ALGEBRA OF PSEUDO-DIFFERENTIAL C*-OPERATORS(*)

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SOMMARIO.- *In questo lavoro si studia l'algebra degli operatori pseudo-differenziali nel contesto delle C*-algebre. In particolare si fa vedere che lo spazio di tutti gli operatori pseudo-differenziali su una varietà compatta è un algebra involutiva.*

SUMMARY.- *In this paper we study the algebra of pseudo-differential operators in the context of C*-algebras. We essentially show that the space of all pseudo-differential operators on a compact manifold is an involutive algebra.*

1. Introduction

Various functional methods have been intensively applied to topology during the last decade. The technique of C^*-algebras is quite useful in a number of problems connected with topological properties of manifolds. In particular, C^*-algebras and their representations play an important role in K-theory and problems associated with it. Several authors have obtained interesting results concerning pseudo-differential operators in the framework of C^*-algebras. Mishenko [5] and [6] interpreted several versions of the theory of pseudo-differential operators in terms of C^*-algebras, that is, elliptic pseudo-differential operators on a compact manifold; pseudo-differential operators on a Euclidian space, operators with almost periodic functions etc. Mishenko and Fomenko [7] derived analogues of the well-known Atiyah-Singer index formulas in this situation.

In this paper we study the algebra of pseudo-differential operators in the framework of C^*-algebras. We essentially prove that every pseudo-differential operator of order m admits an adjoint operator, in this case, which is again a pseudo-differential operator. Consequently, we get that the space of all pseudo-differential operators on a compact manifold is an involutive algebra. This kind of problem has been discussed, in the classical case, by Hörmander [1], Kohn and Nirenberg [2] and Kumano-go [3]

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and [4] among many others. The formulation of such a problem naturally arises when one seeks analogues of the Atiyah-Bott formulas for this case.

Regarding the general theory of pseudo-differential operators we refer to [10].

2. Preliminaries

Let A be any C^*-algebra with an identity and K a right A-module. Recall the definition of a Hilbert C^*-module as in ([9] and [5]).

DEFINITION 2.1. A pre-Hilbert C^*-module is a right A-module K equipped with an inner product $<\cdot, \cdot> : K \times K \to A$ satisfying $\forall x, y, z \in K, a \in A, \lambda \in C$ the following conditions:

i) $<x,x> \geq 0$; $<x,x> = 0$ only if $x = 0$;
ii) $<x,y> = <y,x>^*$;
iii) $<x,y+z> = <x,y> + <x,z>$; $<x,\lambda y> = \lambda <x,y>$;
iv) $<xa,y> = a^* <x,y>$; $<x,ya> = <x,y>a$.

For all $x \in K$, $\|x\|_K^2 = \|<x,x>\|$ defines a norm on a pre-Hilbert C^*-module K. Moreover, it satisfies $\|xa\|_K \leq \|x\|_K \cdot \|a\|$ and $\|<x,y>\| \leq \|x\|_K \cdot \|y\|_K$, $\forall x, y \in K, a \in A$. The inner product gives the homomorphism

$$\varphi : K \to K^* = \mathrm{Hom}_A(K, A)$$

which is not, however, an isomorphism for an arbitrary C^*-algebra A, in contrast to the case where A is the field of complex numbers.

For this reason, a supplementary condition is imposed.

v) the homomorphism $\varphi : K \to K^* = \mathrm{Hom}_A(K, A)$ is an isomorphism. This guarantees the existence of an adjoint to any bounded homomorphism of Hilbert C^*- modules.

