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VISUALIZATION OF CLASSICAL AND QUANTUM TURBULENCE IN CRYOGENIC FLUIDS

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Abstract

Two different projects on the characterization of classical and quantum turbulent flows using frozen particles and cryogenic fluids are reported.

The first project presents the benefits and drawbacks of using liquid nitrogen for fluid dynamics experiments compared to other test fluids. This study is focused on the visualization of high-Reynolds number flow, and presents a new technique that utilizes frozen particles as tracers. The technique would provide a cheap and easy way to produce tracers for visualization in liquid nitrogen, using common hydrocarbons or even atmospheric air.

The technique has been proved viable experimentally, producing micron-sized particles when atmospheric air is injected, and particles as small as 500 nm in diameter with a mixture of nitrogen gas and methane as the seeding gas. These particles have been successfully used as tracers for both particle image velocimetry and particle tracking velocimetry. The size of the particles has been estimated using a Mie scattering model that has been verified with polystyrene latex particles of known size and index of refraction.

A study of the selection of the best seeding gases, which involved estimates of the density and index of refraction of several hydrocarbons,
showed that propylene and propadiene would produce the brightest faithful tracers. An analysis of the parameters attainable using liquid nitrogen as a test fluid for different experiments has been conducted. The experiments considered include grid turbulence, pipe flow, Taylor Couette flow, von Kármán flow and Rayleigh-Bénard convection. A discussion of the feasibility and a comparison with different test fluids state-of-the-art existing experiments is presented.

The second project involves the study of superfluid turbulence in $He_{II}$. Quantized vortices can be visualized using micron-sized solid hydrogen particles as tracers (1). Because of their size, Stokes drag does not allow them to stay trapped on quantized vortices close to the $\lambda$- transition, where the trapping potential is weaker.

A new technique has been discovered to create and visualize sub-micron particles. Several size estimates of these nanoparticles have been made based on both optical and fluid dynamical properties. Being smaller, but not small enough to be influenced by thermal motions, the particles are more passive and are less affected by Stokes drag. Thus they stay trapped closer to transition and on faster moving vortices. The ability to create particles directly into the $He_{II}$ allows the visualization of the vortex dynamics at temperatures lower than ever before.

Particles of different size have been used to study a thermal counterflow. For low heat fluxes, these particles can either trace the motion of the normal component or track quantized vortices when they get trapped on their cores. For high heat fluxes, the increased number of
particle-vortex interactions and scattering events result in a different state in which the particles track neither the vortices nor the normal component. These observations confirm the hypothesis by Sergeev and Barenghi (2) that the discrepancy in the experiments by Paoletti et al. (3) and by Zhang and Van Sciver (4) are due to different regimes of particle-vortex interactions. Analyzing the trajectories of tracers of different size for a wide range of heat fluxes, the particle-vortex interaction mechanism is investigated with a new toy model.
Sommario

Presentiamo due progetti sulla caratterizzazione della turbolenza classica e quantistica in fluidi criogenici usando particelle formate da gas solidificati.

Il primo progetto presenta i vantaggi e gli svantaggi dell’utilizzo dell’azoto liquido come fluido per esperimenti di fluidodinamica, comparandolo ad altri fluidi. Lo studio è focalizzato sulla visualizzazione di flussi ad altro numero di Reynolds, e presenta una nuova tecnica che utilizza particelle di gas solidificati come traccianti. Questa tecnica permetterebbe di produrre in maniera semplice ed economica traccianti per la visualizzazione di flussi di azoto liquido, usando idrocarburi comuni o anche aria.

La tecnica si è dimostrata fattibile sperimentalmente, producendo particelle di circa un micrometro quando si è iniettata aria, e particelle fino a 500 nanometri di diametro con una miscela di azoto e metano allo stato gassoso. Queste particelle sono state usate con successo come traccianti sia per Particle Image Velocimetry che per Particle Tracking Velocimetry. La dimensione delle particelle è stata stimata usando un modello di scattering Mie che è stato testato con particelle di polistirene di nota grandezza e indice di rifrazione.
Uno studio della selezione dei migliori gas, che riguarda la stima della densità dell’indice di rifrazione di vari idrocarburi, ha mostrato che propilene e propadiene produrrebbero i traccianti più luminosi e fedeli al flusso. È stata condotta anche una analisi sui parametri ottenibili usando azoto liquido come fluido-test per diversi esperimenti. Gli esperimenti considerati includono un flusso turbolento generato tramite una griglia, un flusso in un tubo, un flusso di Taylor-Couette, di von Kármán e di Rayleigh-Bénard. Si discute anche la fattibilità di tali esperimenti e si fa un confronto con altri esperimenti che utilizzano altri fluidi, e che rappresentano lo stato dell’arte per le diverse configurazioni.

Il secondo progetto riguarda lo studio della turbolenza nell’elio superfluído. I vortici quantizzati possono essere visualizzati usando come traccianti particelle di idrogeno solido della dimensione dell’ordine del micron (1). A causa della loro dimensione però, la resistenza fluidodinamica di Stokes non permette alle particelle di rimanere intrappolate sui vortici vicino alla transizione lambda, dove il potenziale di intrappolamento è più debole.

È stata scoperta una nuova tecnica che permette di creare e visualizzare particelle più piccole di un micrometro. Varie stime della dimensione di queste particelle è stata fatta, sia da proprietà ottiche che fluidodinamiche. Essendo più piccole, ma non così piccole da essere influenzate dalle fluttuazioni termiche, le particelle sono meno affette dalla resistenza fluidodinamica. Di conseguenza le particelle restano
intrappolate più vicino alla transizione e su vortici che si muovono più velocemente. L’abilità di creare particelle direttamente nell’elio II permette di visualizzare la dinamica dei vortici a temperature più basse di quanto possibile in precedenza.

Particelle di diversa dimensione sono state usate per studiare il controflusso termico. Per bassi flussi di calore, le particelle possono tracciare il moto della componente normale o tracciare il moto dei vortici quantizzati, quando intrappolate nei loro nuclei. Per alti flussi di calore, l’incrementato numero di interazione vortice-particella e di eventi di scattering ha come conseguenza che le particelle non tracciano né i vortici né la componente normale. Queste osservazioni confermano le ipotesi di Sergeev and Barenghi (2) secondo le quali la discrepanza tra l’esperimento di Paoletti et al. (3) e di Zhang and Van Sciver (4) risiedono nell’esistenza di due regimi di interazione vortice-particella. Analizzando le traiettorie dei traccianti di varie dimensioni, e per un ampio range di flussi di calore, il meccanismo di interazione particella-vortice è studiato tramite un nuovo modello.
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\( a.u. \): arbitrary units
\( B \): fidelity factor
\( B_V \): temperature dependent coefficient in the Vinen equation
\( Ca \): cavitation number
\( CCD \): charge-coupled device
\( C_s \): heat capacity
\( C_D \): drag coefficient
\( d \): diameter
\( DNS \): direct numerical simulation
\( E(k) \): Energy density in wavenumber space
\( EM - CCD \): Electron-Multiplying charge-coupled device
\( ETF \): European Transonic Facility
\( f \): rotation frequency
\( f_\psi \): dimensionless modulus of \( \psi \)
\( Fr \): Froude number
\( g \): acceleration of gravity
\( GP \): Gross-Pitaevskii equation
\( GUI \): graphical user interface
\( h \): Planck’s constant
\( h \): \( h/2\pi \)
\( h_t, h_{\text{fluid}}, h_{\text{walls}} \): heat transfer coefficients
\( HeI \): classical fluid state of \(^4\text{He}\)
\( HeII \): superfluid state of \(^4\text{He}\)
\( I \): scattering intensity
i: imaginary unit
k: wavenumber
$k_B$: Boltzmann’s constant
$k_c$: thermal conductivity
$L$: characteristic length scale
$\mathcal{L}$: integral turbulent scale
$t$: line vortex density
LIA: local induction approximation
$LN_2$: liquid nitrogen
$m$: mass
$n$: index of refraction
$N$: number of particles
NTF: National Transonic Facility
$p$: pressure.
$PIV$: particle image velocimetry
$PTV$: particle tracking velocimetry
$PSL$: polystyrene latex
$Pr$: Prandtl number
$q$: heat flux
$r$: position vector
$R_1$: ratio of test fluid viscosity to water viscosity
$R_2$: parameter that compares the surface tension of a fluid with that of water
$R_3$: ratio between the test fluid gas-liquid density ratio with that of air-water
$Ra$: Rayleigh number
$Re$: Reynolds number
$Re_L$: large scale Reynolds number
$s$: distance coordinate along the vortex filament.
$S$: specific entropy
$St$: Stokes number
$S_i$: Mie scattering functions
$SVP$: saturated vapor pressure
STP: standard temperature and pressure

$t$: time

$T$: temperature

$TI$: state of quantum turbulence in a counterflow for ‘low’ heat fluxes

$TII$: state of quantum turbulence in a counterflow for ‘high’ heat fluxes

$T\lambda$: lambda point for helium-4

$U$: characteristic velocity of the flow

$v$: velocity field

$v'$: root-mean-square velocity fluctuation

$v_n$: velocity field of the normal component

$v_s$: velocity field of the superfluid component

$V_0$: potential strength

$We$: Weber number

$x,y,z$: spatial coordinates

$\alpha$: isobaric thermal expansion coefficient

$\Gamma$: circulation

$\Delta$: finitesimal difference

$\delta$: inter-vortex spacing

$\delta_R$: surface roughness

$\delta_V$: quantized vortex core diameter

$\eta$: Kolmogorov length-scale

$\theta$: angle

$\kappa$: quantum of circulation

$\kappa_d$: thermal diffusivity

$\lambda$: Taylor microscale

$\Lambda$: Darcy friction factor

$\lambda_w$: wavelength

$\mu$: dynamic viscosity

$\mu_c$: chemical potential
\( \nu \): kinematic viscosity
\( \xi \): healing length
\( \pi \): \( \pi \) constant
\( \rho \): mass density
\( \rho_n \): mass density of the normal component
\( \rho_s \): mass density of the superfluid component
\( \sigma \): surface tension
\( \Sigma \): quantum stress
\( \tau \): trapping time
\( \phi \): phase of the wavefunction
\( \Psi \): wave function or order parameter
1

Introduction

Fluid mechanics studies the dynamics of liquids, gases and plasmas. As fluids are ubiquitous in the universe, fluid dynamics plays a role both in the basic understanding of nature and in many engineering applications. Originally this discipline was developed to understand the motion of water and air, to solve practical problems in engineering and earth sciences. For these historical reasons, familiarity in everyday life and ease of use, air and water have been the standard test fluids for experiments. A convenient aspect of fluid mechanics has indeed been how easily one can create interesting and insightful experiments, by taking advantage of dimensional analysis and flow visualization with simple apparatuses. While air and water are certainly easy to use, other fluids have properties that are better for certain applications. In particular, cryogenic fluids have a low kinematic viscosity compared to air and water, and so have been considered useful fluids for studying turbulence in compact experiments. Liquid helium, having the lowest viscosity, was thought the best fluid to use for both basic turbulence research and model testing for matching real world parameters (5, 6, 7, 8). While there
are a few successful examples of classical turbulent experiments using cryogenic helium, not many turbulence experiments use liquid helium as a test fluid. Liquid helium has instead been extensively used to study superfluid turbulence.

We present two different projects that characterize classical and quantum turbulent flows using frozen particles and cryogenic fluids.

1.1 Introduction to turbulence

The etymological origin of the word “turbulent” is the Latin verb turbo, turgare, turbavi, turbatus

*disturb, agitate, throw into confusion*

and according to an English thesaurus, in everyday language, these are the synonyms of the word turbulent:

*agitated, boiling, confused, destructive, disordered, disturbed, fierce, foaming, furious, howling, inclement, moiling, noisy, restless, riotous, roaring, rough, ruffled, stirred up, storming, stormy, swirling, tempestuous, thunderous, tremulous, tumultous, unstable, violent, wild.*

As all these words suggest, a turbulent flow is chaotic and disordered.

The word “turbulence”, turbolenza, appears for the first time in Leonardo Da Vinci’s Codex Atlanticus (Fig. 1.1). In this collection of Leonardo’s notes, there are several drawings of his acute and insightful observations of fluid motion. One of the most famous sketches represents the fine whirls in the water created from a jet coming out of a square aperture (Fig. 1.2), underneath which he wrote:
Figure 1.1: First appearance of the word turbulence, *turbolenza*, and basic questions about turbulent flows:

where the turbulence of water is generated

where the turbulence of water maintains for long

where the turbulence of water comes to rest

c. 1508-9 - Codex Atlanticus, Leonardo Da Vinci

Note: Image adapted from Frisch (9)
Observe the motion of the surface of the water, how it resembles that of hair, which has two motions - one depends on the weight of the hair, the other on the direction of the curls; thus the water forms whirling eddies, one part following the impetus of the chief current, and the other following the incidental motion and return flow.

According to Lumley, the Italian Renaissance genius could have prefigured the turbulence decomposition, proposed by Osborne Reynolds four centuries later (10). In reading the passage where Leonardo writes

... the small eddies are almost numberless, and large things are rotated only by large eddies and not by small ones, and small things are turned by both small eddies and large

it is easy to find similarities with the concept of the self-similar cascade, which was introduced by Lewis F. Richardson with a reference to Jonathan Swift’s critique of the self-referentiality of poets (11). Swift’s famous verses present poets as fleas populated by smaller, lilliputian, fleas; Richardson replaces the infinite cascade of smaller and smaller fleas with a cascade of smaller and smaller eddies (12):

Big whorls have little whorls
That feed on their velocity,
And little whorls have lesser whorls
And so on to viscosity.

Leonardo in his notes poses also some general questions regarding the origin, the development and dissipation of turbulence (Fig. 1.1). After more than five centuries, there is a deep knowledge of the subject but several questions about turbulence do not yet have a complete answer. It is even more striking that
Figure 1.2: Studies of Water passing Obstacles and falling, c. 1508-9 - Codex Atlanticus, Leonardo Da Vinci.
among the sharpest scientists of every era have dedicated time and effort to this problem, now dubbed as the last unsolved problem in classical physics.

To illustrate the ubiquity of fluid turbulence we note its presence in the deep interior of the earth, the mixing in the ocean and currents in the atmosphere, the convection in the sun as well as in interstellar medium and accretion disks. Back on the Earth, among the countless engineering problems, we can mention the design of ships, planes, pipes, industrial facilities and energy production plants.

The charm of the simplicity, practical importance and basic questions that poses the understanding on how fluids flows has attracted not only scientists but also engineers and mathematicians.

1.2 Navier-Stokes equation

One of the biggest contributions to the foundations of the theory of fluid dynamics comes from a big engineering failure of a great mathematician and scientist. It all started in the summer of 1749, when the famous Leonard Euler was invited to Potsdam by Frederick the Great for an apparently pedestrian reason: solving the plumbing problem of a fountain (13). For the Prussian king it was a matter of pride that the fountain in his park at Sanssouci be better and higher than the fountain of Versailles. Euler was called after some catastrophic failures on the first attempts to build the piping system of the fountain. The Swiss mathematician studied the problem from first principle and presented a report with practical advice that in the end was neglected. That work was the prelude to his general formulation of hydrodynamic motion presented in the *Principes généraux du mouvement des fluides* in 1755 where appears for the first time what is now
called the “Euler equation”

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\frac{1}{\rho} \nabla p,$$  \hspace{1cm} (1.1)

where \( \mathbf{v} \) is the fluid velocity, \( p \) is the fluid pressure and where the density \( \rho \) has been assumed constant. For a given pressure, the velocity calculated with the equation is much higher than it is found in any real observation, so the equation was not of much practical use.

While the equation written down by Euler is still actively studied after three centuries, the King was definitely not satisfied, and in a letter to Voltaire thirty years after the beginning of the project, wrote:

*Je voulus faire un jet d’eau en mon Jardin; le Ciclope Euler calcula l’effort des roues, pour faire monter l’eau dans un bassin, d’où elle devait retomber par des canaux, afin de jaillir à Sans-Souci. Mon Moulin a été exécuté géométriquement, et il n’a pu élever une goutte d’eau à Cinquante pas du Bassin. Vanité des Vanités! Vanité de la géométrie.\(^1\)*

An important concept missing in the Euler equation is the effect of viscosity \( \nu \), that was introduced by Claude-Louis Navier in 1827 and by George Gabriel Stokes in 1845 in what is now called the “Navier-Stokes equation”

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = \nu \nabla^2 \mathbf{v} - \frac{1}{\rho} \nabla p$$ \hspace{1cm} (1.2)

\[\nabla \cdot \mathbf{v} = 0\] \hspace{1cm} (1.3)

\(^1\)I wanted to make a jet of water in my Garden; the Cyclop\(^2\) Euler calculated the effort of the wheels for raising the water to a basin, from where it should fall down through canals, in order to form a fountain jet at Sans-Souci. My mill was constructed mathematically, and it could not raise one drop of water to a distance of fifty feet from the basin. Vanity of Vanities! Vanity of mathematics. \(^2\) Euler had lost an eye.
where the momentum equation has been supplemented by the incompressibility condition.

The general consensus is that turbulence is a consequence of the Navier-Stokes equations. However, analytical solutions to even the simplest turbulent flows do not exist and a naive application of this equation to practical problems can lead to absurd results. If you try to estimate the flow velocity of a river balancing the gravity force and the viscous drag you find that is $\sim 10^5 \text{ m/s}$! The solution of this problem is in the ratio of the nonlinear to the viscous term, as suggested by Reynolds. This dimensionless number is now called the Reynolds number.

### 1.3 Reynolds number

Fluid dynamics is the discipline that took the most advantage of dimensional analysis for its effectiveness in reducing the number of parameters necessary to characterize complex systems (14). The tight bond between fluid mechanics and dimensional analysis is testified by the countless number of dimensionless parameters, with names of fluid mechanics pioneers, used to describe different flows. A key dimensionless parameter is the Reynolds number ($Re$), which is defined as

$$Re = \frac{UL}{\nu},$$

where $U$ and $L$ are respectively the characteristic velocity scale and length scale of the flow. The Reynolds number expresses the ratio between the inertial and the viscous forces. It is the only parameter necessary in the dimensionless version of the Navier-Stokes equation, and is used to characterize a flow as laminar or turbulent and even to separate sub-disciplines in fluid mechanics.
We focus on high Reynolds numbers ($Re > 10^6$), which characterize astrophysical and geophysical flows, as well as flows in industrial, aerospace and naval applications.

### 1.4 Turbulence: scaling and cascades

To discuss the historical progress of turbulence research is not the purpose of this work and several writings are available (for a brief review see for example (15)). We present just a few milestones of this complex subject that can be found on most books on turbulence (e.g. (9, 16, 17)).

Richardson, besides introducing the important concept of turbulent cascade, realized that the universal properties of turbulence can be found after removing the large scale motion. Hence, he defined the velocity difference $\delta v(r_1, r_2) = v(r_2)v(r_1)$ across a separation $r_{12} = r_2 - r_1$, and the longitudinal velocity increment $\delta v(r_1, r_2) = v(r_1, r_2) \cdot r/r$. The $n$-th moments of $\delta v(r_1, r_2)$ are called structure functions

$$S_n(r) = \langle \delta v(r_1, r_2) \rangle,$$

where $\langle ... \rangle$ is a certain ensemble average. If the probability density function ($PDF$) of the velocity differences were Gaussian, as are the turbulent absolute velocities, it would be completely determined by the second moment $S_2(r)$. In reality, experiments show that the $PDF$ of velocity differences is highly non-Gaussian, with high order moments providing important information about the distribution.

In 1941 Kolmogorov made a breakthrough in the understanding of turbulence introducing the inertial range and the idea of universality. The idea is that for
a stationary turbulent flow, the energy flux from the large scales $L$, where the energy is introduced into the system, is balanced by the energy dissipation rate of the small viscous scales $\eta$. This energy flux is also known as the energy dissipation rate and is denoted by $\epsilon$. Kolmogorov’s first hypothesis is that the small scales are statistically independent from the large scales, and hence the only relevant parameters for the viscous scales are $\epsilon$ and $\nu$. With these parameters one obtains the following scales of length, time and velocity

$$\eta = (\nu^3/\epsilon)^{1/4} \quad \tau_\eta = (\nu/\epsilon)^{1/2} \quad v_\eta = (\nu \epsilon)^{1/4}$$

which are called the Kolmogorov scales (16). The Reynolds number calculated using these scales is equal to one, $\eta v_\eta / \nu = 1$, showing that the small-scale motion is viscous.

