Modelling of port container terminal using the queuing theory

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The paper demonstrates application of the queuing theory in modelling a port container terminal. As a port container terminal is a complex system, it is possible to achieve operational efficiency of the terminal through coordination of particular subsystem capacities, i.e. determination of optimal terminal capacity accommodation. A port container terminal can be considered as a queuing system defined with basic parameters: the ship or container arrival rate and the ship or container service rate, in an observed time unit. Appropriate indices of port container terminal operations are computed on the basis of these parameters. A model of total ship waiting and berth unoccupancy costs has been established through the introduction of costs as optimization criteria thus facilitating decision-making on optimal capacity of a port container terminal.

determined, was presented. The method for determining the berth optimal capacity is presented in paper of M. Noritake and S. Kimura (1983) [12]. Same authors (1990) in paper [13] as well as K. G. Zografos and W. Martinez (1990) in paper [23], have reviewed the method for defining the optimal size of a port within the system of seaports. Through the development of computer technology to a large extent has influenced on application of simulation model of port system, in this paper mathematical models are chosen because of their advantages over simulation in terms of determination the most advantageous, i.e. optimal solution. It should be emphasize that, in former papers, assumption of number of ship arrivals and service time being random variables hadn’t been examined, but only accepted. Also, interdependence of port system parameters and their influence to the system effectiveness, which has great importance in port planning, haven’t been treated enough. Namely, knowing relations between the system elements, efficaciously functioning of port system can be achieved by appropriate changes in model.

2 Defining a port container terminal as a queuing system

A port container terminal is defined as a queuing system with the following structure: entrance units are container ships which form (or not) a queue (depending on the immediate situation) to be serviced (unloading or loading of containers) at the container berths (servicing channels), and leave the system when the service has been performed. The port container terminal as a queuing system is characterised by the following facts:

- It is not possible to anticipate the arrival time of ships at the terminal as it depends on route, speed of ship in knots, weather, organization of maritime transportation processes and other reasons.
- It is not possible either to accurately predict servicing time of the ship, i.e. duration of transshipment operations, as it depends on number and type of container, capacity and
technology of transshipment facilities, weather, organization of port transshipment processes, etc.

The consequence of these facts is the irregular berth employment. If the number of arriving ships is greater than the berth capacity, i.e. the number of ships which can be serviced by the existing berths within an observed time unit, then the ships appear in a queuing line or, conversely, if there are fewer ships, they do not have to wait, however, the berth capacity is not completely used.

Through statistical data analysis on the number of ship arrivals per days and months of a chosen port container terminal, it has been established that no significant dependence exists in the sequence of daily arrivals of container ships, i.e. that ship arrivals are statistically random. An analogous conclusion is obtained by a statistical analysis of the service time duration of container ship. Strength and form of link among observed phenomena has been tested by a statistical method of correlation for data grouping (see more details in doctoral dissertation by Z. Znizerović [24, pp. 47-54.], also in paper of I. Șoșie-V. Serdar [21, pp. 131-136.] and in paper of M. R. Spiegel [20, pp. 296-299.].

From the previous conclusion, it follows that the number of ship arrivals and duration of servicing time can be taken as random variables and, in addition, the empirical distributions of those variables approximated with the appropriate probability distributions, and finally, the queuing theory can be applied in such cases for computing indices of port container terminal operations.

From the queuing theory viewpoint, a port container terminal has the following characteristics:

1. A port container terminal is an open system as the ships are not a component part of the system.
2. A port container terminal is a single or multichannel system (depending on the number of berths) and, in this connection, ships at anchorage form queues for particular berths.
3. The number of ship arrivals as well as the duration of servicing time i.e. duration of ship's stay on the berth are allocated according to certain probability distributions (most often according to Poisson's or Erlang's distribution of the k-order, where k is a natural number). The servicing time of ship, together with the time spent queuing on the berth, represent the time of the ship's stay at the terminal and is one of the more significant indices of port container terminal operations.
4. As regards queuing discipline, a container terminal is a system where servicing is most often carried out according to the FIFO rule (first come-first served) but it is possible that there are certain ships which have priority in servicing.

3 Determination of optimal berth numbers in a port container terminal

It is necessary to define basic parameters for a port container terminal as well as any queuing system. These are: average number of container ships (or containers) which arrive at the terminal in an observed time unit and average number of container ships (or containers) which can be serviced in a same time unit at the terminal. On the basis of these parameters, appropriate indices of port container terminal operations can be computed and using the model of total queuing costs, decisions on optimal capacity of a port container terminal can be made.

3.1 Basic parameters of a servicing process at a port container terminal

The basic parameters of a port container terminal are the ship arrival rate \( \lambda \) and the service rate \( \mu \).

For a chosen container terminal system, parameter \( \lambda \) represents the average number of container ships or containers which arrive at a terminal during an observed time unit (e.g. during a year, month or day).