We shall use the following terminology. A pre-Hilbert C^*-module K which is complete with respect to the norm $\|\cdot\|_K$, together with an inner product satisfying (i)-(iv), is called a Hilbert C^*-module. If (v) is also satisfied, then K is called self-dual. The inner product $<\cdot, \cdot>$ will be called a Hermitian product. Denote by $\mathrm{Hom}_A^*(K_1, K_2)$ the space of A-homomorphisms $T : K_1 \to K_2$ having adjoints, that is, homomorphisms T^* such that

$$<Tx, y> \; = \; <x, T^*y>, x \in K_1, y \in K_2.$$

It is clear that $\mathrm{Hom}_A^*(K_1, K_2) \subset \mathrm{Hom}_A(K_1, K_2)$ is a closed subspace in the operator norm. Also, the space $\mathrm{End}_A^*(K) = \mathrm{Hom}_A^*(K, K)$ is a C^*-algebra. If we denote by $K^{\#} = \mathrm{Hom}_A^*(K, A)$, then the homomorphism $\varphi : K \to K^*$ induced by the inner product $<x, y>$ realizes the isomorphism $\varphi : K \to K^{\#} \subset K^*$.

Thus we have a natural category $\mathcal{M}$ whose objects are Hilbert C^*-modules and whose morphisms are operators in $\mathrm{Hom}_A^*(K_1, K_2)$.

Examples of such objects are numerous. For instance, the C^*-algebra A itself as well as the direct sum A^k of k copies of A, with the Hermitian product given by $<x, y> \; = \sum_{i=1}^{k} x_i^* y_i$, is a Hilbert C^*-module. Also, these modules are self-dual. Furthermore, A^k is projective relative to epimorphisms.

Denote by $l_2(A)$ the space of sequences $x = (x_1, \ldots x_n, \ldots) \; \forall x_n \in A$, which satisfy the condition that $\sum_{n=1}^{\infty} x_n^* x_n$ converges in A. We can define a Hermitian product in $l_2(A)$ by putting

$$<x, y> \; = \sum_{n=1}^{\infty} x_n^* y_n \tag{2.1}$$

and hence the norm by

$$\|x\|^2 = \| \sum_{n=1}^{\infty} x_n^* x_n \| . \tag{2.2}$$

The convergence of the series (2.1) follows from an analogue of Cauchy's inequality for C^*-algebras (see [7]):

$$\| \sum_{n=1}^{\infty} x_n^* y_n \|^2 < \| \sum_{n=1}^{\infty} x_n^* x_n \| \cdot \| \sum_{n=1}^{\infty} y_n^* y_n \| . \tag{2.3}$$

Then $l_2(A)$ is a Hilbert C^*-module ([7]).

For the sake of simplicity we also assume that each Hilbert C^*-module K has at most a countable set of generators, that is, a countable subset whose A-linear span is dense in K.

THEOREM 2.1 ([6]). *Every coutably generated Hilbert C^*-module is a projective object in the category $\mathcal{M}$ relative to epimorphisms.*

If the Hilbert C^*-module K is finitely generated, then K is self-dual, for there is an epimorphism $f: A^k \to K$ having an adjoint. Then by Theorem 2.1, K is a direct summand in the module A^k and is, therefore, self-dual.

3. Vector A-bundles and pseudo-differential operators over C*-algebras

DEFINITION 3.1. Let A be a C^*-algebra. By a vector A-bundle we mean a locally trivial fibre bundle $\xi = (E, p, M, F, G)$ where E is a total space, M is the base of the fibre bundle, $p: E \to M$ is a projection, F is a fibre of the fibre bundle which is finitely generated projective Hilbert C^*-module and G is the structural group equal to $\mathrm{Aut}_A(F)$, the group of A-automorphisms of the Hilbert C^*-module F. If F is a free, k-dimensional Hilbert C^*-module, then $\mathrm{Aut}_A(F)$ is the same as the group of invertible k^{th}- order matrices with coefficients in A.