The second similarity hypothesis of Kolmogorov is that in the inertial range, i.e. for $\eta < r < L$, the only relevant parameter is the energy dissipation rate $\epsilon$. This leads to a prediction for the turbulence statistics. In particular, dimensional arguments yield a prediction for the second order structure function, which in Fourier space becomes the five-third law for the spectral density $E$

$$E(k) = c_K (\epsilon)^{2/3} k^{-5/3},$$

where $k$ is the wavenumber and the Kolmogorov constant $c_K$ is a dimensionless parameter approximately constant ($c_K = 0.5 \pm 0.05$ ) (18). This scaling law has become a cornerstone of experimental turbulence studies, and often assumed as a signature of turbulent phenomena even in non-fluid systems. The other important prediction of Kolmogorov’s theory is the four-fifth law for the third
order structure function

\[ S_3(r) = -\frac{4}{5} \epsilon r. \]

No one has yet been able to find a closed-form expression for structure functions of higher order, but Kolmogorov inferred that the higher order moments scale as \( S_n \sim r^{\zeta_n} \) with \( \zeta = n/3 \). Experimental evidence shows consistent deviations from Kolmogorov’s prediction, showing a dependence on the energy input scale \( L \), also known as integral scale. Such breakdown of scale invariance in the inertial range is called anomalous or multifractal scaling.

It is remarkable that the most important results in turbulence theory do not come from the Navier-Stokes equation.

1.5 Turbulence scales and simulations

As there are no analytical solutions, studying turbulence via the Navier-Stokes equation is only possible by numerical simulations. Direct numerical simulations (DNS) solve the Navier-Stokes equations resolving all the scales of motion, with the appropriate initial and boundary conditions for the flow under study. Since the 1970s, when computational power started to be enough for such demanding computations, DNS has been getting more and more important in supplementing the knowledge gained from experiments. Even if the computational power available has dramatically improved over the years, the main technical challenges of DNS remain the memory and computational speed requirements. The computational cost is mostly determined by the resolution required, and hence by the size of the turbulent scales.
To estimate the ratio of the smallest to the largest scales, it is necessary to estimate the turbulent dissipation rate. Under the assumption that the large eddies lose a considerable amount of energy \( v'^2/2 \) within one turnover time \( t_e = \mathcal{L}/v' \), the dissipation rate becomes \( \epsilon = v'^3/\mathcal{L} \), where \( v' \) is the root mean square turbulent velocity fluctuation. Hence

\[
\eta/\mathcal{L} \sim \text{Re}_\mathcal{L}^{-3/4} \quad \tau/t_e \sim \text{Re}_\mathcal{L}^{-1/2},
\]

where \( \text{Re}_\mathcal{L} = v'\mathcal{L}/\nu \) is the large scale Reynolds number. To evaluate the scales it is also common to define the Taylor microscale \( \lambda \) and the Taylor Reynolds number \( \text{Re}_\lambda \)

\[
\lambda = \left( \frac{15 \nu v'^2}{\epsilon} \right)^{1/2} \quad \text{Re}_\lambda = \frac{v' \lambda}{\nu}
\]
even if they do not have a clear physical interpretation (17).

The number of grid points necessary to resolve both the integral scales and the Kolmogorov scales grows then as

\[
N^3 \sim \text{Re}_\mathcal{L}^{9/4} \sim \text{Re}_\lambda^{9/2}.
\]

The biggest DNS simulation so far was run on the Earth Simulator, with a peak performance of 40 Teraflops, using a 4096\(^3\) grid to simulate a flow with \( \text{Re}_\lambda \approx 1200 \) and \( \eta/\mathcal{L} = O(10^{-3}) \) (19). With the same resolution, an exaflop machine (expected by 2018 according to www.top500.org) could handle 32,768\(^3\) nodes and thus resolve \( \text{Re}_\lambda \approx 4000 \). However, there are still open questions whether scales smaller than \( \eta \) are relevant at high Reynolds numbers and whether resolution requirements should be more stringent. Theory suggests that the smallest dissi-
pative scale is $\eta_s/\mathcal{L} = Re_\mathcal{L}^{-1}$ (20), which would imply that $N^3 \sim Re^3$. In this case an exaflop machine could resolve all the relevant scales only up to $Re_\lambda \simeq 1000$, and resolving scales for $Re_\lambda > 10,000$ could be impossible with expected improvements of computational technologies. While these observations do not imply that computation will not play a big role in the understanding of turbulence in the future, they suggest that high Reynolds number experiments are likely to continue to play a major role in the field.
Superfluid helium and quantum turbulence

2.1 Superfluid helium

The most abundant isotope of helium, helium-4, was liquefied for the first time by Heike K. Onnes in Leiden, in the Netherlands, on July, 10 1908 (21). The liquid was created by cooling the helium gas below its boiling point of 4.2 K, at atmospheric pressure (Fig. 2.1). At this temperature and pressure the helium is called HeI, and is a classical fluid which can be described by the Navier-Stokes equation. At the temperature $T_\lambda = 2.1768 \, K$ and at the saturated vapor pressure of $P_{SVP} = 37.817 \, Torr^1$, a second order phase transition occurs. This transition is called the lambda transition because the specific heat has a spike at the transition temperature reminiscent of the Greek letter $\lambda$ (Fig. 2.2). Below this transition the helium is called HeII and its most remarkable property is superfluidity, i.e. its

---

1$T_\lambda = 2.172 \, K$ and $P_{SVP} = 37.80 \, Torr$ are the traditional values for the lambda transition but, according to the EPT-76 (22) and the ITS-90 (23) temperature scales, the transition occurs at $T_\lambda = 2.1768 \, K$ and $P_{SVP} = 37.817 \, Torr$. As in (24) we adopt such value for our studies.
ability to flow without friction. The term superfluid was introduced by Kapitza in a Nature article published on January 8, 1938 (25), saying

*by analogy with superconductors, ... the helium below the \( \lambda \)-point enters a special state which might be called superfluid.*

In the same issue of Nature, on the next page, an independent work by Allen and Misener (26) reported the properties of liquid helium flows. The submission of these articles in December 1937 is usually recognized as the discovery of superfluidity (27), for which Kapitza was awarded the Noble prize in 1978. However, the special hydrodynamic properties of \( \text{HeII} \) had been reported for the first time by Keesom in 1930 as an experimental irritation because of its capability of leaking through the smallest holes (21). The analogy with Bose-Einstein condensation
Figure 2.2: Plot of the heat capacity as a function of temperature for helium-4 (30). The shape of the curve is reminiscent of the greek letter ‘λ’.

as a macroscopic manifestation of quantum mechanics was given by London in a paper published on April 9, 1938 (28). The analogy with superconductivity was later explained with the Bardeen-Cooper-Schrieffer (BCS) theory of superconductivity in 1957 (29). Another unusual feature of the superfluid phase is the presence of vortices with quantized circulation as predicted by Lars Onsanger in 1949 (see section 2.4).
### 2.2 Two-fluid model

On April 28, 1938, Landau was arrested by the KGB\(^1\) in Moscow during the *Great Purge*. He was released the next year with the help of Kapitza, who wrote a letter to Prime Minister Molotov saying that he had just made a discovery “*in the most puzzling field of the modern physics*” and none except Landau could explain it \((31)\). Indeed, in 1941 Landau did, with the development of the two fluid model \((32)\) for which he was awarded a Nobel prize in 1962. The theory, based on elementary excitations of the system, extended to a new level the previous work by Tisza \((33)\) and London \((34)\). This model describes the *HeII* as a mixture of two interpenetrating fluid components, the normal component and the superfluid component. The superfluid is related to the quantum ground state, and, having zero viscosity and zero entropy \(S\), is similar (but with important distinctions) to a classical inviscid Euler fluid. The normal fluid consists of thermal excitations, and, having nonzero viscosity and entropy, is similar to a classical, viscous Navier-Stokes fluid. The total density of helium II is assumed to be the sum of the superfluid and normal fluid densities,

\[
\rho = \rho_n + \rho_s, \tag{2.1}
\]

where \(\rho_n\) is the normal fluid density and \(\rho_s\) the density of the superfluid component. While the total density is approximately independent of temperature, the relative fractions of normal fluid \(\rho_n/\rho\) and superfluid \(\rho_s/\rho\) are strong functions of

\(^1\)Komitet gosudarstvennoy bezopasnosti / Committee for State Security
the temperature $T$, as shown in Fig. 2.3. It is also assumed that

$$\rho \mathbf{v} = \rho_n \mathbf{v}_n + \rho_s \mathbf{v}_s. \quad (2.2)$$

where $\mathbf{v}_n$ and $\mathbf{v}_s$ are respectively the velocities associated to the flow of the normal and superfluid components.

The presence of the two components have been verified for the first time in 1946 by Andronikashvili (36) using a torsional oscillator immersed in liquid helium. A similar experiment at the molecular scale (37) showed that as few as seven helium atoms are enough to display an inviscid flow (38).
2.3 Gross-Pitaevskii equation

Another model used to describe the uncommon features of the superfluid is based on the microscopic description of superfluidity, as a BEC. At zero temperature the particles of a Bose gas collapse into a single ground state and thus can be described by a single complex order parameter $\Psi = \Psi(x, t)$. The evolution of this complex wavefunction, or order parameter, for weakly interacting Bose gases is given by the the Gross-Pitaevskii equation (GP), also known as the nonlinear Schrödinger equation:

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \Psi + (V_0|\Psi|^2 - \mu_c)\Psi,$$

where $m$ is the mass of the helium atom, $\mu_c$ is the chemical potential, $\hbar$ is Planck’s constant and $\hbar = \hbar/(2\pi)$. The presence of the nonlinear term due to the short range repulsive potential $V_0$ between bosons distinguishes this equation from the linear Schrödinger equation.

Even if the superfluid helium is a strongly interacting quantum fluid with long range quantum order, describing it with the GP equation provides a useful insight into its basic features, but only as a qualitative model.

From the GP it is possible to obtain a fluid dynamical interpretation via the Madelung transformation $\Psi = f_\psi e^{i\Phi}$, where $f_\psi$ and $\Phi$ are the amplitude and phase of $\Psi$ respectively. Substituting into the GP gives a classical continuity equation

$$\frac{\partial \rho_s}{\partial t} + \nabla \cdot (\rho_s \mathbf{v}_s) = 0,$$

(2.4)
and a (quasi) Euler equation

$$\rho_s \frac{\partial v_s}{\partial t} + \rho_s (v_s \cdot \nabla) v_s = -\nabla p + \nabla \cdot \Sigma,$$  \hspace{1cm} (2.5)

where the superfluid density is defined as $\rho_s = mf^2 \psi$ and the superfluid velocity $v_s = \kappa \nabla \Phi$, and where we have introduced the quantum of circulation $\kappa = h/m = 9.97 \times 10^{-8} \text{m}^2/\text{s}$. The pressure $p$ and the quantum stress $\Sigma$ are defined as

$$p = \frac{V_0 \rho_s^2}{2m^2}, \quad \Sigma_{ij} = \frac{\kappa^2 \rho_s}{4} \frac{\partial^2 \ln \rho_s}{\partial x_i \partial x_j}. \hspace{1cm} (2.6)$$

Note that aside from the quantum stress term, the equation is the same as the Euler equation.

### 2.4 Quantized vortices

From the definition of the velocity field $v_s$, it follows that the superfluid flow is described by an irrotational field. This has the consequence that the vorticity, which in a normal fluid can be of any size and strength, is constrained to be on topological defects, the quantized vortices.

Moving around the vortex core along a closed path $C$, the phase of the complex order parameter changes from 0 to a multiple of $2\pi$. If you calculate the circulation $\Gamma$ around a vortex line you find that

$$\Gamma = \oint_C v_s \cdot d\ell = l\kappa$$

In general $l \in \mathbb{N}$, but only vortices with $l = 1$ are stable (30). The velocity of the
flow around the vortex in cylindrical coordinates \((r, \phi, z)\)

\[
v_{s\phi} = \frac{\kappa}{2\pi r}
\]

is the same as that of an ideal line vortex ('irrotational' or 'potential' vortex) in a classical Euler fluid.

The topological nature of these phase defects of the order parameter can be seen also from the superfluid density. The superfluid density, proportional to the amplitude of the order parameter, drops from its bulk value away from the vortex to zero on the axis of the vortex over a characteristic distance of the order of \(1 \text{ nm}\), depending on the temperature. This distance, of the order of the superfluid healing length \(\xi = \hbar/\sqrt{m\mu_c}\), is called the vortex core diameter \(\delta_V\).\(^1\) This hollow core makes the superfluid multiply connected. This is fundamentally different than an Euler fluid, where the constant fluid density leads to a singularity on the axis of the vortices. Moreover the classical Kelvin’s theorem does not hold where the density becomes zero, allowing for a change in circulation and a change in topology while still conserving the energy. Hence the presence of the quantum stress term allows the non-dissipative reconnection of quantized vortices, described in more detail in section 2.6. Still, it is important to note that the ratio between the quantum stress and the pressure term is \(\hbar^2/(m\mu_c L^2)\), where \(L\) is the characteristic length scale of the flow. Hence the quantum stress is important if \(L \sim \xi\), i.e. at scales of the core size.

An evolving disordered set of quantized vortices is called *quantum turbulence*.

---

\(^1\)The size of the vortex core should not be confused with the size of the vortex, in fact the vortex extends to infinity. As the velocity profile is proportional to \(1/r\) it is impossible to define any characteristic size: the velocity profile is scale-free.
and can be easily generated experimentally by driving a turbulent flow in the HeII. As found in experimental and numerical investigations, superfluid turbulence features a disordered tangle of vortices. The higher the turbulence the denser the tangle. To characterize how dense the tangle is we can define two parameters: the line vortex density $\ell$, which is the total vortex length per unit of volume, and the inter-vortex spacing $\delta = 1/\sqrt{\ell}$, the average distance between vortices. At scales bigger than the inter-vortex spacing, where the quantum stress becomes negligible, the system becomes similar to an Euler fluid.

\[ v_s = \frac{\Gamma}{2\pi r} \]
\[ \Gamma = l\kappa \]
\[ l \in \mathbb{N} \]

**Figure 2.4:** Sketch of a quantized vortex.

### 2.5 Vortex filament model

Because of the analogy with an Eulerian fluid, the quantized vortices have been modeled using classical mechanics, the so called ‘vortex-filament model’. This model, developed by Schwartz (39), represents a quantized vortex as a linear filament with a direction given by its vorticity, that can be parametrized by the curve $s = s(\xi, t)$. This allows one to compute the fluid velocity $\mathbf{v}$ due to a vortex
at any point \( \mathbf{r} \) using the Biot-Savart law

\[
\mathbf{v}(\mathbf{r}) = \frac{\kappa}{4\pi} \int_L \frac{(\mathbf{s}_1 - \mathbf{r}) \times d\mathbf{s}_1}{|\mathbf{s}_1 - \mathbf{r}|^3}
\]

where the integral is taken along the filament. This integral diverges for \( r \) approaching \( s \), so to control the divergence one can split the velocity of the filament at the point \( s \) into a local and a non-local contribution. The local contribution results from the section of the line where the vortex can be approximated by circle segments and the nonlocal term represents the contribution from the rest of the vortex line. The localized induction approximation (LIA), used in several works (40), neglects the non-local term, which is often smaller than the local one. While this approximation is much less expensive computationally, it may not always be justified and in some cases may lead to different results than by using the full Biot-Savart law (41). However, simulating the dynamics of one or multiple vortices even using the full Biot-Savart model is much less computationally demanding than using the GP. The GP indeed requires that the scales of the vortex core are resolved, which is often impractical for systems involving more than a few vortices.

### 2.6 Vortex reconnections

The reconnection of vortices is a process which involves a change in the topology, in which two line vortices exchange ends and separate. This phenomenon had been studied in other systems with analogous line defects such as vortices in Bose-Einstein condensates, liquid crystals defects, vortices in superconductors, cosmic strings and, as previously mentioned, classical vortices. The reconnection
of quantized vortices in superfluid helium has been predicted by Feynman in 1955 \(42\), obtained numerically using the GP by Koplik and Levine \(43\) and just recently experimentally observed (see Fig. 2.5) and characterized by Bewley \textit{et al.} \(44\) and Paoletti \textit{et al.} \(45\). The ability to change the topology of the quantized vortices through reconnection is believed to play an important role in the decay of the vortex tangle, and hence in the understanding of quantum turbulence.

![Figure 2.5: Sketch of the reconnection of two quantized vortices. The sequence of images is taken from Paoletti \textit{et al.} \(46\). Each frame is separated by 25 ms.](image)

### 2.7 Quantum turbulence

Turbulence in a superfluid consists of the evolution of a the tangle of quantized vortices \(47, 48\). The discrete nature of the vorticity and the absence of viscosity distinguish quantum turbulence from classical turbulence. Even with such
remarkable differences, quantum turbulence shows a surprising set of similarities with its classical counterpart. In particular it has been reported that it shares with classical turbulence the same Kolmogorov energy spectrum (49), the same pressure drops along pipes (50) and drag on a sphere(51), and very recently the four-fifth law (52). These results come from experiments on length scales considerably bigger that the inter-vortex spacing. A clear difference between classical and quantum turbulence has been reported by Paoletti et al. (3), by measuring the velocity statistics of decaying quantum turbulence. They reported non-Gaussian statistics of the velocity PDF, with $1/v^3$ power tails, in contrast with classical turbulence. These power-law statistics are due to the singular nature of the vortices and it has been reported to disappear for scales bigger than the inter-vortex distance, where the Gaussianity results from the central limit theorem (52, 53).

Another interesting part of the turbulence in superfluid helium is the interplay between the turbulent vortices of the superfluid component and the turbulence of the normal component. While the normal component can be neglected at very low temperatures (often assumed $T < 0.6 K$), it has an important role in the high temperature regime ($0.6 < T < T\lambda$). The dissipation mechanism of the HeII turbulence in the two regimes may be different (47). It has been shown that the vorticities of the normal and superfluid components at the large scales are locked together by mutual coupling (54). Moreover it has been hypothesized by Melotte and Barenghi (55) that the transition to turbulence of the normal component would trigger a different superfluid turbulence regime, which has been observed experimentally (56). These two regimes are called respectively $TI$ and $TII$. Recent experiments by Guo et al. (57) in a counterflow channel seem to confirm such hypothesis.
3 Experimental setup

In this chapter we present the experimental setup and techniques used both in the study of the particles in liquid nitrogen for classical turbulence experiments and the study of the quantized vortex dynamics. The experimental apparatus has been modified and adapted from the one used by Paoletti (46) and Gaff (58), and some parts are inherited from a previous apparatus used by Bewley (59).

3.1 Overview

The cryogenic liquid, either liquid helium or liquid nitrogen, is held in an optical cryostat, which is accessed by an imaging system made of a laser-sheet and a CCD camera. The flow is visualized by tracer particles, which are created in situ by the injection of a gaseous mixture. The injection mechanism is described in section 3.5 while the details about the frozen tracers are given in the Chapter 4. The resulting movies are analyzed with a particle tracking algorithm described in section 3.7. The apparatus, including the instrumentation for control and diagnostics is on a rotating platform. A cell with a heater at the bottom, described in section 3.3,
can be inserted in the test section of the cryostat for driving a heat flux in the fluid. The procedures for running the apparatus with liquid helium are reported in Appendix D. The essential components of the apparatus are reported in the diagram in Fig. 3.1, while Fig. 3.2 shows the visualization setup.
3.2 Cryostat

The cryostat used for the experiments is the Optistat SXM from Oxford Instruments (Fig. 3.3).

The test section of the cryostat has a cylindrical shape with a diameter of

![Visualization setup for the study of quantized vortices. In the drawing are represented the injection system to create the particles, the cryostat, the laser-sheet and the camera. Note that the laser-sheet can be orientated in the vertical or horizontal direction to image from the side windows or from the bottom window of the cryostat. The images are then analyzed with a particle tracking algorithm.](image-url)
Figure 3.3: Oxford Instruments Optistat SXM
49 mm and 645.5 mm long, and can be filled by 1.2 liters of liquid. It is designed to work from 300 K and can reach a temperature as low as 1.6 K by evaporative cooling with a pumping system of 25 m$^3$/hr. The helium reservoir has a capacity of 4 liters and is connected to the test section via a needle valve. The thermal insulation is provided by a vacuum jacket which surrounds the other components of the cryostat. The vacuum in the jacket is guaranteed by the combination of a turbo-molecular pump backed by a roughing pump before any liquid helium is in the reservoir, and by cryo-pumping thereafter. An f/2.2 optical access is provided at 4 radial location and 1 axial location at the bottom of the cryostat by a set of windows. Each of the five sets of windows consist of a window held in place by screws and indium seals on the test section, a window kept in thermal equilibrium by apiezon-N grease with the copper radiation shield, and the windows on the vacuum jacket.

### 3.3 Counterflow cell

The counterflow channel consists of a 2 cm $\times$ 2 cm $\times$ 10 cm rectangular cell made of microscope slides attached with vacuum grease. At the closed bottom a heater to excite the counterflow is made of a Nichrome wire (22 AWG) that is laid in a serpentine pattern in the grooves of the heater plate and held in place using stycast (46).
Figure 3.4: Laser-sheet setup and its characteristics. The two images of the gaussian profile and inhomogeneity in the x-y plane the intensity have been artificially colored inverted and artificially colored in green - the darkest regions represent the brightest intensity.

3.4 Imaging system

3.4.1 Optics and laser-sheet

To visualize the tracers we illuminate them with a laser-sheet generated by a laser and two lenses. The characteristics of the setup are summarized in Fig. 3.4.