It often happens in practice that data on the number of ships within a time unit are not available only the time which elapses between two consecutive ship arrivals. On the basis of these data, an arithmetical mean which represents the average interval between two consecutive ship arrivals (\( i_{\text{arr}} \)) is computed. This interval is, in fact, the reciprocal value of the ship arrival rate:

\[
i_{\text{arr}} = 1/\lambda, \quad \text{or} \quad \lambda = 1/i_{\text{arr}}.
\]

The service rate can be explained by the same analogy. For a chosen container terminal system, \( \mu \) represents the average number of container ships or containers which can be serviced in a time unit at certain berth.

If the number of ships which can be serviced during an observed time unit is unknown and only duration of service time per ship is known, then the arithmetical mean pattern represents the average service time duration per ship (\( i_{\text{serv}} \)) and this time is the reciprocal value of the service rate:

\[
i_{\text{serv}} = 1/\mu, \quad \text{or} \quad \mu = 1/i_{\text{serv}}.
\]

The parameter \( \mu \) represents the accommodative capacity of one berth and multiplicand \( S \cdot \mu \), where \( S \) is the symbol for the number of berths, accommodative capacity of the container terminal as a whole.

The arrival rate and service rate quotient represents the utilization factor or berth occupancy rate \( \rho \):

\[
\rho = \lambda/\mu.
\]

If \( \lambda > \mu \), one berth is insufficient as the utilization factor is greater than 100%. In this event, the number of berths should be increased until the service system stability condition that the utilization coefficient of the system \( \rho/S \leq 1 \) has been satisfied.

In practice, values of the parameters \( \lambda \) and \( \mu \) are determined on the basis of empirical data or assessment depending on the goal and subject of research.
3.2 Operation indices of a port container terminal

Based on a container terminal definition as a queuing system and on basic parameters of a terminal, operating indices of a port container terminal can be computed. These are:

1. Berth occupancy rate \((\rho)\).
2. Container terminal utilization coefficient \((\rho/S)\).
3. Probability that there is no ship at the terminal, i.e. the berth is unoccupied \((P_0)\).
4. Probability that \( n \) ships are at the terminal, i.e. that \( n \) ships are just being serviced or are waiting in a queue to be serviced \((P_n)\).
5. Probability of servicing, i.e. the probability that a ship which arrives at the terminal will be serviced \((P_{sm})\).
6. Probability that all berths are occupied, i.e. that the ship will wait \((P(n=0))\).
7. Average number of ships in queue \((L_0)\).
8. Average number of ships which are just being serviced \((L_{sm})\).
9. Average number of ships at the container terminal, i.e. number of ships in queue and number of ships which are just being serviced \((L)\).
10. Average queuing time of ship, i.e. queuing time of ship before being serviced \((W_0)\).
11. Average servicing time of ship \((W_{sm})\).
12. Average time of ship's stay at the terminal, i.e. queuing time of ship and time of ship's servicing \((W)\).
13. Average number of unoccupied berths \((S-p)\).

The efficiency of the port container terminal, which is very often in practice determined by operating index \(W\), is augmented either with an increase in the number of berths or with curtailment of average servicing time of the ship. However, a growth in the number of berths increases the probability that berths will be vacant which, in turn, means that berth unoccupancy will go up. Similarly, a curtailment in ship service time may affect the quality of service in a negative way thus reducing the number of ship arrivals. That is why the container terminal efficiency can best be determined through the introduction of value indices, i.e. by means of the costs since, in practice, a ship's waiting time has to be paid for and the unoccupancy of the berth can also be expressed in terms of value.

3.3 Queuing cost model of a port container terminal

As with all queuing systems, ship waiting line is notified at the container terminal before the beginning of loading/unloading operations or "waiting", i.e. berth container unoccupancy when there are no ships waiting to be serviced at the terminal.

In order to eliminate waiting at the port container terminal, a great number of berths would have to be constructed to obviate the need for waiting or as many berths which would be permanently employed so that they do not remain unoccupied. These extreme solutions, of course, are not rational, as elimination each participant's waiting period leads to maximum waiting of a second participant in the queuing system.

Due to random arrivals of container ships at the terminal as well as duration of ship service which is also a random variable, from a queuing theory viewpoint, it is not possible in practice to implement such work organization at the port container terminal so that at any one moment the berth capacity is 100% employed and at any one moment a ship arriving does not have to wait for beginning of loading/unloading operations. Since in practice the waiting time and the berth unoccupancy cannot be completely avoided, each terminal strives to reduce waiting time as much as possible, i.e. costs of both participants in the servicing process at the container terminal to carry to a minimum amount.

