Modal approach to elastic and gravity waves in laterally heterogeneous media: from tsunami to acoustic frequency range

Candidate: Davide Bisignano Tutor: Prof. Fabio Romanelli
Abstract

In this thesis work, we presents several applications and developments of the modal method: in particular, imagining a logical path from low to high frequencies, are considered the tsunami waves, seismic waves propagating in structures with strong lateral heterogeneity, to conclude with the application of the method to the acoustic range. The topic treated in the first chapter is the extension of the modal technique to the tsunami wavefield (long period gravity waves), showing how this methodology allows different approaches to the modeling of both heterogeneous structural models and of the seismic rupture process that generates the tsunami. These techniques have been applied in different contexts, both in order to validate the computer codes (Western Mediterranean Sea, the Tohoku earthquake and tsunami of March 2011) and to assess tsunami hazard scenarios for the selected target zones (Vietnam Coasts, Adriatic Sea). The topic of the second chapter is the modeling of seismic waves propagating in strong laterally heterogeneous models, in particular with vertical liquid-solid interfaces. After a short theoretical introduction, we shows the results of some modelling tests adopting several structural models and different frequency ranges (in particular reaching a maximum frequency of 10 Hz). Thus, going to higher frequencies, up to 100Hz, the package of programs originally developed for the seismic waves has been tested and, after appropriate improvements, used to study the propagation of acoustic waves in the oceanic structures. The purpose of these tests, subject of the third and final chapter of this work, is to understand the sensitivity of the acoustic waves to the characteristics of the structure in which they propagate, such as the presence of low velocity sedimentary layers or a low velocity channel in the liquid. In the second and third chapter, the study of the computed signals was complemented through a joint time-frequency analysis (FTAN) which greatly helped to understand their salient features, related to both the source and the structure, further confirming the validity and the polyhedric nature of the modal approach.
Riassunto

Il presente lavoro di tesi presenta diverse applicazioni e sviluppi del metodo modale: in particolare, immaginando un percorso logico dalle basse alle alte frequenze, si sono considerate le onde di tsunami, onde sismiche propagantesi in strutture a forte eterogeneità laterale, fino a giungere all’applicazione del metodo al dominio acustico. Nel primo capitolo l’argomento trattato è quello degli tsunami (onde di gravità a lungo periodo), estendendo la tecnica modale agli tsunami e mostrando come questa metodologia consenta diversi approcci alla modellazione sia delle eterogeneità del modello strutturale che del processo di rottura sismico che genera l’onda di tsunami. Queste tecniche sono state applicate in diversi contesti, sia al fine di validarne i relativi codici di calcolo (Mar Mediterraneo Occidentale, terremoto e tsunami di Tohoku del marzo 2011) che per valutare scenari di pericolosità da tsunami per le zone considerate (costa del Vietnam, Mare Adriatico). L’argomento del secondo capitolo è la modellazione di onde sismiche che si propagano in modelli a forte eterogeneità laterale, in particolare con interfacce verticali liquido-solido. Dopo una breve introduzione teorica, vengono mostrati i risultati di alcuni test di modellazione che utilizzano vari modelli strutturali e diversi domini di frequenza (in particolare si arriverà a una frequenza massima di 10Hz). Quindi, andando a frequenze sempre più alte, fino a 100Hz, il pacchetto di programmi sviluppato originariamente per le onde sismiche è stato testato e, dopo gli opportuni miglioramenti, utilizzato per studiare la propagazione di onde acustiche in strutture oceaniche. Il fine di questi test, argomento del terzo e conclusivo capitolo di questo lavoro, è capire la sensibilità delle onde acustiche alle caratteristiche della struttura in cui si propagano, come ad esempio la presenza di strati solidi sedimentari a bassa velocità o di un canale a bassa velocità nel liquido. Nel secondo e nel terzo capitolo lo studio dei segnali ottenuti è stata approfondita mediante un’analisi congiunta tempo-frequenza (FTAN) che ha notevolmente aiutato a capirne le caratteristiche salienti, correlate sia alla sorgente che alla struttura, confermando ulteriormente la validità e la poliedricità dell’approccio modale in questo genere di studi.
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1 Tsunami Physics and Tsunami Hazard Assessment

1.1 Introduction

1.1.1 Tsunami physics and its modeling

Tsunamis traditionally are modeled using an hydrodynamic approach (e.g. Ben-Menahem and Singh 1981, cap. 9) in which hydrodynamic equations for gravity waves, in particular shallow water gravity waves, are considered and liquid bottom displacement is used as a initial condition (if the tsunamigenic mechanism is an earthquake, this condition is usually given following Okada (1985)). In this approach the sea bottom is considered rigid and, after initial static displacement, not interacting with fluid, that is why this approach is considered partially coupled.

On the opposite, in the modal approach (e.g. Ward 1980, Comer 1984, Panza et al. 2000), topic of this work, tsunami is considered as a long period gravity mode, in which, among restoring forces also gravity is considered: this approach is fully coupled since water and solid layers are in welded contact.

The former approach describes in a complete way the wave propagation but not its generation; in fact, sea bottom uplift is often described in a approximated way. Ward (1981) has shown that the intuitive idea of the bottom uplift as direct cause of water column movement is actually not complete since important frequencies involved in tsunami generation are not those involved in static uplift of sea bottom.

1.1.2 Modal approach for tsunami

Ward (1980) was the first that understood that tsunami could be described as a free-oscillation earth mode. In particular, he considered equations of motion of a spherical earth model surrounded by a liquid layer in which the restoring force is not only elasticity but also gravity. Setting the right boundary conditions he obtained dispersion curves corresponding to those obtained for gravity waves.

Comer (1984) extended this theory from spherical to flat geometry: in this way tsunami mode can be considered as a low velocity and low frequency Rayleigh mode with gravity as
The fully-coupled approach allows to include the tsunami in the framework of the well-know modal theory for seismic waves (e.g. Schwab, 1970; Panza, 1985), with the great advantage that the seismic source can naturally be treated, without the necessity of mapping its action to an initial-boundary condition only. A detailed description of this approach, for vertically layered flat earth models (1D, i.e. with one dimension of heterogeneity), is shown in the following sections. It has to be mentioned that, being an analytical method, the generation of synthetic tsunamigrams demands very short computational times (e.g. just a fraction of a second, on a normal laptop): this feature could be extremely important for the tsunami hazard assement, where a large number of different scenarios have to be considered.

On the other side, this approach is valid only in the linear regime of tsunami: coastal phenomena, like run-up and inundation, can’t be described in this framework. However, the final tsunami heights computed “offshore” can be easily used as an initial condition for detailed, numerically computed inundation models.

In fact, in proximity of the coast several effects can occur, due to the thinning of the liquid layer, local bathymetry and topography of the coast, strongly influencing both travel time, maximum amplitude and final inundation (see fig. 1.1.1).

Figure 1.1.1: Definitions of tsunami off-shore height, inundation height and run-up height.

The major contribution comes from the amplification of the wave approaching the coast due to the progressive thinning of the water layer. The principle of conservation of energy requires that the wave energy, when the tsunami reaches shallow waters, is redistributed into a smaller volume, this results in a growth of the maximum amplitude, called shoaling effect (see fig. 1.1.2). The linear theory gives for the shoaling amplification factor a simple expression, known as Green’s law.
In uniform depth oceans, tsunami propagate out from their source in circular rings (e.g. Figure 9) with ray paths that look like spokes on a wheel. In real oceans, tsunami speeds vary from place to place (even at a fixed frequency) so tsunami ray paths refract and become bent. Consequently, in real oceans, both tsunami travel time and amplitude have to be adjusted relative to their values in uniform depth ones. To propagate tsunami in real oceans, I find it best to keep tsunami 'mode like' versus depth but 'ray like' horizontally. I first transform the various integrals over wavenumber (\(7, 8, 9, \ldots\)) to integrals over frequency because wave frequency, not wave number, is conserved throughout. Using the relations \(u(\omega) = d\omega/dk\) and \(c(\omega) = \omega/k(\omega)\), I find that tsunami vertical motions from (13) for instance, are to a good approximation

\[
u_z\text{surf}(r, t) = \cos(\omega t) d\omega 2\pi k(\omega) u(\omega) J_0(\omega T(r, \omega)) F_0(k(\omega)) G(r) S_L(\omega, r)\]

In (15) the travel time of waves of frequency \(\omega\) has been changed from \(r/c(\omega)\) to \(T(r, \omega) = dr/c(\omega)\int_\omega^\infty\).

Figure 9. Expanding rings of an impact tsunami in a 3D-like style. Note the strong wave spreading due to dispersion.

Figure 11. Effect of shoaling on tsunami eigenfunctions. The shallowing ocean near shore concentrates wave energy into smaller and smaller volumes and tsunami grow in response. (From Ward, 2011).

Typically the shoaling factor ranges from 1 (no growth) up to several units (amplification) depending on the considered domain. Shoaling amplification acts approximately until the wave amplitude is less than half the sea depth (Ward, 2010), then nonlinear phenomena cause the waves to break and eventually turn them backward. In particular, Ward (2010) suggests that due to complications of wave refraction and interference, run-up is best considered as a random process that can be characterised by its statistical properties. Models and observations hint that run-up statistics follow a single skewed distribution spreading between 1/2 and 2 times its mean value. When dealing with very long source-site distances (hundreds of kilometers), an additional effect on tsunami maximum amplitude becomes relevant: the phenomenon of dispersion, i.e. the fact that the components at low frequency of the signal travel faster than the higher ones. After a certain distance the slower high-frequency components tend to migrate at the tail of the wavetrain where they don’t contribute any more to the main peak amplitude.

### 1.2 Vertically layered structural models and equations of motion

The theory, algorithms, and the related computations developed in this chapter are based on the methodology of Panza et al. (2000), where the Thomson-Haskell modal approach is extended to the tsunami mode propagating in models with lateral heterogeneities (2D). In particular, the equations of motion are solved for a vertically layered oceanic halfspace, where the top-most layers are (inviscid) liquid where gravity is included as restoring force. As for seismic case, the first step consists in finding the spectrum (i.e. the eigenvalues-eigenfunctions of the related
differential operator) of the structure, that in the case of tsunami it is made by just one mode, and how it can be excited by a seismic source, represented by an equivalent system of body-forces (e.g. Levshin et al., 1989).

In fig. 1.2.1 it is shown the adopted 1D model: the horizontal layers are described by density, $\rho$, P-wave velocity, $\alpha$ and (for solid layers) S-wave velocity, $\beta$. For what concerns stratification, $z_0$ indicates the solid-liquid interface, $z_{-j}$ the free surface and $z_N$ the last interface over elastic halfspace.

Following Pod’Yapolsky (1968) gravity acceleration is considered active along vertical direction, e.g., $g\rho \hat{k}$; variation of density is due to density gradient linked to fluid compressibility, i.e., $\frac{\Delta \rho}{\rho} = -\nabla \cdot \mathbf{u}$.

The equations of motion for liquid and solid layers will be respectively eq. 1.2.1 and 1.2.2.

$$\alpha^2 \nabla (\nabla \cdot \mathbf{u}) - g \hat{k} \nabla \cdot \mathbf{u} = \frac{\partial^2 \mathbf{u}}{\partial t^2}$$  \hspace{1cm} (1.2.1)

$$\alpha^2 \nabla (\nabla \cdot \mathbf{u}) - g \hat{k} \nabla \cdot \mathbf{u} - \beta^2 \nabla (\nabla \times \mathbf{u}) = \frac{\partial^2 \mathbf{u}}{\partial t^2}$$  \hspace{1cm} (1.2.2)

These equations can be obtained also from equation 8.59 of Aki & Richards (1980), considering constant gravity acceleration, $g_0$, and gravity potential $U = -g_0 \rho_0 z$, and its perturbation $dU = -g_0 \rho_0 w$.

Figure 1.2.1: Model adopted for tsunami mode computation (From Panza et al., 2000).

The pressure in the fluid layers has both an elastic and a gravitational component and must vanish at the free surface, at first order it can be written as
\[ p[z_{-1} + w(z_{-1})] = p(z_{-1}) + \frac{dp}{dz} |_{z_{-1}} w_{-1} = -\rho_{-1} \alpha_j^2 \mathbf{V} \cdot \mathbf{u} + \rho_{-1} g w_{-1} |_{z_{-1}} = 0. \] (1.2.3)

At the interface between two liquid layers, for example \(-j\) and \(-(j+1)\), since shear stresses vanish, only pressure and vertical displacement are continuous; in particular, these condition should be evaluated at perturbed interface, i.e. \(z = z_{-j} + w(z_{-j})\), but for displacement it’s equivalent to evaluate the condition at unperturbed one, i.e. \(z = z_{-j}\).

Continuity condition between layers \(-j\) and \(-(j+1)\) will be

\[ w_{-j}(z_{-j}) = w_{-j-1}(z_{-j}) \] (1.2.4)

and

\[-\rho_{-j} \alpha_j^2 \mathbf{V} \cdot \mathbf{u}_{-j} + \rho_{-j} g w_{-j} |_{z_{-j}} = -\rho_{-j-1} \alpha_{j-1}^2 \mathbf{V} \cdot \mathbf{u}_{-j-1} + \rho_{-j-1} g w_{-j-1} |_{z_{-j}}. \] (1.2.5)

At the liquid-solid interface, vertical displacement and pressure have to be continuous, expressed respectively by eqs.1.2.6 and 1.2.7, while shear stress in the solid should vanish, as can be seen in eq. 1.2.8

\[ w_{-1}(z_0) = w_1(z_0) \] (1.2.6)

\[-\rho_{-1} \alpha_1^2 \mathbf{V} \cdot \mathbf{u}_{-1}(z_0) = \sigma_1(z_0) \] (1.2.7)

\[ 0 = \tau_1(z_0). \] (1.2.8)

In equation 1.2.7 the gravitational terms have been removed, since at this interface the vertical displacement is very small compared to the one in the liquid; in equation 1.2.2, the gravitational part can be neglected and in the solid layers only the elastic forces are considered.

At the interface between two solid layers all components of the stress and of displacement are conserved: for example between layers \(m\) and \(m+1\) boundary conditions are:

\[ w_m(z_m) = w_{m+1}(z_m) \] (1.2.9)

\[ u_m(z_m) = u_{m+1}(z_m) \] (1.2.10)
1.2.1 Solutions of the equations of motion

The solutions of equation of motion in the \( j \)–th liquid layer, considering both gravitational and elastic forces, in terms of an harmonic wave propagating along x axis with phase velocity \( c \) and angular frequency \( \omega \), are:

\[
\begin{align*}
\sigma_m(z_m) &= \sigma_{m+1}(z_m) \\
\tau_m(z_m) &= \tau_{m+1}(z_m)
\end{align*}
\] (1.2.12)

for \( 1 \leq m \leq N - 1 \).

where \( k \) is horizontal wave-number and

\[
\begin{align*}
\eta_1(-j) &= -\omega \psi_j - \frac{g}{2\alpha_j} \\
\eta_2(-j) &= \omega \psi_j - \frac{g}{2\alpha_j} \\
\psi_j^2 &= \frac{\alpha_j}{c^2} - 1 + \frac{g^2}{4\alpha_j \omega^2}
\end{align*}
\]

in the solid layers the solutions are given by

\[
\begin{align*}
u_m(x,z,t) &= \left\{ \frac{i\alpha_m^2}{\omega c} C_m \exp\left( -\frac{\eta_{2m} z}{\alpha_m} \right) + D_m \exp\left( -\frac{\eta_{1m} z}{\alpha_m} \right) \right\} \\
&\quad + \frac{-ir_m \beta_m}{\omega} \left\{ E_m \exp\left( -\frac{\omega \beta_m z}{\beta_m} \right) + F_m \exp\left( \frac{\omega \beta_m z}{\beta_m} \right) \right\} \exp[i(\omega t - kx)]
\end{align*}
\] (1.2.15)
w_m(x, z, t) = \left\{ \begin{array}{l}
\alpha_m^2 \left( \eta_{1m} + \frac{g \beta_m^2}{\alpha_m (\alpha_m^2 - \beta_m^2)} \right) C_m \exp \left( -\eta_{2m} \frac{z}{\alpha_m} \right) + \\
- \left( \eta_{2m} + \frac{g \beta_m^2}{\alpha_m (\alpha_m^2 - \beta_m^2)} \right) D_m \exp \left( -\eta_{1m} \frac{z}{\alpha_m} \right) + \\
- \frac{\beta_m^2}{\omega c} \left[ E_m \exp \left( -\frac{\omega r_m}{\beta_m} z \right) - F_m \exp \left( \frac{\omega r_m}{\beta_m} z \right) \right] \right\} \exp \left[ i(\omega t - kx) \right] 
\end{array} \right. 
\tag{1.2.16}
\]

where

\[ r_m = \left\{ \begin{array}{l}
i \left[ 1 - (\beta_m/c)^2 \right] & c > \beta_m \\
(\beta_m/c)^2 - 1 & c < \beta_m 
\end{array} \right. \tag{1.2.17} \]

As stated before, in the solid layers the gravity effect can be neglected, and thus the wave function can be written as

\[ u_m(x, z, t) = \left\{ \begin{array}{l}
i \frac{\alpha_m^2}{\omega c} \left[ C_m \exp \left( -\frac{\omega r_m}{\alpha_m} z \right) + D_m \exp \left( \frac{\omega r_m}{\alpha_m} z \right) \right] + \\
- \frac{i r_m}{\omega} \frac{\beta_m}{\alpha_m} \left[ E_m \exp \left( -\frac{\omega r_m}{\beta_m} z \right) + F_m \exp \left( \frac{\omega r_m}{\beta_m} z \right) \right] \right\} \exp \left[ i(\omega t - kx) \right] 
\tag{1.2.18} \]

and

\[ w_m(x, z, t) = \left\{ \begin{array}{l}
\frac{r_m \alpha_m}{\omega} \left[ C_m \exp \left( -\frac{\omega r_m}{\alpha_m} z \right) - D_m \exp \left( \frac{\omega r_m}{\alpha_m} z \right) \right] + \\
- \frac{\beta_m^2}{\omega c} \left[ E_m \exp \left( -\frac{\omega r_m}{\beta_m} z \right) - F_m \exp \left( \frac{\omega r_m}{\beta_m} z \right) \right] \right\} \exp \left[ i(\omega t - kx) \right]. \tag{1.2.19} \]

To find the dispersion relation \( F = c(\omega) \) and to fix layer constants \( A_{-j}, B_{-j}, C_m, D_m, E_m, F_m \) the boundary conditions 1.2.3 - 1.2.12 are applied.