The laser source is a green ($\lambda = 532 \text{ nm}$) laser pointer with a power output of 4 $mW$, modified to run with a power supply. To create the sheet, the laser beam is first expanded in the $y$ direction with a cylindrical diverging lens, and then is both collimated in the $y$ direction and focused in the $z$ direction by a spherical
biconvex lens. The collimation in the $y$ direction prevents further expansion in the $y$ direction, while the focusing in $z$ direction guarantees a thin laser-sheet that has measured $1/e^2$ width of $175 \mu m$, thinner than the depth of field of the camera lens. An aperture cuts the laser-sheet to roughly 1 cm to avoid unnecessary light into the test section that would cause reflections and heat the system, and to fully illuminate the $8.2mm \times 8.2mm$ field of view of the camera lens. The measured characteristics of the laser sheet are reported in Appendix C. We note that the power of the laser-sheet is limited to few milliwatts to minimize the perturbation of the flow when using liquid helium caused by the heat generated on the windows (46). In order to detect small particles illuminated by a dim laser we use a low-light camera, described in the next section.

3.4.2 Camera

The imaging setup consists of a Princeton Instruments ProEM camera and a f/2.8 Micro-Nikkor 105 mm lens with 1:1 magnification (see Fig. 3.5). The camera is set on a mount that we built, which can be adjusted with micrometer precision in the $y$ and $z$ directions. Thanks to the high quantum efficiency and very low noise, the camera’s cooled electron-multiplying CCD (EM-CCD) can detect signals of few photons. The ProEM can record up to 30 frames per seconds (fps) at a full frame resolution of $512 \times 512$ pixels (pixel size = $16 \mu m$), and up to several hundred fps sub-framing. The maximum dynamic range is 16 bit which is translated on a greyscale image in 1-65536 Analog-to-digital units (ADU). More details on the camera and on how an EM-CCD works are given in Appendix A.
Figure 3.5: Drawing of the lens and the camera used for the experiments along with some of their most important characteristics, such its extremely low noise that can be effectively reduced to less that one $e^-$ rms using the on-chip multiplication gain enabled. In the plot of the Quantum Efficiency are evidenced the wavelengths of the laser diodes that we tested (405 445 532 650 nm).
3.4.3 Image acquisition software

To control the ProEM camera image acquisition we use WinView, a commercial software developed by Princeton Instruments, and MicroManager, an open source software package for controlling automated microscopes that works with ImageJ\(^1\). We use WinView to take preliminary data, testing and when a specific mode of the camera is needed, while we use Micro-Manager when long continuous data acquisitions are necessary. Micro-Manager can indeed take a continuous series of images, saving the images directly the hard disk. In contrast, WinView can record a sequence of frames limited by the RAM memory available. Moreover MM can be easily controlled via Matlab, allowing to be integrated with the control system / data acquisition system (see section 3.6.2) of the rest of the experiment.

3.5 Injection apparatus

In this section we describe the apparatus to inject a gaseous mixture into the cryogenic fluid to create the particles, while in Chapter 4 we have a in depth description of the characteristics of the particles and the techniques.

The gaseous mixture consists of a seeding gas that freezes when injected into the cryogenic liquid, diluted in either nitrogen or helium gas depending on our test fluid. The dilution of the seeding gas is a key factor in the creation of small particles as will be evidenced in Chapter 4. The mixture ratio, starting from one part in fifty to one part in several thousands, can be created in a controlled way in a small pressurized tank. The tank is set, using a regulator, to a pressure of

\(^{1}\text{ImageJ is a Java-based open source image processing program developed at the National Institutes of Health}
about 20 psi (≈ 1.4 bar) and is connected through a 0.125 in copper tube to an injection tube inserted in the test section of the cryostat. The 0.25 in stainless steel injection tube is just 0.01 in thick to reduce thermal conduction between the room temperature air and the cryogenic fluid in which is immersed. The end of the pipe is right above the test inner cell and roughly 2 cm above the field of view. The flow of the mixtures is let into the cryostat by a solenoid valve controlled through the Matlab interface via a National Instruments data acquisition control board (Ni-DAQ). The amount of gas injected can be controlled by the time for which the valve is opened, which can be as low as 10⁻² s, by the differential pressure between the test section and the tank, and by regulating the flow using two needle valves positioned along the copper tubing. Note that with the installation of new valves is now possible to flush the line before any injection, to get rid of possible unwanted gases that could have leaked in the line and that would substantially contaminate highly diluted mixtures.

3.6 Controls and diagnostics

3.6.1 Temperature control

The temperature of the liquid helium in the test section and in the bath is controlled via the evaporative cooling. Pumping is possible independently in the two sections, but usually the temperature is kept at the same value. The temperature is measured in the test section with a sensor on the wall by the Oxford Instruments ITC503 temperature sensor, and with a LakeShore CX-1050 Cer-nox Resistor placed above the counterflow cell using a lock-in amplifier. The temperature is regulated by modulating the pumping rate via a pneumatic valve
The valve is closed for pressures less than 20 psi and completely open for a pressure greater than 60 psi. The pressure is controlled by a current-to-pressure transducer. The transducer has been set up such that an applied voltage of 1 V or less the valve is closed, while for a 5 V or more is closed. The voltage, and hence the aperture of the valve, is controlled via the National Instruments data acquisition control board.

Temperature stability is guaranteed by a proportional-integral-derivative (PID) control system implemented in Matlab, and tuned according to the modified Ziegler-Nichols rules (60). We note that the maximum temperature stability is obtained through a fine manual tuning of the pumping rate, which can avoid the temperature oscillations due to the PID control of the valve. We automated also the cooling process of the liquid helium from 4.2 K to 2.2 K, to minimize helium consumption.

The temperature can also be varied using two sets of heaters. The heater at the bottom of the cell, described in section 3.3, is used to drive the counterflow during the helium experiment. The heater on the test section wall, part of the original configuration of the cryostat, can dissipate up to 80 W at the maximum voltage of 40 V.

### 3.6.2 User interface and data logging

The the instrumentation control and the data logging of the experiment, that we rebuilt completely, are controlled though a graphical user interface (GUI) (see Fig. 3.6). The effort has been toward automating tasks to reduce possible errors and logging every parameter to facilitate debugging. In particular are measured and recorded both the temperatures in the test section, the voltage that control
Figure 3.6: Image of the experiment control interface developed in Matlab. The interface displays the temperatures in the test section and inside the cell, the helium level, the pressure in the vacuum jacket, the voltage applied to the heater and the size/scattered intensity distribution of the particle in the field of view. Moreover it allows to control the injection via the solenoid valves, the pumping rate and the heaters.
the pneumatic valve, the helium level in the bath, the voltage applied to both the heaters, the pressure in the vacuum jacket, the time and length of injections. The control dashboard shows in real time the temperature trace of the sensor on the test section wall and inside the cell. The GUI can also show the distribution of intensity (in ADU) scattered by each particle in the field of view to adapt the camera setting to maximize signal to noise and/or dynamic range. Instead that displaying the ADUs, the GUI can display the size distribution of the particle, but as explained later, this distribution should be interpreted with care. This has been possible interfacing the camera to the Matlab via MicroManager and integrating in the GUI the code for the size estimate described in section 4.5 and Appendix B, and a particle detection algorithm.

3.7 Particle tracking velocimetry

Particle Tracking Velocimetry (PTV) (61) is a method to measure the fluid velocity based on the images of the tracers. Compared to the more widely used Particle Image Velocimetry method (PIV) (62) which gives an Eulerian velocity field of the flow, the PTV method is a Lagrangian method. Instead of computing the velocity field for a region of the flow, it calculates the velocity of each particle that can be detected. Following the trajectories of each individual particle the PTV algorithm is suitable for the erratic motion of particles trapped on vortices and in general when the velocity field is not smooth (see Fig. 3.7). Moreover, by providing the complete trajectory of every particle, it allows to compute higher order derivatives using the information of several frames and hence a more accurate computation of accelerations.
The PTV technique is based on two major parts: the feature tracking part, and the trajectory linking. The feature tracking part finds the position of the particle at each frame. The trajectory linking algorithm identify the locations in different frames that correspond to the same particle and they link them in a trajectory. This is done through minimizing a cost function for the possible associations.

Over the course of the study we used different PTV algorithms and different implementations with different accuracy and performance. We have used both the IDL and Matlab implementation of the original algorithm by Crooker and Grier (61). We then used an algorithm that allows the detection of lower signal-to-noise particles developed by the MOSAIC group (63): we used its Java implementation as Image J plugin for its convenience and in its C implementation for better performance. The latest code we used is a C++ highly parallelized algorithm developed and implemented by Ouellette (64) which is much faster than all the aforementioned algorithms which has been optimized to detect low signal to noise particles using the feature tracking procedure adopted in (63). The fully vectorized Matlab code we developed, allows for data visualization, exploration and analysis of the tracks.

![Figure 3.7](image)

**Figure 3.7:** Example of particle trajectory obtained using a PTV algorithm. The trajectory represents a particle in a counterflow. It is clear how the PTV can track sudden changes in directions and non-smooth velocity fields.
4

Particles for flow visualization and velocimetry

In this chapter we review and characterize the frozen tracers for visualization and velocimetry in liquid helium and liquid nitrogen.

4.1 Tracer particle

Compared to other velocimetry techniques like hot wires or pitot tube probes, the PIV and PTV techniques are directly based on the two fundamental dimensions of the velocity: space and time. However, velocimetry based on imaging techniques are indirect measures of the flow because they measure the velocity of the tracers and not the velocity of the fluid. For this reason, the physical characteristics of the particles are very important so as to not have big differences between the measured velocity of the particle and the velocity of the flow. The best tracers are of course those that more faithfully follow the velocity of the fluid. As a note of caution it is important to realize that even a molecule of the fluid is far from
being a perfect tracer. Indeed the diffusion rate of the molecule almost prevents its use in a measure of the small scales of turbulence. This apparent paradox is due to the difference between the motion of the molecules and the motion of the fluid, for which the continuity hypothesis is assumed (65).

4.2 Frozen particles as fluid tracers

Tracers for visualization and velocimetry of air and water are very different due to the different properties of the fluid. Because of the many practical and industrial applications, there are standard techniques and many publications on how to generate tracers for both (e.g. see (62)). Common tracers in water are oils droplets, oxygen bubbles, or solid particles such as hollow glass spheres, metal flakes or plastic particles usually ranging from a few to hundreds of microns. Tracers for gas flows require smaller particles, usually from hundreds nanometers to several microns, because of the low density of the fluid. The options are broad, with companies specialized in making the best tracers for different specific applications. The situation for cryogenic liquids is very different. The low temperature environment, the relatively scarce literature on the subject (reviewed in section 4.6), makes the search for good fluid tracers much more challenging. In particular, finding faithful tracers in liquid helium is not a simple task (59). A successful technique developed by Bewley (66) in liquid helium consists in creating micron-sized hydrogen particles. We have improved this technique and extended it to create frozen tracers in liquid nitrogen. The technique is described in section 4.6.
4.3 Dynamic of a particle in a fluid

The forces acting on a particle in an unsteady flow (67) and the study of its fidelity as a tracer have been studied in detail (68). We use a simplified approach described in (69). We define the fidelity based on the Stokes and the Froude number

\[
St = \frac{\tau_p}{\tau_f} \quad Fr = \frac{u_p}{u_f}
\]

respectively as the ratio of the particle response time \(\tau_p\) to the timescale of the flow \(\tau_f\) and the ratio of the particle settling velocity \(u_p\) to the characteristic velocity of the flow \(u_f\). The Stokes number quantifies the inertia of the particle while the Froude number the settling. If \(St, Fr \simeq 0.3 - 0.5\) measurements of the integral scales are accurate within \(5 - 10\%\), so we define that a tracer is faithful if \(St < 0.05\) and \(Fr < 0.05\). We then can define the fidelity factor \(B\) as the ratio between the maximum and minimum Reynolds number flow for which a particle can be considered a faithful tracer (66):

\[
B = \frac{Re_{\text{max}}}{Re_{\text{min}}} = \frac{\rho_f^2 \nu^{8/3}}{\rho_p^{2/3} (\rho_p - \rho_f)^{4/3} g^{4/3} d^{-4}}.
\]

The fidelity of the particles is a strong function of the particle size. Hence smaller particles are better fluid tracers. On the other hand, smaller particles scatter less light.

4.4 Light scattering by a particle

The amount of light scattered by a particle is an important factor in the choice of the tracers, as it determines how well imaging can see it. In particular, the
scattering of light limits the time resolution of the velocimetry: dim particles need long exposure times. In the next section we discuss the light scattered by a spherical particle according to the Mie scattering model (70).

In general, when the light wavelength is similar to the particle diameter, light interacts with the particle over a cross-sectional area larger than the geometric cross section of the particle. The Mie calculation provides this scattering cross section and the Mie-Scattering functions $S_1$ and $S_2$ allow one to compute the scattering intensity $I$ at a distance $R$ and angle $\theta$ in this general case

$$\frac{I_i}{I_0} = \frac{S_j(\theta, \lambda, w, d, n)^2}{k^2 R^2},$$

where $I_0$ is the intensity of the incident light, $k = 2\pi/\lambda$ is the wave number of the light, and where $j$ represents the polarization, $j = 1$ orthogonally polarized and $j = 2$ parallel polarized. If the incident radiation is unpolarized then the scattered radiation exhibits partial polarization, with the degree of polarization depending on the angle of observation.

In the limit of $d_p << \lambda_w$

$$S_2(\theta) = S_1(\theta) \cos(\theta) = -\frac{i k^2 d^3}{8} \frac{n^2 - 1}{n^2 + 2},$$

and the Mie theory becomes the Rayleigh theory for which it is possible to explicitly write the scattering intensity

$$\frac{I}{I_0} = \frac{1 + \cos^2 \theta}{2R^2} \left( \frac{2\pi}{\lambda_w} \right)^4 \left( \frac{n^2 - 1}{n^2 + 2} \right)^2 \left( \frac{d}{2} \right)^6$$
and the scattering cross section of the particle

\[ \sigma = \frac{2\pi^5}{3} \frac{d^6}{\lambda_w^4} \left( \frac{n^2 - 1}{n^2 + 2} \right)^2. \]

The large power on \( d \) and \( n \) shows the importance of the particle size and the index of refraction on the scattering intensity.

### 4.5 Size estimates

As we already stressed, an important parameter in evaluating the fidelity of a tracer is its size. In Chapter 7 it will be shown that size is even more important for the particles that track the velocity of quantized vortices. Measuring the particle size is important in countless applications and there are several reviews on the subject (71, 72). Unfortunately most of the standard techniques are not suitable with our current apparatus for various reasons. The configuration is particularly challenging: the particles are confined in a space relatively far from the limited optical access, they are moving in the fluid, and their molecular motion is low because of the low temperature environment. We present a terminal velocity and optical size estimates that we have developed and used for particles both in liquid helium and liquid nitrogen. We tested this estimate, revealed its limitations and proposed possible improvements. The estimates for the particles in liquid helium have been compared and cross-checked with other estimates over possible.

#### 4.5.1 Terminal velocity estimate

Assuming a particle is spherical, it is possible to calculate the diameter based on its terminal velocity according to the following relation
Figure 4.1: Terminal velocity of solid nitrogen particles of different sizes in liquid helium as a function of temperature

\[ D = C_D \frac{3 \rho_f U_T^2}{4g |\rho_p - \rho_f|} \]

and where \( C_D \) is the drag coefficient. The drag coefficient of a spherical particle can be calculated for example with the relations:

\[ C_D1 = \frac{24}{Re} \left[ 1 + 0.15 Re^{0.687} \right], \]
which for $\text{Re}<800$ gives deviations of $+5$ to $-4\%$,

$$C_D^2 = 2 + 24/\text{Re},$$

which for $\text{Re}<10$ gives deviations of $-3$ to $-5\%$ and

$$C_D^3 = 0.28 + 6/\text{Re}^{1/2} + 21/\text{Re},$$

which for $0.1<\text{Re}<4000$ gives deviations of $+7$ to $-6\%$ \cite{73}. For our estimates we took the average of these values and, with a recursive function, we estimated the particle diameter. For the estimates in liquid helium, we estimated the viscosity and density of liquid helium interpolating the values reported in \cite{74}. In Fig. 4.1 are reported the terminal velocity of solid nitrogen particles of different sizes in liquid helium as function of the temperature. We note that these estimates are meaningful for relatively big particles and/or particles with a substantial density mismatch.

### 4.5.2 Optical size estimate

We developed an optical estimate to calculate particle size from the light scattered: knowing the intensity of the laser-sheet and the particle index of refraction, we assume their size to be that of the biggest particle which, in the center of the laser-sheet, would scatter a given amount of light, based on the Mie scattering model.

The details on how we calculated the scattering intensity and on how we related the ADUs for each particle to the incident light are given in Appendix B.
To verify the reliability of the optical estimate, we tested it in water with spherical mono-disperse polystyrene latex (PSL) particles with known index of refraction ($1.59 - 1.6$). We used the particles of 50 nm, 200 nm, 500 nm and 1 µm diameter from the Polybead® Sampler Kit III. The particles of 50 nm could be seen just as a mist but the 200 nm particles could instead be seen individually. Note that the index of refraction mismatch of the PSL in water $n_m = 1.6/1.33 \simeq 1.20$ is lower than the index of refraction mismatch of solid nitrogen particles ($n = 1.2646$) in liquid helium ($n_{He} = 1.0298$) and of any hydrocarbon in liquid nitrogen, except for methane. This confirm that 200 nm frozen particles are possible to be seen with our setup.

**Figure 4.2:** (A) distribution of luminosity on the CCD for each PSL particle in the field of view (B) distribution of particle size estimated with the Mie model from the luminosity distribution. (C) Distribution of size, estimated using a monotonic 4th order polynomial to approximate the Mie curve. The cases considered are for 200 nm (A1, B1, C), 500 nm (A2, B2) and 1 µm (A3, B3).
The scattered intensity distribution observed in Fig. 4.2 coincides with the expected distribution of mono-disperse particles in a gaussian laser-sheet with a certain (measured) non-uniform distribution, see Fig. 4.3. Fig. 4.2 shows also the size estimates obtained, where the gaps and peaks are an artifact of oscillations in the Mie model.

These estimates cannot give any information on the size of an individual

\[ \text{Figure 4.3: In this figure is generated using synthetic data and represented the distribution of the light scattered by a collection mono-disperse particles in a laser-sheet with gaussian profile in the } z \text{ direction; in A the lasersheet is uniform in } x - y; \text{ in B the intensity of the laser-sheet in } x - y \text{ reproduce the non-uniform measured profile of the laser in use in our system. The distribution in B, shows no peak and is similar to the measured intensity distribution of the particles in Fig. 4.2} \]
particle because of the unknown position in the laser sheet in the direction (z) orthogonal to the imaging plane, but can give an estimate in a statistical sense. The estimates lose their meaning for particles bigger than a few microns, where the scattered intensity greatly oscillates as function of diameter. Compared to the test particles, the frozen particles are not mono-disperse and may have complex shapes, but this doesn’t preclude us from obtaining a rough estimate of their size.

Note that one of the biggest problem is the gaussian profile in the z direction of the laser-sheet. A uniform profile with a sharp decay would allow us to estimate the size of single particles. We investigated this problem but there are no easy way to do so (See Appendix C for details). An alternative to a sharp profile would be the use of multiple laser-sheets partially overlapping of different wavelengths and/or different polarization in order to identify the position of the particle in the z direction. A completely different viable approach to measure the particle size would be to use in-line holography.

4.6 Tracers for cryogenic nitrogen

In liquid nitrogen there are no standard seeding techniques as in water or air (75). Previous experiments in liquid nitrogen by White et al. (69, 76) and Bewley et al. (59, 66) with commercial micron-sized particles reported problems with particles clumping together because of the lack of an available surfactant. They also reported problems in injecting the particles without also injecting atmospheric air that resulted in large atmospheric ice particles.

We present a technique that instead uses these frozen atmospheric particles as tracers. The technique is an adaptation of the one used in liquid He: instead of
injecting a controlled amount of a dilute mixture of helium and hydrogen gas, we inject a mixture of nitrogen and a seeding gas. The first mixture tested has been atmospheric air, which is *de facto* a mixture of nitrogen and other components. The particles resulting from the injections are water and argon ice, with water most prevalent. While using atmospheric air is a simple way to create tracers, we investigate the possibility of using controlled mixtures for different seeding gases to improve tracer quality, and better understand frozen particles formation. This study may also be helpful for visualization purposes in cryogenic wind tunnels. Working in a cryogenic environment, techniques used in room temperature tunnels are not suitable: pressure sensitive paints and oils cannot be used without serious risks of contamination. Several other ways of visualizing the flow have been attempted, including pigments, paints and soap bubbles (77), schlieren, shadowgraph, moire deflectometry (78), holographic interferometry (79), liquefied propane (80) (81), vapor screen (82), ‘nitrogen smoke’ using a mixture of liquid nitrogen and steam-bearing air (83), and condensing/freezing gases on localized areas of the surface of the models (84). Doppler global velocimetry has been employed using ice crystals injected in a mixture of nitrogen/ water vapor (85), but recent efforts (86) reported problems in creating water ice particles for particle image velocimetry studies, and declared the intention to study different seeding gases. At the same time, several of the well-studied measurement techniques used in cryogenic nitrogen gas, confirmed by the extensive literature on the subject (87), can be adapted to work in liquid nitrogen.
4.6.1 Selection of seeding gases in liquid nitrogen

The best tracers are neutrally buoyant and small. As the limiting factor for the size of the tracers is the amount of light they scatter, the index of refraction is the most important factor in selecting the particles. Hence, we considered several compounds with a freezing point higher than 77 K, and studied their density and index of refraction at 77 K.

