}{M_W} + \frac{\delta m_b}{m_b} \frac{\delta \cos \beta}{\cos \beta}$

$$ + \frac{1}{2} \sum_j c_j \delta Z_{2j} + (qSV) + (V qq) + (qVS) + (SF'f) $$

$$ - \left( \frac{2}{9c_W^2} \right) - \left( \frac{\tilde{m}_b^2}{s_W^2 M_W^2} \right) - \left( \frac{1 + 26c_W^2}{36s_W^2 c_W^2} \right) - \left( \frac{\tilde{m}_b^2 + \tilde{m}_b^2}{4s_W^2 M_W^2} \right) $$

$$ - \left( \frac{5}{2s_W^2} \right) + \left( \frac{1}{2c_W^2} \right) - \left( \frac{1 + 2c_W^2}{4s_W^2 c_W^2} \right) + \left( \frac{3(m_t^2 + m_b^2)}{4s_W^2 M_W^2} \right) $$

$$ + \left( \frac{1 + 26c_W^2}{36s_W^2 c_W^2} \right) + \left( \frac{1}{3c_W^2} \right) - \left( \frac{1 + 2c_W^2}{2s_W^2 c_W^2} \right) + \left( \frac{\tilde{m}_t^2 + 3m_b^2}{4s_W^2 M_W^2} \right) $$

$$ - \left( \frac{3}{4s_W^2 M_W^2} m_t^2 - m_b^2 \tan^2 \beta \right) - \left( \frac{3\tilde{m}_b^2}{4s_W^2 M_W^2} \right) $$

$$ + \left( \frac{1}{6c_W^2} \right) - \left( \frac{4}{18c_W^2} \right) + \left( \frac{1}{12c_W^2} \right) + \left( \frac{3}{4s_W^2} \right) + \left( \frac{1 + 2c_W^2}{2s_W^2 c_W^2} \right) \rightarrow 0 $$

(B.16)

2. In the $s$-channel, factorize $\frac{a}{4\pi}N_{1L}^{Born} \Delta$:

- Left-side diagrams: $\delta Z_R^h + (V qq) + (S qq) + (\chi qq)$

$$ - \left( \frac{2}{9c_W^2} \right) - \left( \frac{\tilde{m}_b^2}{s_W^2 M_W^2} \right) + \left( \frac{1}{9c_W^2} \right) + \left( \frac{\tilde{m}_b^2}{2s_W^2 M_W^2} \right) + \left( \frac{1}{9c_W^2} \right) + \left( \frac{\tilde{m}_b^2}{2s_W^2 M_W^2} \right) \rightarrow 0 $$

(B.14)
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- Upper diagrams: $\delta Z_R^b + (ffV) + (ffS) + (SSf)$

\[
- \left( \frac{2}{9c_W^2} + \frac{\bar{m}_b^2}{s_W^2 M_W^2} \right) + \left( \frac{1}{9c_W^2} + \frac{\bar{m}_b^2}{2s_W^2 M_W^2} \right) + \left( \frac{1}{9c_W^2} + \frac{\bar{m}_b^2}{2s_W^2 M_W^2} \right) \to 0
\]

(B.17)

- Lower diagrams: $\frac{1}{2}\delta Z_R^b + \frac{1}{2}\delta Z_L^b + \delta g - \delta M_W^2 + \frac{\delta m_b}{m_b} \frac{\delta \cos \beta}{\cos \beta}$

\[
+ \frac{1}{2} \sum_j \frac{\delta \cos \beta}{\cos \beta} Z_{ji} + (V S q) + (q' q V) + (S V q) + (f' f S)
\]

\[
\frac{1}{2} \sum_j \frac{\delta \cos \beta}{\cos \beta} Z_{ji} + (V S q) + (q' q V) + (S V q) + (f' f S)
\]

4. In the $u$-channel, factorize $\frac{\alpha}{4\pi} N_{2R}^{Born} \Delta$:

- Upper diagrams: $\delta Z_L^b + (ffV) + (ffS) + (SSf)$

\[
-2 \left( \frac{2}{9s_W^2 c_W^2} + \frac{\bar{m}_b^2 + \bar{m}_b^2}{36s_W^2 c_W^2 M_W^2} \right) + \left( \frac{1}{2s_W^2 c_W^2} + \frac{\bar{m}_b^2 + \bar{m}_b^2}{4s_W^2 c_W^2 M_W^2} \right) + \left( \frac{1}{2s_W^2 c_W^2} + \frac{\bar{m}_b^2 + \bar{m}_b^2}{4s_W^2 c_W^2 M_W^2} \right) \to 0
\]

(B.19)

- Lower diagrams: $\frac{1}{2} \delta Z_L^b + \frac{1}{2} \delta Z_R^b + \delta g - \delta M_W^2 + \delta m_b - \frac{\delta \cos \beta}{\cos \beta}$

\[
+ \frac{1}{2} \sum_j \frac{\delta \cos \beta}{\cos \beta} Z_{ji} + (V S q) + (q' q V) + (S V q) + (f' f S)
\]

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\[ -\frac{1 + 26c_W^2}{36s_W^2c_W} - \left( \frac{\tilde{m}_t^2 + \tilde{m}_b^2}{4s_W^2M_W^2} \right) - \left( \frac{1}{9c_W^2} \right) - \left( \frac{\tilde{m}_b^2}{2s_W^2M_W^2} \right) \\
- \left( \frac{5}{2s_W^2} \right) + \left( \frac{1}{s_W^2} \right) - \left( \frac{1}{2c_W^2} \right) + \left( \frac{1 + 2c_W^2}{4s_W^2c_W^2} \right) + \frac{3(m_t^2 + m_b^2)}{4s_W^2M_W^2} \\
+ \left( \frac{1 + 26c_W^2}{36s_W^2c_W^2} \right) + \left( \frac{1}{3c_W^2} \right) - \left( \frac{1 + 2c_W^2}{2s_W^2c_W^2} \right) + \left( \frac{\tilde{m}_t^2 + 3\tilde{m}_b^2}{4s_W^2M_W^2} \right) \\
- \frac{3(m_t^2 - m_b^2 \cot^2 \beta)}{4s_W^2M_W^2} - \frac{3\tilde{m}_b^2}{4s_W^2M_W^2} + \left( \frac{1 + 2c_W^2}{12s_W^2c_W^2} \right) + \frac{1}{2s_W^2} \\
- \frac{2}{9c_W^2} + \left( \frac{1}{6c_W^2} \right) + \left( \frac{1 + 2c_W^2}{2s_W^2c_W^2} \right) \rightarrow 0 \]
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