Suppose that the base M is a compact smooth manifold (of class C^∞). Denote by $\Gamma(\xi)$ the space of continuous sections of the bundle ξ. The space $\Gamma(\xi)$ is endowed with the natural structure of Hilbert C^*-module, coinciding with the structure of the Hilbert C^*-module in each fibre F when $\Gamma(\xi)$ is restricted to F. Each A-bundle ξ admits a fibre Hermitian product with values in A. Thus if $u_1, u_2 \in \Gamma(\xi)$ are two sections, then a continuous function $<u_1, u_2> \in C(M, A)$ is defined, $C(M, A)$ being the algebra of continuous functions on M with values in A. Without loss of generality we can assume that the gluing functions of the vector bundle are smooth sections (of class C^∞), and denote by $\Gamma^\infty(\xi)$ the space of smooth sections (of class C^∞) of the bundle ξ. We can then choose the fibre Hermitian product to be smooth, i.e. for any two sections $u_1, u_2 \in \Gamma^\infty(\xi)$, the function $<u_1, u_2> \in C^\infty(M, A)$, where $C^\infty(M, A)$ is the space of smooth functions (of class C^∞) with values in A.

We now want to introduce Sobolev norms in $\Gamma^\infty(\xi)$. First we consider the local situation.

Let $X \subset R^n$ be a bounded open set, and $C_0^\infty(X, A)$ the ring of smooth functions (of class C^∞) with compact supports and with values in A. Let

$S(\mathbf{R}^n, A)$ denote the space of C^∞-functions whose derivatives decrease faster than any power of $\|x\| = \left(\sum_{i=1}^{n} x_i^2\right)^{1/2}$ as $\|x\| \to \infty$.

For $u \in S(\mathbf{R}^n, A)$ we define the Fourier transform $\hat{u}(\xi)$ by

$$\hat{u}(\xi) = \int e^{-ix\cdot\xi} u(x)dx, \tag{3.1}$$

where $\xi \in \mathbf{R}^n, x \cdot \xi = x_1\xi_1 + \dots + x_n\xi_n$ and dx is the Lebesgue measure on $\mathbf{R}^n$.

The inverse Fourier transform can be defined as

$$u(x) = \int e^{ix\cdot\xi} \hat{u}(\xi) đ\xi, \tag{3.2}$$

where $đ\xi = (2\pi)^{-n} d\xi$.

We denote by Δ the operator

$$\Delta = -\sum_{i=1}^{n} \frac{\partial^2}{(\partial x_i)^2} .$$

For $u \in C_0^\infty(X, A)$ we put

$$\|u\|_s^2 = \| \int_X ((1 + \Delta)^s u^*(x))\, u(x)dx \|. \tag{3.3}$$

where s is any real number, and denote by $H_0^s(X, A)$ the completion of $C_0^\infty(X, A)$ relative to the Sobolev norm (3.3). Then

LEMMA 3.1 ([7]). *The space $H_0^s(X, A)$ is isomorphic to $l_2(A)$ as a Hilbert C^*-module.*

Note that a Hermitian product in $H_0^s(X, A)$ is given by

$$\langle u, v \rangle_s = \int_X ((1 + \Delta)^s u^*(x))\, v(x)dx. \tag{3.4}$$

For $s = 0$, $H_0^s(X, A)$ equals $L_2(X, A)$ where $L_2(X, A)$ denotes the space of such measurable functions (i.e. classes) f, for which the integral $\int_X f^*(x)$

$f(x)\,dx$ converges. Likewise, in $L_2(X, A)$ we have a Hermitian product defined by

$$<f, g> = \int_X f^*(x)\, g(x)\, dx, \tag{3.5}$$

and denote the induced norm by $\|\cdot\|_{L_2}$.

Let E be trivial A-bundle on the domain X with fibre P, where P is a finitely generated projective Hilbert C^*- module with a nondegenerate, positive definite inner product with values in A. For the sake of simplicity, we assume that P is A^k, where A^k is a direct sum of k-copies of A. Analogous to (3.3) we define Sobolev norms in the space $\Gamma_0^\infty(X, A^k)$ of sections with compact support by

$$\|u\|_s^2 = \| \int_X < (1 + \Delta)^s u(x), u(x) > dx \| . \tag{3.6}$$

The completion of the space relative to the Sobolev norm is denoted by $H_0^s(X, A^k)$. Then it follows trivially from Lemma 3.1 that the Hilbert C^*-module $H_0^s(X, A^k)$ is isomorphic to the module $l_2(A^k)$, the direct sum in the module $l_2(A) \times \ldots \times l_2(A)$.