We focused on hydrocarbons, the substances with density closer to liquid nitrogen at 77 K and with the highest index of refraction; see Table 4.1.

In particular alkanes are among the most polarizable molecules (97), and hence with highest index of refractions. For convenience, we limit the search to nontoxic substances with boiling points lower than room temperature, i.e. gases.

While toxicity, boiling and freezing temperature are easily available in literature for most elements, density and index of refraction at liquid nitrogen temperature are hard to find. For most of the substances, for which real measures in the solid phase were not available, we estimated the index of refraction using the Lorentz-Lorentz (Clausius-Mossotti) relation

\[
\frac{n^2 - 1}{n^2 + 2} = K_p \rho,
\]

where \(K_p\) is a constant that depends on the polarizability of the molecules. We computed the constant \(K_p\) from the index of refraction \(n\) and density \(\rho\) where data is available, often in the liquid phase. Then we estimated the index of refraction from the solid density at 77 K. We note that the Lorentz-Lorentz relation is valid if the absorption is negligible, a condition that we assumed valid for all our cases. The Lorentz-Lorentz relation has been extensively used to estimate index
of refraction or the density when just one of the two is known.

To cross-check our estimates, we calculated the index of refraction using the molar refractivity calculated according to (98) using chemicalize.org. The estimates are in agreement within a 2% difference, except for benzene.

If measures of solid density at 77 K were not available we estimated it using a simple relationship (99)

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<th>$T_B$</th>
<th>$T_M$</th>
<th>$\rho_{77}$</th>
<th>$\rho_m$</th>
<th>$T_m$</th>
<th>ref</th>
<th>$n_{77}$</th>
<th>$n_m$</th>
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<th>phs</th>
<th>ref</th>
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<td>1.39</td>
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</tbody>
</table>

Table 4.1: The table reports for several hydrocarbons and other substances: the boiling temperature $T_B(K)$, the melting temperature $T_M(K)$, the density $\rho_{77}(kg/m^3)$ at 77 K, the index of refraction at 77 K estimated from the measure $n_{77}$ and the index of refraction $n_r$ calculated from the refractivity. The table reports also the references (ref) and the measures on which the estimates are based (in grey): the measured density$\rho_m(K)$, the measured index of refraction $n_m(K)$, the temperature of the measure $T_m(K)$ and the phase (phs, liquid L or solid S). 1 (88) 2 (89) 3 (90) 4 (91) 5 (92) 6 (93) 7 (94) 8 (95) 9 (96)
\[ \rho_S(T) = \left( 1.28 - 0.16 \frac{T}{T_t} \right) \rho_L(T_t) \]

between the density of organic compounds in the solid phase \( \rho_S \) and their density at the triple point \( \rho_L(T_t) \). Finding the density at the triple point for organic compounds is easy given the comprehensive data available because of the interest in the chemical industry.

The uncertainty in the density gives an uncertainty on the index of refraction of

\[ \frac{\delta n}{n} = \frac{3K \rho}{2(K_p \rho - 1)(2K \rho + 1)} \frac{\delta \rho}{\rho}. \]

An uncertainty in the solid density estimate of 6 % leads to an uncertainty of the order of 2% in the index of refraction. For propane and butane, we have an error of 13% and 16% which would lead to uncertainty of 6-7% in the index of refraction.

Knowing the index of refraction, to select the best tracers we considered the amount of scattered light as a function of a fidelity factor \( B \). Despite the fact that for high Reynolds flow the Stokes number is more important than the Froude number, we have chosen the fidelity factor because it is independent of the particular parameters of the flow.

To estimate the light scattered by ice particles of different compounds, assumed to be spherical, we used the Mie scattering model.

The result, shown in Figure 4.4, shows that the best gases would be propylene, also known as propene, and propadiene which are almost buoyant in liquid
nitrogen at 77 K and have estimated indices of refraction of 1.5 and 1.49, respectively.

4.6.2 Creation of frozen particles in liquid nitrogen

We have successfully demonstrated the possibility of creating faithful frozen tracers in liquid nitrogen using different gases. These tracers allow the study of liquid nitrogen flow using particle tracking and particle image velocimetry as shown in Fig. 4.5.

Using a mixture of 1 part of methane in 50 parts of nitrogen we estimated, using the Mie model, that particles of the order of 500 nm were created, as shown in Fig. 4.6.

We estimated that the particles resulting from injections of air were of the

![Graph showing scattering intensity as a function of particle size and fidelity factor B](image)

**Figure 4.4:** Table showing the density and index of refraction at 77 K and 1 atm of several hydrocarbons. Properties of other substances are reported for comparison. The plot shows the scattering intensity as a function of particle size, and as a function of the fidelity factor B for tracers made of these substances, up to 5 µm in diameter.
order of 1 \(\mu m\) or larger. We also used propane as a seeding gas, but could not create particles suitable for a visualization experiment: right after the injection we often observed a mist that lasted for a few seconds before disappearing. We do not understand the reason for this behavior compared to methane or water, but it could be due to its lower melting temperature (or the size of the molecule). Methane injections resulted in good tracers even if its melting point is just 14 \(K\) above the liquid nitrogen temperature. This gives good reason to believe that the other compounds with higher melting points would create good tracers. At present, we do not have adequate experience with other seeding gases under different parameters to confirm this hypothesis and to characterize the particle formation process.

The experimental setup, adapted from the helium experiment, turned out to be not completely adequate for liquid nitrogen. The biggest problem has

![Figure 4.5: Liquid nitrogen flows seeded with methane particles: (A) PIV vectors (B) trajectories of the tracers obtained using the particle tracking algorithm described in (64)](image)
been lack of a filtering system to keep the liquid nitrogen clean. The bigger density mismatch between particles in liquid helium, while affecting the fidelity, guaranteed that the big particle aggregates would either float to the top of the cryostat, in the case of the lighter hydrogen, or fall to the bottom, in the case of the heavier gases like nitrogen or neon. As this condition is not satisfied in liquid nitrogen, particle aggregates continue to float in the bulk of the fluid, confusing the measurements. Moreover, the liquid nitrogen was often contaminated with ice particles from the supplier. This reduced our ability to conduct multiple tests changing dilutions and seeding gases. To solve this problem a filtering system is necessary. Two possible solutions are charcoal filters, and an electrostatic cleaning device reported effective in liquid nitrogen (100).
4.7 Particles in liquid helium

Attempts to create hydrogen particles in liquid helium started in 1957, when Chopra and Brown (101) generated millimeter sized frozen particles injecting a mixture of hydrogen and deuterium\(^1\). In 1989 Murakami and Ichikawa (102) reported creating micron-sized hydrogen particles in superfluid helium, but there are no size estimate to support their claim. In 2002 Celik and van Sciver (103) generated frozen traces using neon, and they estimated their diameter to be about 10 µm by their terminal velocity. An important advancement in the technique has been the dilution of the hydrogen (or hydrogen-deuterium mixture) with helium gas. This technique has been developed independently by Boltnev et al. in 2002 (104) to embed impurities in liquid helium and by Bewley in 2006 (59) to create tracers for velocimetry studies. The technique allows the creation of micron-size solid hydrogen particles injecting the hydrogen mixture right above the lambda transition. Injections directly into the superfluid create much bigger particles that could not be used as tracers. The particles are slightly buoyant since the solid hydrogen density is 88 kg/m\(^3\) while the density of helium is roughly 146 kg/m\(^3\) depending on the temperature. The resulting particles have been successfully used by Bewley (59) and Paoletti (46) to trace the motion of the quantized vortices as it will be explained in section 7.2. Compared to their setup, the new EMCCD camera, allows one to detect smaller tracers. These smaller particles have been observed for the first time as spontaneous particles. An examination revealed that these particles were coming from a small leak in the top part of the cryostat, and hence were mostly frozen nitrogen particles. The atmospheric air, leaking into the

\(^1\)The deuterium was added so that the frozen tracers would be neutrally buoyant in liquid helium
top part of the cryostat was mixing and diluting into the cold helium gas and then fall in a sort of ‘snow’ on the surface of the liquid helium, to then reach the field of view because of the higher density (1026 kg/m$^3$ at 14 K (105)). A test revealed that such particles were invisible to the previous camera used by Paoletti (46), a Princeton Instruments ICCD camera. Because to study the quantized vortex dynamics is particularly important to have even some small particles, regardless of having a mono-disperse distribution, we estimated the size of single particles. In particular we observed that some particles appeared to be trapped on vortices at temperature less than 10 mK from the transition, compared to the micron-sized hydrogen tracers that could be observed trapped not closer than 50 mK from $T_\lambda$. Such observation can gives us an estimate on the particle size because of a threshold velocity over which particles cannot stay trapped $v_{max} = \rho_s k^2/(9\pi \mu d)$. The mechanism is described in section 7.2 and a plot of the threshold velocity is presented in Fig. 7.4. It is important to notice the dependance of $v_{max}$ on the particle size, and on the superfluid density, that in the critical region scales as $\rho_s \sim (1 - T/T_\lambda)^{-2/3}$ (106). Based on these assumptions, the particles trapped on vortices at 5 mm/s at 6 – 7 mK below the lambda transition are estimated to be smaller than 100 nm. We selected and studied 11 of these particles trapped close to the lambda transition. We estimated their size with the trapping model and the optical estimate (using the particle intensity at its brightest frame) and we found an agreement within a factor 2. We also compared the optical estimate with the terminal velocity estimates for micron-sized particles to find roughly the same level of agreement. While the injection mechanism using the atmospheric leak it is not yet controllable, sub-micron particles can now be created also using dilution rations of hydrogen in helium up to 1:100,000.
We note that whenever two particles get in contact they clump together, so the size distribution of the particles evolves in time. In particular, the presence of the vortices increases the clumping rate, as the particles collects on their cores (see section 7.2). As the particles get bigger, they either float to the free surface of fall to the bottom of the cryostat. Hence, usually, by the time the helium is cooled below 2 K few tracers are available. However, by injecting a highly dilute mixture of helium and atmospheric air we succeed in creating sub-micron particles injecting directly in the HeII. The results of these injection are quite different than those above the lambda transition. In particular, the particles do not spread uniformly across all the volume, but they remain bundled together in sheet-like structures that resembles those created by injecting dye into still water (see Fig. 4.7). An explanation on this different behavior could be that when in HeI, the particles are mixed by the stirring motion provided by the bubbles. For this reason, injections in HeII are not as effective in creating particles distributed across the fluid and not always provide a consistent result. However, the creation
of particles in \textit{HeII} allows one to study the dynamics of the vortices at low temperature as it was not possible before.
5

Liquid nitrogen: analysis of future prospects

5.1 High Reynolds number experiments

While direct numerical simulations can be used to study moderate Reynolds number flows \((Re < 10^4)\), they are impractical for the study of flows with parameters that match natural flows, or the study the universal scaling ranges thought to occur in turbulence. This underscores the importance of high Reynolds number experiments \((6, 107, 108, 109)\).

Going to higher Reynolds numbers in a controlled environment is challenging. Increasing the Reynolds number by increasing the size of the experiment, such as large tow tanks or large wind tunnels, results in a steep increase in costs and in a severe reduction of operational flexibility. Increasing the fluid velocity \(U\), beside being expensive, makes measurements and visualization more difficult, is limited by compressibility effects and drastically reduces the turbulent time scales, which are proportional to \(\nu^{1/2}U^{3/2}\). Moreover the velocity is limited by the power
available, and by the cooling necessary to maintain a constant temperature.

For these reasons, several recent experiments focused on increasing the Reynolds number by decreasing the viscosity of the fluid. This allowed basic turbulence research without large scale facilities. This approach has also been adopted to do model testing for aerospace and naval applications: cryogenic wind tunnels are now considered a better solution than large room temperature facilities. There are at least 20 cryogenic wind tunnels operating in 9 countries: the European Transonic Facility in Köln and the National Transonic Facility at the NASA Langley Research Center can respectively reach Reynolds number per meter \((Re/m = U/\nu)\) of \(Re/m = 2.2 \times 10^8\) and \(Re/m = 4.8 \times 10^8\). Despite the large number of experiment that have adopted cryogenic nitrogen gas, very few fluid dynamics experiments adopted liquid nitrogen as test fluid. We discuss the merits of using liquid nitrogen for high Reynolds number experiments, and introduce a new technique to create frozen particles in liquid nitrogen. These particles can be used for flow visualization and for velocimetry, using techniques such as particle tracking (61), particle image velocimetry (PIV) (62) or laser Doppler velocimetry (LDV).

## 5.2 Cryogenic helium

Liquid helium, having the lowest viscosity \((2.5 \times 10^{-8} \text{ m}^2 \text{ s}^{-1} \text{ at } 4.2 \text{ K})\), was thought the best fluid to use for both basic turbulence research and model testing for matching real world parameters (5, 6, 7, 8). Successful examples of classical turbulent experiments using cryogenic helium (liquid and gas) include a Von Kármán flow (110) in Paris, grid turbulence experiments (TSF/TSF-f) in Grenoble
jet flow in Grenoble (112) and at CERN (113), and convection experiments in Grenoble (114) and in Trieste (formerly in Oregon) (115). A few large helium experiments are now under development, such as an 80 cm diameter Von Karmann cell (SHREK) in Grenoble, a convection cell in Brno (116), the jet flow (GreC II) and the pipe flow HePipe at CERN. Still, not many turbulence experiments use liquid helium as a test fluid. The low temperature environment requires expertise and facilities that are not common in the fluid dynamics community, and a different set of diagnostics. We note that even using cryogenic nitrogen gas has been considered a challenge by wind tunnel experts because of the new expertise required (117). Additional problems come from the fact that helium’s extremely small turbulence scales (owing to its exceptionally small viscosity) are difficult to resolve. Further, its specific heat ratio $\gamma$ being different from that of air, as it is for heavy gases like freon-12 or sulfur hexafluoride, complicates model testing for aerospace applications (117). Moreover, the visualization technique for liquid He flows, either using solid particles (76), frozen deuterium (102), neon (118) or hydrogen particles (66) are not suitable to resolve the length scales of high Re flows. For a review of advantages and disadvantages of using liquid helium and cryogenic helium gas see (119) and (120).

5.3 Liquid nitrogen in fluid mechanics

While liquid helium has the lowest viscosity of all the fluids, liquid nitrogen has a kinematic viscosity at 77 $K$ which is about one-fifth (and at 104 $K$ and 10 $bar$, about one-tenth) of the viscosity of water at 25$^\circ C$. 
Liquid nitrogen still requires low temperature facilities but the working temperature of 77 K imposes far fewer requirements, bypassing most of the drawbacks of using liquid helium while keeping the advantages of cryogenic environment: low thermal radiation, low thermal noise, pure fluids, fast sensor response time and efficient thermal regulation. It is worth noting that, at the present time, having delivered liquid nitrogen from commercial suppliers in dewars of 100 liters, costs less than $0.35 per liter (2012), 20 times cheaper than liquid helium. For a large scale facility, the price of liquid nitrogen can drop to less than $0.10 per liter (2012), more than 50 times cheaper than liquid helium. Moreover, nitrogen is a renewable resource, while helium is not.

Figure 5.1: Properties of liquid nitrogen along the coexistence curve. From webbook.nist.gov using as source (121). The markers identify the states at ★ 1 bar, ● 5 bar, ■ 15 bar, considered later in the paper. The properties at the critical point $P = 33.978 \text{ bar}$ and $T = 126.192 \text{ K}$ are $\rho = 313.3 \text{ kg/m}^3$ and $\nu = 5.99 \times 10^{-8} \text{ m}^2/\text{s}$. 
For flow visualization, liquid nitrogen has the edge over helium and, to some extent, even over water. Its thermal expansion coefficient, two orders of magnitude bigger than helium, allow much more powerful light sources to be used without perturbing the flow. Its density of 808 kg/m makes it much easier to find buoyant tracers.

Compared to water ($n_{H_2O} = 1.33$), its low index of refraction $n_{LN_2} = 1.205$

**Figure 5.2:** Ratio of scattering intensity between particles of different index of refraction in liquid nitrogen and in water. In this case, the scattering intensity has been calculated using the Rayleigh scattering model.
makes tracers brighter (see Fig 5.2): in liquid nitrogen 1 \( \mu m \) PSL particles scatter as much as twice the light at a 90° angle as in water. This fact allows one to reduce the imaging exposure and hence allows for a higher time resolution.

5.4 Liquid nitrogen experiments

We consider possible high-Reynolds experiments using liquid nitrogen for different flow configurations, and compare them with other test fluids. As reference fluids we considered water, liquid helium and pressurized sulfur hexafluoride. Water at room temperature and atmospheric pressure is the most common test fluid for high-Reynolds experiments and liquid helium right above the superfluid transition is the classical fluid with the lowest viscosity. Pressurized gases are an alternative to cryogenic fluids as low viscosity fluid. The choice of \( SF_6 \) at 15 bar comes from the fact that is used in the Göttingen Turbulence Facility (122). Sulfur hexafluoride has one of the lowest kinematic viscosity among gases because of its high density. The gas is extensively used by the electric industry and at an average price of roughly $20 per kilogram, is relatively expensive and requires particular handling procedures (123). Sulfur hexafluoride, while not toxic, is subject to several regulations because it is a highly potent greenhouse gas: a pound of \( SF_6 \) that has the same global warming impact of 11 tons of \( CO_2 \) (124). The properties of the test fluids are summarized in Table 5.1, while a detailed review of the fluid and thermodynamic properties of nitrogen in comparison with \( H_2O, He \) and \( SF_6 \) can be found in (125).

We note that along the coexistence curve, as pressure increases, the liquid nitrogen density and viscosity decrease as shown in Figure 5.1. The viscosity in
particular scales as $\nu \propto P^{-0.29}$. Hence, at higher operating pressure the power requirements to obtain a fixed $Re$ are greatly reduced. At the same time, the higher temperature of the test fluid allows the use of liquid nitrogen at 77 K as an effective coolant. Achieving an increased Reynolds number by either increasing size or pressure require a similar increase in mass of the apparatus: increasing the $Re$ by a factor of 2 requires the mass of the container to increase by a factor of 8 by increasing its size, and by factor of 10 by increasing the pressure. While a pressurized vessel is required to meet several stringent specifications compared to a room temperature apparatus working at atmospheric pressure, the additional requirements necessary for a cryogenic apparatus to work with pressurized fluids are minor, especially if the pressure is limited to a few bar. Varying the pressure also allows considerable flexibility via the density of liquid nitrogen. This characteristic could be useful in studying inertial particles and would allow the tuning of the density of the liquid to obtain neutrally buoyant tracers.

We note that, with an appropriate design, a liquid nitrogen experiment can be adapted to run with cryogenic nitrogen gas, increasing the flexibility of the apparatus and broadening its scope beyond basic turbulence research. In this case,
creating a pressurized vessel would be even more important as the kinematic viscosity of nitrogen gas decreases, along the coexistence curve, as \( \nu \propto P^{-0.80} \). In the following examples, we chose liquid nitrogen at 94 \( K \), assuming it to be a good compromise between cost and flexibility. The fluid is considered for simplicity at the saturated vapor pressure of 5 \( \text{bar} \), but in a practical application the pressure should be increased to avoid cavitation problems (see section 6.4.2). We assume the tracers to be 1 \( \mu m \) in diameter and with a density of 900 \( \text{kg/m}^3 \).

5.4.1 Grid turbulence

A common way to generate a flow which resembles the idealized homogenous isotropic turbulence, is forcing a fluid flow through a grid, or pulling a grid through a stationary fluid. In this section we consider the stationary grid case, which allows one to obtain a stationary turbulent flow, while the towed grid is considered

![Figure 5.3: Possible configuration for controlling the temperature with liquid nitrogen at 77K surrounding the vessel containing pressurized liquid nitrogen.](image-url)
Grids can be either passive as in the early work in the late 60’s by Comte-Bellot and Corrsin (126), or active as in the work by Makita (127), where agitator winglets are mounted on the grid. The purpose of an active grid is to enhance the turbulent intensity allowing to achieve higher $Re_\lambda$ and $Re_D$ in a facility of a give size. In particular an active grid allows one to obtain a turbulence intensity up to $v'/U \simeq 20\%$ compared to the 2% that is common with passive grids, and it increases the integral scale beyond the mesh size $L_{mesh}$ (128).

