1 Introduction
Traffic systems are generally quite complex. For this reason, in creating their theoretical mathematical model, we would have to take into account an extremely large number of variables and their interrelationships. However, with methods of logical and methodological decomposition, a traffic system may be divided into a finite set of simpler subsystems which are then studied and analysed separately [8]. Each of the subsystems represents an independent system while other subsystems represent its more or less relevant environment. Such a scientific approach is based on the axiomatic principle of describing a traffic system with the set of elementary statements which are true on the basis of evidence of the traffic science theory. In this way consistency, completeness and minimality of the traffic system structure are guaranteed while scientific precision and generality are maintained.

Air traffic can be defined as an undivided and extremely complex dynamic system. When referring to air traffic as a system, we understand its inner structure, i.e. the set of subsystems, elements and their interrelationships as well as their relationships with the environment. An air traffic system is divided and interrelated in vertical and horizontal directions [9]. In vertical analysis five strata are found. They behave as technical, technological, organizational, economic and legal subsystems. In the horizontal direction, the system of air traffic is a synthesis of three subsystems of activities: transport, disembarkation/embarkation and flight control. In this part we are interested in the airport subsystem within the framework of the air traffic system. We are going to deal with it as a system of aircraft, passenger, cargo, luggage and post operations. All the necessary and relevant activities are carried out by airports, which are organized as business companies.

Rapid development of air traffic has emphasized the importance of airports, which have in many places become the bottleneck in the process of air traffic dynamics. In most cases this bottleneck is felt as lack of capacity. Airports have become complex technological and organizational structures which follow the laws of dynamic systems. Therefore technical and technological modernization alone cannot provide satisfying results in optimal exploitation of existing capacities and planning the new ones. For this reason we have to adopt a scientific approach to managing airports as complex dynamic systems.

2 The properties of an airport system
All real systems refer directly to their environment and consist of separate closely linked subsystems. Strict boundaries between the subsystems, the system itself and the environment are not accurately defined, but they depend on the concrete approach to the problem regarding the desired aim. When speaking about the airport system, its technology, i.e. the organization of technological activities at the airport, is its technological subsystem. In this context the relative environment is the whole air traffic system. The air traffic system acts upon the characteristics of traffic flows taking place at airports as well as technological processes that enable these flows. Apart from the structure of the system itself its operation also depends on how the environment is structured and on the intensity with which its elements influence the components of the system. The system environment are all the elements which are not a constituent part of the system but are nevertheless directly linked with its elements. Typical of airport systems is a numerous set of influential elements from the environment. These can be divided into the following groups [2]:

- normative elements,
- economic elements,
- technical elements,
- market elements,
- geographical - climatic elements,
- ecological - social elements.

In the airport system, each group exerts influence on separate subsystems with different intensity. Traffic systems (and all their subsystems) are stochastic, as there are always random variables in their operation. Stochasticity of technological subsystems of a traffic system is conditioned by the very nature of traffic. Traffic system controllers' interest is to study these random processes intensely, and to make them - to a certain degree of accuracy - statistically predictable. In an air traffic system the airport as its subsystem has known infrastructure and terminal capacity, but traffic flow in this system is stochastic. Stochasticity of passenger and aircraft flows causes periodical overloading of facilities, and here a basic question arises of how to solve this problem as on average the capacities are not surpassed by demand. Optimal control has to depend on a clearly defined function of the goal which will be defined in structuring the mathematical model of control.
All real life problems change in time and are therefore dynamic, and so are traffic systems and all their subsystems including the airport system. According to the type of input and output signals, dynamic systems can be continuous or discrete. Artificial systems, such as traffic systems with all their subsystems, are discrete by nature as the events taken as input functions do not take place continually in strict mathematical sense. But if the rule determining the function defined over a given time interval \([0,T]\) is such that it can be applied at almost any point of this interval (definition area), we can replace it with continuous function. In this article we shall make up and solve the mathematical model of system control for continuous functions.