Similarly one can verify that, $L_2(X, A^k)$ is the same as $H_0^s(x, A^k)$ for $s = 0$, in which the Hermitian product is given by

$$<f, g> = \int < f(x), g(x) > dx; \; < f(x), g(x) > \; = : \sum_{i=1}^{k} f_i^*(x)\, g_i(x). \tag{3.7}$$

We now define pseudo-differential A-operators in spaces of sections of A-bundles E_1 and E_2. Let $\pi : T^* X \to X$ be the natural projection of a cotangent bundle. Consider the pre-images of the bundles $\pi^*(E_i)$, $i = 1, 2$, taking account of the fact that the inner product in each fibre is induced by the inner product in the fibres of E_i. Without loss of generality, we assume that $E_i = E$, $i = 1, 2$, with fibre A^k. We consider the A- homomorphisms of the bundles $a : \pi^*(E) \to \pi^*(E)$ as a family of A-homomorphisms

$$a(x, \xi) : A^k \to A^k$$

parametrized by points of the cotangent bundle $(x, \xi) \in T^*X$. Suppose that $a(x, \xi)$ satisfy the following conditions:

(a) $$\| \partial_\xi^\alpha \partial_x^\beta a(x, \xi) \| \leq C_{\alpha\beta} < \xi >^{m-|\alpha|} \tag{3.8}$$

where $<\xi>$ stands for $(1 + \sum_{i=1}^{n} \xi_i^2)^{1/2}$, α, β are multi-indices of non-negative integers and m being any real number.

(b) $a(x, \xi)$ have compact support in the variable x, that is, $\pi(\text{supp } a) \subset X$ is a compact set.

We shall call the homomorphism satisfying conditions (a) and (b) the *symbol of the pseudo-differential A*-operator T, which is defined by:

$$T u(x) = \int e^{ix \cdot \xi} a(x, \xi) \hat{u}(y) \, đ\xi, \tag{3.9}$$

where $u \in \Gamma_0^\infty(X, A^k)$.

The number m is called the *order of the operator T*. We shall denote by S^m the class of symbols satisfying the properties (a) and (b) as given above.

THEOREM 3.1. *The operator T defined by* (3.9) *is a continuous map from $\Gamma_0^\infty(X, A^k)$ into itself.*

The proof of Theorem 3.1 is an easy computation (see [8]).

By means of Theorem 3.1, we can extend the operator T as a continuous map of $S(R^n, A^k)$ into $S(R^n, A^k)$.

THEOREM 3.2. ([7]). *Every pseudo-differential A-operator T of order m:*

$$T : H_0^s(X, A^k) \to H_0^{s-m}(X, A^k)$$

is a bounded operator in the Sobolev norms.

It now follows trivially from Theorem 3.2 the following:

THEOREM 3.3. (L_2-continuity). *A pseudo-differential A- operator T of order zero can be extended to a bounded map of $L_2(R^n, A^k)$ into $L_2(R^n, A^k)$, i.e., there exists a constant $C > 0$ such that*

$$\| Tu \|_{L_2} \leq C \| u \|_{L_2} , \; u \in \Gamma_0^\infty(X, A^k) .$$

Next we consider the adjoint of a pseudo-differential A-operator. Note that

$$< u, v > \; = \int < u(x), v(x) > dx,$$

where $u, v \in \Gamma_0^\infty(X, A^k)$ and $< u(x), v(x) >$ equals$\sum_{i=1}^{k} u_i^*(x)\, v_i(x)$.

THEOREM 3.4. *Every pseudo-differential A-operator T of order m admits an adjoint operator, T^*, given by*

$$< Tu, v > \; = \; < u, T^* v >, u, v \in \Gamma_0^\infty(X, A^k), \tag{3.10}$$

which is again a pseudo-differential A-operator.