We report estimates for possible grid turbulence experiments produced in small-, medium- and large-scale recirculating apparatuses of 0.1, 0.5, 3 m diameter

<table>
<thead>
<tr>
<th>Grid flow for $d = 0.5m$ and $v=15m/s$ at different T and p</th>
<th>Grid flow with LN at 94 K and 5 bar for different sizes</th>
</tr>
</thead>
<tbody>
<tr>
<td>T(K)</td>
<td>p (bar)</td>
</tr>
<tr>
<td>77</td>
<td>1</td>
</tr>
<tr>
<td>94</td>
<td>5</td>
</tr>
<tr>
<td>110</td>
<td>15</td>
</tr>
</tbody>
</table>

| 3  | 20  | 1.2×10⁻⁷  | 723  | 2.1×10⁻⁶  | 5.8×10⁻¹ | 3.7×10⁻⁵ | 1.5×10⁻⁴ | 9.2×10⁷  | 1.3×10⁸  | 1.9×10⁴  | 0.015 |
| 0.5| 15  | 1.2×10⁻⁷  | 723  | 1.7×10⁻⁶  | 2.7×10⁻¹ | 2.4×10⁻⁵ | 9.1×10⁻⁴ | 1.1×10⁷  | 1.6×10⁸  | 6.9×10⁵  | 0.025 |
| 0.1| 15  | 1.2×10⁻⁷  | 723  | 1.2×10⁻⁶  | 4.1×10⁻¹ | 1.1×10⁻⁵ | 4.1×10⁻⁴ | 4.3×10⁷  | 3.1×10⁸  | 3.1×10⁵  | 0.05 |

**Table 5.2:** Table with flow parameters for different grid turbulence experiments. The integral scale is assumed to be $L = d/4$, where $d$ is the diameter of the apparatus, the turbulence intensity $v'/U = 0.2$ and the drag coefficient $C_D = 0.6$ for corners, 0.2 for honeycomb, 2.2 for screens (129). We assume a grid with a solidity of 0.34 and a drag coefficient of 4.5 as in (126). The power requirements depend on the number of screens necessary to have a uniform flow approaching the grid.
(see Table 5.2). For the medium case, we compare parameters with water, SF$_6$ at 15 bar and liquid helium (see Table 5.4).

The Kolmogorov scale of all the proposed liquid nitrogen experiments is bigger than 1 $\mu$m. The tracers would have a Stokes number of less than 0.05 (i.e. are faithful tracers), and could resolve both the inertial scales and the dissipative scales $l_D > 8\eta$ (17). The timescale of the small eddies, of the order of $\tau_\lambda$, can be resolved with current high-speed cameras. For comparison, we report in Table 5.3 the data for the Göttingen Turbulence Facility, the current state of the art grid turbulent facility, and the possible parameters attainable with liquid nitrogen at the same operating pressures and using the same assumptions. The difference in the values of the parameters with the other estimates comes from a different set of characteristics.

Beside cost, one of the biggest advantage of liquid nitrogen over SF$_6$ is in the fidelity of the tracers. It is difficult to make a direct comparison because there is no established technique to create tracers in SF$_6$, but the density mismatch between the tracers and the gas is practically unavoidable, while for liquid nitrogen it is possible to have tracers which are almost neutrally buoyant. On the other hand, the higher density of liquid nitrogen requires more driving power to reach the same Reynolds number. As usually the limitations are not in the amount of driving power available but on the cooling power available, the higher thermal conductivity of liquid nitrogen ($k_{LN} = 146 \text{ mW/mK}$ at 77 K and $k_{LN} = 82 \text{ mW/mK}$ at 110 K and 15 bar) compared to SF$_6$ ($k_{LN} = 14 \text{ mW/mK}$ at room temperature and 15 bar (130)) compensate for the higher amount of power to dissipate. Moreover, for pressurized liquid nitrogen the cooling could be done using liquid nitrogen at 77 K as coolant. Even for the highest power
<table>
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<th>Fluid</th>
<th>T(K)</th>
<th>p (bar)</th>
<th>d (m)</th>
<th>v (m/s)</th>
<th>ν (m²/s)</th>
<th>ρ (kg/m³)</th>
<th>η (m)</th>
<th>ντ (s)</th>
<th>P (W)</th>
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<td>106.6</td>
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<td>1.7 ×10⁵</td>
<td>2.1 ×10⁷</td>
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<td>8.9 ×10⁻⁶</td>
<td>623</td>
<td>4.7 ×10⁻⁶</td>
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<td>9.8 ×10⁵</td>
<td>3.8 ×10⁷</td>
<td>11000</td>
</tr>
</tbody>
</table>

**Table 5.3:** Table with the attainable parameters by the Göttingen Turbulence Facility under the assumption that the integral scale is \( L = 0.45d \) and the turbulence intensity is 20%. \((122)\). For comparison the parameters attainable with liquid nitrogen in the same conditions are reported.

Within the requirements of the estimates reported in the table, it would be possible to have a heat flux lower than \( 2 \times 10^5 \text{W/m}^2 \), which corresponds to the transition from the nucleate-boiling to the less efficient film boiling regime, allowing heat transfer rate of the order of \( 5 \times 10^2 \Delta T^{2.5} \text{W/m}^2\text{K}^{-2.5} \) \((131)\). For example, 1 MW of power would boil liquid nitrogen at a rate of 5 liters per second; for comparison the liquid nitrogen tank volume of the 0.3 m in diameter Pilot Transonic Wind tunnel is 212 m³, and that of the National Transonic Facility is 946 m³ \((132)\). Moreover, the power requirements and boiling rate can be substantially reduced by slightly decreasing the Reynolds number. Using liquid nitrogen both as test fluids and coolant has the additional advantage of simplifying the operations and design of the apparatus.

### 5.4.2 Pipe flow

Pipe flow has been traditionally a standard configuration where to study turbulence since the pioneering work of Reynolds \((133)\), and it still an active area of research. To discuss pipe flow experiments we introduce the viscous velocity \( u_τ \), the viscous length \( l_* \) and the Kármán number \( R^+ \):

\[
u_τ = \sqrt{\frac{\tau_w}{\rho}} \quad l_* = \nu u_τ \quad R^+ = \frac{u_τ d}{2\nu},
\]
where \( \tau_w \) is the wall shear stress and \( d \) the pipe diameter. Note that the Kármán number is nothing but the ratio of the viscous scale to the large scale \((d/2)\), and for a pipe flow is a unique function of Reynolds number \((134)\). To estimate the wall shear stress \( \tau_w = \frac{\Lambda \rho U^2}{8} \), we calculate the Darcy friction factor \( \Lambda \) solving the Colebrook-White equation (see (135) or (136))

\[
\frac{1}{\sqrt{\Lambda}} - 2 \log \left( \frac{\delta_R/d}{3.7} + \frac{2.51}{Re\sqrt{\Lambda}} \right),
\]

and assuming a surface roughness of \( \delta_R = 2.5 \mu m \).

From the estimates reported in Table 5.4, it would be possible to reach Reynolds number of \( 1.7 \times 10^7 \) with a \( 0.1 m \) diameter pipe and \( 8.3 \times 10^7 \) in a facility of \( 0.5 m \) in diameter. For comparison, the CICLoPE air facility with \( 0.9 m \) diameter and \( 115 m \) length and 400 kW of cooling power, described in \((107)\), is designed to reach \( Re = 2.3 \times 10^6 \), \( R+ = 4.0 \times 10^4 \) at \( 38 m/s \) with a viscous scale \( 11 \mu m \), and \( Re = 4.3 \times 10^6 \) at maximum power.

We note that with less than 50W of power, it would be possible to reach, in the \( 0.1 m \) diameter pipe using liquid nitrogen at 15 bar, an \( Re = 2 \times 10^6 \) and an \( R+ = 4.6 \times 10^4 \), with a viscous scale larger than \( 1 \mu m \), i.e. larger than the tracers described in Chapter 4. Hence it is possible to reach parameters high enough to have both a well developed overlap region and a \( k^{-\frac{5}{3}} \) region \((107)\), in a liquid nitrogen facility which is a tenth of the size and utilizing 1000 times less power. The spatial scales are, of course, smaller—as the viscous scale decreases roughly with the size of the apparatus and the inverse of the Kármán number—but the time scales are larger, allowing better time resolution. Moreover, using liquid nitrogen, the fidelity of the particle is much better than for air. As far as other
diagnostics, it is worth mentioning that commercial hot wire probes can be used in liquid nitrogen via a simple calibration (137), while they cannot be used in helium (138). As the heat capacity of materials at cryogenic temperatures is lower than at room temperature, the temporal response of hot wires in liquid nitrogen is limited by the thermal and momentum diffusion times across the boundary layer. These time scales vary as the size of the sensor divided by the velocity of the flow (139), and hence are limited by how small the sensor can be made. It has been suggested that sensors can be fabricated to sub-micron scales (140), hence allowing one to have high both spatial and temporal resolution.

5.4.3 Taylor-Couette flow

The possibility of reducing the viscosity of the fluid, keeping the size and speed constant, is even more crucial in rotating experiments. Taylor-Couette flow is a well studied flow of a fluid between two concentric rotating cylinders. In most of the experiments the flow is driven by rotating the inner cylinder of radius $r_i$ with the outer cylinder of radius $r_o$ stationary. This configuration has been extensively studied for transitional phenomena and instabilities. Recent efforts have extended to cases with both cylinders rotating, to better understand astrophysical phenomena. If we define the dimensionless torque as

$$G = \frac{T_o}{\rho\nu^2H},$$

where $H$ is the height of the cylinder and $T_o$ is the torque on one of the cylinders, it can be predicted that asymptotically $G \sim Re^\alpha$ with $\alpha = 2$. In this case the Reynolds number is calculated as $Re = (r_o - r_i)\Omega r_i/\nu$. It has been found
<table>
<thead>
<tr>
<th>fluid phase</th>
<th>N₂</th>
<th>N₂</th>
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<th>H₂O</th>
<th>He</th>
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<tr>
<td>T(K)</td>
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<td>5</td>
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<tr>
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<td>146</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>grid flow d = 0.5 m \ ν = 15 m/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>η (10⁻⁶ m)</td>
</tr>
<tr>
<td>λ (10⁻⁴ m)</td>
</tr>
<tr>
<td>τₙ (10⁻⁵ s)</td>
</tr>
<tr>
<td>τₙ (10⁻⁵ s)</td>
</tr>
<tr>
<td>P (10⁴ W)</td>
</tr>
<tr>
<td>Re (10⁶)</td>
</tr>
<tr>
<td>Reₙ (10³)</td>
</tr>
<tr>
<td>Stₜₜₜ</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>pipe flow d = 0.1 m \ ν = 20 m/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>lₜ (10⁻⁷ m)</td>
</tr>
<tr>
<td>η (10⁻⁶ m)</td>
</tr>
<tr>
<td>τₙ (10⁻⁵ s)</td>
</tr>
<tr>
<td>P (10⁴ W)</td>
</tr>
<tr>
<td>Re (10⁶)</td>
</tr>
<tr>
<td>Reₙ (10³)</td>
</tr>
<tr>
<td>R+ (10⁵)</td>
</tr>
</tbody>
</table>

**Table 5.4:** Table with flow parameters for different test fluids for a grid experiment and a pipe flow experiment. The integral scale is assumed to be $L = d/4$, the turbulence intensity $v'/U = 0.2$ and the drag coefficient $C_D = 0.6$ corners, 0.2 for honeycomb, 2.2 for screen (129). The grid apparatus is assumed to be 100d long and with a static grid with drag coefficient $C_D = 4.5$. The pipe flow is assumed to be 100d long.
experimentally by Lathrop et al. (141) that $\alpha$ was monotonically increasing with $Re$, and for their highest Reynolds number ($Re \sim 10^6$) they found that $G \simeq 0.161 Re^{1.87}$. Hence the power dissipated in a Taylor-Couette apparatus for a fixed Reynolds number is approximately cubic in the viscosity $^1$

$$P = T_o \Omega \propto \nu^3 Re^{\alpha+1}.$$  

As an example we consider the experiment at the University of Maryland described in (142), that could reach $Re$ up to $4.4 \times 10^6$ using water at $90^\circ C$ ($\nu = 3.2 \times 10^{-7} m^2/s$). To reach $Re = 1.5 \times 10^7$ it would require almost $150 kW$ of power with an impractical differential rotation velocity of $67 Hz$, while using liquid nitrogen at $15$ bar and $110 K$ it would need just $2 kW$ and a rotation velocity of less than $19 Hz$.

To dissipate the power in liquid nitrogen the easiest option would be to allow the liquid nitrogen to boil. This solution would need just an insulation jacket on the outer cylinder and an exhaust system on the inner cylinder, where the boiling would occur, to get rid of the nitrogen gas. If a free surface is allowed this method would limit the running time because of the evaporating fluid - we note that $2 kW$ of power would boil the liquid nitrogen at $110 K$ and $15 bar$ at $1$ liter per minute. If no free surface is allowed it would be necessary to have a reservoir above the cylinder or refill the liquid nitrogen constantly through a junction.

Another possibility to dissipate the power could be to allow the nitrogen to boil or to cool the outer cylinder with liquid nitrogen at $77 K$. The whole system

$^1$This scaling, valid for the outer cylinder stationary, is an upper bound for the power dissipated for different rotation rates combinations of the inner and outer cylinders (for fixed Reynolds number) (142).
Figure 5.4: Schematics of liquid nitrogen towing tank. Several relevant parameters are compared with an analogous facility using water.

could be immersed in a liquid nitrogen bath or the outer cylinder could have an outer jacket where liquid nitrogen could boil and be refilled. Also in this case, the power dissipated on the wall would allow cooling in the nucleate-boiling regime.

5.4.4 Towing tank

A towing tank allows one to study surface waves and the motion of surface ships in a controlled environment. To match the relevant fluid dynamics parameters of big ships, large facilities are necessary. For example, the NACA Tow Tank at the NASA Langley research center, VA, USA and the David Taylor Model Basin at the Naval Surface Warfare Center in Bethesda, MD, USA, which can reach $Re \sim 10^8$, are respectively $902m \times 8.5m \times 8m$ and $904m \times 4.9m \times 6.4m$. We study the parameters attainable by a liquid nitrogen facility that would allow to severely reduce the size of the facility while still allowing accurate testing.
We define the Froude number

$$Fr = \frac{U}{(gL)^{1/2}},$$

as the ratio of the characteristic velocity and the gravitational wave velocity, and characterize the resistance of a moving object by wave generation. To keep $Fr$ constant, the ratios of the $Re$ of real and model scales as

$$\frac{Re_E}{Re} = \left(\frac{L_E}{L}\right)^{3/2} \frac{\nu}{\nu_E},$$

while the Weber number

$$We = \frac{\rho U^2 L}{\sigma},$$

which characterize surface tension effects, scales as

$$\frac{We_E}{We} = \frac{\rho_E \sigma}{\rho \sigma_E} \left(\frac{L_E}{L}\right)^2,$$

where $\sigma$ is the surface tension. We can then define three ratios to compare the modeling (denoted by the subscript $E$) to the real ship:

$$R_1 = \frac{\nu_E}{\nu}, \quad R_2 = \frac{\rho E \sigma}{\rho E \sigma}, \quad R_3 = \frac{\rho_{air} \rho_{liquid}}{\rho_{H_2O} \rho_{gas}},$$

where $Re_3$ compares the gas-liquid density ratio of the test fluid with that of water-air. We report the values of the ratios for liquid nitrogen and liquid helium in Table 5.5.

To have a concrete example we consider a surface ship 50 m long moving at 32 knots (16.5 m/s) and a model scaled by a factor 10. Liquid nitrogen, matching
Table 5.5: Parameters for the towing tank experiment using as test fluids liquid nitrogen at $77\,K$ and 1 bar, liquid nitrogen at $94\,K$ and 15 bar, and liquid helium at $2.178\,K$ and saturated vapor pressure.

<table>
<thead>
<tr>
<th></th>
<th>LN 77 K</th>
<th>LN 94 K</th>
<th>He 2.178</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>5</td>
<td>8.3</td>
<td>60</td>
</tr>
<tr>
<td>R2</td>
<td>6.4</td>
<td>9.7</td>
<td>35.5</td>
</tr>
<tr>
<td>R3</td>
<td>0.25</td>
<td>0.045</td>
<td>0.16</td>
</tr>
</tbody>
</table>

the $Fr$ number, matches the $Re = 8.25 \times 10^8$ within a factor 6.3 (3.8 using LN at 5 bar and 94 K) the $We$ within a factor 15.6 (10) instead that 31.5 and 100 using water. For such a model, a tank 5 m wide and 50 m long would allow a run time of 9 seconds (see Fig. 5.4).

For comparison it is worth mentioning the 41 m long -8 m wide Helium towing tank proposed in (143). According to the estimates such impressive facility would be able to match the Reynolds number and the Froude number of a 200 m long ship moving at 32 knots, as well as to match the gas to liquid density ratio of air to water. However, such parameters are possible to reach only in the superfluid phase, where dynamics can be quite different than a classical fluid. Using $HeI$, it would be possible to match gas to liquid density ratio of air to water at best within a factor 6.
6

Cryogenic nitrogen convection experiment

In this chapter we investigate the design and feasibility of a large scale pressurized cryogenic nitrogen facility to study Rayleigh-Bénard convection. We consider a tank 6 m in diameter and 5 m tall for pressurized cryogenic nitrogen up to 40 bar.

6.1 Overview

The main goal of the pressurized cryogenic nitrogen cell under study is to investigate turbulent convection at extreme Rayleigh numbers (up to $10^{18}$). However, as proposed for the 10 m tall helium convection cell (144), several inserts could increase the flexibility of the apparatus (see Fig. 6.1).

For example, towed grid, von Kármán flow, or Taylor-Couette inserts could potentially generate flows with the highest Reynolds number ever for such configurations, with turbulent scales big enough to be resolved by current measurements techniques.
Figure 6.1: The drawings are a concept design for the pressurized cryogenic nitrogen cell. In particular the drawings represents a model in scale 10:1 that would work as prototype for the big cell. At the bottom are represented possible inserts to drive the turbulent flows.
Other inserts, such as a cryogenic wind-tunnel and a liquid nitrogen towing tank, could broaden the scope of the cell to applied sciences and model testing. Such inserts, while could not match the parameters of the largest dedicated facilities (which have budgets of hundreds millions of dollars), could still provide interesting testing capabilities and allow to explore different parameter spaces. Indeed, the ability to change the pressure up to the critical point of nitrogen, allows to have large variation of several physical parameters.

A concept design of the experimental apparatus is presented in Fig. 6.1; technical drawings and the full engineering details are beyond the scope of this work. The apparatus consists of a pressure vessel, surrounded by a jacket for liquid nitrogen at atmospheric pressure and $77\, \text{K}$ to act as refrigerant. This jacket is in turn enclosed in a vacuum jacket to improve insulation. The top and bottom of the cylinder are closed by the modular inserts, which are secured to the cell via flanges. In the case of motorized inserts, a pressure rotary seal or magnetic shaft coupler can be used for the rotating shaft, or the motor can be enclosed in a pressurized gas environment. All solutions have their own challenges and require some particular attention. Several ports on the cell would allow installation of different diagnostics and/or have optical access. Optical windows resistant up to $40\, \text{bar}$ are necessary on the pressure vessel, but their size can be much smaller than the whole port. Standard windows can be installed on the vacuum jacket or, depending on the application, the windows on the pressure vessel can be left in contact with room temperature air.
6.2 Rayleigh-Bénard convection

Rayleigh-Bénard convection consists of a flow driven by buoyancy effects, generated by heating a fluid from below and cooling it from above. When the temperature difference exceeds a certain threshold, the flow of hot fluid rising and cold fluid falling becomes turbulent (see Fig. 6.2) (see for example Tritton (145)). Turbulent convection occurs in several natural and industrial processes. In particular, convection plays a key role in the dynamics of the outer layer of the sun, giant planets, Earth’s outer core, atmosphere, and oceans.