Condition at free surface, \( z = 0 \) is described by eq. 1.2.3 that can be written also

\[ \rho_{-1} \alpha_{-1}^2 \nabla \cdot u_{-1} - \rho_{-1} g w_{-1} = 0. \tag{1.2.20} \]

Divergence of \( u \) is:
\[
\frac{\partial u_{-l}}{\partial x} = -\frac{\alpha_{-l}^2}{c^2} \left[ A_{-l} \exp \left( -\frac{\eta_{2(-l)}}{\alpha_{-l}} z \right) - B_{-l} \exp \left( -\frac{\eta_{1(-l)}}{\alpha_{-l}} z \right) \right] \exp \left[ i \left( \omega t - kx \right) \right]
\] (1.2.21)

and

\[
\frac{\partial w_{-l}}{\partial z} = \frac{1}{\omega^2} \left[ -\eta_{1(-l)} \eta_{2(-l)} A_{-l} \exp \left( -\frac{\eta_{2(-l)}}{\alpha_{-l}} z \right) + \eta_{1(-l)} \eta_{2(-l)} B_{-l} \exp \left( -\frac{\eta_{1(-l)}}{\alpha_{-l}} z \right) \right] \exp \left[ i \left( \omega t - kx \right) \right]
\]

that considering

\[
\eta_{1(-l)} \eta_{2(-l)} = \omega^2 \left( 1 - \frac{\alpha_{-l}^2}{c^2} \right)
\]

becomes

\[
\frac{\partial w_{-l}}{\partial z} = \left[ \left( \frac{\alpha_{-l}^2}{c^2} - 1 \right) A_{-l} \exp \left( -\frac{\eta_{2(-l)}}{\alpha_{-l}} z \right) + \left( 1 - \frac{\alpha_{-l}^2}{c^2} \right) B_{-l} \exp \left( -\frac{\eta_{1(-l)}}{\alpha_{-l}} z \right) \right] \exp \left[ i \left( \omega t - kx \right) \right].
\] (1.2.22)

1.2.2 Displacements and stresses in liquid layers in matrix form

Eq. 1.2.20 can be written in matrix form in the following way

\[
S_{-l} \begin{pmatrix} A_{-l} \\ B_{-l} \end{pmatrix} = 0
\] (1.2.23)

with

\[
S_{-l} = \begin{pmatrix} \rho_{-l} \alpha_{-l}^2 \left( -1 - \frac{g\eta_{1(-l)}}{\alpha_{-l} \omega^2} \right) & \rho_{-l} \alpha_{-l}^2 \left( +1 + \frac{g\eta_{1(-l)}}{\alpha_{-l} \omega^2} \right) \end{pmatrix}
\]

At an interface between two liquid layers the vertical velocity and pressure are continuous: for the vertical component of the velocity unperturbed interface can be considered and the same assumption is valid for the pressure with its components’s variation \( \rho_{-j} \alpha_{-j}^2 (\nabla \cdot u_{-j}) \), the elastic, and \( gw_{-j} \left( \rho_{-j-1} - \rho_{-j} \right) \) the gravitational one.

So one can assume that the quantity conserved at interface is \( \rho_{-j} \alpha_{-j}^2 (\nabla \cdot u_{-j}) - gw_{-j} \rho_{-j} \).
This last expression can be written in matrix form as

\[
\begin{pmatrix}
  kw_{-j} \\
  p_{-j}
\end{pmatrix} = L_{-j} K_{-j} \begin{pmatrix}
  A_{-j} \\
  B_{-j}
\end{pmatrix}
\]  

where

\[
L_{-j} = \begin{pmatrix}
  -\frac{\alpha_{-j} \psi_{-j}}{c} - \frac{g}{\alpha_{-j} \omega^2} & -\frac{\alpha_{-j} \psi_{-j}}{c} + \frac{g}{\alpha_{-j} \omega^2} \\
  \rho_{-j} \alpha_{-j}^2 (1 - \frac{g \eta_{-j}}{\alpha_{-j} \omega^2}) & \rho_{-j} \alpha_{-j}^2 (+1 + \frac{g \eta_{-j}}{\alpha_{-j} \omega^2})
\end{pmatrix}
\]

is the matrix that link stress-displacement vector to coefficients \( A_{-j} \) and \( B_{-j} \) in every layer and

\[
K_{-j} = \begin{pmatrix}
  \exp\left(-\frac{H_{-j} \eta_{j-1}}{\alpha_{-j}}\right) & 0 \\
  0 & \exp\left(-\frac{H_{-j} \eta_{j}}{\alpha_{-j}}\right)
\end{pmatrix}
\]

is the vertical propagation matrix inside the \( j \)th layer, being \( H_{-j} \) its thickness.

Consequently, considering the stress-displacement vector at \( z = z_{-j} \) one can write

\[
\begin{pmatrix}
  kw_{-j} \\
  p_{-j}
\end{pmatrix} = L_{-j} \begin{pmatrix}
  A_{-j} \\
  B_{-j}
\end{pmatrix}
\]

while in \( z = z_{-j+1} \) the following expression is valid

\[
\begin{pmatrix}
  kw_{-j} \\
  p_{-j}
\end{pmatrix} = L_{-j} K_{-j} \begin{pmatrix}
  A_{-j} \\
  B_{-j}
\end{pmatrix}.
\]

From this last equation follows that

\[
\begin{pmatrix}
  A_{-j} \\
  B_{-j}
\end{pmatrix} = K_{-j}^{-1} L_{-j} \begin{pmatrix}
  kw_{-j} \\
  p_{-j}
\end{pmatrix} \bigg|_{z_{-j+1}},
\]

from which it is possible to connect the stress-displacement vector at two interfaces.

In fact, the following expression holds

\[
\begin{pmatrix}
  kw_{-j} \\
  p_{-j}
\end{pmatrix} = D_{-j} \begin{pmatrix}
  kw_{-j} \\
  p_{-j}
\end{pmatrix} \bigg|_{z_{-j+1}}
\]

where the matrix \( D_{-j} \), named layer-matrix, is given by

\[
D_{-j} = L_{-j} K_{-j}^{-1} L_{-j}^{-1}.
\]

Using the conditions 1.2.4 and 1.2.5, the coefficients of the top layer can be linked to those of the last liquid layer in the following way

\[
\begin{pmatrix}
  A_{-l} \\
  B_{-l}
\end{pmatrix} = L_{-l}^{-1} D_{-l} D_{-l+1} \cdots D_{-2} L_2 \begin{pmatrix}
  A_{-1} \\
  B_{-1}
\end{pmatrix}.
\]

Considering the condition 1.2.20 at free surface it is possible to write
\[ S_{-1}L_{-1}^{-1}D_{-1}D_{-1+1}D_{-2}L_{2} \left( \begin{array}{c} A_{-1} \\ B_{-1} \end{array} \right) = M \left( \begin{array}{c} A_{-1} \\ B_{-1} \end{array} \right). \] (1.2.30)

At the solid-liquid interface, the gravitational component of the stress is negligible and pressure can be written as
\[ p_{-1} = \rho_{-1} \alpha_{-1}^{2} \nabla \cdot u_{-1} = R \left( \begin{array}{c} A_{-1} \\ B_{-1} \end{array} \right) \] (1.2.31)
with
\[ R = \left( \begin{array}{c} -\rho_{-1} \alpha_{-1}^{2}, \rho_{-1} \alpha_{-1}^{2} \end{array} \right), \]
while the vertical displacement can be written as
\[ k w_{-1} = \frac{\alpha_{-1}}{c \omega_{-1}} \left( \eta_{1(-1)} - \eta_{2(-1)} \right) K_{-1} \left( \begin{array}{c} A_{-1} \\ B_{-1} \end{array} \right) = P \left( \begin{array}{c} A_{-1} \\ B_{-1} \end{array} \right). \] (1.2.32)

### 1.2.3 Matrix form of displacement and stresses in the solid layers

Neglecting gravity, the solutions of the equations of motion in solid layers are
\[ u_{m}(x, z, t) = \left\{ \frac{i \alpha_{m}^{2}}{\omega c} \left[ C_{m} \exp \left( -\frac{\omega r_{am} \alpha_{m}}{\omega m} z \right) + D_{m} \exp \left( \frac{\omega r_{am} \alpha_{m}}{\omega m} z \right) \right] \right. + \] \[ \left. - \frac{ir_{m} \beta_{m}}{\omega} \left[ E_{m} \exp \left( -\frac{\omega r_{bm} \beta_{m}}{\omega m} z \right) + F_{m} \exp \left( \frac{\omega r_{bm} \beta_{m}}{\omega m} z \right) \right] \right\} \exp [i(\omega t - kx)] \]
and
\[ w_{m}(x, z, t) = \left\{ \frac{r_{am} \alpha_{m}}{\omega} \left[ C_{m} \exp \left( -\frac{\omega r_{am} \alpha_{m}}{\omega m} z \right) - D_{m} \exp \left( \frac{\omega r_{am} \alpha_{m}}{\omega m} z \right) \right] \right. + \] \[ \left. - \frac{\beta^{2}_{m}}{\omega c} \left[ E_{m} \exp \left( -\frac{\omega r_{bm} \beta_{m}}{\omega m} z \right) - F_{m} \exp \left( \frac{\omega r_{bm} \beta_{m}}{\omega m} z \right) \right] \right\} \exp [i(\omega t - kx)]. \]

In the \( m-th \) elastic layer the stresses are given by the general formula
\[ \sigma_{mij} = \lambda_{m} (\nabla \cdot \mathbf{u}_{m}) \delta_{ij} + 2 \mu_{m} e_{mij} \] (1.2.33)
where \( e_{mij} \) is the strain and it is defined by
\[ e_{mij} = \frac{1}{2} \left( \frac{\partial u_{mi}}{\partial x_{j}} + \frac{\partial u_{mj}}{\partial x_{i}} \right); \]
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given $\alpha_m = \sqrt{\frac{2\mu_m + \lambda_m}{\rho_m}}$ and $\beta_m = \sqrt{\frac{\mu_m}{\rho_m}}$ one can write $\mu_m = \rho_m \beta_m^2$ and $\lambda_m = \rho \alpha_m^2 - 2\mu_m = \rho_m (\alpha_m^2 - 2\beta_m^2)$.

The derivatives of these expressions are

\[
\frac{\partial u_m}{\partial x} = \left\{ \frac{\alpha_m^2}{c^2} \left[ C_m \exp \left( - \frac{\omega r_{am}}{\alpha_m} z \right) + D_m \exp \left( \frac{\omega r_{am}}{\alpha_m} z \right) \right] + \right.
\]

\[
- \frac{r_{\beta_m \beta_m}}{c} \left[ E_m \exp \left( - \frac{\omega r_{\beta_m}}{\beta_m} z \right) + F_m \exp \left( \frac{\omega r_{\beta_m}}{\beta_m} z \right) \right] \exp[i(\omega t - kx)]
\]

\[
\frac{\partial u_m}{\partial z} = \left\{ \frac{i\alpha_m}{c} \left[ - C_m \exp \left( - \frac{\omega r_{am}}{\alpha_m} z \right) + D_m \exp \left( \frac{\omega r_{am}}{\alpha_m} z \right) \right] + \right.
\]

\[
- \frac{i\beta_m^2}{c^2} \left[ E_m \exp \left( - \frac{\omega r_{\beta_m}}{\beta_m} z \right) - F_m \exp \left( \frac{\omega r_{\beta_m}}{\beta_m} z \right) \right] \exp[i(\omega t - kx)].
\]

\[
\frac{\partial w_m}{\partial x} = \left\{ \frac{r_{am}^2}{c} \left[ - C_m \exp \left( - \frac{\omega r_{am}}{\alpha_m} z \right) - D_m \exp \left( \frac{\omega r_{am}}{\alpha_m} z \right) \right] + \right.
\]

\[
- \frac{\beta_m r_{\beta_m}}{c} \left[ - E_m \exp \left( - \frac{\omega r_{\beta_m}}{\beta_m} z \right) - F_m \exp \left( \frac{\omega r_{\beta_m}}{\beta_m} z \right) \right] \exp[i(\omega t - kx)].
\]

Normal stress in the $m$-th layer is $\sigma_{zz} = \lambda_m (\nabla \cdot \mathbf{u}_m) + \mu_m \frac{\partial w_m}{\partial z}$ and it can be written as $\sigma_{zz} = \rho_m \left( \alpha_m^2 - 2\beta_m^2 \right) \frac{\partial u_m}{\partial x} + \alpha_m^2 \frac{\partial w_m}{\partial z}$.

that, in explicite form, omitting the exponential part in $(x,t)$ becomes

\[
\sigma_{mcz} = \left[ \left( \alpha_m^2 - 2\beta_m^2 \right) \frac{\alpha_m^2}{c^2} - \alpha_m^2 r_{am} \right] C_m \exp \left( - \frac{\omega r_{am}}{\alpha_m} z \right) + \right.
\]

\[\left. + \left( \alpha_m^2 - 2\beta_m^2 \right) \frac{\alpha_m^2}{c^2} - \alpha_m^2 r_{am} \right] D_m \exp \left( \frac{\omega r_{am}}{\alpha_m} z \right) + \right.
\]

\[\left. + \left[ \left( \alpha_m^2 - 2\beta_m^2 \right) \frac{r_{\beta_m \beta_m}}{c} + \alpha_m^2 r_{\beta_m \beta_m} \right] E_m \exp \left( - \frac{\omega r_{\beta_m}}{\beta_m} z \right) + \right.
\]

\[\left. + \left[ \left( \alpha_m^2 - 2\beta_m^2 \right) \frac{r_{\beta_m \beta_m}}{c} + \alpha_m^2 r_{\beta_m \beta_m} \right] F_m \exp \left( \frac{\omega r_{\beta_m}}{\beta_m} z \right) + \right.\]
\[ + \left( - \left( \alpha_m^2 - 2\beta_m^2 \right) \frac{r_{\beta m}}{c} + \frac{\alpha_m r_{\beta m} \beta_m}{c} \right) F_m \exp \left( \frac{\omega r_{\beta m}}{\beta_m} z \right) \] 

and is it possible to write

\[ \sigma_{mzz} = \left[ -\rho_m \alpha_m^2 (\gamma_m - 1) \right] C_m \exp \left( -\frac{\omega r_{\alpha m}}{\alpha_m} z \right) + \left( -\rho_m \alpha_m^2 (\gamma_m - 1) \right) D_m \exp \left( \frac{\omega r_{\alpha m}}{\alpha_m} z \right) + \] 

\[ + \left[ 2\rho_m \beta_m^2 \frac{r_{\beta m} \beta}{c} E_m \exp \left( -\frac{\omega r_{\beta m}}{\beta_m} z \right) + 2\rho_m \beta_m^2 \frac{r_{\beta m} \beta}{c} F_m \exp \left( \frac{\omega r_{\beta m}}{\beta_m} z \right) \right] \]  

with \( \gamma_m = \frac{\beta_m^2}{c^2} \).