Mathematical models describing long-term time dependence of the airport system operation, can be created to show the balance of circular passenger flows at airports. An important feature of passenger flows is their circularity demonstrated by the fact that one part of the circular flow are passengers beginning their journeys (outgoing passengers) and the other part those who are finishing them (incoming passengers). The general assumption is that the number of passengers in both parts of the circular flow is the same over a long period of time as passenger air transport does not represent mass migrations but mostly return trips. Apart from circular incoming and outgoing passenger flow, transit airports comprise the circular flow of transit passengers as well, and on average this is also balanced. The circular passenger flow consists of four pairs of flows: entrance - embarkation, exit - disembarkation, exit - embarkation and entrance - exit. The incoming and outgoing transit passengers are regarded the same passenger at departure and arrival. Mathematically, this can be expressed as the sum of integrals on a closed path (curve).

\[
N_P(t) = \frac{dN_P(t)}{dt}
\]

We denote:
- the total number of passengers in the unit of time

\[
N_{PD}(t) = \frac{dN_{PD}(t)}{dt}
\]
- the number of passengers arriving in the unit of time

\[
N_{PO}(t) = \frac{dN_{PO}(t)}{dt}
\]
- the number of passengers departing in the unit of time

Then it is

\[
\int_{T} N_P(t) dt = \int_{T} N_{PD} dt + \int_{T} N_{PO} dt = 0 \quad (01)
\]

or

\[
\int_{T} N_{PD} dt = \int_{T} N_{PO} dt \quad (02)
\]

Equations (01) and (02) show that airports are generally studied as systems with balanced passenger flows at departure and exit.

Dynamic system is linear when the transformation of a linear combination of input functions is the same as linear transformation of these input functions. On these grounds traffic systems can be viewed as linear systems. In the case of an airport system the requirement for linearity means solving the problem of passenger flow with optimization of capacity for handling the passengers and aircraft. The critical point in passenger handling is the function of check-in which regulates the entropy of passenger flow (passenger and luggage flow). The linearity of the system enables us to determine the optimum number of check-in counters, so that the flow is coordinated with the timetable and with the operations on the platform. According to the definition of the goal, which represents criterion of optimality, we are calculating optimum control of the airport system. This means that with the existing timetable and with optimal exploitation of available facilities we expect a minimum of delays or minimum total costs and make sure that traffic flow goes on undisturbed.

Linear dynamic systems, including traffic systems with all their subsystems, are time-independent (invariant) when the structure and duration of an output function (signal) do not depend on the chosen start of observation. Weather dimension can move uniformly in one direction depending on the input which in the system produces an equally formed output signal in real time. Linear dynamic time-independent systems are called stationary systems. As everything that happens in traffic systems (and their subsystems) takes place in real time, these systems are stationary. In conditions of stationariness the change in the total number of passengers in a unit of time can be defined as the derivative of the function \(N_P(t)\) if this function is known and differentiable. It is in the interest of the airport to determine its maximum, which means that its derivative must equal zero:

\[
N_P(t) = \frac{dN_P(t)}{dt} = 0 \quad (03)
\]

Function \(N_P(t)\) can be defined empirically from a large number of measurements and statistical surveys. Its form is the criterion of the efficiency of the airport.

### 3 The structure, characteristics and functions of an airport system

In the framework of an air traffic system airports perform two key functions: the first one is infrastructure which allows aircraft to take off and land, and the second one is traffic function which allows passengers to board and disembark the planes. In this sense the airport is divided into two basic functional parts: infrastructure or air part and terminal or ground part. For this reason airports as systems are studied from the point of view of traffic infrastructure and from the point of view of traffic processes. The operation of an airport, and studying it as a subsystem in the system of air traffic, is based on two fundamental functions[2]:

1. Technical function of an airport (ramp handling) which provides the infrastructure and machinery necessary for lan-
ding, parking, servicing and taking off. These functions are usually identified as handling the aircraft.

2. Traffic function of an airport (traffic handling) which refers to technological procedures of embarkation and disembarkation of passengers, luggage, cargo and post.

The technological processes at the airport are determined as a form of traffic-production function to which all the other functions are subordinated. This means that in the theoretical mathematical model it will be necessary to take into account all the data which are essential and relevant for the optimal regulation of traffic, and therefore necessary in planning of the development of traffic technology.

From the point of view of traffic-production functions, the airport system elements are formed into two interrelated groups of technical and technological strata. The technological subsystem consists of elements of organization, information technology and operations. The technical subsystem consists of the following elements: air infrastructure, land infrastructure and fixed and mobile technical equipment. The term ‘airport technology’ denotes airport service processes which are carried out by means of specific airport equipment.