To characterize convection we define the dimensionless temperature difference as the Rayleigh number $Ra$, the dimensionless heat transfer coefficient as the Nusselt number $Nu$, and the ratio of viscous to thermal dissipation as the Prandtl number $Pr$:

$$Ra = \frac{\alpha g \Delta T L^3}{\nu \kappa}, \quad Nu = \frac{h L}{\kappa}, \quad Pr = \frac{\nu}{\kappa_d},$$

where $\alpha$ is the isobaric thermal expansion coefficient, $\kappa_d$ is the thermal diffusiv-

Figure 6.2: The image on the right shows a diagram of a Rayleigh-Bénard convection cell. The structures and thermal plumes that characterize the flow are visible in the rendering of a numerical simulation (146) in the middle and in the shadowgraph image (147) on the right.
ity, \( \Delta T = T_b - T_t \) is the temperature difference across the hot bottom \( T_b \) and the cold top \( T_t \), \( h \) is the heat transfer coefficient and \( k_c \) is the thermal conductivity of the fluid. The Rayleigh number, which can be interpreted as the ratio of buoyancy to viscous and thermal dissipation, is of the order \( Ra \sim 10^{20} \) in the ocean, \( Ra \sim 10^{21} \) in the sun, and \( Ra \sim > 10^{20} \) for most astrophysical phenomena. However, studying flows with such high \( Ra \) is challenging: numerical simulations and experiments cannot reach such parameters, and extrapolating the behavior from the dynamics at lower \( Ra \) is not an options, as theories suggests that the dynamics can be very different. Among the several open issues on convection (for an introduction see for example Ahlers (148)), indeed one of the most debated is related to the ‘so called’ ultimate regime. In 1962 Kraichnan (149) predicted that, when the the Rayleigh number exceeds a certain value, there is a change in heat transport mechanism and the Nusselt number scales as \( Nu \sim Ra^{1/2}/(\log Ra)^{3/2} \) instead that \( Nu \sim Ra^{1/3} \) (or \( Nu \sim Ra^{2/7} \) as some experiments suggest) for the classical regime. The experimental confirmation of the ultimate regime is controversial, as different experiments that achieved very high \( Ra \), by using cryogenic helium, had different outcomes (115, 150, 151). The results published this year (2012) (152) from the pressurized \( SF_6 \) ‘U-Boot’ experiment, show a transition for \( Ra \simeq 5 \times 10^{14} \), but many questions are still open. For example it is not yet clear which are the right logarithmic corrections to the \( Ra^{1/2} \) scaling among those predicted by several theories. Hence the need for a facility that would reach Rayleigh number higher that the current setups.

In Table 6.1 we report the parameters attainable for the convection cell under study, with diameter \( D = 6 \, m \) and height \( L = 5 \, m \). To satisfy the Boussinesq
Figure 6.3: Parameters attainable with the convection experiment of diameter $D = 6\, m$ and height $L = 5\, m$ using pressurized cryogenic nitrogen assuming $\Delta T = 0.05/\alpha$. The values are for nitrogen along the coexistence curve.
Table 6.1: Table with flow parameters for different test fluids for a convection experiment of diameter $D = 6\, m$ and height $L = 5\, m$. The temperature difference across the plates has been assumed to be $\Delta T = 0.05/\alpha$.

<table>
<thead>
<tr>
<th>fluid phase</th>
<th>N$_2$</th>
<th>N$_2$</th>
<th>SF$_6$</th>
<th>H$_2$O</th>
<th>He</th>
</tr>
</thead>
<tbody>
<tr>
<td>T(K)</td>
<td>126.09</td>
<td>110</td>
<td>298</td>
<td>298</td>
<td>2.2</td>
</tr>
<tr>
<td>p (bar)</td>
<td>33.79</td>
<td>15</td>
<td>15</td>
<td>1</td>
<td>0.053</td>
</tr>
<tr>
<td>$\nu$ ($10^{-6}$m/s)</td>
<td>6</td>
<td>9</td>
<td>16</td>
<td>100</td>
<td>1.8</td>
</tr>
<tr>
<td>$\kappa$ ($10^{-6}$m/s)</td>
<td>0.12</td>
<td>4.8</td>
<td>17</td>
<td>15</td>
<td>2.5</td>
</tr>
<tr>
<td>$\alpha$ ($10^{-4}$K)</td>
<td>$3.4 \times 10^3$</td>
<td>14.3</td>
<td>6.6</td>
<td>3.3</td>
<td>9.6</td>
</tr>
<tr>
<td>$\Delta T(K)$</td>
<td>0.015</td>
<td>3.5</td>
<td>7.6</td>
<td>15</td>
<td>5.2</td>
</tr>
<tr>
<td>Pr</td>
<td>52</td>
<td>1.9</td>
<td>0.95</td>
<td>6.6</td>
<td>0.72</td>
</tr>
<tr>
<td>Ra ($10^6$)</td>
<td>$8.9 \times 10^{17}$</td>
<td>$1.4 \times 10^{16}$</td>
<td>$2.3 \times 10^{15}$</td>
<td>$4.0 \times 10^{14}$</td>
<td>$1.4 \times 10^{17}$</td>
</tr>
</tbody>
</table>

approximation\(^1\), on which most of convection theories relies on, there are some limitations on the maximum $\Delta T$ that can be used to drive the flow. While it is standard to assume $\Delta T < 0.3/\alpha$ (153), to be conservative we assume $\Delta T = 0.05/\alpha$ (119). As for cases studied in Chapter 5, we compare liquid nitrogen with other test fluids. We consider also liquid nitrogen close to the critical point ($T_C = 126.192\, K$ and $P_c = 33.958\, bar$ ), where there is a strong temperature dependance of the physical properties, important in the study of Rayleigh-Bénard convection (154).

The Oregon $0.5\, m$ diameter convection cell using helium close to the critical point (115) could span the Rayleigh number over 11 orders of magnitude, reaching $Ra \sim 10^{17}$ for $\Delta T = 0.2/\alpha$. Under the same condition, the proposed liquid nitrogen tank could reach $Ra \sim 3 \times 10^{18}$, with a Kolmogorov scale of the order of $\eta \sim 10^3\, \mu m$, where we assumed $\epsilon = \frac{\nu^3}{\eta}(Nu - 1)RaPr^{-2}$ (155) and $Nu =$

\(^1\)In the Bussinesq approximation the density is assumed constant except that in the buoyancy term
0.124Ra^{0.309} (115). For comparison, the maximum Rayleigh number attainable by the U-Boot experiment using SF$_6$ up to 19 bar, in the weakly non-Boussinesq regime, is Ra $\sim 10^{15}$ (156).

6.3 Inserts

6.3.1 von Kármán flow

A possible way to reach very high Reynolds number in the cell would be to generate a von Kármán flow, i.e. drive the turbulent flow via two counter-rotating impellers.

Using as guideline the measured scale in Voth et al. (157), we assume an integral scale to be one sixth of the diameter ($L = D/6 = 1$ m) and the RMS turbulent fluctuation velocity to be one half of the tangential blade speed $v' = U/2$.

Assuming two propellers of 6 m in diameter spinning at 0.5 Hz ($U \simeq 10$ m/s), the flow would have a Reynolds number up to about $10^9$, however, the heat generated would be too high to be dissipated through either steel or aluminum walls (see section 6.4.3). Hence for the estimates we assume $f = 0.4$ Hz. The resulting flow could have a maximum Reynolds number $Re = HU/\nu = 6 \times 10^8$, an integral Reynolds number $Re_L = 6 \times 10^7$ and a Taylor Reynolds number as high as $Re_\lambda = 30,000$, with a Kolmogorov scale around 1.5$\mu$m. As reference, field measurement in a tidal channel (158) and atmospheric measurements (159) recorded Taylor-Reynolds numbers below 20,000. It is worth mentioning also the two von Kármán helium flows, by Maurer et al. (160) who reached a $Re_\lambda = 2100$ and the new 0.8 m diameter SHREK experiment in Grenoble, that will reach
Beside reaching larger parameters, in comparison with the helium experiment, the liquid nitrogen flow would have bigger turbulent scales that are possible to resolve with current experimental techniques.

6.3.2 Towed grid

To generate turbulence more homogeneous and isotropic than in a von Kármán flow, we consider a towed-grid insert. To study a towed grid flow one cannot use Taylor’s frozen hypothesis and hence time averaging, as with a stationary/wind-tunnel grid. However, a towed-grid is an excellent configuration to observe the turbulent decay, which is instead limited by wall-side effects contamination in the case of a stationary grid.

Assuming a grid with 20 meshes moving at a maximum speed of $U_{\text{grid}} = 5 \, \text{m/s}$ and 0.44 solidity, we estimate that it would be possible to reach $Re_\lambda = 4000 - 10000$ and $Re_{\text{mesh}} = U_{\text{grid}}L_{\text{mesh}}/\nu = 5 - 25 \times 10^6$ (the range is given for different operating temperature and pressures along the coexistence curve of nitrogen) with a Kolmogorov scale $\eta = 2 - 10 \, \mu\text{m}$. At times 50 and 5000 $L_{\text{mesh}} U_{\text{grid}}$ (i.e. 3 seconds and 5 minutes) after pulling the grid the Taylor Reynolds numbers are expected to be respectively $Re_\lambda = 2500 - 6500$ and $Re_\lambda = 700 - 1600$. For such estimate, we assumed that $E \propto t^{-6/5}$ and $l \propto t^{2/5}$ (161).

6.3.3 Wind tunnel

A cryogenic wind tunnel insert would have the merit to broaden the scope of research to model-testing.

While any wind tunnel insert could not compete with the biggest facilities
Figure 6.4: The plot on the left shows the power dissipated by a high Re von Kármán flow \((Re_\lambda>10,000)\) in pressurized liquid nitrogen or cryogenic nitrogen gas, driven by propellers spinning at 0.4 Hz. The plot shows also the maximum heat dissipation through a 2 cm stainless steel or 6 cm aluminum wall. The stainless steel wall could not dissipate all the heat generated by the liquid nitrogen flow while, for operating pressures higher than 20 bar (temperatures higher than 115 K), the aluminum wall could. On the contrary, both stainless and aluminum would be able to dissipate the flow generated in cryogenic nitrogen gas for all the pressures above roughly 3 bar (90 K). The two plots on the right show the turbulent parameters attainable by such flows, and in particular that it would be possible to obtain \(Re_\lambda > 25,000\) with a Kolmogorov scale bigger than 1 \(\mu m\).
such as the NTF or ETF in size and capabilities, it would allow to reach higher pressures and lower temperatures, opening up a unique set possibilities to be explored. For comparison the $2.5\,m$ test section of the NTF can reach $9\,bar$ and $110\,K$, and the $0.3\,m$ test section of the NASA Transonic Cryogenic Tunnel can reach $78\,K$ and $5\,bar$.

Most of the cryogenic wind tunnels are closed-circuit, for obvious reasons, with the ARA-Bedford facility in England being an exception. Such facility uses an open return system in a cryogenic room. The $6\,m \times 5\,m$ cell would act as a cryogenic room, where both a open-circuit or closed-circuit tunnel could fit. More research is needed in whether an open-circuit or close circuit would be the best option for such facility.

6.4 Preliminary feasibility study

6.4.1 Materials

We present a few gross estimates to verify the feasibility and to understand the size of the project. A careful engineering of the vessel design should be done elsewhere.

The material for the pressure vessel should be a good thermal conductor and be strong across all the cryogenic temperature range. The choice of materials which are also lightweight and not very expensive is relatively limited. For example, carbon steels are good structural materials at room temperature but become brittle at low temperatures. The deterioration of mechanical properties at low temperature is common of all the body-centered-cubic (b.c.c) metals, such as chromium, iron, molybdenum, vanadium, ferritic stainless steels and nickel steels.
The best metals for structural parts for cryogenic applications are those with a face-centered-cubic (f.c.c) structure, since they have a high fracture toughness over the entire cryogenic temperature range. These metals include Aluminum, Copper, noble metals and austenitic stainless steels. In particular, the AISI 300 stainless steels are popular for cryogenic applications (131). For example, types 301 and 310 are considered metallurgically stable for all cryogenic conditions, and they are used for applications where maximum stability and toughness is required, such as liquid oxygen and liquid hydrogen tanks for aerospace applications. Type 304 and 304L are used (annealed) for tubing, pipes, and valves to transfer of cryogens, storage tanks, and components that don’t require high strength (162). Still, the strength of these steels strongly depend upon their treatments: for example the yield strength\(^1\) at 77 K of annealed 304 steel is roughly 400 MPa (163) while that of the cold-rolled hardened version is higher than 1400 MPa (162). While cold reduced 310 steel has a better yield strength than 304 steel, 304 steel has a better thermal conductivity (Fig. 6.5). To maximize the thermal conductivity, instead of using steels, one can use aluminum. In particular aluminum 2014-T651 precipitation-hardened has an high yield strength (550 MPa at 77 K (131) and 414 MPa at room temperature) and a thermal conductivity more than an order of magnitude bigger than steels.

Assuming a maximum pressure of 40 bar and a maximum stress equal to half the yield strength at room temperature, we estimated a wall thickness for the 6 m pressure vessel of about 2 cm using 304 steel and about 6 cm using Aluminum 2014-T651 precipitation-hardened\(^2\). In both cases the weight of the outer wall

\(^1\)The yield strength is defined as the stress necessary to permanently deform the material by 0.2%.

\(^2\)A study on fatigue, in particular of aluminum, is also necessary.
Figure 6.5: Properties of metals at cryogenic temperature.
References Thermal conductivity: 304 and 310 (164) - Al2014-T651 (165)
References Yield Strength: \(^1\) (162) - \(^2\) (131) - \(^3\) (166)
would be about 15 tons\(^1\). As we will present in section 6.4.3, the choice of the material depends upon the choice of the flows that we will generate. For example, the lower thermal conductivity of stainless steel is not an issue unless we want to generate extreme Reynolds number von Kármán flows.

6.4.2 Cavitation

Cavitation is the process of vapor formation in a liquid subject to reduced pressure, i.e. corresponds to the process of boiling as a result of a change in pressure rather than heating. In general cavitation should be avoided to maintain a controlled environment and avoid structural damages to the experiment. Of course the problem doesn’t exist when working in the gas phase.

The cavitation number \( Ca \) is defined as the ratio between the difference of the fluid pressure \( p \) with respect to the vapor pressure \( p_V \) and the head loss:

\[
Ca = \frac{\frac{1}{2} \rho U^2}{p - p_V}.
\]

The larger the cavitation number, the less likely the event of cavitation. However, estimating the cavitation inception is not straightforward and depends case by case. For our estimate we assume that \( Ca > 10 \) is safe to avoid cavitation, as often the cavitation occurs for \( Ca \sim 1 \) (167). Hence, the propeller moving at 10 m/s, which gives a pressure head of \( 4 \times 10^4 Pa \) (\( 2.5 \times 10^4 Pa \) with liquid nitrogen at 20 bar), would require to pressurize the fluid 4 bar above the vapor pressure to avoid cavitation problems. From this preliminary analysis it follows

\(^1\)As reference the price per ton of 304 steel is roughly $1000 and $5000 for the raw material in the standard and hardened version respectively (2012).
that cavitation issues in such facility could be addressed without particular problems.

6.4.3 Heat transfer calculations

To estimate if it is possible to dissipate the heat generated by the turbulent flows, we consider the heat transfer from the cryogenic nitrogen that acts as a test fluid, to the liquid nitrogen acting as a coolant. To estimate the heat transfer from the test fluid to the pressure vessel wall $h_{\text{fluid}}$, we assume turbulent heat transfer over a flat plate. We consider the following relation for the Nusselt number $Nu = \frac{1}{2} C_f Re Pr^{1/3}$ that is considered valid for $Pr > 0.5$ (168). We estimate the skin friction coefficient $C_f$ using the empirical relation $C_f = (2\log_{10} Re - 0.65)^{-2.3}$, which is valid up to $Re = 10^9$ (136).

The heat transfer coefficient across the wall $h_{\text{wall}}$ is simply the metal thermal conductivity divided by the wall thickness. We estimated that $h_{\text{wall}} = 0.4 \times 10^3 W/m^2K$ for the 2 cm 304 steel and $1.5 \times 10^3 W/(m^2K)$ for the 6 cm Al 2014-T651 wall.

To estimate if the evaporative cooling system would be able to dissipate the heat generated by turbulent dissipation, we estimate first the heat flux per unit area on the pressure vessel\textsuperscript{1}. Hence we estimate the temperature difference necessary between the pressure vessel wall and the cooling nitrogen at 77 K to dissipate all that heat. By knowing the minimum temperature of the pressure vessel achievable for such power and knowing the total heat transfer coefficient $h_{\text{total}} = 1/(1/h_{\text{fluid}} + 1/h_{\text{wall}})$, we calculate the maximum amount of heat that is

\textsuperscript{1}It is safe to assume to operate in the nucleate-boiling regime as the transition to film boiling for such facility would occur for a power exceeding 20 MW.
possible to transfer from the test fluid to the refrigerant liquid nitrogen. As an example we report in Fig. 6.4 the value for a von Kármán flow, which is probably close to the most turbulent flow that it is possible to generate in such apparatus.
Visualization of vortices in a counterflow

7.1 Visualization of quantized vortices

The extremely small difference in index of refraction between the atomic sized hollow cores and the surrounding helium makes quantized vortices practically invisible. The only way to visualize them is using tracer particles. The first visualization of vortex lines has been done using ions by Williams and Packard in 1974 (169). These so-called ions are semi-macroscopic objects, much larger than the bare ion, with radii of the order of 1nm. The negative ion consists of a bubble that surrounds the electron. The cavity is due to the short-range repulsive force between the electron and the electron clouds of the He. The positive ion is instead made of a cluster of helium atoms bounded to the central ion (170). These tracer particles were imaged after accelerating them onto a phosphor screen located at the free surface of the fluid.

Another technique developed by Guo et al. (171), relies on the visualization
of the electron bubble by increasing its size up to 10 µm by hitting it with an acoustic pulse. By synchronizing a flash lamp with the ultrasonic transducer it is possible to take pictures of these bubbles. The zigzagging trajectories of some of the bubbles suggest that the bubbles were trapped on the vortices (See Fig. 7.1A).

As described in Chapter 4, in 2006 the Maryland group developed a new technique (1) to visualize the quantized vortices using micron-sized hydrogen ice particles (see Fig 7.3). Following the successes of this technique, new groups have started to use it.

A new approach that is under development uses excited He molecules as tracers (172). The technique has already been used successfully to track the normal component by McKinsey et al. (173) but not yet to study the dynamic of the vortices. Indeed the particles are too small to stay trapped on the vortices for the temperatures explored so far, as it will be explained in the next section, and they will need to be in liquid helium below 1K to track the vortices.

While it does not really visualize the vortex dynamics, it is worth mentioning a recent technique by Lebedev et al. (174): by ablating gold nanoparticles from a target into the He II that became trapped on the vortices, they observed the creation of gold nano-wires (See Fig. 7.1B). The detailed mechanism of filaments creation is still unclear, but in the next section we describe the creations process of frozen particles filaments which may be analogous.

### 7.2 Particle trapping

The basic particle trapping mechanism, that can be applied to all the aforementioned tracers, was explained by Parks and Donnelly in 1966 (175). The main
role in the trapping is given by the Bernoulli pressure around the vortex

\[ P = -\frac{\rho_s \kappa^2}{8\pi^2 r}. \]

While the pressure forces are the same as in a classical line vortex, since superfluid flow has zero viscosity, the particle is not dragged around the vortex. Hence, in a superfluid, all particles, independently of their density are drawn to the quantized vortices, while in a viscous fluid denser particles are expelled to the boundaries. When a particle is trapped on the core, decreases the energy of the vortex line by a factor

\[ \Delta \epsilon = \frac{\rho_s \kappa^2}{4\pi} d \ln \left( \frac{d}{2\delta_V} \right), \]
where $\delta_V \sim \xi$ is an effective core size and $d$ is the diameter of the particle. The decrease of energy is equal to the modulus of the trapping force times the diameter of the particle. Hence, the trapping force per unit volume decreases rapidly as the diameter increases.

Beside the trapping force, a particle trapped on a quantize vortex is subjected to both the Stokes drag from the normal component and gravity. For large velocities, the Stokes drag can pull the particle out of the vortex. The balance between the trapping force and the Stokes drag gives rise to a threshold velocity $v_{max} = \rho_s \kappa^2 \frac{\pi \mu d}{9\pi \mu d}$ over which the particle cannot stay trapped. This relation suggests that smaller particles stay trapped on faster moving vortices. Through the dependence of $\rho_s$ on $T$ it implies also that smaller particles stay trapped closer to transition (see Fig. 7.4). This last statement is not true if the diameter is so small that $k_B T > \Delta \epsilon$, where $k_B$ is the Boltzmann’s constant, because then the thermal energy can release the particles (176) (this is the case with positive ions above a temperature of roughly 1K or the excited helium molecules).

Particles accumulate on vortices until the filaments are completely covered in ‘ice’. Qualitatively we can distinguish three stages (see Fig. 7.2): a sparse-particles stage when few, sparse particles are trapped on a vortex; a dotted-line stage where several particles are trapped on a vortex and they appear often to be equally spaced (as reported already by Bewley (59) and Paoletti (46)); a frozen-filament stage when the vortex is completely covered in ice, and there is no visual separation among particles. These frozen-filaments do not behave any longer as
\[ V_{s\Phi} = \frac{\kappa}{2\pi r} \]

\[ P_{Ber} = -\frac{\rho_s V_{s\Phi}^2}{2} \]

\[ F_{\text{trap}} = \int_V \nabla P_{Ber} \, dV \]

**Figure 7.2:** Diagram of the mechanism by which the tracers get trapped on the quantized vortices.
Figure 7.3: Several images of frozen particles trapped on vortex lines. Each image is about $2.5 \, mm \times 2.5 \, mm$. 
Figure 7.4: Plot of the threshold velocity over which the particle cannot stay trapped on the vortex as a function of particle diameter for different temperatures of HeII.

quantized vortices but wander into the fluid as a structure who’s dynamics is dominated by the Stokes drag of the normal component. Note that we observed vortices covered by a thin continuous layer of ice still able to reconnect, so more than the inter-particle spacing on the vortex, the total ice mass and thickness matters in affecting the ability of a filament to reconnect. Sparse-particles vortices and dotted lines are able to reconnect with free vortices and among each other. However, froze-filaments are not able to reconnect among each other and instead they attach together when in contact. Moreover, they cannot reconnect even with dotted lines, but they stick to them. In particular, we believe that this
mechanism is the main responsible to the formation of complex multi-branched structures. Indeed, while frozen-filaments can get in contact with each other, it is much more likely that they get in contact with dotted lines, which travel across the fluid space due to reconnection events. The movement of a vortex after a reconnection increases dramatically the likelihood of getting in contact with another vortex/filament structure. For the same reason, vortices that had just reconnected are much more likely to reconnect again. Hence we believe that reconnections events are crucial in the formation of these complex-filaments structures. The same mechanism can apply to the nanowires formation described in the previous section. A photo-sequence of a prototypical event of a structure-formation is reported in Fig. 7.5.