Shear stresses are defined by \( \tau_{mzx} = \rho_m \beta_m^2 \left( \frac{\partial u_m}{\partial z} + \frac{\partial w_m}{\partial x} \right) \) and they can be expressed as

\[ \tau_{mzx} = \rho_m \beta_m^2 \left[ -2i \frac{\alpha_m r_{\alpha m}}{c} \right] C_m \exp \left( -\frac{\omega r_{\alpha m}}{\alpha_m} z \right) + \left( 2i \frac{\alpha_m r_{\alpha m}}{c} \right) D_m \exp \left( \frac{\omega r_{\alpha m}}{\alpha_m} z \right) + \] 

\[ + \rho_m \beta_m^2 \left[ i (\gamma_m - 1) E_m \exp \left( -\frac{\omega r_{\beta m}}{\beta_m} z \right) - i (\gamma_m - 1) F_m \exp \left( \frac{\omega r_{\beta m}}{\beta_m} z \right) \right]. \]  

Thus, from eqs. 1.2.18, 1.2.14, 1.2.34 and 1.2.35, like for the liquid case, stress-displacement vector can be obtained

\[
\begin{pmatrix}
  k u_m \\
  k w_m \\
  \tau_{mzx} \\
  \sigma_{mzz}
\end{pmatrix}
= L_m K_m
\begin{pmatrix}
  C_m \\
  D_m \\
  E_m \\
  F_m
\end{pmatrix}
\]  

(1.2.36)

where

\[
L_m = \begin{pmatrix}
\frac{\alpha_m^2}{c^2} & -\frac{\alpha_m^2}{c^2} & -\beta_m r_{\beta m} & -\beta_m r_{\beta m} \\
\alpha_m r_{\alpha m} & -\alpha_m r_{\alpha m} & -\beta_m r_{\alpha m} & -\beta_m r_{\alpha m} \\
-\frac{\alpha_m^2}{c^2} & \frac{\alpha_m^2}{c^2} & \frac{\beta_m^2}{c^2} & \frac{\beta_m^2}{c^2} \\
-\rho_m \alpha_m^2 (\gamma_m - 1) & -\rho_m \alpha_m^2 (\gamma_m - 1) & \mu_m (\gamma_m - 1) & \mu_m (\gamma_m - 1)
\end{pmatrix}
\]  

(1.2.37)

and
Therefore, applying conditions 1.2.9, 1.2.10, 1.2.11, 1.2.12 and following the same procedure adopted for liquid layers one can write

\[
L_m K_m \begin{pmatrix}
C_m \\
D_m \\
E_m \\
F_m
\end{pmatrix} = L_{m+1} \begin{pmatrix}
C_{m+1} \\
D_{m+1} \\
E_{m+1} \\
F_{m+1}
\end{pmatrix}
\]

(1.2.39)

and then

\[
\begin{pmatrix}
k u_m \\
k w_m \\
\tau_{m\infty} \\
\sigma_{mz}
\end{pmatrix} = D_m \begin{pmatrix}
k u_{m+1} \\
k w_{m+1} \\
\tau_{(m+1)\infty} \\
\sigma_{(m+1)z}
\end{pmatrix}
\]

(1.2.40)

with

\[
D_m = L_m K_m^{-1} L_m^{-1}.
\]

(1.2.41)

### 1.2.4 Boundary conditions at the solid-liquid interface and dispersion relationship

Expressions 1.2.31 and 1.2.32, after applying the boundary conditions at the liquid-solid interface, and isolating second and third components of 1.2.36, 1.2.6 and 1.2.7, can be written as

\[
\begin{pmatrix}
\frac{r_0 a_1 a_0}{c} \\
\frac{r_0 a_1 a_2}{c} \\
\rho_1 a_1^2 (1 - \frac{\beta_1^2}{c^2}) \\
\rho_1 a_2^2 (1 - \frac{\beta_1^2}{c^2})
\end{pmatrix} - \frac{r_0 a_1 a_0}{c} - \frac{r_0 a_1 a_2}{c} - \frac{\beta_1^2}{c^2} - \frac{\beta_1^2}{c^2} \frac{2\mu_1 \beta_1 r_0}{c} = \begin{pmatrix}
C_1 \\
D_1 \\
E_1 \\
F_1
\end{pmatrix} = \begin{pmatrix}
p_1 \\
p_2 \\
r_1 \\
r_2
\end{pmatrix} \begin{pmatrix}
A_{-1} \\
B_{-1}
\end{pmatrix}.
\]

(1.2.42)

From condition 1.2.30, two expressions for the coefficients of the first liquid layer are obtained:
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\[ A_{-1} = -m_2 K \]  \hspace{1cm} (1.2.43)

and

\[ B_{-1} = m_1 K \]  \hspace{1cm} (1.2.44)

where \( K \) is a multiplicative constant to be fixed; substituting 1.2.43 and 1.2.44 in 1.2.42 the boundary condition at the solid-liquid interface becomes

\[ \mathbf{F} K = \begin{pmatrix} f_1 \\ f_2 \end{pmatrix} = \begin{pmatrix} -p_1 m_2 + p_2 m_1 \\ -r_1 m_2 + r_2 m_1 \end{pmatrix}. \]  \hspace{1cm} (1.2.45)

Now, in analogy with the Thomson-Haskell method for seismic waves (Haskell, 1953), a dispersion relationship can be found: it consists in a function of phase velocity, frequencies and of different layers constants posed equal to zero or to a constant.

To this aim, one can reduce 1.2.42 to one row using 1.2.45 and, as other boundary conditions for shear stress, eq. 1.2.8.

In this way, an expression formally equal to that considering a free surface condition of a solid structure is obtained, that is

\[ \mathbf{C} \begin{pmatrix} C_1 \\ D_1 \\ E_1 \\ F_1 \end{pmatrix} = 0 \]  \hspace{1cm} (1.2.46)

where \( C \) is a 2x2 matrix given by

\[ C = \begin{pmatrix} \frac{r_{a_1} \alpha_{a_1}}{c} & -\frac{r_{a_1} \alpha_{b}}{c} & -\beta_1^2 \frac{2 \mu_1 \beta_1 r_{b_1}}{c} & \beta_1^2 \frac{2 \mu_1 \beta_1 r_{b_1}}{c} \\ \rho_1 \alpha_{a_1}^2 (1 - \gamma_1) & \rho_1 \alpha_{a_1}^2 (1 - \gamma_1) & 2 \mu_1 \beta_1 r_{b_1} & 2 \mu_1 \beta_1 r_{b_1} \end{pmatrix}. \]  \hspace{1cm} (1.2.47)

Considering eqs. 1.2.39 and 1.2.40 and vertical matrix propagation until the halfspace, \( m = N \), the following equation can be written

\[ \begin{pmatrix} C_1 \\ D_1 \\ E_1 \\ F_1 \end{pmatrix} = L_1^{-1} D_2 ... D_{N-1} L_N \begin{pmatrix} C_n \\ E_n \end{pmatrix}, \]  \hspace{1cm} (1.2.48)

considering that \( D_n \) and \( F_n \) go to zero in the halfspace, since there is no component propagating from bottom to the top. It follows the condition
\[ F \left( \begin{array}{c} C_n \\ E_n \end{array} \right) = 0 \]  \hspace{1cm} (1.2.49)

where

\[ F = C L_1^{-1} D_2 \ldots D_{N-1} L_N F \]  \hspace{1cm} (1.2.50)

is a 2x2 matrix whose determinant shall be posed equal to zero such that the homogeneous system of equations represented by 1.2.49 has non null solutions; then defining

\[ F(c, \omega) = \det (F) \]  \hspace{1cm} (1.2.51)

one obtains the dispersion relationship

\[ F(c, \omega) = 0. \]  \hspace{1cm} (1.2.52)

Starting from a reference value of the phase velocity for deep water gravity waves, i.e. \( c = \sqrt{gh} \), the program computes, by recursive approximations, the value of \( c \) that satisfy 1.2.52 for every frequency point finding in this way the pairs \((c, \omega)\) that form the dispersion curve of the tsunami mode.

In fig. 1.2.2 the phase and group velocities, computed for different liquid thicknesses are shown.

![Figure 1.2.2: Phase and group velocities computed for different liquid thicknesses.](image)

1.2.5 Eigenfunctions computation and energy integral computation

Once that pairs \((c, \omega)\) are found, it is possible to calculate the eigenfunction coefficients and perform separate computations for liquid and solid layers. Starting from 1.2.23 and using 1.2.29
we determine in a recursive way coefficients for every liquid layer; for solid layers we start from 1.2.49 and proceed recursively with 1.2.48.

Then, since different normalizations for liquid and solid layers have been used, the boundary conditions at sea bottom have to be satisfied. In particular, the continuity of vertical displacement and pressure is requested, leading in this way to a common normalization.

Once that eigenfunctions are known it is possible to compute the energy integral $I_1$ defined by e.g. eq.7.74 in Aki&Richards (1980):

$$I_1 = \frac{1}{2} \int \rho \left[ (u^*)^2 + w^2 \right] dz$$  \hfill (1.2.53)

that, for a structure composed by homogenous layers, becomes

$$I_1 = \frac{1}{2} \sum_{i=1}^{n+l} \rho_i \left( \int (u_i^*)^2 + w_i^2 \right) dz.$$  

This last expression can be calculated analytically using expressions 1.2.13, 1.2.14, 1.2.15 and 1.2.16, since only hyperbolic and trigonometric functions are present (Schwab et al., 1984).

### 1.2.6 Excitation by a seismic source

Once that eigenvalues and eigenfunction are computed for a structural model, one has to study how they can be excited by a seismic source.

The representation theorem (Aki and Richards, 1980) demonstrates that the seismic wavefield radiated by a pure tectonic earthquake, i.e. by a shear dislocation (or by a slip vector on a fault with a direction perpendicular to the fault normal), is identical to the one generated by an opportune equivalent system of body forces with a total resultant and moment equal to zero. This body force distribution is not unique but if, the medium is isotropic, it can always be represented by a distribution of double couples on the fault surface. If the receiver is far enough from the source, i.e. source-receiver distance is much greater than the source dimensions (far-source condition), then the source can be considered as a point and represented by a double couple oriented in an proper way. A double couple can be considered as a combination of two force couples, $M_{ij}$, where index $i$ represents the direction of the two opposite forces of intensity $f$ and index $j$ is the direction along which this two forces are separated by the distance $d$.

In general a force distribution corresponding to different point source mechanisms can be represented by a moment tensor $\mathbf{M}$ whose component are nine couples of forces.

$$\mathbf{M} = \begin{pmatrix}
M_{xx} & M_{xy} & M_{xz} \\
M_{yx} & M_{yy} & M_{yz} \\
M_{zx} & M_{zy} & M_{zz}
\end{pmatrix}$$ \hfill (1.2.54)
For example, a left strike-slip mechanism on a fault placed on the $y-z$ plane has an equivalent body force $M_{xy} + M_{zy}$ that can be represented by the following moment tensor

$$
M = \begin{pmatrix}
0 & M_0 & 0 \\
M_0 & 0 & 0 \\
0 & 0 & 0 \\
\end{pmatrix} = M_0 \begin{pmatrix}
0 & 1 & 0 \\
1 & 0 & 0 \\
0 & 0 & 0 \\
\end{pmatrix}
$$

(1.2.55)

The scalar seismic moment $M_0$ is the moment of each of the two couples that form the equivalent double couple, and is defined as $M_0 = \mu A \bar{u}$ where $\mu$ is the rigidity of the medium, $A$ is the fault area and $\bar{u}$ the average slip.

The moment tensor of an earthquake represents also its geometrical characteristics (e.g. fault-plane orientation) while the scalar moment is also measure of its dimensions. Kanamori (1997) has defined a magnitude, $M_w$, based on scalar seismic moment, i.e.

$$
\log M_0 = 1.5M_w + 9.1
$$

(1.2.56)

where $M_0$ is expressed in $N \cdot m$.

Moment magnitude has different advantages in comparison to other magnitude scales: it doesn’t saturate and it’s directly connected to rupture process. An estimate of $M_0$ from observation is relatively simple: in fact this quantity is proportional to the amplitude of ground motion at low frequencies.

### 1.2.7 Tsunami mareograms for 1D and 2D models

In order to obtain a tsunami signal generated by a given seismic source, its magnitude and focal mechanism, via the equivalent moment tensor, have to be included in the formulation. In the present work this operation is done following the theory adopted in the modal approach for seismic waves (e.g. Levshin et al., 1989); actually, this natural inclusion of the seismic source is one of the main advantages in adopting the modal method for tsunami.

In particular the following formula is used for 1D models

$$
U = \frac{\exp(-i\pi/4) \exp[i\omega(t - X/c)]}{\sqrt{8\pi}} \frac{R\chi}{\sqrt{\omega c \sqrt{v_g I_1}}} \frac{u}{\sqrt{v_g I_1}}
$$

(1.2.57)

where: $X$ is the epicentral distance, $u$ is the eigenfunction at the receiver, $I_1$ is the energy integral $v_g$ is the group velocity, $R = R(\omega)$ is the Fourier transform of the source time function and $\chi$ represents the radiation pattern, function of the focal mechanism and source depth, $h_s$.

The tsunami mareogram (or tsunamigram) is first computed in the frequency domain and then, using the inverse Fourier transform, expressed in time domain.
For 2D models, i.e. models that presents also lateral heterogeneities in the liquid layer thickness in the direction of propagation of the wave, the problem has been solved by Panza et al. (2000). In particular, they consider the configuration schematized in fig. 1.2.3

![Figure 1.2.3: Structure with lateral heterogeneity.](image)

and in this case formula 1.2.57 becomes

$$ U = \frac{\exp(-i\pi/4)}{\sqrt{8\pi}} \exp[i\omega(t-\tau)] \frac{R\chi}{\sqrt{\omega c \sqrt{v_g I_1}}} |_{s} \frac{u}{\sqrt{v_g I_1}} |_{r} $$

(1.2.58)

where the quantities with index s are referred to the source structure, while those with index r are computed at the receiver structure; \( \tau \) and \( J \) are the arrival time and the geometrical spreading, replacing \( X/c \) and \( X \) in expression 1.2.57, whose expressions are

$$ \tau = \int_{0}^{X} \frac{dx}{c(x)\sqrt{1-p^2(x)c^2(x)}} $$

(1.2.59)

$$ J = \frac{\partial Y}{\partial \theta} \cos \theta $$

(1.2.60)

where \( \theta \) is the angle between the direction of propagation of the ray and the propagation axis (\( X \) in this case), and \( Y \), as one can see in fig. 1.2.4, can be expressed, according to the ray theory, with

$$ Y = \int_{0}^{X} \frac{p(x)c(x)}{c(x)\sqrt{1-p^2(x)c^2(x)}} dx $$

(1.2.61)

in which \( p(x) \) is the ray parameter, that remains constant along the entire trajectory and equal to \( \sin \theta/c \).
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Figura 1.2.4: Geometrical spreading.

Considering the phase velocity, \( c(x) \), as a linear function of the distance, i.e. \( c = c_0(1 + \varepsilon x) \), the following expression holds

\[
Y = \int_0^X \frac{\sin \theta_0 (1 + \varepsilon x)}{\sqrt{1 - \sin^2 \theta(1 + \varepsilon x)^2}} \, dx = \frac{(\cos \theta_0 - \cos \theta)}{\varepsilon \sin \theta_0} \quad (1.2.62)
\]

\[
\tau = \int_0^X \frac{1}{c_0(1 + \varepsilon x) \sqrt{1 - \sin^2 \theta(1 + \varepsilon x)^2}} \, dx = \frac{[\cosh^{-1}(1/\sin \theta) - \cosh^{-1}(1/\sin \theta_0)]}{\varepsilon c_0} \quad (1.2.63)
\]

\[
J = \frac{(\cos \theta_0 - \cos \theta)}{\varepsilon \sin^2 \theta_0} = \frac{Y}{\sin \theta_0} \quad (1.2.64)
\]

if one assumes that wave propagates along x axis, then \( \theta_0 \to 0 \) and \( \theta \to 0 \) and equations 1.2.63 and 1.2.64 become

\[
\tau = \frac{\log(1 + \varepsilon x)}{\varepsilon c_0} \quad (1.2.65)
\]

and

\[
J = \left(1 + \frac{\varepsilon X}{2}\right) X. \quad (1.2.66)
\]

Considering \( c(x) \) as a piecewise linear function, expressions 1.2.65, 1.2.66 or, 1.2.63 and 1.2.64, become valid for adjacent sectors of the structure; thus, to compute the travel time and geometrical spreading along the entire path one has

\[
\tau = \begin{cases} 
\sum_{i=1}^n \frac{\cosh^{-1}(1/\sin \theta_i) - \cosh^{-1}(1/\sin \theta_{i-1})}{\varepsilon_i c_{i-1}} & \theta_0 \neq 0 \\
\sum_{i=1}^n \frac{\log(1 + \varepsilon x_i)}{\varepsilon_i c_{i-1}} & \theta_0 = 0 
\end{cases} \quad (1.2.67)
\]

\[
J = \begin{cases} 
\frac{\cos \theta_0 \cos \theta_i}{\sin \theta_i} \sum_{i=1}^n \frac{y_i}{\cos \theta_i \cos \theta_{i-1}} & \theta_0 \neq 0 \\
\sum_{i=1}^n \frac{c_{i-1}}{c_0} \left(1 + \frac{\varepsilon x_i}{2}\right) x_i & \theta_0 = 0 
\end{cases} \quad (1.2.68)
\]

In order to justify the assumption that \( \theta_0 \) can be considered zero, one can compare \( \tau \) and \( J \) using both the complete formulas 1.2.63 and 1.2.64 and the simplified ones 1.2.65 and 1.2.66 finding that their difference is quantitatively very small.
In particular, for geometrical spreading one has a variation of about 5% for $\theta_0 = 46^\circ$ and of 10% for $\theta_0 = 64^\circ$; for arrival times the difference is 100s for a take-off angle of 25° and 1000s for angles of 72°. These variations have very little influences on the final signals. Following these considerations, it is possible to assume the variation of velocity only in the direction of propagation; in particular one can assume that $c$ has a linear variation along structures bounded by vertical planes that are parallel to y-z plane, see fig. 1.2.3.