Organization of technological processes is the basic function of an airport. It must take care that all the aircraft land and take off safely and in time, and that they are provided with timely and quality service. It should provide adequate loading and unloading of passengers, luggage and cargo, and all the necessary procedures to ensure the operation of all the complementary activities. If an airport does not fulfill these conditions, negative consequences occur, such as late arrivals and delays in air traffic, inadequately used airport capacities, poor airport services and financial loss. However, in the operation of an airport unavoidable problems often crop up. These are late arrivals and departures due to bad weather or technical malfunctions, overloading during peak hours and security related measures in exceptional circumstances.

During the operation of a dynamic system, processes can take place on several levels. As a rule, each level has its partial objective and with this its local criterial function of control. Different partial objectives in the same system can be either consistent or conflicting. The efficiency of such systems depends on coordinating and synchronizing the operation of different parts of the system. An airport system operates on three levels which comprise its functional activities: strategic, administrative-organizational or coordinate and operational levels. Characteristics of each level differ according to the relationships with the system environment, time dimension, requirements and expectations criteria, and decision making technique (Fig. 1). Airports as dynamic systems are therefore studied, analysed and optimized according to their basic functions from the point of view of these hierarchical levels.

The operational level of an airport system is the same as the operational level of most production systems. It consists of three interactive subsystems [5]:

1. the subsystem of demand, expressed as a flow of passengers, luggage, cargo and aircraft;
2. the subsystem of production, expressed as a technological process;
3. the subsystem of facilities, expressed as infrastructure, terminals and other vital airport facilities. A subsystem of technological resources or facilities plays the same role in an airport system as stock does in a “classical” production system.

This means that the operative level of the airport system consists of the elements that characterize these subsystems (Fig. 2) i.e. customers (users), traffic flow, technical facilities, technological operations and services. These elements are intertwined with each other and influence one another.

A special feature of all traffic systems is the coincidence of pairs of elements: technological operations and services, and traffic flow and users. In an airport system the users are airlines, passengers and luggage and cargo owners. Based on these traffic flows, airport systems organize and carry out all the necessary technological operations. These operations result in airport services (production) which, in the dynamics of the traffic flow, occur in phases. Offering airport services involves the use of proper infrastructure and technical facilities. In traffic technological potential acts as stock as these usable resources are made to meet the demand at all times, including peak times. All the above components of the airport

<table>
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<th>Levels</th>
<th>Aims</th>
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<tr>
<td>strategic</td>
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<td>open</td>
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Fig. 1: the theoretical model of the functional structure of airport system levels
system operative level are interrelated. Groups of elements can be classified as subsystems specific to the airport system:

1. the subsystem of traffic flow represents the subsystem of demand,
2. the technological subsystem represents the production subsystem whose organization manifests itself as definition of production parameters,
3. the technical subsystem represents the subsystem of production facilities/stock.

The subsystem of traffic flows (demand) consists of the following set of elements: users, passenger flows, luggage and aircraft, services (separate phases of the flows). The technological subsystem within the airport system consists of the following elements: flows, technological operations, services (phases of the flows). The technical system consists of the related group of elements: flows, technical capacities, operations.

The coordinative level of the airport system consists of two subsystems: the sales and administrative subsystems. Of course, each of them can be studied and analysed independently (Fig. 3). The sales subsystem consists of the following elements: the market, users, services, income and calculations. The administrative subsystem consists of the following elements: operations, services, income, costs and net income.

At strategic level a subsystem of planning is found which consists of the following elements (Fig.4): operations, business effects, investments and capacities.

4 A theoretical model of the airport system control

The components of air traffic, i.e. airports, airlines and passengers are functionally connected via airports. These connections appear in pairs as the following interrelationships: airport-airlines, airport-passengers and airlines-passengers.

Airports offer their infrastructure and terminals, airlines offer aircraft and lines while passengers themselves take care that they arrive at and leave the airport. So the following parameters of an airport can be defined [1]:

1. Capacities of the infrastructural sector of an airport, which depend on the equipment and maintenance of airport infrastructure, and technical characteristics of the ground operative.
2. Capacities of the terminal sector of an airport, which depend on input-output terminal units and technical equipment of the terminal.
3. Supply and demand of air transport services, which depend on the number and category of passengers, the number and type of aircraft, and the number and type of lines.