7.3 Thermal counterflow

A much studied configuration in quantum turbulence is the so called thermal counterflow. The configuration is made by a pipe or channel open on one end and closed on the other by a heater which dissipates a heat flux $q$ (Fig. 7.6). Because

![Figure 7.5: Photo-sequence of a prototypical event of frozen filaments structure formation. In the first image vortices 1 and 2 reconnect. Vortex 1 and two separate one from the other. In the last image, vortex 2 gets caught by the frozen-filament structure 3.](image-url)
all the entropy and heat are carried by the normal component, the imposed heat
flux \( q = \rho T S v_n \) is carried away by the normal component (177). To conserve
mass the superfluid velocity \( v_s = -v_n \rho_n / \rho_s \) align oppositely to the heat flux.
The motion of the two components excites the formation of a vortex tangle,
which moves in the direction of \( v_s \) with velocity \( v_t \). Since the pioneering work of
Vinen (178), several studies have been done on the decay of vortex tangle and the
line vortex density in counterflow turbulence (for a review see for example (179)
(180)). In the late ’50, Vinen and Hall showed that the two fluid equations, while
in good agreement with the experiments in the laminar region, cannot define the
dynamic state of the turbulent superfluid helium, because the equations do not
include the local interactions between the normal component and the quantized
vortex lines. The magnitude of the force by the interaction of the normal fluid
on one vortex line is

\[
F_L = \frac{B_V \rho_s \rho_n \kappa}{2 \rho} |v_n - v_s - v_L|
\]

where \( B_V \) is a temperature dependent coefficient, \( v_L \) is the velocity of the line
and where we neglected geometric factors. It can be shown that the two fluid
equations can the be rewritten as

\[
\rho_s \frac{\partial v_s}{\partial t} + \rho_s (v_s \cdot \nabla) v_s = -\frac{\rho_s}{\rho} \nabla p + \rho_s S \nabla T + \frac{\rho_n \rho_s}{2 \rho} \nabla (v_n - v_s)^2 - F_{ns} \quad (7.1)
\]

\[
\rho_n \frac{\partial v_n}{\partial t} + \rho_n (v_n \cdot \nabla) v_n = -\frac{\rho_n}{\rho} \nabla p - \rho_s S \nabla T - \frac{\rho_n \rho_s}{2 \rho} \nabla (v_n - v_s)^2 + F_{ns} + \nu \nabla^2 v_n \quad (7.2)
\]
where, under the assumption of homogeneity of vortex lines, the mutual friction term which takes into account of $F_L$ is

$$F_{ns} = \frac{B_V \rho_s \rho_n \kappa}{2 \rho} \ell_0 v_{ns}. \quad (7.3)$$

\(\ell_0\) is the line vortex density at steady state and \(v_{ns} = |v_n - v_s|\). Under the same hypothesis of homogeneity Vinen modeled respectively the growth and destruction of vortex line density as \((d\ell/dt)_p = \chi_1 B_V (\rho_n/\rho) \ell^{3/2} v_{ns}/2\) and \((d\ell/dt)_d = \chi_2 \kappa \ell^{2}/2\pi\), where \(\chi_1\) and \(\chi_2\) are two phenomenological parameters. The change in line vortex density is then given by the Vinen equation

$$\frac{d\ell}{dt} = \chi_1 B_V (\rho_n/\rho) \ell^{3/2} v_{ns}/2 - \chi_2 \kappa \ell^{2}/2\pi. \quad (7.4)$$

In the steady state case the line vortex density grows as the relative velocity squared

$$\ell_0 = \gamma^2 v_{ns}^2 \quad (7.5)$$

where \(\gamma = \pi B_V \rho_n \chi_1 / \kappa \rho \chi_2\).

### 7.4 Counterflow visualization

Previous visualization experiments of counterflow have shown different results which do not agree.

In the work by Zhang and Van Sciver (4) it is reported that the velocity of the particles is roughly one half the predicted velocity of the normal component. The authors conducted a PIV study with polymer spheres with a diameter of 1.7\(\mu\)m and density of 1100\(kg/m^3\) as tracers particle. Over the explored param-
eter range $110 \leq q \leq 1370 \text{mW/cm}^2$, they observed that the velocities of the particles, after taking into account for the velocity due to density mismatch, was roughly half the predicted velocity of the normal component. This observation was roughly independent of the temperature. This difference with the predicted velocity was attributed by the author to a body force to model the exchange of momentum between the particles and the quantized vortex lines, that reproduce the temperature independence of their observations.

In contrast, the following work by Paoletti et al. (3), for a range of heat fluxes between 12 and 91 mW/cm$^2$, shows a different result. Using the micro-sized hydrogen particle technique described in section 4.7, they could observe two different motions: the motions of the particles dragged on the normal component,
and the motion of the particles trapped on the vortices. Measuring vertical velocity distributions they found a bimodal distribution, with the two peaks of the distribution corresponding to these two different motions. The velocities at which the peak for the particles moving upward, dragged by the normal component, is in agreement within experimental error with the predicted velocity of the normal component according to the two fluid model. The vertical velocity of associated to the peak of the particles trapped on the vortices was found to be consistently lower than the superfluid velocity. This discrepancy increases with heat flux. Note that while the velocity of the tangle is slightly lower than the superfluid velocity, this difference does not account for the experimental discrepancy. Another effect that partially explains the discrepancy is that the particles could slide on the vortices, as confirmed using a vortex filament model by Adachi and Tsubota (181). Paoletti also observed a temperature dependance of the velocity distributions: higher peaks in the distribution corresponding to the particles dragged by the normal component and lower broader peaks for the particles trapped on the vortices for higher temperatures. This observation is consistent with the fact that the trapping force decreases for temperature approaching $T_\lambda$.

In (3) the discrepancy with the experiment by Zhang is attributed to different analysis techniques. Indeed, because of the nature of the PIV technique, it is not possible to separate the effects of the particles moving with the normal component by the erratic motion of the particles trapping on the vortices. Other hypotheses are based on the particle-vortex interaction mechanism, investigated in several numerical and analytical works (182, 183, 184, 185, 186, 187). In particular in the review by Sergeev and Barenghi (2) the difference is explained by the existence of two different regimes in the particle-vortex interaction process, with the the ratio
of particle diameter to the intervortex spacing $d/\delta$ as the controlling parameter.

In the very recent experimental counterflow observation (188), Chagovets and Van Sciver report a bimodal distribution of the measured vertical velocity “for most velocities of the flow”, reproducing the results by Paoletti. The counterflow reported were for an applied heat flux from $0.7$ to $100\, \text{mW/cm}^2$ for temperature from $1.55$ to $2\, \text{K}$. The bimodal distribution is attributed, as in (3), to the particles dragged upward by the normal component and the particles interacting with the vortices. For the first set they state that the particles have straight trajectories, and that such behavior “does not change with increasing power applied at the heater”. They keep the distinction between particles dragged by the normal component and particle interacting with the vortices, even if for higher heat fluxes they could not observe any particle moving according to the velocity of normal component. They explained the absence to the particles by assuming that the particles left the field of view faster than the camera could collect enough images. For the second group of particles, they observe that for low heat fluxes the particles are coupled with the velocity of the superfluid component. With an increase of heat flux, the particles start moving slower up to a point when they change direction by 180 degrees, moving in the direction of the normal component, but slower than the first group of particles. In the paper they also report events where a particle that is first trapped on a vortex becomes un-trapped and becomes dragged by the normal component. The explanation given lies in the balance between the Stokes drag and the vortex trapping force explained in section 7.2. Still, the observations for these un-trapping events start to occur at velocities of the normal component about an order of magnitude lower than the predicted transition. A hypothesis to explain such discrepancy is the fact
that the trapped particles are bigger than the average size due to agglomeration caused by the vortices, but further studies on the subject are suggested. In the next section, we report some newer counterflow studies which suggest a different interpretation of the aforementioned observations.

### 7.5 New counterflow observations

In order to visualize the counterflow for a broader spectrum of heat fluxes, overlapping the values explored in (3, 4), we used the glass cell described in section 3.3. The purpose of the cell is twofold: the liquid helium surrounding it acts as a thermal bath for the channel and allows one to reach higher heat fluxes without boiling helium; at the same time the cell isolates the system from heat leaks and unwanted flows allowing for a ‘quieter’ system to measure lower heat fluxes. To understand qualitatively the evolution and the change in behavior of the particles

![Figure 7.7](image_url)

**Figure 7.7:** The plots represents the time-delayed plot of the vertical velocity for $q = 40 \text{ mW/cm}^2$ and $q = 60 \text{ mW/cm}^2$. In this case $dt = 30 \text{ ms}$. The difference of the two plots is an evidence of the more erratic behavior of the particle at higher heat fluxes.
at different heat fluxes, we performed some counterflow experiments ramping the heat flux. The ramps ranges from 0 to roughly $200 \text{mW/cm}^2$ and temperature from $T = 2 \, K$ to $T = 2.1 \, K$ during periods of $10 - 60 \, s$. Initially all the particles are at rest. Then a few particles get trapped on the vortices, while most particles are dragged by the normal component. As the heat flux keeps increasing, more and more particles get trapped (and obviously moving faster) and some particles get un-trapped from the vortex, as reported also in (188). Gradually, more and more particles start interacting with the vortices and, at the same time, getting dislodged from the core. This can be seen from the difference in the time-delayed plot of the vertical velocity between $q = 40 \, \text{mW/cm}^2$ and $q = 60 \, \text{mW/cm}^2$ in Fig. 7.7. Increasing $q$ further, it looks like all the particles are interacting with the vortices and moving in an erratic motion. The tracks of this irregular motion of the particles is shown in Fig. 7.8. According to our observations, there are no

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure7.8.png}
\caption{Particle trajectories for $q = 40 \, \text{mW/cm}^2$ and $q = 120 \, \text{mW/cm}^2$ and $T \simeq 2 \, K$. For the high heat flux case, the tracks display an erratic motion of the particles.}
\end{figure}
particles in this regime moving just with the normal component, as hypothesized by Chagovets and Van Sciver(188). Indeed, the two peaks of the bimodal distributions for the vertical velocity of the particles becomes less and less pronounced as $q$ increases, up to a point where they merge into a single peak (Fig. 7.9). Moreover, we observe a large number of events where the particles have a sudden change of direction, that can be interpreted as a particle-vortex interaction. As evidence, Fig. 7.10 shows the tracks where particles have accelerations bigger than a threshold, set to $1.5 \text{ cm/s}^2$.


**Figure 7.10:** The image represents the trajectories for $q = 28\text{ mW/cm}^2$ and $q = 114\text{ mW/cm}^2$ of particles with an acceleration above a threshold (in this case 0.015 $\text{m/s}^2$). The red dots represent the end of the particle trajectory, and the black circles the location where the particle acceleration passed the threshold.

### 7.5.1 Discussion and toy model

Based on these observations, we believe that for low heat fluxes it is possible to make a clear distinction: some particles are dragged by the normal component unaffected by the tangle, while other particles are moving on a vortex for all the time that they are visible (before going out of the laser-sheet or out of the field of view). For moderate heat fluxes, roughly $q = 50\text{ mW/cm}^2$, this distinction becomes fuzzier: particles ‘jump’ from one category to the other by trapping and un-trapping events. For heat fluxes above roughly $q = 100\text{ mW/cm}^2$, we think that the distinction becomes fictitious, as all the particles in the observed field of view are moving in an erratic motion, shaking up and down. Instead of this distinction between trapped and un-trapped, we can use a more general approach to explain the observations. We introduce two parameters: the number of interactions $N$, which depends upon the line vortex density (which is a function
of the heat flux $q$ and temperature $T$), and the trapping time $\tau$ which depends
on the trapping force (which is a function of the heat flux, the temperature and
the diameter of the particle). Increasing $q$, and keeping the other parameters
constant, $N$ increases but $\tau$ decreases. We can then interpret the observation
assuming that for low $q$ the particles look either trapped or un-trapped on the
vortices because of the few interactions but long trapping time, while for high $q$
there are a lot of interactions but for a short time (Fig.7.11), up to a ‘scattering’
regime.

The results seems to confirm that the discrepancies should be attributed to the
particle interaction process in different regimes as predicted in (2).

The large number of interactions also lead us to believe that the un-trapping
mechanism is closely related to the interaction with other vortices. A vortex
passing nearby a particle trapped on another vortex can, in principle, ‘kick’ the
particle out of the potential well and make it untrapped. This mechanism would
explain why the simple balance between the Stokes drag and trapping force in
(188) predicts that the particles become un-trapped for much higher velocities
than the observed ones do. In particular, if the Stokes drag is the main un-
trapping mechanism, then the smaller particles, less affected by the drag, should
smoothly track the normal component for the heat flux range explored. This
seems not to be the case: some of the sub-micron particles are strongly affected
by the increase in heat flux, displaying a vigorous ‘shaking’ motion with a large
number of ‘un-trapping events’. We believe that the Stokes force is the main
un-trapping mechanism when the line vortex density is low, but it is just part of
the whole picture when the tangle is dense. Using the data reported in (189) we
estimated that for $T = 2 K$, in our cell the $TI - TII$ transition would occur for
Figure 7.11: Schematic diagram explaining the toy model developed to explain the counterflow observations. On the left the track of a particle that becomes trapped and untrapped twice.

$q$ of about 1-2 $mW/cm^2$, implying that that all of our measures are in the $TII$ state. Hence for $T = 2K$ we used the value $\gamma \approx 300 s/cm^2$ (189) and estimated that the intervortex spacing is roughly $200 \mu m$ for $q = 20 mW/cm^2$ and $42 \mu m$ for $q = 100 mW/cm^2$. Note that these values are higher than the values used to simulate the particles vortex interaction both in (183) and (190).

Additional observations and a more in-depth analysis are necessary to prove the model and role of the interactions in the un-trapping events.
Conclusions

8.1 Summary

In this thesis we presented the study of the use of frozen particles for visualization and velocimetry of turbulent flows in cryogenic liquids.

In Chapter 1 we presented a brief historical introduction to turbulence and to the Navier-Stokes equation to motivate our work. To understand the difference between classical and quantum turbulence we reviewed in Chapter 2 the theoretical and experimental background on superfluid helium and quantized vortices.

In Chapter 3 we described the experimental setup, adapted from the one described in (46) and (58), used to visualize both liquid nitrogen flows and superfluid helium flows using frozen tracers. The experimental control and data acquisition setup, entirely rebuilt and expanded by the author, and the particle tracking velocimetry method are also presented.

In Chapter 4, after a brief introduction on the fluid dynamics of tracers, we discussed the creation and characteristics of these frozen particles. We presented our new technique to produce frozen tracers in liquid nitrogen in a cheap
and easy way using common hydrocarbons or even atmospheric air. We gave a demonstration on the creation of sub-micron particles used for both particle image velocimetry and particle tracking. The analysis of different seeding gases shows that propylene and propadiene would create the brightest faithful tracers. Such analysis, based on our estimates on the index of refraction and density of several hydrocarbons, could be useful even for visualization in cryogenic wind tunnels.

We presented an improved technique to generate frozen tracers in liquid helium as small as 200 nm, compared to the micron-sized tracers previously reported. Such smaller particles, being less affected by Stokes drag, can track the dynamics of quantized vortices that are moving faster and closer to the lambda transition. Moreover, we reported the creation of tracers directly into the $HeII$, which allows one to visualize the vortex dynamics at temperatures lower than ever before. The chapter includes the estimates developed by the author to characterize these tracers, and the tests conducted to verify their validity.

In Chapter 5, after reviewing the use of cryogenic fluids in high Reynolds number experiments, we presented the prospect of using liquid nitrogen. The analysis, conducted by the author, includes the advantages and disadvantages of liquid nitrogen compared to other test fluids, and the study of several possible liquid nitrogen experiments for different flow configurations. For such experiments we evaluated their feasibility and compared the parameters attainable with existing experiments. The flows considered include a grid flow at the resolution of the dissipation scale at Reynolds number $10^7$ and $10^8$, a pipe flow, a Taylor-Couette and a towing tank experiment. The analysis suggests that liquid nitrogen would be a good choice for simple experiments, when it is necessary to reach high
Reynolds numbers and size is a constraint. Liquid nitrogen is inexpensive and relatively easy to handle and it could reduce the size of the apparatus by factors of 5-10 compared to water. Existing experiments could be adapted to run with liquid nitrogen and new high Reynolds number experiments could be possible due to the much smaller size of the projects.

In Chapter 6 we analyzed the feasibility and merit of a pressurized cryogenic nitrogen cell 6\,m in diameter and 5\,m tall. The cell would allow one to reach Rayleigh numbers up to $10^{18}$, a value never reached before in a controlled environment.

In Chapter 7 we reported the work on quantum turbulence. After reviewing the different techniques to visualize quantized vortices, and the mechanism by which tracers get trapped on vortex cores, we described a new model proposed by the author, that explains new and old counterflow observations, and that sheds light on the interaction between particles and vortices.

8.2 Future work

8.2.1 Liquid helium

The next step is to test the toy model with additional data and a more extensive analysis to conclude the work on the counterflow observations.

Possible future investigations include include the study of near transition phenomena using the sub-micron particles, the study of the $TI - TII$ transition and the use of fluorescents nanoparticles as tracers.
8.2.2 Liquid nitrogen

In order to build a large scale facility using pressurized liquid nitrogen as a test fluid, several steps are recommended.

The first one is building a small scale facility as proof of concept, in order to conduct velocimetry using pressurized liquid nitrogen and test the tracers. An important goal would be a detailed experimental study of the frozen tracers in liquid nitrogen at different temperature and pressure, using the different seeding gases proposed in this thesis.

The ideal experimental setup for this goal has the following characteristics:

- a pressure vessel good up to 35 bar to allow the use of liquid nitrogen up to the critical point,
- a precise injection system to test various mixtures using different seeding gases at different dilutions
- a working filtering system to remove the particles from the fluid
- an optical setup that would allow an accurate measure of the particle size
- a system to drive a turbulent flow (towing grid or propellers)
- a surrounding jacket to fill with liquid nitrogen at 77 K to test the temperature control system

The same system could be used with pressurized cryogenic nitrogen gas at various temperature to find the best tracers to use in cryogenic wind tunnels.

After proving that using pressurized liquid nitrogen is possible and convenient, a precise large scale facility should be identified as the most suitable to be built. The characteristic to evaluate are the possible impact of the research, the cost,
the risk, the logistics, the flexibility and longevity of the apparatus, as well as the expertise required.

A particularly promising project that will be further investigated is the large convection cell described in Chapter 6.
Appendix A

Imaging camera

A.1 Charge-coupled device

A simple analogy to understand how a charge-coupled device (CCDs) works is the "water bucket" idea, where the buckets (the pixels) arranged on a field (the CCD) collect the raindrops (the photons). Once the rainstorm is over, each bucket, moving along conveyor belts, is emptied in a metering bucket (output electronics). By measuring the water coming from each bucket it is possible to provide a 2D record of the rainfall (the CCD image) within the field. In reality the pixels of the CCD collect electrons generated by photoelectric effect by the incident photons. The percentage of photons that hitting the pixel create an electron-hole pair is called quantum efficiency. Each pixel have a series of electrodes, called gates, that allow to hold the electrons in a potential well. By manipulating the voltages on the gates, at the end of each exposure, the electrons are moved from one pixel to the next one, along columns. On each cycle, all the pixels shift down one column at the same time, with the bottom row shifted to the pixels of the output register. These pixels, not exposed to the light, are then
shifted to the output electronics where the charge of the electrons is amplified and measured as voltage. The voltage in the converted to a number, called 'analog to digital units' (ADUs), by the ADC (Analog to Digital converter). The number of electrons necessary to produce 1 ADU is called "analog gain", and it is important in having a quantitative estimate of the photons hitting the CCD.

To increase the device speed and frame rate, frame transfer devices basically have two separate CCDs attached together: the image array, which is exposed to light, and the storage array, which is masked. At the end of each exposure, the image is rapidly shifted from the active to the storage array. While the storage
array is being read by the output node, the image array integrates charge for the next image (191).