The shoaling factor can be obtained from a comparison between the formula for a 1D, 1.2.57, and that for a 2D , 1.2.58 models. This factor accounts for amplification of the wave with the thinning of sea layer thickness.

In particular, considering the ratio for two vertical amplitudes at two different receivers, using the 2D formula one has:

$$\left| \frac{W(X_2,0,\omega)}{W(X_1,0,\omega)} \right| = \left| \frac{w(0,\omega)}{w(0,\omega)} \right| \frac{2\sqrt{v_g I_1}}{\sqrt{v_g I_2}} \sqrt{\frac{J_1}{J_2}}. \quad (1.2.69)$$

Using another approximation, ie. considering a rigid bottom and an incompressible fluid, the eigenfunction for liquid layers, expression 1.2.1 assumes the classical form of a gravity wave

$$w(z,\omega) = w(0,\omega) = \frac{\sinh[k(H-z)]}{\sinh(kh)} \quad (1.2.70)$$

and

$$u(z,\omega) = u(0,\omega) = -iw(0,\omega) \frac{\cosh[k(H-z)]}{\cosh(kh)}. \quad (1.2.71)$$

where $H$ is the depth of liquid layer.

Using this two expressions the energy integral becomes

$$I_1 = \frac{\rho w^2(0,\omega) \sinh(2kh)}{\sinh^2(kh)} \cdot 2k, \quad (1.2.72)$$

and, at long periods, where the shallow water approximation holds,

$$c \approx v_g \approx \sqrt{gH} \quad (1.2.73)$$

$$I_1 = \frac{\rho g w^2(0,\omega)}{\omega^2}. \quad (1.2.74)$$

The first part of right side term of equation 1.2.69 becomes

$$\sqrt{\frac{H_1}{H_2}} \quad (1.2.75)$$
that, neglecting geometrical spreading effects, is the Green’s law for tsunami propagating in models with varying bathymetries.

1.2.8 Radiation pattern

In equation 1.2.57, $\chi$ represents the azimuthal dependence of the excitation factor: for a double couple it is given by Ben-Menhaem and Harkrider (1964) and is equal to

$$\chi(h_s, \phi) = d_0 + i(d_1\sin\phi + d_2\cos\phi) + d_3\sin2\phi + d_4\cos2\phi \quad (1.2.76)$$

with

$$d_0 = \frac{1}{2}B(h_s)\sin\lambda\sin2\delta \quad (1.2.77)$$

$$d_1 = -C(h_s)\sin\lambda\cos2\delta$$

$$d_2 = -C(h_s)\cos\lambda\cos\delta$$

$$d_3 = A(h_s)\cos\lambda\sin\delta$$

$$d_4 = -\frac{1}{2}A(h_s)\cos\lambda\sin2\delta$$

where, as shown in fig. 1.2.5, $\phi$ is the strike-receiver angle (measured in anticlockwise direction), $h_s$ is the focal depth, $\delta$ is the dip angle and $\lambda$ is the rake angle.
Functions $A(h_s)$, $B(h_s)$ and $C(h_s)$ depend on the eigenfunction values at the hypocenter and they assume the following expressions:

$$A(h_s) = -iku(h_s)$$

$$B(h_s) = -iku(h_s) \left(3 - 4 \frac{\beta^2(h_s)}{\alpha^2(h_s)}\right) - 2 \left[\frac{\partial w}{\partial z}\right]_{h_s} -iku(h_s) \left(1 - 2 \frac{\beta^2(h_s)}{\alpha^2(h_s)}\right)$$

$$C(h_s) = \frac{\partial u}{\partial z}\Bigg|_{h_s} -ikw(h_s).$$

Any radiation pattern can be obtained as a combination of three different elementary configurations e.g. Aki&Richards (1980): a strike slip mechanism on a vertical fault, a dip-slip mechanism on a vertical fault and a thrust mechanism on a fault with dip equal to $\pi/4$.

**1.2.9 Source modeling**

In the sections 1.2.6 and 1.2.8 a brief description of the seismic point source representation, and its inclusion in the modal method framework is given. However, the seismic source mod-
elling processes can take different steps. The most common approach consists in adopting a kinematic model of the seismic source; the kinematic description is merely phenomenological but, also the simplest versions (e.g. Haskell, 1964) are able to describe the gross features of the rupture process by simply using five source parameters: the fault dimensions (length, L, and width, W), the amount of slip of any point of the fault, the rise-time, and the rupture velocity. In order to describe the source microstructure, i.e. its roughness, one can use a stochastical or a deterministic (or a combination of both) distribution of barriers and asperities on the fault surface resulting in a non-uniform distribution of slip. Thus, at a given site the motion is dependent on the size and the duration of each of the sub-sources, and on their distribution in space and in time. Furthermore, at epicentral distances comparable with the dimensions of the fault, the relative positions of the sub-sources with respect to a receiver can play a fundamental role in the interference of the different wavetrains, resulting in the so-called directivity effect. For an earthquake with a given radiated energy, the decay of the source spectrum at high frequencies shows a strong azimuthal dependence, since the corner frequency at a given receiver in the near-field is a function of all the kinematic source parameters.

The time histories are simulated using (1) Size Scaled Point Source (SSPS), (2) Extended Source (ES) and (3) Space and Time Scaled Point Source (STSPS) models. In the first case, the point source (approximation valid when the receiver is at distances greater than the source dimensions) is scaled for its dimensions using a relatively simple spectral scaling law, with zero phase. In the ES case, the seismic waves due to an extended source are obtained by approximating it with a rectangular plane surface, corresponding to the fault plane on which the main rupture process is assumed to occur. The source is represented as a grid of point subsources, and their seismic moment rate functions are generated considering each of them as realizations (sample functions) of a non-stationary random process. Specifying in a realistic way the source length and width, as well as the rupture velocity, one can obtain realistic source time functions, valid in the far-field approximation. Finally, to calculate the ground motion at a receiver, Green’s functions are computed with the highly efficient and accurate modal summation technique, for each subsource-receiver pair, and then convolved with the subsource time functions and, at last, summed over all subsources. Furthermore, assuming a realistic kinematic description of the rupture process, the stochastic structure of the accelerograms can be reproduced, including the general envelope shape and peak factors. The extended seismic source model allows us to generate a spectrum (amplitude and phase) of the source time function that takes into accounts both the rupture process and directivity effects, also in the near source region. The third case (STSPS), where a mixture of extended and point sources is used, is not part of this work, since due to its characteristic long periods, tsunami is not strongly affected by source time function.
**SizeScaledPointSource(SSPS)**

In this case, we represent the finite fault by using the source spectra scaling laws of Gusev (1983), as reported in Aki (1987). We have chosen these curves for several reasons: as compared to $\omega^{-2}$ spectra (e.g. Joyner, 1984; Houston and Kanamori, 1986) Gusev curves, that are based on a solid statistical analysis, in the range from 2s to 0.1s are more conservative for the worst possible scenario; they give often the correct corner frequency in order to fit with the synthetic seismograms the observed amplitudes (Panza et al., 2001).

The point source model is a rough approximation of the physical source process when a large earthquake is considered in the calculation of synthetic seismograms or tsunamigrams at distances of the same order of the fault dimensions: the adoption of a given spectral scaling law corresponds to an average on the directivity function and on the regional variations due to different tectonic regimes.

**ExtendedSource**

When using the extended source model, the earthquake source is considered to be a dislocation over a planar rectangular area. Slip varies in space and increases weakly monotonously in time, so that the dislocation rate is non-negative. Its unit moment tensor (defined by slip direction and fault-normal direction) does not vary over this area or in time. Therefore, the description of the source in space-time is essentially scalar, in terms of the distribution over the fault area of the seismic moment (and its time rate). To specify temporal and spectral properties of the simulated sources, the equivalent point source (SSPS) moment rate time history $M_0(t)$, its Fourier transform $M_0(f)$ and corresponding amplitude spectrum $|M_0(f)|$ (“source spectrum”) have been widely used (e.g. Panza et al., 2001). The detailed space-time history of a simulated source can be described as shown by Gusev (2010). As a final step, for each subsource, its skeleton time history is shifted in time, by a delay corresponding to the rupture propagation kinematics, and then convolved with the common unit pulse, to produce the moment rate time function of this subsource. The scheme of the source model is shown in fig. 1.2.6
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Figure 1.2.6: Schematic representation of an extended source (ES model) embedded in a laterally heterogenous medium (I refers to the source structure and II to the receiver structure). Each cell represents an instantaneous double-couple, and at the sites of interest the Green’s functions are calculated by means of modal summation theory.

1.3 The Boumerdes earthquake: tsunami modeling

The approach we adopt to model tsunami wavefield generated by offshore earthquakes is the extension, performed by Panza et al. (2000) to the case of tsunami propagation, of the well-known modal theory, as shown in detail in sections 1.1 and 1.2.

In the modal approach the equations of motion are solved for a vertically multi-layered structural model; the set of equations is converted into a matrix problem whose eigenvalues and eigenfunctions should be calculated (e.g. Haskell, 1953; Knopoff, 1964; Schwab and Knopoff, 1972). In general, the modal theory gives a solution corresponding to the exact boundary conditions, and it can be extended to models with slightly varying thickness of the water layer.

The modal method allows to calculate synthetic tsunamigrams both for laterally homogeneous (1D) and laterally heterogeneous (2D) models; these signals are computed using equations 1.2.57 and 1.2.58 respectively.

For the 2D case, the structural model is parameterized by a number of 1D structures put in series along the profile from the source to the receiver site as shown in fig. 1.2.3.

The parameterization of the bathymetry could be important for the longer source-site paths, since it can strongly influence travel times. However, in this case, in order to compute a large dataset of signals with a fast computational time, we have modified the existing code in such a way that it permits a simplification of laterally heterogenous media (2D).

In particular, we have chosen for every source-receiver path only two different structures: one relative to the source and one to the receiver site. This choice, combined to that of another parameter that quantify the slope of bathtimetry along the wave path, in many cases permits to reproduce quite well the results obtained with a full 2D model. Some examples of these results
will be shown in the following sections.

One of the benefits of this approximation consists in the possibility to perform very fast and automatic calculation of 2D signals, especially computed at a grid of receiver for extended target areas.

**The extended source model**

As mentioned in section 1.2.9, the point-source approximation holds for source-receiver distances greater the fault length; this assumption is not always valid and a modelling of an extended source is requested. In this work, to obtain mareograms for the extended source we have developed a code that uses the synthetic slip distribution along the fault obtained by a stochastic procedure using program PULSYN jointly developed at the ICTP with Gusev (Gusev, 2010). This code discretizes the fault and assigns a value of the slip and of rupture time to each subsource (see figure 1.3.1). The characteristic of each subsource is then used as an independent source to compute the relative signal; the sum of all the signals obtained gives us a final mareogram for the extended source model.

![Figure 1.3.1: An example of 2D final slip function and rupture history obtained with Pulsyn (e.g. Gusev and Pavlov, 2006).](image)

Another original contribution of this work is the possibility to use together in a combined way 2D bathymetry models and extended source models: as it will be shown in section 1.4 for the Tohoku tsunami this combination gives results that are in good accordance with the measured signals.

### 1.3.1 Newly implemented algorithms: computational examples

As a first application of these new algorithms we have decided to work on Boumerdes event of the 21th of May 2003, and consequent tsunami. This event killed 2200 People and generated
a tsunami that was recorded in the Mediterranean region, especially along the Spanish coasts. Algeria is a zone subjected to elevated seismicity but, historically, the number of dangerous tsunami registrated is quite low (see Tab. 1.1); among these events the most important occurred in 1856 in Jijel and created waves of 5 m height. In detail, the fault parameters are (see fig. 1.3.2): strike equal to 54°, dip equal to 50° with a reverse mechanisms (near most tsunamigenic configuration), a length of 54 km and a width of 24 km (Meghraoui et al., 2003; Delouis et al., 2004).

![Figure 1.3.2: Slip model on the fault at the origin of the earthquake of 2003 at Boumerdes (Meghraoui et al., 2004).](image)

<table>
<thead>
<tr>
<th>Year</th>
<th>Source</th>
<th>Magnitude</th>
<th>Tsunami area</th>
<th>Max (m)</th>
<th>Distance (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1773</td>
<td>Tangier (Morocco)</td>
<td>*</td>
<td>Algiers</td>
<td>1.8</td>
<td>799.4</td>
</tr>
<tr>
<td>1856</td>
<td>Jijel</td>
<td>*</td>
<td>Jijel</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>1885</td>
<td>Algeria</td>
<td>*</td>
<td>Algerian shores</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>1980</td>
<td>El Asnam</td>
<td>7.3</td>
<td>Algerian shores</td>
<td>0.7</td>
<td>*</td>
</tr>
<tr>
<td>1989</td>
<td>Tipasa</td>
<td>5.9</td>
<td>Tipasa</td>
<td>22.8</td>
<td></td>
</tr>
<tr>
<td>2003</td>
<td>Zemmouri</td>
<td>6.8</td>
<td>Zemmouri</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1.1: Tsunami catalogue for Algeria.

As a first step we have chosen 11 sites corresponding to important cities (see fig 1.3.5) in the Western part of Mediterranean Sea; then, we have performed a parametric study to select the right parameters for simplified 2D model using as a reference the results of Lamara (2007).

In figures 1.3.3 and 1.3.4 the comparison between our results, obtained with simplified 2D model, and the results of Lamara (2007), computed using a full 2D model is shown.
We have then computed the signals for the chosen receivers and we extrapolated the maximum amplitude and its travel time for different magnitude-depth pairs. As an example, in table 1.2 we have listed our results corresponding to M=7.5 and depth=10 km using the simplified 2D program. In figs.1.3.6-1.3.16 the results for all the receivers comparing 1D signals and 2D signals, both for point and for extended source model are shown.
In particular, the last combination, i.e. a 2D bathymetry model combined with an extended source, represents an element of innovation in comparison with other similar works (e.g Lamara, 2007; Bisignano and Romanelli, 2009).

![Map of North-African coast and Balearic Islands](image)

**Figure 1.3.5:** Source (red pin) and selected receivers (yellow pins).

<table>
<thead>
<tr>
<th>Location</th>
<th>Distance from source (km)</th>
<th>Maximum amplitude (m)</th>
<th>Arrival time (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algeri</td>
<td>50</td>
<td>320</td>
<td>12</td>
</tr>
<tr>
<td>Genova</td>
<td>890</td>
<td>45</td>
<td>115</td>
</tr>
<tr>
<td>Gibraltar</td>
<td>750</td>
<td>50</td>
<td>120</td>
</tr>
<tr>
<td>Gulf of Palmas</td>
<td>490</td>
<td>70</td>
<td>70</td>
</tr>
<tr>
<td>ibiza</td>
<td>270</td>
<td>140</td>
<td>45</td>
</tr>
<tr>
<td>Jijel</td>
<td>165</td>
<td>145</td>
<td>35</td>
</tr>
<tr>
<td>Marseille</td>
<td>705</td>
<td>60</td>
<td>100</td>
</tr>
<tr>
<td>Palma de Maiorca</td>
<td>270</td>
<td>100</td>
<td>65</td>
</tr>
<tr>
<td>Salerno</td>
<td>1000</td>
<td>40</td>
<td>150</td>
</tr>
<tr>
<td>Tetouane</td>
<td>760</td>
<td>50</td>
<td>115</td>
</tr>
<tr>
<td>Valencia</td>
<td>385</td>
<td>75</td>
<td>90</td>
</tr>
</tbody>
</table>

Table 1.2: Amplitudes and arrival times for selected receivers: with M=7.5 and source depth equal to 10 km.

In figs. 1.3.17 and 1.3.18 we show the computed hazard maps (for M = 7.5 and depth = 10 km) for a part of North-African coast and for the zone in front of Balearic Islands.
The use of simplified 2D model permits a very fast computation of tsunami hazard maps for a grid of receivers. The low computational cost of this method permits to repeat these calculations for different sources and source models, obtaining in this way a complete signals database that can be used in presence of a real-time data to confirm or close a pending alert. These results can be used also as realistic input data for the tsunami hazard assessment at a given site, or at a coastal region, in terms of expected amplitudes and arrival times associated to different earthquake scenarios (see also section 1.5). At the same time, they can be combined with hydrodynamic modelling in order to obtain inundation maps (considering also coastal non linear effects) for the studied zones.

Figure 1.3.6: Computed signals for Algiers receiver.
Figure 1.3.7: Computed signals for Genova receiver.

Figure 1.3.8: Computed signals for Gibraltar receiver.
Figure 1.3.9: Computed signals for Ibiza receiver.

Figure 1.3.10: Computed signals for Jijel receiver.
Figure 1.3.11: Computed signals for Marseille receiver.

Figure 1.3.12: Computed signals for Gulf of Palma receiver.
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Figure 1.3.13: Computed signals for Palma De Maiorca receiver.

Figure 1.3.14: Computed signals for Salerno receiver.
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Figure 1.3.15: Computed signals for Tetouane receiver.