In this way the system of air transport can be decomposed into elements which are directly connected with the airport system: aircraft, lines, airport infrastructure, airport terminals, road access to airports, passenger and cargo traffic.

Technological-production processes in all traffic systems including airport systems are specific in that the production and consumption of traffic services are simultaneous. One special characteristic of traffic systems is that pairs of elements within the systems occur together: technological operations - services and traffic flow - users. The operational technological-production level of an airport system consists of three subsystems which are typical of all production systems: the subsystem of demand, which is shown as the flow of passengers, luggage and cargo, the subsystem of production, which is shown as a technological process, and the subsystem of stock - facilities which is shown as infrastructure, terminals and technical resources.

In the creation of the regulation circuit of this system we take into account the fact that the simultaneousness of production and consumption of traffic services is typical of technological-

**Fig. 2: flowchart representation of the operative subsystem within the airport system**
production process in the airport system [7]. In this process there is no stock in the classical sense, as traffic services cannot be produced in advance for a known customer or stock built up for unknown customers. The demand of traffic services is neither uniform in time nor known in advance. It varies, has its ups and downs and it can only be met by installing and activating proper technological capacities. Because of this, the function of stock in traffic belongs to all the technological potential which is large enough to meet periods of extra demand. The demand of traffic services is not given and known explicitly in advance. With market research we can only learn about the probability of our expectation of a certain intensity of demand. The demand is not given with explicitly expressed mathematical function, we only know the shape and type of all the family of functions. The demand is a random process for which all the statistical indicators are known. The system input represents the demand for products/services which a given subject offers. They are airport system services, which eventually mean air transport of passengers, goods and/or cargo. Let demand be a stationary random process with known statistical characteristics - mathematical hope and autocorrelation [6]. Any given demand should be met with
current and standard services according to transport timetables and order. The difference between current capacity of services and demand is the input function for the object of control. The output function measures the amount of (un)transported passengers, cargo or luggage. When this difference is positive, i.e. when the airlines capacity exceeds the demand, the aircraft will be partly empty, certain flights will be cancelled etc. When the difference is negative, i.e. when the demand surpasses the capacities extra aircraft will have to be added. Otherwise, there will be delays, queues etc. In the new cycle there will be a system regulator which will contain all the necessary data about the true state and which will, according to given demand, provide basic information for the production process. In this way the regulation circuit is closed (Fig. 5). With optimal control we will understand the situation where all the passengers, luggage and cargo are transported with the minimum involvement of additional facilities. On the basis of the described regulation circuit we can set up a mathematical model of airport control - this is a system of differential equations for continuous/discrete systems.

5 Conclusion
For the study of structure, interrelationships and operation of a phenomenon with system characteristics, the best method is the general systems theory, and within the latter, the systems regulation theory. When we refer to traffic technology as a synthesis of organization, information technology and operations, we have to consider, in creating a mathematical model, its dynamic dimension. As each such complex phenomenon makes up a system, the traffic technology in this article is again dealt with as a dynamic system. Elements of the technological system make up an ordered entity of interrelationships and thus allow the system to perform production functions. Because of condition of linearity, response functions of the system are, with reference to the type of traffic, either continuous or discrete. Generally speaking, airports, airlines and passengers are the component parts of air traffic. Their functional relationships via airport appear in pairs: airport-airlines, airport-passengers and airlines-passengers. For air traffic operation many conditions have to be fulfilled, such as highly developed infrastructure, the use of up-to-date transport technologies, market operations, reliable operation of integral information system etc. During the control process a great deal of information must be processed, which can only be done if transparent and properly developed information system is available. During the operation of the airport an enormous amount of data is used which can only be processed into information for control if high quality software and powerful hardware are available. Communications also play a major role, as it is necessary to contact and use a number of international databases interactively. Models of optimum control can also be used in the airport system. The mathematical model with which we are describing the system can be analytically more or less complex, but generally the procedure always follows the same rule. With appropriate computer tools, an algorithm which has not been presented in this article can be used for concrete numerical examples.

REFERENCES