Total waiting costs can be observed separately: costs from the shipowner's viewpoint and from the container terminal viewpoint. A clash of interest exists between the shipowner and the container terminal: it is in the shipowner's interest to have the ship wait as short a period as possible and in the port container terminal interest to handle as much traffic as possible in an observed time unit with the least number of berths. Nevertheless, the servicing process at the container terminal should be resolved taking into consideration total waiting costs as the interests of both, shipowner and port, are mutually interwoven: the port container terminal is not indifferent to the long waiting time of the ships even through it has high berth utilization, as this waiting is expensive and can divert the ships to other ports; in the event of a short waiting time, the supposition is that the container berths are poorly employed and this may result in an increase of port service.
costs, which, in turn, is not in the shipowner’s interest. If costs are taken as optimization criteria, then the servicing process solution at the container terminal will represent the optimum number of berths for which total expenses of ship waiting time and expenses of berth unoccupancy are minimum in an observed unit of time. In this regard, the optimal variant will be that one which will reduce to a minimum losses resulting from waiting. The total queuing costs \( C \) include:

1. ship queuing costs \( C_w \), and
2. berth unoccupancy costs \( C_b \).

Total ship queuing costs and berth unoccupancy costs are computed as follows:

Ship queuing costs

\[ C_w = c_w \cdot L_Q \cdot t \quad , \quad (1) \]

Unoccupied berth costs

\[ C_b = c_b \cdot (S \cdot \rho) \cdot t \quad , \quad (2) \]

Total queuing costs

\[ C = C_w \cdot L_Q \cdot t + C_b \cdot (S \cdot \rho) \cdot t \quad , \quad (3) \]

where:
- \( C \) is the amount of total costs expressed in currency units in an observed time unit (example: in USD/hour),
- \( L_Q \) is the average number of container ships in the queue,
- \( S \) is the number of container berths,
- \( \rho \) is the berth occupancy rate; \( \rho = \lambda/\mu \),
- \( t \) is the length of time period for which costs are computed (e.g. day, month, year),
- \( c_w \) is the amount of costs caused by waiting of ship, expressed in currency units for an observed time unit (e.g. in USD/hour/ship),
- \( c_b \) is the amount of costs arising from unoccupancy of berth, expressed in currency units for an observed time unit (e.g. in USD/hour/berth).

Since container traffic is expressed in TEU and not in number of ships, and a cost unit \( c_w \) relates to the container ship queuing cost unit, it is necessary that \( L_Q \) is converted into the number of ships taking into consideration the number of containers which, on average, are loaded/unloaded at the terminal.

From the queuing theory, it is known that:

\[ L_Q = \sum_{n=S+1}^{\infty} (n \cdot S) \cdot P_n \quad , \quad (4) \]

\[ (S \cdot \rho) = \sum_{n=S+1}^{\infty} (S \cdot n) \cdot P_n 
\]

so that total cost function can be written in the form:

\[ C = [c_w \cdot \sum_{n=S+1}^{\infty} (n \cdot S) \cdot P_n + c_b \cdot \sum_{n=S+1}^{\infty} (S \cdot n) \cdot P_n] \cdot t \quad , \quad (6) \]

or, if is taken into consideration that \( W_Q = L_Q/\lambda \), i.e. \( L_Q = \lambda \cdot W_Q \) formula (3) assumes the form:

\[ C = [c_w \cdot \lambda \cdot W_Q + c_b \cdot (S \cdot \rho)] \cdot t \quad , \quad (7) \]

where \( W_Q \) is the average time of ship spent in the queue.

Model of ship queuing costs and berth unoccupancy costs, or total queuing model costs is applied in such a manner that utilising formulae (3), (6) or (7), the amount of ship queuing costs and berth unoccupancy costs for a certain number of container berths are computed; then, through a change in the number of berths, the service process at the container terminal is programmed and on the basis of the results obtained for various values of berth numbers, the optimal number of berths can be determined, i.e. the number of berths for which the amount of total queuing costs is minimal.

The model of total queuing costs shown can serve as a basis for making appropriate business decisions during analysis of existing employment capacity or planning the development of future port container terminal capacity.

4 Conclusion

The port container terminal is a complex system composed of several subsystems: the quay with berths, stacking area for containers, traffic network for internal transportation as well as loading and unloading zone for land vehicles. So as to facilitate operational efficiency of the port container terminal, it is necessary to coordinate mutually the capacity of all subsystems in such a manner that exit from one subsystem represents entry to the following subsystem. This is possible to achieve through determination of the optimal number of port container terminal berths using the queuing theory. The port container terminal is a queuing system for which the appropriate indices can be computed; e.g.: the probability of berth unoccupancy, the anticipated number of ships in the queue, waiting time of ship, etc. On the basis of those indices and through the introduction of costs as optimization criteria, a model of total ship queuing costs and berth unoccupancy costs has been fixed. It is possible with this model to determine the optimal number of berths and, based on that result, to calculate the required capacity and the other subsystems of the port container terminal.
LITERATURE