Figure 1.3.16: Computed signals for Valencia receiver.
1.4 The Tōhoku-Oki earthquake: tsunami modelling

The Tōhoku-Oki earthquake (M=9.0) of the 11th of March 2011 has been the strongest ever recorded in Japan and the 7th on a planetary scale. As everyone knows, the main damages, especially those inherent to Fukushima nuclear plant, were caused by tsunami. The great amount of energy released by this event could have moved earth rotation axis of about 17cm and produced a vertical slip of about 4 meters of the Japan coasts.
This event has provided an unprecedented opportunity to utilize Japan’s monitoring networks (e.g. GPS, seismic and DART buoys) to gather data. The implications of the new observations, especially about the dynamical properties of tectonic faults, need to be further explored and integrated in sound physical models of earthquakes to go towards better quantification of the related hazards. This devastating event caused over 20000 people to die and the damages at Fukushima nuclear plants are still on the chronicles in these days. These and other catastrophes call for an increased attention in dealing with tsunami disasters, both on alert systems (e.g. DART systems) and hazard maps. In particular the unexpected magnitude of the latest event and the consequent inefficiency, particularly for tsunami barriers, impose us to put new consideration on the concept of “maximum credible earthquake” and on the hazard-scenarios based on it. The Headquarters for Earthquake Research Promotion (HERP) admitted that the size of the source area, which covers the offshore areas of central Sanriku, Miyagi Prefecture, Fukushima Prefecture, and Ibaraki Prefecture, the consecutive rupturing, and the magnitude 9 were beyond expectation (Earthquake Research Committee, HERP: The evaluation of the Tōhoku District - Off the Pacific Ocean Earthquake released on March 11). Moreover, in contrast to the fact that the rupture spread from the hypocenter to the shallow area of the plate boundary, and slip amount was above 20m, it was assumed that the shallow plate boundary along the Japan trench in the offshore area of Miyagi Prefecture was not able to store a large amount of strain energy, because the area was assumed to be creeping. Some experts, however, commented that the area was strongly coupled, the strain energy has hence been stored for a long time, and the rupturing off the coast of Miyagi Prefecture became the trigger for this earthquake. An interesting insight comes from earthquake dynamics: Ide et al. (2011) showed that strong spatial variation of rupture characteristics in the moment magnitude (Mw) 9.0 Tōhoku-Oki megathrust earthquake controlled both the strength of shaking and the size of the tsunami that followed; a combination of a shallow dipping fault and a compliant hanging wall may have enabled large shallow slip near the trench. These results are confirmed by the tsunami waveform inversion performed by Fujii et al. (2011): ocean bottom pressure and GPS wave gauges recorded two-step tsunami waveforms, gradual increase in water level (~2 m) followed by an impulsive tsunami wave (3 to 5 m). The slip distribution estimated from 33 coastal tide gauges, offshore GPS wave gauges and bottom pressure gauges show that the large slip, more than 40 m, was located along the trench axis (see fig. 1.4.1). This offshore slip, similar but much larger than the 1896 Sanriku “tsunami earthquake”, is responsible to the recorded large impulsive peak. The large slip on the plate interface at southern Sanriku-oki (~30 m) and Miyagi-oki (~17 m) around the epicenter, similar location with larger slip than the previously proposed fault model of the 869 Jogan earthquake, is responsible to the initial water rise and presumably large tsunami inundation in Sendai plain.
Figure 1.4.1: From left to right: a) Recorded tsunami signals (red curves) by: sea-bottom pressure sensors in open ocean (TM-1, TM-2), GPS wave gauges, coastal tidal gauges. The blue and green curves are computed tsunami waveforms from the large near trench (blue) and deeper interplate slip (green), as reported in c). b) Slip Distribution estimated by the tsunami waveforms inversion. c) Sea floor deformation computed from the estimated slip distribution. Modified by Fuji et al. (2011).

1.4.1 Tsunami computation and data comparison: validation of the modal method.

To show the validity of the modal technique for tsunami we have performed some tests: we have adopted the finite source model by Gavin Hayes from USGS (http://earthquake.usgs.gov/earthquakes/eqinthenews/2011/usc0001xgp/finite_fault.php) and, using a simplified 2D propagation model, we have computed signals at three DART buoys in the ocean (buoy 21418, buoy 21401, buoy 21413) and at the two buoys towards the Japan coast located near Fukushima and Kamaishi, respectively. The model of slip distribution on the fault is shown in fig. 1.4.1 and the map with receivers position is shown in fig. 1.4.2.
The results for DART buoys 21418, 21401 and 21413 are shown in figs. 1.4.3, 1.4.4 and 1.4.5 respectively: considering the arrival time and the amplitude of the main peak the fit with recorded data is quite good.

Figure 1.4.2: Slip distribution on the fault and receivers position.

Figure 1.4.3: Computed and recorded signals for 21401 DART buoy.
In figs. 1.4.6 and 1.4.7 are shown the results for Kamaishi and Fukushima buoys and in these cases the agreement is poorer: while the source-receiver paths in the previous case were mainly in the Pacific Pit, now they follow a more complicated bathymetry. It should also be noted that the real signals stop abruptly in both cases, probably due to damages on the buoy for the large amplitude of the wavetrain, and the only available data that we can infer in this case is the arrival
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time of the first phases.

![Figure 1.4.6: Computed and recorded signal for Kamaishi buoy.](image1)

![Figure 1.4.7: Computed and recorded signals for Fukushima buoys.](image2)
1.4.2 Parametric studies: rupture model and bathymetric approximation.

In order to get a better evaluation of the adopted bathymetry approximation we made a comparison of results obtained with the full 2D model and the simplified one. The results for Kamaishi and Fukushima receivers are shown in figs. 1.4.8 and 1.4.9, respectively; blue lines represent full 2D model while red lines represent the simplified one.

Figure 1.4.8: Comparison of the signals obtained considering full and simplified bathymetry for Kamaishi receiver.
Figure 1.4.9: Comparison of the signals obtained considering full and simplified bathymetry for Fukushima receiver.

The results show that the bathymetric approximation adopted for Kamaishi receiver is quite acceptable, while for the Fukushima site the recorded peak values are underestimated: in order to obtain a better agreement the simplified parameterization of bathymetry should be changed.

In evaluating a tsunami hazard the slip distribution on the fault plane is usually unknown. The algorithm PULSYN can also generate random slip models on a given fault and it can be used to quantify how different slip distributions affect the final signals. In this case, a comparison has been made between different random slip distributions and the one similar to the adopted one shown in fig.1.4.1 that is characterized by greater values on the top part of the fault. Intuitively, a configuration like this has a major tsunamigenic potential and this feature is very important when evaluating hazard scenarios on a conservative bases.

This statement is confirmed by the next numerical experiment: in figs. 1.4.10 and 1.4.11 the comparison between the chosen distribution of slip and a random one (others random distribution give similar results) is shown for Kamaishi and Fukushima receivers.
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Figure 1.4.10: Signals obtained from the initial slip distribution and a random one for Kamaishi receiver.

Figure 1.4.11: Signals obtained from the initial slip distribution and a random one for Fukushima receiver.
1.5 Tsunami Hazard Analysis

Tsunami phenomenon has reached a global attention with the 26 December 2004’s event in Sumatra, when 300000 people were killed. This and other similar episodes in recent years (e.g. Samoa Island of the 29-th of September 2009, Chile of the 27-th of February 2010, Tohoku of the 11-th of March 2011) have imposed a renewed attention in dealing with tsunami disasters: both on alert systems (e.g. DART systems) and on hazard maps.

Anyway, after the tragic consequences of the 11 March 2011 Tohoku earthquake, the road toward safety is still long: in particular, the unexpected magnitude of this event, and the consequent inefficiency of tsunami defences, push the scientific community to put under a new consideration the concept of “maximum credible earthquake” and the related hazard-scenarios.

Modelling an hazard scenario has the main purpose to assess the maximum threat expected from a studied phenomenon in a certain area and to give specific directives to the local authorities in order to prevent and mitigate serious consequences on the population, the infrastructures and the environment. To build scenario-based tsunami hazard maps for a specific coastal area one has first to characterize the seismic sources (or other tsunamigenic events, not considered in this work) and select the earthquake scenarios that can drive the hazard. By means of the modelling then the maximum amplitude of the vertical displacement of the water particles on the sea surface and the travel time of the maximum amplitude peak are computed: in fact they are the most relevant aspects of the tsunami wave and they are often the only characteristics always recorded in the chronicles and therefore in catalogues. The horizontal displacement field is calculated too, and, on average, it exceeds the vertical one by an order of magnitude (this accounts for the great inundating power of tsunami waves with respect to wind driven ones).

It is important to point out that the extremely efficient analytical modeling techniques (computation times are of the order of seconds and are bound to decrease with the natural rate of improvement of computers) for real time simulations can be very useful also for integration in a Tsunami Warning System, since they can be compared with real time incoming open-sea level data, in order to validate, or close, an impending alarm. Following the guidance provided by the Intergovernmental Coordination Group for the Indian Ocean Tsunami Warning and Mitigation System (ICG/IOTWS 2007), a tsunami hazard earthquake scenario is built up by specifying the various characteristics of a potentially tsunamigenic seismic source. Schematically, for a scenario based THA the necessary steps are: a) building a database of potentially tsunamigenic earthquake Source Zones, b) each Source Zone includes an active tectonic structure with a Maximum Credible Earthquake and a typical fault, c) provide information on the expected tsunami impact (e.g. height and arrival times) onto the target coastline. The procedure can include additional stages: building a unique aggregated scenario by combining together all of the computed ones (selection of the maximum value of a given physical variable such as e.g. height); subjectivity, and the related uncertainties, can be treated by performing a sensitivity
analysis. One of the critical aspects is in step b) and it is related to the fact that the uncertainties when recurrence interval is long and the historical record is short cannot be overcome, since magnitude and location from historical intensity data can be inaccurate and the compilation of existing data is unacceptable or misleading. Thus, one of the key points is that the procedure should be progressively updated as knowledge (theoretical and experimental) of earthquake source advances.

1.6 Tsunami Hazard assessment for Vietnam coasts

This application has been developed in the context of a long-time collaboration between the Department of Mathematics and Geosciences of University of Trieste and the Institute of Geophysics of VAST (Bisignano and Romanelli, 2010; Bisignano et al., 2011).

1.6.1 Recorded earthquakes and tectonics systems in South East Asia region

From the analysis of various earthquakes catalogues (ASC, NEIC, NOAA, and the local one by Cao, 1997) 74526 earthquakes had been counted in South East Asia region starting from 1900; among them 98 reached values of magnitude bigger than 7. None of them, as one can see in Fig. 1.6.1, took place in Vietnam territory.

Figure 1.6.1: Historical seismicity of South East Asia region.
From a tectonic point of view, this region represents the South-Eastern part of the Eurasian Plate and it is surrounded by a subduction zone of depth ranging from 200 to 500-600 km that can be seen as a large reverse fault zone. This fault extends from Myanmar through Nicobar, Java, Timor to East Philippines; its westward part has a relatively gentle dip and a rather great depth and separates Eurasian Plats from Indo-Australian one. In this zone also auxiliary faults with strike-slip mechanism are present. In the eastern part, closing South China Sea, there is Philippine trench in which Philippine micro plate is subducted beneath the South-East part of Eurasian plate. In general this zone presents an high level of complexity that reveals itself also on bathymetry. The remaining part of South East Asia, including Vietnam, during Late Cenozoic was an intraplate section which was spitted by several second order faults that divided this zone into 3 micro plates: South China-Borneo, Shan-Indonesia and Malaysia-Sumatra. The movements among these micro plates create reverse and strike-slip faults.

A report from USGS (Kirby et al, 2005) has assessed the tsunami hazard for the entire Pacific subduction zone; this study has identified three main areas of interest: Philippine trench, Ryukyu trench and North Sulawesi trench. Nevertheless South China Sea is not a well studied zone and many uncertainties still remain about the possible tsunamigenic sources. Moreover, because of its topographic characteristics and its elevated degree of anthropization, the Vietnam coasts could be subjected to elevated risk also with waves of few meters.

The choice of the potential tsunamigenic sources has been done considering both earthquake’s catalogue and seismotectonic setting described by the work of Cao (1997) and it is shown in fig 1.6.2. For these reasons, following Cao (1997) and Pham and Thai (2007), we have selected six potential tsunamigenic source zones within the South China Sea (sources 1-6 in fig 1.6.3). Because of the low level of knowledge about source mechanism, we have decided, on a conservative basis, to use the most effective tsunamigenic mechanism, that is a reverse dip-slip dipping at 45° (e.g. Ward, 1981; Okal, 1988). Then, we have considered different scenario earthquakes that can be classified, according to: a) magnitude, b) occurrence period, Tm, to be intended solely for an engineering analysis, and c) risk level:
1 Tsunami Physics and Tsunami Hazard Assessment

Figure 1.6.2: Fault description of South China Sea.

Figure 1.6.3: Selected sources (red pins).

- Magnitude 6.5 Frequent (Tm\approx70-80 years)
- Magnitude 7.0 Occasional (Tm\approx120-140 years, Strong)
- Magnitude 7.5 Sporadic (Tm\approx200-250 years, Very Strong)
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- Magnitude 8.0 Rare (Tm ≃ 500 years, Disastrous)
- Magnitude 8.5 Exceptional (Tm ≃ 1000 years, Catastrophic)

For the last case we have considered only Source 1 in the south-west part of Philippine trench, with a focal depth of 25 km.

Then, following for example Liu et al. (2007) and considering its high level of seismic activity, we have decided to include in the analysis also a source in the northern Philippine Trench (Source 7 in fig. 1.6.3) that turns out to be the most effective among the tsunami sources that may affect Vietnam coasts.

Tsunami signals are computed at the nodes of a grid with step 0.2° that covers the considered region. Since the aim of this work is the regional average definition of tsunami hazard, a point seismic source model has been adopted.

Concerning the bathymetry, we have used the bathymetric map of South China Sea available from the Mississipi University (http://www.southchinasea.org/maps/bathymetry.jpg). As an example of these results, four different snapshots of the propagation of tsunami generated by a $M_w = 8$ event in Source 1 are shown in figure 1.6.4.

![Tsunami wave height snapshots](http://www.southchinasea.org/maps/bathymetry.jpg)

Figure 1.6.4: Snapshots of the tsunami wave height for a M=8 event located at Source 1.
The maps showing the distribution of the tsunami wave peaks along the Vietnam coasts computed aggregating the results from the first six sources are reported in Figures 1.6.5, 1.6.6, 1.6.7 and 1.6.8, for $M_w = 6.5, 7.5$ and 8, respectively.

Figure 1.6.5: Hazard scenario for M=6.5.

Figure 1.6.6: Hazard scenario for M=7.
Finally, we have computed the historical and maximum credible scenarios considering also Source 7: the first is calculated from maximum magnitude earthquakes as reported in the last 100 years, while for the second case we have assigned a maximum credible event for every source, referring not only to information from the statistical analysis of earthquake catalogues, but also to any available information about the tsunamigenic potential of fault systems.
The results are shown in figures 1.6.9 and 1.6.10: in both cases it is evident that the main contribution is given by the source in Manila Trench. In particular, the results from the maximum credible scenario (where we have assigned $M = 9$ to that source), giving about 10 meters wave height on the central part of Vietnam coast, are in accordance with the simulations of Ca and Xuyen (2008) and with paleotsunamis traces.

The values of historical hazard scenario are in agreement with historical chronicles, which do not report records of important tsunami related to seismic sources in Vietnam in the recent past.

![Figure 1.6.9: Historical scenario.](image)
However, considering the low level of monitoring of the South China Sea and the advanced anthropic setting of the Vietnam coasts, and their high level of vulnerability, these kind of results can be used as a preliminary knowledge basis to: design early warning systems, reduce tsunami risk and plan land-use for the Vietnam coasts.

1.7 Tsunami hazard assessment for Adriatic Sea

This study can be considered a development of the work of Paulatto et al. (2007), where they have used the modal method for 2D models to compute tsunamigrams from selected sources, performing parameters tests on focal mechanism and magnitude, to receivers that corresponds to important coastal cities along the Adriatic coasts. These results have been used as a starting point to compute additional and more complete tsunami hazard scenarios. As an initial step, we have reconsidered some results computed by Paulatto et al. (2007) but adopting a simplified bathymetry model. This operation can be considered as an optimization parametric test on the proper choice of the two structures adopted in the simplified model, and on the parameters describing the general characteristics of the bathymetry along the path from the source to receiver. The simplified models have then been adopted for the speditive computations, switching from the traditional single source-receiver configuration to a receivers on a grid one. This allows to consider a general overview of the tsunami hazard for a target area and, as
Tsunami Physics and Tsunami Hazard Assessment

a possible development, to compute the boundary and initial conditions for a hybrid approach devoted to numerical coastal inundation models.

We have then performed some tests for the algorithm, and the related original code, that computes tsunamigrams in 3D bathymetric models. This code, originally written for seismic waves, has been adapted to tsunami waves allowing also computation of tsunami ray propagation with curved ray paths. First, we performed some validity tests in simple 2D models where the results can be validated with the ones coming from the 2D code (see section 1.2.7).