Proof (see also [8]): We can write for $u, v \in \Gamma_0^\infty(X, A^k)$,

$$< Tu, v > \; = \iiint e^{-i(x-y)\cdot\xi}\, u^*(y)\, a^{\#}(x, \xi)\, v(x)\, dy đ\xi dx.$$

$$= \int u^*(y) \left\{ \iint e^{i(y-x)\cdot\xi}\, a^{\#}(x, \xi)\, v(x)\, dx đ\xi \right\} dy \;,$$

where $a^{\#}(x, \xi) = (a_{ji}^*(x, \xi))$, the matrix of the symbol $a(x, \xi)$ being $a_{ij}(x, \xi)$. Hence it follows that

$$T^* v(x) = \iint e^{i(x-y)\cdot\xi} a^{\#}(y, \xi)\, v(y)\, dy đ\xi, \tag{3.11}$$

which is evidently a pseudo-differential A-operator. One can see easily that (3.10) determines T^* uniquely.

Thus a pseudo-differential A-operator is a morphism in the category $\mathcal{M}$, described in the preceding section.

THEOREM 3.5. *Let T be a pseudo-differential A-operator with symbol $a(x, \xi)$ and let T^* be its adjoint operator. Then the symbol of T^* has the following expansion:*

$$a^*(x, \xi) \sim \sum_{\alpha} \frac{1}{\alpha!}\, \partial_\xi^\alpha\, D_x^\alpha\, a^{\#}(x, \xi) \tag{3.12}$$

where the asymptotic sum runs over all multi-indices α.

THEOREM 3.6. (Composition). *Let S and T be pseudo-differential A-operators with symbols* $a(x, y) \in S^{m_1}$ *and* $b(x, \xi) \in S^{m_2}$, *respectively. Then* $R = ST$ *is a pseudo-differential A-operator with symbol* $r(x, \xi) \in S^{m_1+m_2}$, *and one has the analogue of Leibniz's formula:*

$$r(x,\xi) \sim \sum_{\alpha} \frac{1}{\alpha!}\ \partial_\xi^\alpha b(x,\xi) \cdot D_\beta^\alpha a(x,\xi)\ . \tag{3.13}$$

Proof of Theorem 3.6.: One can write

$$R\,u(x) = \int\int e^{i(x-y)\xi} b(x,\xi) \int e^{iy\cdot\eta} a(y,\eta)\,\hat{u}(\eta)\,đ\eta dy đ\xi$$

$$= \int r(x,\eta) e^{ix\cdot\eta} \hat{u}(\eta)\,đ\eta,$$

where

$$r(x,\eta) = \int\int e^{i(x-y)\cdot(\xi-\eta)} b(x,\xi)\,a(y,\eta)\,dyđ\xi.$$

It can be shown easily, as in the classical case, that $r(x, \xi) \in S^{m_1+m_2}$, and by Taylor's formula

$$r(x,\eta) = \int\int \left(\sum_{|\alpha|\le N} \frac{1}{\alpha!}\ \partial_\eta^\alpha\ b(x,\eta)(\xi-\eta)^\alpha \right) \cdot a(y,\eta) e^{i(x-y)(\xi-\eta)} dyđ\xi\ + r_N.$$

Integrating first with respect to y and then with respect to ξ, one verifies that

$$\int\int e^{i(x-y)(\xi-\eta)} a(y,\eta)(\xi,\eta)^\alpha dyđ\xi = D_x^\alpha\, a(x,\eta).$$

Hence

$$r(x,\eta) = \sum_{|\alpha|\le N} \frac{1}{\alpha!}\ \partial_\eta^\alpha\ b(x,\eta)\ D_x^\alpha\, a(x,\eta)\ +\ r_N.$$

It remains to show that $r_N \in S^{m_1+m_2-N}$. This can be done in a similar way as the corresponding assertion in the case of classical pseudo-differential operators, provided we take into consideration the standard estimates while applying to a C^*-algebra.

The proof of Theorem 3.5. uses Taylor's formula as above, and can be proved as in the classical case. Therefore, we omit the proof.