Electron-Multiplying CCD (EM-CCD) have an extended portion of the CCD’s serial register with high clock voltages, that allow to multiply the electrons via impact-ionization processes. This charge multiplication can raise the signal above the read noise of the output amplifier, allowing for low light detection of few photons.

A.2 ProEM camera

To measure the analogue gain we have taken several couples of flatfield images at different apertures. Then we plotted the variance of the difference of the two images vs the average signal see Fig. A.2. The gain is the slope of the linear fit (more details can be found at http://www.mirametrics.com/tech_note_ccdgain.htm). The gain value of 2.79 e-/ADU is 15% less than the value stated in the manual.¹

¹Using a different technique that uses just two frames to measure the gain we have got even lower values.
Figure A.2: ProEM camera measured analogue gain. The gain is represented by the value of the slope of the linear fit.
Appendix B

Size estimates

We estimate the scattered intensity ratio $I/I_0$ of a particle of diameter $d$ and index of refraction $m$ at $90^\circ$ and $15$ cm calculated from the Mie coefficients $S_i$. We estimate the flux of photons $I_0$ and we calculate the number of photons hitting the CCD during one exposure time. Knowing the camera characteristics and settings, we can estimate total number of ADUs due to the light of the particle (see the Matlab code ADU_gauss.m for details). We then calculate an array with the numbers of ADUs that would generate particles for $40,000$ particles of diameters from $1$ nm to $10$ µm.

We then measure the total number of ADU due to light scattered by the particle under study. This can be manually done adding the ADU from each pixels in which the particle is present, after removing the ADUs due to the background. To do it automatically we used a particle detection algorithm that after identifying the particle, calculate the 'mass' of the particle in ADUs. Finally to estimate the size, we compare the number of ADUs of the particle with the array generated.
% calculate intensity level on camera (ADU) from scattering intensity
% It assumes that the particle is in the center of the lasersheet
%
% input : intensity , gain , exposure time , laser power , wavelength
% output : ADU from orthogonal component , from parallel component , total

% NOTE: % Use mieinten.m for the intensity
% Distance from lens taken into account in mieint
% (15cm from sheet to lens)

% Enrico Fonda – enrico.fonda@gmail.com – 2010

function out=ADU_gauss(intensity,gain,expo,lp,lambda)

if nargin <5, lambda = 532; end % (nm)
if nargin <4, lp = 3; end % (mW) laser power
if nargin <3, expo = 58; end % (ms) exposure time
if nargin <2, gain = 0.8 % EM Gain
    contgain = 0.8; end % controller gain 3 – low noise 0.8
if gain==0.8
    gain=1; % EM Gain
    contgain=0.8;
else
    contgain=3;
end

h = 10^-34*6.62606896; %(J*s) Planck constant
%c = 299792458; % m/s speed of light
shx = 0.82; % (cm) sheet length
shy = 0.82; % (cm) sheet height
rho0 = 90; % 1/e^2 radius of the waist
windows = 0.85; % losses due to windows (4% each surface)
arealens = pi*(1.8*10^-2)^2; % (m^2) area of the lens / d=3.6cm

switch lambda
    case 532, qe = 0.95;
    case 405, qe = 0.6;
    case 650, qe = 0.93;
    otherwise, disp('error')
end
eph = c*h/(lambda*10^(-9)); % energy of each photon at wavelength l
ash = sqrt(pi)*rho0*10^(-6)*shy*10^(-2); % (m^-2) area laser sheet normalized to gaussian
Win2 = (1p*10^(-3))/ash; % (W/m^-2) incident intensity i.e. power per surface area

% if particles on center of the lasersheet

phos = Win2/eph; % (photons/(s*m^-2)) incident photons per unit time and unit area

10 = phos*expo/(1000); % (m^-2) photons per m^-2 during one exposure time

photons(1) = arealens*10*intensity(1);
photons(2) = arealens*10*intensity(2);
out(1) = windows*qe*photons(1)*gain/contgain;
out(2) = windows*qe*photons(2)*gain/contgain;
out(3) = out(1)+out(2);

codes/ADU_gauss.m
Appendix C

Laser-sheet

Compared to previous setup using AAA batteries, the new laser-sheet setup allows for uninterrupted illumination and a more constant power output. Some data has been taken with a 50 $mW$ green laser (and laser diodes of 405 $nm$ and 650 $nm$ has also been tested) but, if not stated otherwise, the data refers to the 4 $mW$ green laser.

To measure the intensity of the laser-sheet we used a Molectron EPM 1000. The output of laser is highly fluctuating after being turned on but it becomes quite stable after 30 minutes, and it becomes stable at its maximum power after about an hour of operation. The stability depends on the voltage at which the specific laser diode operates, so after replacing a laser it is suggested to re-tune the power supply to match the optimal voltage for maximum stability. With the current setup, after one hour after being turned on, the laser power at the tip of the pointer varied between 4$mW$ and 4.4$mW$ over one hour period. After passing through the two lenses and the aperture the power measured has been measure between 1.8 and 2.4 $mW$. After passing through the 6 windows of the cryostat
and then 2 microscope slides of the cell, the power was attenuated roughly by another 25% to 1.4-1.8mW. We can then assume that, after a break-in period of an hour of the laser and power supply, the power in the test section is between 1.5 and 2.1mW.

Note: the light at 532 nm is generated by frequency doubling the output of 1064nm of a Nd:YVO4 laser crystal, pumped by an high power infrared diode at 808nm. For this reason we expected the output light, even if usually is filtered by a dielectric mirror, may have some IR component that would have just the unnecessary consequence of heating the system. For this reason we tested the power output of the laser with a IR filter (Tiffen standard hot mirror) and with a 1064 mirror. The measured power output of the light has been reduced by 10% by passing through each of these components.

To measure the spatial inhomogeneity of the laser-sheet we filled a glass cell with water and 50 nm polystyrene latex (PSL) particles and image the resulting pattern at an 90°. The resulting pattern is shown in Fig. 3.4. The difference in luminosity from the darkest (16 bit greyscale 2233) from the brightest area (16 bit greyscale 5128) is within a factor 4 after subtracting the baseline. This pattern is fixed in time but potentially, during a real run, particles getting stuck on the test section window or on the glass cell wall can create additional dimmer areas. The width of the laser-sheet can be estimated using Gaussian beams optics calculations but we measured it to have a direct observation of the actual sheet. In fact, as several possible definitions of width are possible (192), we measured the $1/e^2$ width (193).

The setup is similar to the one used for measuring the inhomogeneity in the $x-y$ direction with the exception that the sheet has been rotated 90°. We images
the light scattered by the particles and using a gaussian fit, we estimated the $1/e^2$ width to be 175 $\mu m$.

A particle which is 50 (100) $\mu m$ from the center has an intensity of 0.6 (0.13) times that of the center.

Assuming that the laser-sheet in uniform in $x$ and $y$, we can compute the intensity of the light as

$$I(x, y) = I(y) = \frac{\gamma_{tot}}{(x_2 - x_1)^2 + y^2} e^{-\frac{x^2}{\sigma_0^2}}.$$ 

We considered the possibility of cutting the tails of the gaussian distribution using a slit to make ‘squared’ laser-sheet. To see if it would be possible we performed a fresnel diffraction calculation and looked at the profile of the laser-sheet for different position of the slit. Setting a slit on the inner cell it would be 14.1 $mm$ from the end of the field of view. Such configuration, as shown in Fig. C.1 would not provide much benefit compared to the current configuration. The profile stay

![Figure C.1: The figure present the simulated profile of the laser-sheet at 14.1mm and at 1mm from a 150$\mu m$ slit - the gaussian profile of the current setup is also presented for comparison.](image-url)
squared just for a few millimiters after the slit, so its use could be beneficial just if the field of view and the channel is reduced to few millimiters in size.
Appendix D

Standard operating procedures
(March 2012)

In this section we describe the procedure to run the experiment using liquid helium and a list of troubleshooting solutions for common problems. The procedure has been adapted from the one reported in Gaff’s thesis (58), with some parts left unchanged.

D.1 List of procedure’s abbreviations

The following abbreviations will be utilized:
cryo = cryostat
ILM = Intelligent Level Meter - Oxford Instruments liquid helium level sensor
ITC = Intelligent Temperature Control - Oxford Instruments temperature control unit
HD = Hard Drive
pneumo = pneumatic
TS = test section
t.t. = transfer tubes
turbo = turbo molecular pump
VJ = vacuum jacket (listed as OVC in Oxford manuals)
D.2 2-3 days before running

1. Pump down the VJ.
   • If the VJ is at 1 atm (i.e. it has recently been opened to atmosphere for repairs), then
     – Open all valves from the cryostat to the roughing pump (tall vacuum jacket needle valve on cryostat V-VJA, V-VJB, valve to turbo pump V_{turbo}). The cryostat and all lines should be at 1 atm.
     – Turn on VJ roughing pump. Wait until pressure is < 100 mTorr.
     – Turn on turbo pump.
     – Let the system pump down until the VJ pressures is 10^{-4} Torr.
   • If the VJ is under partial vacuum (i.e. it has not been opened to atmosphere since the last experiment), then
     – measure its pressure with \( P-VJ \) Pirani gauge by closing \( V-VJB \) and opening \( V-VJA \).
     – If pressure is below about 10^{-2}Torr start pumping VJ with turbo pump.
     – If pressure is higher, start pumping with roughing pump first.

2. Check that there is sufficient liquid nitrogen, liquid and gaseous helium
3. Check laser and optics.
4. Backup data and free HD space.

D.3 1 day before running

System Status:
• VJ at 10^{-4} or below \(^1\)
• VJ roughing and turbo pumps still pumping
• Bath and and test sections still at STP.

Precool the cryostat with liquid nitrogen

1. Setup
   • Plug in Oxford temperature sensor (ITC) (black cord to top of cryostat labeled "SENSOR")
   • Turn on ITC temperature sensor
   • Plug in 24V DC power supply for solenoid injection valves electronics box

\(^1\)While current gauge can show pressure as low as 10^{-4}Torr, the pressure inside the VJ is expected to be much lower.
• Turn on control PC
• Start Matlab GUI
• Check all valves are closed, including pneumatic valve

2. Fill & Flush procedure

• Check that input port seal on bath is tight
• Turn on TS roughing pump (plug in into one of the large grounded silver outlets)
• Open He gas tank - regulator to \( \approx 20 \text{psi} \)
• Check He floor copper tubing gas lines are open
• Open bath and test section valves \((V_{TS}, V_{bath}, V_{cap})\)
• Stream He gas through bath and test section by opening solenoid valve \(V_{injection-He}\) until it comes out from pressure release valve and
• Wait a few minutes
• Stop streaming He gas and pump it out with TS-pump. Track pressure on P-TS gauge
• Repeat until pressure is consistently < 10 mTorr (usually >5 cycles)

3. Last filling cycle, keep streaming helium and insert L-tube into into input port, and turn off TS roughing pump (unplug).

4. Fill bath with liquid nitrogen

• Connect nitrogen Dewar to L-tube (use long insulated copper pipe)
• Close all cryo valves \((V_{TS}, V_{bath}, V_{cap})\)
• Stop streaming He gas
• Fill the bath with liquid nitrogen (takes 10 min)

5. End of the day procedure

• Leave black hose attached until morning
• Unplug 24V DC power supply for silver electronics box
• Make sure gas bottles are closed
• Leave VJ pumps running

D.4 Day of run

System status 1 - 1st day of running:

• Vacuum jacket at \(10^{-4} \text{ mTorr}\) or below
• VJ roughing and turbo pumps still pumping
• Bath section filled with liquid nitrogen (77 K)
• TS temperature = 280 K
System status 2 - subsequent/consequent days of running:
- VJ off (See Notes regarding Corrupted Vacuum Jacket for alternative comments)
- VJ roughing and turbo pumps off
- Bath and TS filled with remnant He gas, boiled off from previous day’s run
- TS temperature \( \approx 140 - 150 \, K \)

Running day procedure

1. Initial Setup
   - Turn on ITC temperature sensor
   - Plug in 24V DC power supply for solenoid injection valves electronics box
   - Turn on He gas to 20 psi

2. If system status 1, cool Test Section with Liquid Nitrogen
   - Fill & Flush the TS with He to remove all atmosphere (water vapor).
   - Open capillary valve (open \( V_{TS} \) \( V_{bath} \) \( V_{cap} \))
   - When \( T = 90 \, K \) (about 10 minutes later) close all cryo valves (\( V_{TS} \) \( V_{bath} \) \( V_{cap} \)) to avoid having \( LN_2 \) in TS
   - Pressurize TS with He gas
   - Disconnect black hose from L-tube
   - Push L-tube all the way down into cryostat until it hits the bottom
   - Reopen just capillary valve to blow off remaining nitrogen from bath with helium streaming through capillary only Use safety glasses
   - Direct air out of L-tube by holding finger over brass nozzle
   - Test section should be completely empty of nitrogen when finished
   - Put on (or hold) welding gloves
   - Loosen input port o-ring seal completely
   - Heat L-tube and remove - o-ring will come off too
   - Place cap back on input port (even with no o-ring if no spare one)
   - Heat the L-tube and get o-ring (and dry it)
   - Place cap back on input port
   - Heat (if necessary) and close all valves

3. Set up for Liquid Helium Filling (to be done in parallel with Fill Flush procedure)
   - Position helium Dewar on forklift and secure with ratchet rope
   - Teflon tape Dewar gas input nozzle
   - Open clear, flexible plastic tubing helium gas line (black knob near TS pump)
   - Fully attach flexible tubing to helium Dewar via large copper nozzle
   - Slightly close pressure release valve on helium Dewar
• Slide brass nozzle to end of long leg of t.t.
• Position ladder and stool

4. Fill & Flush so that nothing except He is in the TS and bath
   • Turn on TS-pump
   • Proceed as previously described until pressure below 5 mTorr
     – as long as there is LN₂ in bath, the pressure doesn’t go below few Torr
     – if pressure doesn’t decrease after several flushes after cooling with nitrogen, consider reinserting L-tube
   • Keep streaming He gas - NOTE: Proceed quickly to avoid rise in temperature
     – turn off TS-pump
     – close all cryo valves ($V_{TS}, V_{bath}, V_{cap}$)

5. Beginning transfer liquid helium
   • Turn on ILM and set rate to Fast
   • Climb ladder and insert long leg of t.t. into Dewar (open Dewar lever)
   • Pressurize helium Dewar by opening and closing the helium gas input lever until gaseous helium will flow out of the short leg of the t.t.
   • While regularly pressurizing the Dewar to maintain a constant flow of helium gas out the t.t., slowly lower the long leg into the Dewar in 6-inch increments.
   • As soon as the gas flowing out of the short leg becomes cold, insert the short leg all the way into the bath, stopping 1–2/cm from the bottom.
   • Re-tighten the input port seal
   • Open bath section valve (sideways knob) so excess gas can flow out of brass nozzle
   • Open test section and capillary valves so helium can flow into and cool test section
   • Adjust flexible tubing knob and/or pressure release lever on Dewar so that helium gas steadily streams out of brass nozzle

6. Liquid helium filling
   • Continue pressurizing Dewar (‘open-close gas’ input lever) so that gaseous stream out of brass nozzle remains strong and steady
   • Lower transfer tubes in 6-inch increments
     – raise Dewar using forklift as necessary
     – try to keep t.t. connecting hose parallel to the floor
   • Track cooling rate
     – track test section temperature and liquid level
     – if lots of ice appear on transfer line, the system is cooling too quickly
     – if no visible/audible stream of helium gas coming out of brass nozzle, the system is cooling too slowly
- adjust the Dewar pressure release level or the flexible tubing knob accordingly.

• As soon as the first drop of liquid helium registers on the ILM
  - Close vacuum jacket valve
  - Turn off turbo pump
  - Turn off VJ-pump after 10-20 minutes (allow turbo to spin down)
  - Close turbo pump valve

• Continue filling with helium until full (ILM = 101.1 consistently)
  - As $T = 40 \, K$, the specific heat drops dramatically and the temperature will begin fall much more rapidly
  - Re-pressurize Dewar whenever ILM level begins to fall
  - Continuing pressurizing the Dewar until $T = 4.2 \, K$
  - Eventually, ILM should reach 101.1%, indicating the bath section is full of liquid helium
  - Check that liquid helium is visible in test section (Should see boiling/bubbles)
  - Once the above stipulations are met, make sure that the level does not fall between ILM fast rate readings. Any gross fluctuations in level indicate a strong thermal gradient, which should be avoided.

7. Transfer line removal
   • Heat valves and input port
   • Put on welding gloves
   • Close all valves on cryo and open pressure release valve on Dewar
   • Loosen input port seal completely so that o-ring doesn’t break and instead remain attached to t.t.
   • Remove transfer tube from cryostat
   • Remove transfer tube from dewar with assistance to lower forklift
   • Cap input port seal (even without o-ring if a second o-rig is not available)
   • warmup with heat-gun the t.t. to remove o-ring and cap
   • Replace input port cap and tighten seal
   • Close top valve/lever on Dewar
   • Change ILM rate to Slow
   • Close flexible tubing knob and turn off helium gas tank

8. Cooling Helium from from $4.2 \, K$ to $T_{\lambda}$ (about 30-40 minutes)
   • Turn on pneumatic valve/ TS roughing pump
   • Open test and bath valves (but not capillary)
   • press Autocool button for automated cooling at a rate of about $1 \, mK/s$
   • prepare visualization setup
– Turn on laser (block it to avoid heating the cryostat windows)
– Turn on camera power supply and open WinView32 or ImageJ camera software so that CCD may begin cooling
  * Ensure that the shutter is closed
  * Open ’Setup - Detect System Temperature’ to check CCD temperature
  * Temperature lock should be set to $T = -70^\circ C$
  * No acquisition mode (Focus nor Acquire) should be activated until the temperature has been achieved and locked for at least 20 minutes

9. Take data!

10. End of the day procedure
  • Close all cryo valves ($V_{TS}$, $V_{bath}$, $V_{cap}$)
  • Disconnect 24V power supply for injection valves
  • Close He gas tank
  • Measure liquid He level
  • Backup data

D.5 Troubleshooting

D.5.1 Pump

• Pump doesn’t work
  – check all that all the valves on the pumping line are open
  – check pump oil
• Pump is grumbling
  – check if exhaust line valve is open
  – check if pumping too much gas (e.g. pumping to open atmosphere or helium out of the tank)
  – check pump oil

D.5.2 Vacuum jacket

If the VJ doesn’t hold vacuum try to understand if it is a real leak or a virtual-leak/outgassing (see Fig. D.3)

• if outgassing problem
  – keep pumping longer
  – backfill VJ with $N_2$ gas
• use long stainless steel pipe
• warm up the cryostat with band-heaters

• if suspect leak
  – leakcheck with alcohol
  – leakcheck with He gas and leakchecker
    • if found leak proceed to fix the problem
    • if no leak is found the best bet is to replace the indium seals of the inner window

D.5.3 Cryostat and helium

• Temperature starts to rise in test section
  – check liquid He level in TS - if empty refill from bath
  – check if injection lines are closed
  – check if all heaters are off
  – check for leaks
• Cannot refill TS
  – check level He in bath
  – capillary valve may be plugged
    • open and close capillary several times
    • close bath valve to increase pressure
• Fog on cell’s windows
  – if before liquid helium is in TS, try fill and flush with He (unlikely to solve the problem)
  – consider aborting experiment and be more careful in flushing the TS with He
• Test section cannot be pumped down to 5mTorr
  – check injection lines swagelock connections
  – leakcheck with alcohol

D.5.4 Laser

• Laser is dead
  – check if all wires are plugged
  – check if button is pressed (underneath tape)
  – check power supply voltage
  – replace laser
• laser is flickering
  – adjust voltage via potentiometer on power supply

D.5.5 Camera

• Frame is dark
  – check if laser is on and that laser-sheet is coming out of the cryostat
  – check if shutter is open
  – check that problem is not just lack of particles
Figure D.1: Temperature trace and liquid helium level from an experimental day using liquid helium. The different parts of the procedure represents those of a typical running day (Note that in this case, after the liquid nitrogen filling, the temperature raised to 154K - while usually reach about 120K). In (A) we report an example of the temperature trace during the ‘data acquisition part’ of the experimental run. In (B) and (C) are respectively reported the temperature and the helium level in the bath for the whole process of an ‘experimental day’. In (D) are shown several examples of temperature traces of the overnight warming up of the test section after the end of the experimental run.
Figure D.2: Drawing of main components of the experimental apparatus.
Figure D.3: The plot shows the outgassing / leak rate in the vacuum jacket for several measures for several conditions of the cryostat. The leak rate is computed dividing the pressure in mTorr at which the VJ raised (from $P < 0.1$ mTorr) by the number of minutes the VJ has been closed. The diameter of the dots is proportional to the cube root of the number of minutes. The blue dot represents the outgassing rate when the system was working properly and its value is marked with a blue line. This value should be use just indicatively as a reference as the outgassing rate depends on how long the system was pumped on.
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