New results for the city of Trieste have been obtained in the framework of a project involving the FVG Protezione Civile and The Abdus Salam International Centre for Theoretical Physics (ICTP) named “Sviluppo di un nuovo approccio di modellazione della struttura della Terra e delle sorgenti sismiche finalizzato alla definizione ed al regolare e continuo aggiornamento della pericolosita’ dipendente dal tempo mediante la previsione a medio termine degli eventi sismici nella regione Friuli Venezia Giulia” (Report in press).

We have computed the signals for a receiver located near Trieste considering the possible tsunamigenic sources near Split and Ancona: with this configuration a straight ray path could not reach Trieste due to the presence of the “optical” obstacle represented by the Istria peninsula.

The considered tsunamigenic zones are shown in fig. 1.7.1. and the results will be discussed in the next sections, they suggest that a tsunami with maximum amplitude up to few meters can be expected also in the Adriatic sea, in agreement with the number of historical events reported in the catalogues (e.g Paulatto et. al., 2007) and as confirmed by Tiberti et al. (2009).

Figure 1.7.1: Map of selected sources in Adriatic Sea (Paulatto et al., 2007).

1.7.1 Zone 1: Eastern Central Adriatic Sea and coast of Croazia

The first considered source zone is located in front of Split, Zone 1 in fig. 1.7.1. This zone includes the South-East area of the central Adriatic pit (or Jabula pit) and the Croatian coasts
from Zadar to the island of Hvar. The seismicity of the coastal region is determined by the subduction of Adriatic plate under the Dinarides (ZS9) while the seismicity of the central Adriatic area is of intra-plate type. The typical fault mechanism are thrust or strike-slip and the focal depth ranges from 10 to 25 km. Most of the zone is underwater and thus the macroseismic data are not abundant.

The maximum reported historical magnitude is $M = 6.1$. According to the most pessimistic estimates, earthquakes with magnitudes lower than 6.0 generate tsunamis with maximum magnitudes of few centimeters: therefore events with much higher values of magnitude (i.e. 6.5, 7.0 and 7.5) have been studied. Three values of focal depth are used in the calculation: 10, 15, and 25 km. The focal mechanism is fixed for all simulations: a thrust with dip angle equal to 45 degrees. The location of the epicenter is fixed for all simulations at the point of coordinates 43.20°N, 15.21°E, near the central Adriatic pit, in the correspondence of the 29 March 2003 earthquake of magnitude $M = 5.5$. The thickness of the water layer above the source is 200m. Paulatto et al. (2007) have chosen four receivers in correspondence of the cities of Durres, Ortona, Split and Venice. The complete bathymetries for these four sites and the relative bathymetries used for full 2D modelling are shown in fig.1.7.2.
The full 2D model results have been used as a reference to select the right parameters for the simplified 2D model; in tables 1.3, 1.4 and 1.5 there are the results (peak amplitude and its arrival time) for full 2D, simplified 2D and 1D models, respectively, for magnitude $M = 6.5$ and focal depth $h_f = 10km$.

<table>
<thead>
<tr>
<th>Location</th>
<th>Peak Amplitude (cm)</th>
<th>Arrival time (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Durres</td>
<td>2.0</td>
<td>110</td>
</tr>
<tr>
<td>Ortona</td>
<td>7.0</td>
<td>20</td>
</tr>
<tr>
<td>Split</td>
<td>6.0</td>
<td>30</td>
</tr>
<tr>
<td>Venice</td>
<td>3.0</td>
<td>190</td>
</tr>
</tbody>
</table>

Table 1.3: Amplitude and arrival times computed for the Zone 1 using the full 2D model (from Paulatto et al., 2007).
1 Tsunami Physics and Tsunami Hazard Assessment

<table>
<thead>
<tr>
<th></th>
<th>Peak amplitude(cm)</th>
<th>Arrival time (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Durres</td>
<td>2.5</td>
<td>108</td>
</tr>
<tr>
<td>Ortona</td>
<td>7.0</td>
<td>43</td>
</tr>
<tr>
<td>Split</td>
<td>6.0</td>
<td>37</td>
</tr>
<tr>
<td>Venice</td>
<td>3.5</td>
<td>178</td>
</tr>
</tbody>
</table>

Table 1.4: Amplitude and arrival times computed for the Zone 1 using the simplified 2D model.

<table>
<thead>
<tr>
<th></th>
<th>Peak amplitude (cm)</th>
<th>Arrival time min</th>
</tr>
</thead>
<tbody>
<tr>
<td>Durres</td>
<td>1.0</td>
<td>110</td>
</tr>
<tr>
<td>Ortona</td>
<td>1.5</td>
<td>21</td>
</tr>
<tr>
<td>Split</td>
<td>3.0</td>
<td>31</td>
</tr>
<tr>
<td>Venice</td>
<td>1.0</td>
<td>190</td>
</tr>
</tbody>
</table>

Table 1.5: Amplitude and arrival times computed for the Zone 1 using the 1D model.

As one can see the agreement between the full 2D and the simplified 2D signals is largely acceptable, while this does not applies to the 1D ones. Thus, with a proper tuning in the parameter selection towards a conservative choice (i.e. choosing the receiver model in order to maintain the arrival time of the main arrival and increase its amplitude), and considering the required computing time, the simplified 2D approach should be preferred for speditive tsunami hazard scenarios evaluation.

In figs. 1.7.3 and 1.7.4 the examples of hazard scenarios along Adriatic coasts are shown: with magnitude 6.5 and $h_s = 10km$, and magnitude 7.5 and $h_s = 15km$, respectively.
Figure 1.7.3: Scenario for the Zone 1 computed using the simplified 2D model and for M=6.5.
1.7.2 Zone 2: Eastern italian coasts

This zone comprehends the Adriatic coasts of Central Italy, from Ravenna to San Benedetto del Tronto. The seismicity is determined by the passive subduction of the Adriatic plate under the Northern Apennines. The focal mechanisms are mainly thrust and strike slip with focal depths ranging from 10 to 25 km. The maximum magnitude reported on historical catalogues is \( M = 6.0 \). For the simulations values of 6.0, 6.5 and 7.0 are chosen. Three values of focal depths are used in calculations: 10, 15 and 25km. The location of the representetive epicenter used for the modelling is chosen offshore, at the point of coordinates 43.65°N, 13.55°E, in corrispondence of the epicenter of the 1972 earthquake of magnitude 5.1, located about 10 km far from the coast of Ancona. The thickness of the water layer above the source is 50m.
The receivers are chosen in correspondence of the cities of Durres, Ortona, Venice and Zadar. With the same procedure adopted for Zone 1, we performed parametric tests on simplified 2D structures trying to fit as much as possible the results obtained by Paulatto et al. (2007). The results, for $M = 6$ and $h_s = 10\text{km}$ are shown in tabs. 1.6 and 1.7, while in tab. 1.8 the results for a 1D model are shown.

<table>
<thead>
<tr>
<th></th>
<th>Peak Amplitude (cm)</th>
<th>Arrival time (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Durres</td>
<td>0.1</td>
<td>165</td>
</tr>
<tr>
<td>Ortona</td>
<td>0.8</td>
<td>40</td>
</tr>
<tr>
<td>Venice</td>
<td>0.4</td>
<td>135</td>
</tr>
<tr>
<td>Zadar</td>
<td>0.5</td>
<td>85</td>
</tr>
</tbody>
</table>

Table 1.6: Amplitude and arrival times computed for the Zone 2 using the full 2D model (from Paulatto et al., 2007).

<table>
<thead>
<tr>
<th></th>
<th>Peak Amplitude (cm)</th>
<th>Arrival time (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Durres</td>
<td>0.1</td>
<td>168</td>
</tr>
<tr>
<td>Ortona</td>
<td>0.3</td>
<td>45</td>
</tr>
<tr>
<td>Venice</td>
<td>0.4</td>
<td>137</td>
</tr>
<tr>
<td>Zadar</td>
<td>0.4</td>
<td>90</td>
</tr>
</tbody>
</table>

Table 1.7: Amplitude and arrival times computed for the Zone 2 using the simplified 2D model.

<table>
<thead>
<tr>
<th></th>
<th>Peak Amplitude (cm)</th>
<th>Arrival time (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Durres</td>
<td>0.1</td>
<td>170</td>
</tr>
<tr>
<td>Ortona</td>
<td>0.3</td>
<td>37</td>
</tr>
<tr>
<td>Venice</td>
<td>0.3</td>
<td>172</td>
</tr>
<tr>
<td>Zadar</td>
<td>0.4</td>
<td>90</td>
</tr>
</tbody>
</table>

Table 1.8: Amplitude and arrival times computed for Zone 2 using the 1D model.

For the receivers located in Venice, Durres and Zadar the agreement between the 2D models is quite good while for Ortona receiver the simplified assumption is not acceptable, due to the complexity of the bathymetry along the source-receiver path. Also in this case, the simplified 2D model gives results much more precise than the 1D one.

After the choice of correct bathymetry parameteritazion the hazard scenario has been computed and the results are shown in figs. 1.7.5 and 1.7.6 for a magnitude $M = 6.5$ and $h_s = 10\text{km}$, and for a magnitude $M = 7.0$ and $h_s = 15\text{km}$, respectively.
Figure 1.7.5: Hazard scenario for the Zone 2 computed using the simplified 2D model and for $M=6.5$. 
1.7.3 Zone 4: Northern Albanian coast.

This zone includes the coastal region of Albania, from the border with Montenegro to latitude 40.50°N. The seismicity is determined by the subduction of the Adriatic plate under the Albanides. The typical focal mechanisms are thrust and strike-slip, the maximum historically reported magnitude is 7.3, the strongest in the Adriatic region. The focal depth ranges from 10 to 30 km. The values of magnitude chosen for the computations are 6.5, 7.0 and 7.5, the values of focal depth are 10 km, 20 km and 30 km. The representative epicenter is located at the point of coordinates 41.50° N, 19.00°E, in correspondence with the epicenter of the 346 AD earthquake of magnitude 7.3. The thickness of the water layer above the source is 180 m. The receivers are
chosen in correspondence of the cities of Ancona, Bari, Durres and Dubrovnik. The comparison between the results obtained using a full 2D bathymetry model and those with simplified model is shown in tabs 1.9 and 1.10, magnitude is $M = 7.0$ and focal depth $h_s = 10km$. In tab.1.11 the amplitude and arrival times obtained using a 1D model are shown.

<table>
<thead>
<tr>
<th></th>
<th>Peak amplitude (cm)</th>
<th>Arrival time (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ancona</td>
<td>5</td>
<td>150</td>
</tr>
<tr>
<td>Bari</td>
<td>10</td>
<td>40</td>
</tr>
<tr>
<td>Dubrovnik</td>
<td>25</td>
<td>30</td>
</tr>
<tr>
<td>Durres</td>
<td>35</td>
<td>25</td>
</tr>
</tbody>
</table>

Table 1.9: Amplitude and arrival times computed for the Zone 4 using the full 2D model (from Paulatto et al. 2007).

<table>
<thead>
<tr>
<th></th>
<th>Peak amplitude (cm)</th>
<th>Arrival time (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ancona</td>
<td>10</td>
<td>150</td>
</tr>
<tr>
<td>Bari</td>
<td>15</td>
<td>40</td>
</tr>
<tr>
<td>Dubrovnik</td>
<td>15</td>
<td>35</td>
</tr>
<tr>
<td>Durres</td>
<td>73</td>
<td>17</td>
</tr>
</tbody>
</table>

Table 1.10: Amplitude and arrival times computed for the Zone 4 using the simplified 2D model.

<table>
<thead>
<tr>
<th></th>
<th>Peak Amplitude (cm)</th>
<th>Arrival Time (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ancona</td>
<td>7</td>
<td>160</td>
</tr>
<tr>
<td>Bari</td>
<td>5</td>
<td>35</td>
</tr>
<tr>
<td>Dubrovnik</td>
<td>4</td>
<td>27</td>
</tr>
<tr>
<td>Durres</td>
<td>20</td>
<td>26</td>
</tr>
</tbody>
</table>

Table 1.11: Amplitude and arrival times computed for Zone 4 using the 1D model.

For this source the agreement between the two 2D models is not very good, and this result can be easily understood looking at the quite complicated bathymetry in front of Durres (see fig. 1.7.1) that cannot be well reproduced by simplified 2D model. However, also in this case, it is evident that the simplified 2D model is far more precise than the 1D one: considering the results shown in the tables one can compute hazard scenarios to have a general idea of the level of threat of the zone. For the zones near Ancona and Bari, the level of the wave can be overestimated: this can be acceptable adopting a conservative criterion in hazard evaluation, while for the zones near Dubrovnik and Durres this amplitude can be strongly underestimated because of the bathymetric complexity.

In figs 1.7.7 and 1.7.8 hazard scenarios for $M = 6.5$ with $h_s = 10km$, and $M = 7.5$ with $h_s = 20km$ are shown.
Figure 1.7.7: Hazard scenario for the Zone 4 computed with the simplified 2D model and for $M=6.5$. 
1.7.4 Methodology for 3D computations

Theory

As mentioned before, another possible development in the use of modal method for tsunami is the computation of signals for a model with a full 3D lateral heterogeneity. In particular, the heterogeneity is not considered only along source-receiver path but also in the other points of the structural model: this means that one can consider also curved raypaths, following Fermat principle, and waves bending around obstacles.

The theory adopted for this algorithm is fully explained by La Mura et al. (2011) for its application to the seismic wavefield; its extension to tsunami modeling is quite straightforward. This
approach allows to handle structures that exhibit smooth lateral heterogeneities, in particular compared to vertical ones.

More precisely a lateral variation in the thicknesses of the layers and in the elastic parameters characterizing each layer, can be expressed as a perturbation, by a parameter $\varepsilon$, of the original properties in such a way that, if $\varepsilon = 0$, the medium is a laterally homogeneous layered structure. When a perturbation in $\varepsilon$ is small within a wavelength, an approximate solution corresponding to the surface wave field can be obtained by mean of a procedure based on the ray method (Woodhouse, 1974; Levshin et al., 1989).

The fundamental quantities considered in this ray method are, as in the case of the 2D modelling seen in section 1.2.6, the travel time and the geometrical spreading. Treating the propagation of the waves from the point of view of the ray theory requires that the minimum wavelength involved in the problem must be larger than the lateral gradient (a step in our case) between the structures. If the lateral heterogeneous model is made up of two or more vertically heterogeneous structures justaposed each other, two adjacent structures have to be very close in the parameter’s space where they are determined by a point, or, analogously, they have to be very similar, in the sense of the elastic parameters, to allow the application of the WKBJ approximation. From a mathematical point of view, the condition of validity of this approximation can be expressed as $|\nabla \perp p| \ll \frac{\omega}{c} p$, where $p$ indicates elastic parameters, or, considering wavelength $|\nabla \perp p| \ll \frac{2\pi}{\lambda_{\max}} p$. Thus, the smooth condition involves the horizontal gradient of each elastic parameter.

In tsunami computation the main parameter in computing the dispersion function (e.g. phase velocity) is the thickness of the liquid layer, more precisely its root square (valid for shallow water waves) $c = \sqrt{gh}$, and this method can work for bathymetries with weak gradients for distances comparable to the wavelength only.

**The 3D model and the ray tracing**

The model can be represented with a grid like the one shown in fig. 1.7.9: a vertically layered structure is assigned to every point of the grid; the colours are used to represent the different spectral characteristic of every structure; the star represents the source, the triangle the receiver.
Once that the grid and the numerical parameters are set (depending on the maximum frequency, on the kind of heterogeneity and on the level of accuracy required), a bilinear interpolation is used to compute in each point of the rectangular area the phase velocity as a weighted average of the values at the corners of the cell the point belongs to.

The rays leaving from the source and reaching the receiver must then be computed: this procedure is called ray tracing and here is performed by shooting method. In two-point ray tracing, we seek the ray that connects two fixed points S and R. The initial direction of each ray is defined at point S as the direction of the straight line connecting the two points. With this initial condition, the shooting method is used to reach the point R: the ray path is determined by treating it as an initial-value problem with a specified starting point and a trial propagation direction, and then iteratively adjusting the propagation direction until the target end point, R, is reached within a pre-assigned tolerance.

The ray tracing problem is formalized as a Cauchy problem of ordinary non-linear differential equations and is performed by the Runge-Kutta algorithm to determine the value of the azimuth corresponding to the ray that effectively does reach the receiver point. The solution of the system defines a ray at the discrete points (dots) shown, as example, in fig. 1.7.10. In order to be sure that the wave front, that is orthogonal to the ray at each point of it, has passed through the receiver point, the angles $\gamma'$s are computed starting from a point of the ray that corresponds to the distance $r$ (see fig. 1.7.10) defined at step 2. These points, for which the $\gamma'$s are analyzed, are red coloured.
To ascertain that the wavefront passed through the receiver point we proceed as follows:

1. at the point of the ray corresponding to the length $r$, the angle $\gamma_1$ is defined, and at the next point the angle $\gamma_2$ is defined

2. if the cosines of $\gamma_1$ and $\gamma_2$ have the same sign, the procedure goes on to the next pair of points;

3. if the cosines result to be opposite in sign (this circumstance occurs when $\gamma_1 < \frac{\pi}{2}$ and $\gamma_2 > \frac{\pi}{2}$, as shown in fig.1.7.10), this means that the wavefront passed the receiver point and the Runge-Kutta computation of the ray can stop;

4. if the tolerance threshold, $\delta$ (fixed by the user), is exceeded then the azimuth, $\alpha_A$, of the line connecting the point $A'$ and the source is computed; a new Cauchy problem is solved with a new initial condition for the azimuth given by $\alpha_s^{(1)} = \alpha_{A'}$ (new ray); for the new ray steps 1 to 3 are repeated until convergence is reached

5. the iterative computation of the ray is terminated when the distance between the receiver and the end point of the ray, defined at 3 (see point $A'$ in 1.7.10) is less than the tolerance parameter.