As a consequence of Theorems 3.2., 3.4., 3.5. and 3.6., the class $S^{\infty} = \bigcup_{m \in R} S^{m}$ makes an involutive algebra in the sense that if $T_i \in S^{m_i}$, $i = 1, 2$, then $T_1 + T_2 \in S^m$ for $m = \max(m_1, m_2)$ and $T_1 \cdot T_2 \in S^{m_1+m_2}$.

In order to define a pseudo-differential A-operator in sections of the A-bundles E_1 and E_2 on a compact manifold M, we consider the atlas $\{X_\alpha\}$ of charts of the manifold M, in each of which the bundles E_1 and E_2 are trivial. Let $a : \pi^*(E_1) \to \pi^*(E_2)$ be a A-homomorphism satisfying (3.8). Let $\{\varphi_\alpha\}$ be a partition of unity subordinated to the covering $\{X_\alpha\}$, and let ψ_α be functions such that $\psi_\alpha | \operatorname{supp} \varphi_\alpha \equiv 1$ and $\operatorname{supp} \psi_\alpha \subset X_\alpha$. We then put

$$Tu(x) = \sum_\alpha [T_\alpha(\varphi_\alpha u)](x); \tag{3.14}$$

where $u \in \Gamma^\infty(M, E_1)$ and T_α is the pseudo-differential A-operator defined by (3.9) in the chart X_α by means of the symbol $a_\alpha(x, \xi) = a(x, \xi)\, \psi_\alpha(x)$.

Using the partition of unity φ_α and (3.4), we define Sobolev norms in the space of sections $\Gamma^\infty(M, E_i)$. Let $u_1\ u_2 \in \Gamma^\infty(M, E_i)$ be arbitrary sections. We put

$$<u_1, u_2>_s = \sum_\alpha \int ((1 + \Delta_\alpha)^s \varphi_\alpha(x)\, u_1(x), \varphi_\alpha(x)\, u_2(x))\, dx,$$

$$\|u\|_s^2 = \|<u, u>_s\| . \tag{3.15}$$

The completion, relative to the Sobolev norm, of the space of sections $\Gamma^\infty(M, E_i)$ will be denoted by $H^s(M, E_i)$. In general, if an operator $T : H^s(M, E_1) \to H^{s-m}(M, E_2)$ is bounded for all s, then we shall say that the operator T is of order m. We now formulate some necessary propositions for later use.

THEOREM 3.7. *The pseudo-differential A-operator* $T : H^s(M, E_1) \to H^{s-m}(M, E_2)$ *of order m defined by* (3.14) *is bounded in the Sobolev norms.*

THEOREM 3.8. *The definition of pseudo-differential A-operator* (3.14) *does not depend on the partition of unity φ_α, the functions ψ_α, or the local coordinates system to within operators of smaller order.*

THEOREM 3.9. *Let a_1 and a_2 be symbols of the pseudo- differential A-operator T_1 and T_2 respectively. Then the operator T_3 and $T_2 T_1$, where $a_3 = a_2 a_1$, (i.e., composition of symbols), differ by an operator of lower order.*

The proof of Theorems 3.7.-3.9. is entirely analogous to that of the same propositions for the case of classical pseudo-differential operators, provided account is taken of the special features in applying the standard estimates in case of a C^*-algebra, for instance, which were introduced in the proof of Lemma 3.2. [7].

Likewise, with obvious modifications one can extend Theorem 3.4. to a compact manifold:

THEOREM 3.10. *Every pseudo-differential A-operator T,*

$$T : H^s(M, E_1) \to H^{s-m}(M, E_2)$$

of order m has an adjoint, T^*, which is also a pseudo-differential A- operator.

Summarizing all this, we have

THEOREM 3.11. *The class $S^\infty = \bigcup_m S^m$ of pseudo-differential A-operators on a compact manifold M, is an involutive algebra. Here S^m denotes the class of pseudo-differential A- operators $T : H^s(M, E_1) \to H^{s-m}(M, E_2)$ of order m.*

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