Once that the correct ray is calculated, it is easy to derive the two quantities, geometrical spreading $J$ and travel-time $\tau$, necessary to compute the tsunamigram via eq. 1.2.58.

**Validation**

Before using this methodology in the context of tsunami hazard assessment for the Adriatic Sea, we performed some validation tests on it. In particular, we have tested the 3D code in a 2D
The following computations have been performed using: $M = 6.5$, $h_s = 10km$, strike-receiver angle = 45°, dip = 45° and rake = 90°.

As a starting step, we have tested the two codes in a laterally homogeneous model with a liquid thickness equal to 100m. The results are shown in fig. 1.7.11 and, as one can expect, they are identical.

![Figure 1.7.11: Results for 2D and 3D computation for 1D model.](image)

We have then performed the computation using the two codes in laterally heterogeneous models. The first adopted model is shown in fig. 1.7.12 and the related signals are shown in fig.1.7.13, and, as one can expect, the results are in a perfect agreement.
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Figure 1.7.12: First adopted laterally heterogenous model.

Figure 1.7.13: Results for 2D and 3D computation for model of fig.1.7.12.

The second 2D heterogenous model is shown in fig. 1.7.14 and
1 Tsunami Physics and Tsunami Hazard Assessment

Figure 1.7.14: Second adopted lateral heterogenous model.

the related results are shown in fig.1.7.15. In this case the agreement is very good but some slight differences are present, probably due to the different interpolation of bathymetry points adopted in the computations.

Figure 1.7.15: Results for 2D and 3D computation for model of. 1.7.14.
1.7.5 Examples of 3D tsunami computations for Source zone 1 and 2 and receiver in front of Trieste

We have used this methodology to evaluate a possible tsunami generated by Zone 1 striking the Gulf of Trieste. This situation cannot be reproduced by 2D modeling due to the presence of Istria peninsula, that would “optically” cover the receiver along the straight source-receiver path.

In tab. 1.12 the peak amplitudes for three values of magnitude, 6.5, 7, 7.5, and three different focal depths are shown, while in fig. 1.7.16 the signal computed for $M = 7.5$ and $h_s = 10\ km$ is plotted as an example.

<table>
<thead>
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<td>2.5 cm</td>
<td>14.5 cm</td>
<td>81 cm</td>
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<td>H=15 km</td>
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<td>3.5 cm</td>
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<td>H=25 km</td>
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Table 1.12: Peak amplitudes for the signal computed along Split-Trieste path.

![Figure 1.7.16: 3D signal computed along Split-Trieste path.](image)

The same procedure has been applied to the case of Zone 2 (near Ancona), and for a receiver in front of Trieste. The results are given in tab.1.13 for three different magnitudes and focal depths, while in fig. 1.7.17 the signal computed with $M = 7$ and $h_s = 10\ km$ is shown.
1 Tsunami Physics and Tsunami Hazard Assessment

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Table 1.13: Peak amplitudes for the signals computed along Ancona-Trieste path.

Figure 1.7.17: 3D signal computed along Ancona-Trieste path.
2 Seismic waves in laterally heterogeneous oceanic-continental structures

2.1 Introduction

Among the methods that attempt to solve equations of motion in flat laterally heterogeneous media with analytical techniques we can distinguish two main complementary classes: methods based on ray theory and methods based on mode coupling.

Ray methods are based on the principles of classic geometrical optics. The synthetic signal is built up as a superposition of rays, reflected and transmitted according to the Snell law. The modal approaches, that will be more extensively analyzed, share the idea that the unknown wavefields are built up as a superposition of the normal modes characteristic of the medium. The choice between the two physical representations of the wavefield depends upon the kind of data that one wants to model. It depends on whether one is interested in the dispersive features of the complete signal or in the study of the arrival times of some early phases. The number of rays necessary to model the late arrivals in one seismogram becomes huge and difficult to handle. Furthermore, at long periods, ray theory can no longer be applied, being rigorously defined at infinite frequency, and problems arise for peculiar transition zones, known as caustics. On the other hand the number of modes necessary to describe adequately the first arrivals in a seismogram could be too large to be efficiently handled. The duality between rays (P-SV, SH-waves) and modes (Rayleigh, Love) is evident also from the formal point of view: any of the two representations can be written as the Fourier Transform of the other. Whenever the source-receiver configuration is in the far-field condition, surface waves (fundamental and first few higher modes) represent the longest and strongest portion of a seismic signal generated from an earthquake. These waves constitute the dominant part of the seismogram and thus supply the data with the most favorable signal/noise ratio. Therefore, analysis of surface waves is crucial for gaining knowledge of the elastic and anelastic properties of the areas crossed by the waves, and for seismic hazard studies, with engineering implications.

Surface waves cannot be modelled easily with methods based on ray theory, because of com-
Seismic waves in laterally heterogeneous oceanic-continental structures

computational problems: it is not a theoretical limitation but a practical one. There is no doubt that the modal summation is the most suitable technique for modelling the dominant part of seismic ground motion. The key point of the technique is the description of the wavefield as a linear combination of given base functions: the normal modes characteristic of the medium. In the case of the Earth, the modal summation technique is an exact method, since for a finite body the normal modes form a complete set. If we approximate the Earth with a flat layered half-space, the completeness of normal modes is no longer satisfied, since they are associated only with the discrete part of the wavefield spectrum. Extension of the modal summation technique (whose milestones can be found in Haskell (1953), Knopoff (1966), Schwab (1970), Schwab and Knopoff (1970), Panza (1985), for example), to laterally heterogeneous media can be performed following different procedures. The choice of the most suitable procedure must take into account the geometry and the physical properties of the medium.

The main assumption of WKBJ method, widely used in seismology (Woodhouse, 1974), is that the lateral variations of the elastic parameters are regular (compared to the wavelength). Once this hypothesis is satisfied, we can assume that the energy carried by each mode in a given structure is neither reflected nor transmitted to other modes. In other words, modes are not coupled; each mode propagates with a wavenumber driven by the local structure. The amplitude is not changed, while phase perturbations are computed by averaging the whole source-receiver path, neglecting the horizontal position of the heterogeneities. An example of this approach has been already discussed in Section 1.7.4, but its applicability is limited: tectonic scenarios like boundaries between ocean and continent or deep faults cannot be treated. A different approach is based on the idea that the unknown wavefield generated by the lateral heterogeneities is written as a linear combination of base functions representing the normal modes (Love and Rayleigh) of the considered structure, therefore the problem reduces to the computation of the coefficients of this expansion. If we consider a heterogeneous medium made of two layered quarterspaces in welded contact, the traditional method (e.g. Alsop, 1966) assumes that at a given frequency the set of eigenfunctions is complete for each of the two quarterspaces. If this condition is satisfied, then the unknowns of the problem, i.e., the transmission and reflection coefficients, can be computed assuming the proper continuity conditions at the vertical interface. There are two problems with this approach: (1) at a given frequency the discrete spectrum of the eigenfunctions is not complete and the continuous spectrum should be included, and this requires the cumbersome computation of branch-line integrals; and (2) expansion in series of the base functions can be carried out for a finite number of terms, so that a control over the approximations introduced becomes necessary.

In a modal approach alternative to the original Alsop’s method, that will be followed in this section, the coupling coefficients for the modes transmitted and reflected at the vertical interface are computed, and the outgoing (inhomogeneous) surface waves are obtained as a
superposition of homogeneous and inhomogeneous waves using Snell’s law at each section (supposed infinite) of the vertical interface. The main objection to this approach is that the horizontal boundary conditions are no longer satisfied and therefore some diffracted waves, near the vertical interface, are not properly taken into account. Nevertheless it is possible to estimate the severity of the approximation introduced by checking the energy balance between the incoming and the outgoing wavefields. The main developments of this methodology can be found in Alsop (1966), Alsop et al. (1974), Gregersen and Alsop (1974), Vaccari et al. (1989), Romanelli et al. (1996), Romanelli et al. (1997).

The basic model treated in this Section consists of two layered quarterspaces in welded contact, and it is shown in fig. 2.1.1, but the formalism can be extended to any laterally heterogeneous structure using a series of 1D layered structures in welded contact at the vertical interfaces.

The original contribution of this work is the inclusion in the formalism, and in the relative codes, of the computation of the transmission coefficient for the liquid-solid, the solid-liquid and the liquid-liquid boundaries. Obviously, these extensions refer only to the algorithm for the Rayleigh modes. This upgrade permits to compute more realistic hazard scenarios for the coastal zones, considering in the proper way the submarines sources. Some computational examples will be shown, in particular focusing on the differences between the signals computed with this new algorithm and those computed with the 1D models.

The structure of this algorithm allows also to isolate the single inter-coupling and intra-coupling coefficients helping the comprehension of how the interface affect the wave propagation. This can be useful not only for theoretical aspects but also from a practical point of view, for example to study not well understood phases traveling along liquid-solid paths (e.g. Butler & Lomnitz, 2002, Rovelli et al., 2004).

Figure 2.1.1: Example of discontinuity interface.
2.2 Theory

2.2.1 The seismic wavefield in laterally heterogeneous anelastic models

In the description of this method we follow the general lines of the work Vaccari et al. (1989). The expression describing the displacement due to surface waves, propagating in a perfectly elastic and vertically stratified halfspace has been generalized, following the approximate ray theory for surface waves (Levshin, 1985), to the case of two stratified quartespaces in welded contact. For this formulation to be valid the source and the receiver should be placed far from the vertical interface in comparison to the biggest wavelength of interest. For elastic media, neglecting second-order terms, the radial component of the displacement spectrum can be written

\[
(u^r) = R(\omega) e^{-i(3\pi/4)} e^{-i\omega(l/c+l'/c')} \frac{\omega}{2\pi} \frac{\epsilon'}{2c'U''} \Gamma_{nn'} \chi(h_s, \phi) \sqrt{2c'2U''} (2.2.1)
\]

while the vertical one is related to radial one by ellipticity \( \epsilon' \)

\[
u^z = [\epsilon' e^{i\pi/2}]^{-1} u^r.
\]

Eq. 2.2.1 represents the radial displacement, indicated by index \( r \), carried by the \( n - th \) Rayleigh mode generated by a point source, at distance \( l \) from the vertical interface, transmitted through the boundary between the two media, and propagated to the receiver at a distance \( l' \) from the same vertical interface on the surface of second structure as mode \( n' \).

For a more detailed description of this formula one can see Vaccari et al. (1989); the unprimed quantities refer to the source structure (I) and the primed ones to the receiver one (II).

The term \( \Gamma_{nn'} \) is the coupling coefficient and is a function of the frequency, of the incoming mode, \( n \), and of the outgoing mode \( n' \), i.e. \( \Gamma_{nn'} = \Gamma_{nn'}(\omega, n, n') \). This quantity is described in detail in section 2.2.2.

2.2.2 The coupling coefficient definition

The starting point is the stress-displacement system of the incoming mode, that can be described by the stress-displacement vector defined by

\[ \mathbf{A}_I = (u_I, v_I, w_I, p_{x1}, p_{y1}, p_{z1}) \]

where \( p_{ij} \) is the \( j-th \) component of the stress acting across the plane normal to the \( i-th \) axis. In this case only the \( p_{xj} \) (with \( j = x, y, z \)) components are considered since, due to the geometry of the problem, only stresses acting on the vertical plane \( x = 0 \) are involved. Similarly a stress-displacement vector \( \mathbf{A}_II \) is defined for the medium with the receiver.

In particular considering Rayleigh waves propagating along the \( x \) axis (normal incidence) the stress-displacement vector can be written as
\( \mathbf{A}_I = (u_I, 0, w_I, p_{xzl}, 0, p_{xzI}) \)

and its components can be decomposed into P-components and SV-components.

Thus, referring to the fig. 2.1.1, the problem can be solved for every single section of on the vertical interface as it would be infinite, using the well-known formulae based on Snell’s law and continuity conditions of displacement and tractions at the boundary.

These conditions change for a liquid-solid interface, in fact in a liquid layer both the shear stresses and the S-waves components of displacement vanish.

Since the sections are actually limited, the effects arising at the corner between the horizontal and the vertical interfaces are neglected: in particular these effects can be imagined as a system of diffracted waves. With this approximation, a transmitted stress-displacement system \( \mathbf{A}_T \) can be determined.

The final goal of this method consist in the determination of how each of the modes of the receiver medium is excited by the modes in the incoming wavefield or, in other words, how the transmitted system is redistributed among the normal modes of the receiver medium. In the following, the term "intracoupling" refers to the coupling of a mode with itself, while "inter-coupling" indicates the coupling of a mode with another one.

This operation can be performed computing the projection of a stress-displacement vector onto another using the scalar product suggested by the Herrera’s orthogonality relation (Herrera, 1964):

\[
\langle \mathbf{A}_I, \mathbf{A}_II \rangle = \frac{1}{2i} \int_{0}^{\infty} dz [u_I \bar{p}_{xII} + v_I \bar{p}_{yII} + w_I \bar{p}_{zII} - p_{xII} \bar{u}_I - p_{yII} \bar{v}_I - p_{zII} \bar{w}_I] \tag{2.2.2}
\]

where bar denotes the complex conjugate.

So, once that the stress-displacement vectors \( \mathbf{A}_I, \mathbf{A}_II \) and \( \mathbf{A}_T \) (for a detailed evaluation of \( \mathbf{A}_T \) (see the appendix A of Vaccari et al., 1989), and the scalar products are defined, the coupling coefficients can be computed.

The quantity

\[
\gamma_{nn'} = \frac{\langle \mathbf{A}_T^{(n)}, \mathbf{A}_II^{(n')} \rangle}{\langle \mathbf{A}_I^{(n)}, \mathbf{A}_I^{(n)} \rangle^{1/2} \langle \mathbf{A}_II^{(n')}, \mathbf{A}_II^{(n')} \rangle^{1/2}} \tag{2.2.3}
\]

provides the amplitude of the mode \( n' \) of the outcoming wave due to the mode \( n \) of the incoming wave (and this energy transfer is guided by the \( \mathbf{A}_T \) vector).

If we prefer to consider an incoming mode with unit surface amplitude we need to define a normalization coefficient and the following quantity must be used, \( \Gamma_{nn'} \) of the eq. 2.2.1, defined by:

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\[ \Gamma_{nn'} = \gamma_{nn'} \cdot \langle A^{(n)}_I, A^{(n)}_I \rangle^{1/2} = \langle A^{(n)}_T, A^{(n')}_{II} \rangle \langle A^{(n')}_{II}, A^{(n')}_{II} \rangle^{1/2} = \langle A^{(n)}_T, A^{(n')}_{II} \rangle \langle A^{(n')}_{II}, A^{(n')}_{II} \rangle^{1/2}. \] (2.2.4)

The approximations used to calculate \( \Gamma_{nn'} \) are:

- The Rayleigh modes for the two media are evaluated assuming that each medium is a halfspace instead of a quarterspace; this is a reasonable assumption at a few wavelengths from the interface.

- A system of diffracted waves arising at the corners of the sections is neglected; this is a good approximation for a small contrast in the elastic parameters characterizing the two quarterspaces (Gregersen and Alsop, 1974).

In the section 5 of Vaccari et al. (1989) the reversibility theorem, that is a quantitative criterion to check the validity of this last approximation, is given.

Once that the expressions for the various stress and displacement components are assigned, the integrations indicated in eq. 2.2.2 and in the other scalar products must be performed over the semiaxis \( z > 0 \). These integrals can be evaluated as a sum of integrals over each section (see fig. 2.1.1) and take the form

\[ \int_0^\infty dz \to \sum_{s=0}^{S-1} \int_{H_s}^{H_{s+1}} dz + \int_{H_S}^\infty dz. \]

The calculation of these integrals (Vaccari et al., 1989) can be performed analytically, avoiding problems of consuming time machine (especially at the higher frequencies) and of loss of precision.

On the other hand, the analytical solutions are easy to calculate because of the separation of P and SV components that leads to integrals that are of elementary types, i.e. sinusoidal and hyperbolic functions.

In the computation of transmission coefficients for oceanic-continental or continental-oceanic interfaces the formal scheme of the method is the same as for the continental-continental boundaries, but the relations for the amplitude of outgoing P and SV (when present, just in the liquid-solid case) components are different. The ones we implemented in the codes follow the matrix form that can be found in Ben-Menahem and Singh (1981) or in Pujol (2003).
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2.3 Computational examples

2.3.1 Towards scenario based seismic hazard assessment

For the first computational example we have chosen two structures, one oceanic and one continental, representative of the Iberian region. This choice was motivated by the fact that the author has been involved in the international project “MitigAtion of Seismic Risk through Integrated Scenarios of Crustal Deformation, Earthquake Hazard, Earthquake Damage and Losses, and Real Time Ground-shaking at Regional and Urban Scales (MARSH)”, a multidisciplinary research project focusing on risk assessment and mitigation of damage and losses induced by earthquakes in Spain. Within this framework, his role was devoted to the development of scenario based seismic hazard assessment, that is based on modeling techniques, developed from knowledge of the seismic source process and of the propagation of seismic waves, that can realistically simulate the ground motion due to an earthquake by means of synthetic seismograms. At regional scale, a set of sources is defined in the tectonically active areas of the considered region. From those sources, and once the physical properties of average structural models have been defined, wave propagation is efficiently modeled with the modal summation technique (Panza, 1985; Florsch et al, 1991) and broad-band synthetic seismograms are generated at the free surface on a predefined grid of points covering the study region. Such kind of large set of realistic time series constitutes a valuable database that can be used by civil engineers for the reliable definition of the seismic input when designing seismo-resistant structures. A summarizing view of the expected ground shaking can be easily produced for the considered region by mapping the peak values extracted from the synthetic seismograms in terms of displacements, velocities and accelerations. Examples of the application of this methodology to a regional and local scale can be found in Panza et al. (2012), while its extension to tsunami hazard has been discussed in Chapter 1. However, when the hazard can be driven by earthquakes occurring in tectonic settings like oceanic-continental subduction zones (e.g. Iberian peninsula), it is necessary to extend the methodology to treat laterally heterogeneous models with sharp discontinuities.

The adopted structural models are shown in fig. 2.3.1, and the computations are performed with a cutoff frequency $F_{max} = 1$. The main differences between the two models are in the top of the structures and they are not limited to the presence of the liquid layer, involving also the solid layers.

The first test is devoted to the validation of the codes and we used the only other available analytical algorithm for 2D models with a liquid layer, i.e. the one developed by Its and Yanovskaya (1985) based on the elastodynamic Green’s function approach, that we have properly updated and re-coded (the acronym NIY is used in the following).

The comparison is shown in fig 2.3.2 for different path configurations. The seismic source is
fixed offshore, 13 km under the sea-bottom, the oceanic-continental interface is 50 km far from the epicenter and the receiver is placed inland at different distances from the interface: from the top to the bottom in fig. 2.3.2 are shown the signals for 50 + 50km, 50 + 100km, 50 + 150km, 50 + 200km, 50 + 250km.

In the first column are plotted the signals for the 1D continental structure, in the second the results obtained with the new code and in the third the results computed by NIY one. The computations are performed considering all the available modes of the structures, including also the intercoupling contributions for the 2D cases.

Two main results are evident from this figure: the good agreement between the two different 2D methods and their great difference, both in the shape and in the amplitude, between the computation performed with the 1D model. For receivers nearer to the interface the difference is more evident but also for the most distant ones (i.e. at 200 and 250 km from interface) the shapes of the signals clearly change. In particular it seems that many phases are strongly attenuated in amplitude: this can be due to the strong horizontal gradient in velocity between the liquid and the solid layer. In fact, this kind of gradients can imply a very low angle of critical incidence and this causes the conversion from homogeneous in to evanescent waves in the solid layer.
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Figure 2.3.1: Oceanic structure (on the left) and continental structure (on the right).
Figure 2.3.2: Comparison of signals computed at 1Hz with three different models (from the left to the right: 1D oceanic model, 2D oceanic-continental model by NIY method, 2D oceanic-continental model by mode coupling method) at different source-receiver distances (from the top to the bottom: 100km, 150km, 200km, 250km, 300km; for the 2D models the source-interface distances is fixed to 50km).

To check if such a strong gradient could affect the final signal in an unrealistic way we have performed another numerical experiment: we have splitted the path in the water in three adjacent structures to smooth the heterogeneity at the interface. The chosen path is shown in fig.2.3.3 while the results, with the receivers at the same distances from the oceanic-continental interface of the previous case, are in fig. 2.3.4. In the first column from the left there are the 1D signals, in the second signals computed using the 2D model with just one oceanic structure, while in the right column the signals computed with the path model indicated in fig. 2.3.3 are shown.

It is important to note that the two different 2D models give results that are very similar and
both strongly differ from the 1D one. Thus, we can say that, in this case, the first approximation adopted, with just one oceanic structure, is acceptable in order to reproduce the main characteristics of a oceanic-continental path on the wavefield.

Figure 2.3.3: Bathymetry model for three coupled oceanic structures.
Figure 2.3.4: Comparison of signals computed at 1Hz with three different models (from the left to the right: 1D model, 2D oceanic-continental model using 2 structures in series, 2D oceanic-continental model in fig. 2.3.3) at different source-receiver distances (from the top to the bottom: 100km, 150km, 200km, 250km, 300km, for the 2D models the source-interface distances is fixed to 50km).

In fig. 2.3.5 the signals relative to the same source-receiver configuration but considering only the first three modes (also with the intercoupling coefficients) are plotted. The differences found in the signals computed with all the available modes are evident also in this case: the strong difference in amplitude can be related to attenuation phenomena at the boundary (i.e. over critical incident angle). This is confirmed by fig. 2.3.6 where the relative coupling energies of the first three modes in themselves are plotted: it is evident the loss of the efficiency of coupling mechanism beyond about 0.2Hz. In this case these losses are not compensated by the intercoupling coefficients that (not plotted here) are an order of magnitude smaller than the
Intracoupling ones.

Figure 2.3.5: Comparison of signals computed at 1Hz with two different models (from the left to the right: 1D model, oceanic-continental model) at different source-receiver distances (from the top to the bottom: 100km, 150km, 200km, 250km, 300km; for the 2D models the source-interface distances is fixed to 50km) using the first three modes of the incoming wavefield coupled with the first three modes of the outgoing wavefield.
We have then performed another test, computing the signals for a continent-ocean-continent configuration, in which the two solid structures are the same.

The source has been placed in the first solid structure and its distance from the first interface is 50 km, the path in the liquid is 50 km and the receivers are (shown from the top to the bottom of fig. 2.3.7) at: 50 km, 100 km, 150 km, 200 km, 250 km from the liquid-solid interface. In fig. 2.3.7 the signals for the same source-receivers distances but computed with the 1D solid model and with this 2D model are shown in first and second column respectively. The differences between these groups of signals highlight the effect of the part of the path in the oceanic structure, since in this case the structure at the source and at the receivers is the same. In fig. 2.3.8 are shown the relative coupling energies transmission coefficients for the continental-oceanic interface.
Figure 2.3.7: Comparison of signals computed at \(1Hz\) with three different models (from the left to the right: 1D model, 2D continental-ocean-continental model) at different source-receiver distances (from the top to the bottom: 150\(km\), 200\(km\), 250\(km\), 300\(km\), 350\(km\); for the 2D model the distance from the source to the continent-ocean interface distances is fixed to 50\(km\) and the path in oceanic structure is fixed to 50\(km\)) using the first three modes of the incoming wavefield coupled with the first three modes of the outgoing wavefield.
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Figure 2.3.8: Relative coupling energy for the continent-ocean interface for the fundamental mode of the first structure coupled with the fundamental mode (red line) and the first higher mode (blue line) of the second one.

2.3.2 Towards the acoustic range

To extend the computations to higher frequencies we have chosen two structural models that differ just for the top layer that in the oceanic structure it is a 400m thick liquid layer while in the continental one is solid. The structures are shown in fig. 2.3.9.

In the first test, after the validation with the NIY approach, we computed the signals for a oceanic-continent configuration with the cut-off frequency equal to 1Hz and the results are shown in fig.2.3.10. The source is located in the oceanic structure with the epicenter at 50km from the vertical interface, and the receiver is placed at different distances from it (from the top to the bottom 50, 100, 150, 200 and 250km). The signals computed with the 1D continental, 1D oceanic and 2D models are shown in the first, second and third columns, respectively. In this case, the signals for the three different configurations are not very different, since the structural differences are confined in the top 400m. As concerns the amplitudes, we can see that the 2D model gives values that are more or less intermediate between those obtained with the two different 1D models (except for the source-receiver distance equal to 200km).
Seismic waves in laterally heterogeneous oceanic-continental structures

Figure 2.3.9: Oceanic structure (on the left) and continental structure (on the right).
Figure 2.3.10: Comparison of signals computed at 1Hz with three different models (from the left to the right: 1D continental, 1D oceanic, 2D oceanic-continental models) at different source-receiver distances (from the top to the bottom: 100km, 150km, 200km, 250km, 300km; for the 2D models the source-interface distances is fixed to 50km) using all the modes.

In fig.2.3.11 the relative coupling energies are shown: the red line indicates the coupling energy of the fundamental mode of the first structure with the fundamental mode of the second structure, while the blue line represents the intercoupling energy between the fundamental mode of the first structure and the first higher mode of the second structure. Up to a frequency of about 0.6Hz, the energy of the incoming mode excites mainly the fundamental mode of the solid structure and for higher frequencies the intercoupling processes begin. The sum of the two coupling energies is near to 1 (except for higher frequencies), demonstrating that, for this case, there are few attenuation scattering processes at the interface.
When we push the computation of the coupling coefficients, and of the signals, up to 10Hz we highlight some the effects due to the presence of the vertical interface. In particular in fig.2.3.12 the results for the 1D continent (first column), 1D oceanic (second column) and the 2D models (third column) are shown. The distance from the epicenter to the interface is 25km while the interface-receiver distances are 25km (first row), 50km (second row) and 75km (third row). It is evident that the 2D signals have amplitudes that can be considered an average of the 1D ones.
Figure 2.3.12: Comparison of signals computed at 10Hz with three different models (from the left to the right: 1D continental, 1D oceanic, and 2D oceanic-continental models, at different source-receiver distances (from the top to the bottom: 25km, 50km, 75km; for the 2D model the source-interface distances is fixed to 25km).

In this case the effect of the intercoupling processes between the different modes is stronger than in the other computations. As an example, in fig. 2.3.13 we show the relative coupling energies for the first higher mode of the oceanic structure with: the fundamental mode (blue line), the first higher mode (red line), and the second higher mode (green line) of the continental structure, respectively.
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Figure 2.3.13: Relative coupling energy for the oceanic-continental for the first higher mode of the first structure and the fundamental mode (blue line), first higher mode (red line) and second higher mode (green line) of the second one.

2.3.3 FTAN analysis

Frequency-Time Analysis (FTAN) (e.g. Levshin et al., 1989) is an interactive group velocity-period filtering method that uses multiple narrow-band Gaussian filters and maps the waveform record in a two-dimensional domain: time (group velocity)-frequency (periods). It allows an accurate study of the dispersion of surface waves propagation characteristics. In fact surface waves, showing not impulse nor quasi-harmonic behavior, are difficult to be studied in time or spectral domain, since their principal feature, dispersion, is described by a function rather than a single parameter.

The FTAN has the property of separating signals in accordance to their dispersion curve, since a visual picture requires a function of two variables. Let us consider a signal in time \( x(t) \) and its Fourier transform, \( X(\omega) \), and let it pass through a system of parallel relatively narrow-band filters \( H(\omega - \omega^H) \) with varying central frequency \( \omega^H \). The combination of all the signals at the output of all the filters is a complex function of two variables:

\[
S(\omega^H, t) = \int_{-\infty}^{+\infty} H(\omega - \omega^H)X(\omega)e^{i\omega t}d\omega \tag{2.3.1}
\]

A contour map of \( |S(\omega^H, t)| \) is called FTAN map, and it is used to visualize the dispersion curves, since, for frequency fixed, a “mountain ridge” (increased amplitudes) appears. The frequency-time region of a signal is that part of the \((\omega^H, t)\) plane occupied by the relevant
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ridge, and the statement “the energy of a signal concentrates around its dispersion curve” has a clear meaning.

In fig. 2.3.14 an example of FTAN map is shown.

![Figure 2.3.14: FTAN map: dashed line is a dispersion curve \((t, \omega^H)\)](image)

The function \(S(\omega^H, t)\) is not a property of the original signal alone, since it involves also the filter characteristics \(H(\omega - \omega^H)\), chosen by the investigator: we have different classes of signal representations that differ in the filter choice.

When the shape of \(H(\omega - \omega^H)\) is known, the function \(x(t)\) or \(X(\omega)\) can be recovered:

- \(X(\omega)\) from infinitesimally small filters = \(\delta(\omega - \omega^H)\)
- \(x(t)\) from infinitely broad filters = \(1/(2\pi)\) with the advantage that the noise can be more easily separated, for surface wave identification.

The choice of \(H(\omega - \omega^H)\) is guided by the typical properties of the signal to be processed. For surface waves two simple rules have to be followed:

- No phase distortion \((H\text{ has to be real valued})\)
- Best resolution and the optimal choice is found to be a Gaussian filter, described by two parameters: central frequency, \(\omega^H\), and width of the frequency band, \(\sigma\).

In the following pictures (2.3.15, 2.3.16 and 2.3.17), we show the application of FTAN (XF-TAN2012, provided by Franco Vaccari, personal communication) to the last row of the signals plotted in Figure 2.3.12, i.e. computed with the 1D continental, 1D oceanic and 2D structural models. The results obtained by the FTAN analysis are in the following figures and, in each one, different quantities are shown: on the top-left there is the signal in time domain, and on the top-right its amplitude spectrum, on the down-left its FTAN map and on the right the the \(V_S\) of the structural model versus depth. The analysis can be enriched by using the spectrogram representation (figs. 2.3.18), i.e. the horizontal axis represents arrival time, the vertical axis is frequency in the Fourier domain and the third dimension indicates the amplitude of a particular frequency at a particular time represented by the intensity or colour.
From the combined analysis it is evident that all the signals show a common feature up to a frequency of about 0.5Hz. At higher frequencies, corresponding to relatively smaller wavelengths, the differences in the structural models cause a different evolution of the propagating wavefield, explained by mode scattering in the 2D case.

Figure 2.3.15: FTAN results for the signal computed at 10Hz using the 1D continental model at a source-receiver distance of 50km.
Figure 2.3.16: FTAN results for the signal computed at 10Hz using the 1D oceanic model at a source-receiver distance of 50km.
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Figure 2.3.17: FTAN results for the signal computed at 10Hz using the 2D oceanic-continental model at a source-receiver distance of 50km and a path of 25km in the oceanic structure and 25km in the continental structure.

Figure 2.3.18: Spectrograms of the signals computed at 10Hz using (from left to right) 1D continental, 1D oceanic, 2D oceanic-continental models, with a source-receiver distance of 50km; for the 2D model the source-interface and the interface-receiver distances are 25km.
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3.1 Introduction

In the first two chapters of this work the modal method has been used to study the propagation of tsunami and seismic wavefield in different structural models. Those phenomena involve different physical mechanisms and frequency ranges: for tsunami the computations are performed at frequencies in the range from $0.0005\text{Hz}$ to $0.01\text{Hz}$, while for seismic waves we have shown the results of the computations performed from $0.005\text{Hz}$ to $10\text{Hz}$.

In this chapter we extend this method, and the related codes, to the acoustic range, in particular studying the propagation of seismo-acoustic waves in a oceanic structure for frequencies up to $100\text{Hz}$.

If we consider a vertically layered oceanic structure the seismo-acoustic wave propagation can be treated with the same formalism of Section 1.2, neglecting gravity and extending the computational domain to higher frequencies, following the early example of Costa and Panza (1994), who adapted the seismic multimode technique and the related codes to this purpose.

The study of the acoustic waves in water has various important applications. For example the study of sound propagation in the fluids permits, without invasive actions to the system, an observation of the density gradient: this can be very important to control the sea water conditions. One can study, for example, the signal produced by an acoustic source ($100−200\text{Hz}$) in the water and its variations when the physical parameters of the water layers change.

Other developments are more related to seismology, and in particular to the study of sedimentary layers on the sea bottom. The ability to construct reliable models of the shear velocities of marine sediments down to $100m$ or more beneath the sea floor is important in a number of disparate disciplines. For exploration seismologists, for example, these models would help to improve the shear-wave static correction needed in oil and gas exploration; for geotechnical engineers, these models would help to constrain the shear modulus for investigations of foundation vibrations, slope instabilities, and expected earthquake effects. Also, knowledge of sedimentary acoustic properties is needed to understand acoustic wave loss, which is important for sonar propagation, particularly in shallow water. Over the past few decades, the shear or geoacoustic properties of marine sediments have been extensively studied in the laboratory and in situ in a variety of marine environments from near the shore to the deep oceans. The greatest
advances (see Ritzwoller and Levshin (2002) for a comprehensive review and theoretical foundations) appear to have been in the use of interface waves to estimate shallow shear velocities: in a marine setting, the waves trapped near the solid-fluid interface are sometimes called Scholte waves (Scholte, 1958), in contrast with Stoneley waves near a solid-solid interface or Rayleigh waves near the air-solid interface. All of these waves, however, are dispersive with phase and group velocities that are sensitive primarily to shear velocities at depths that are inversely related to frequency. One of the best methods for their analysis is based on the measurement of the velocities of the fundamental and first overtone with multiple-filtering methods, called FTAN (see Section 2.3.3), and it is the one that will be used in the remaining sections, where a virtual explosion will be adopted as wavefield source.

3.2 Spectral analysis of an oceanic structure

The first step is the computation of the spectral characteristics of a vertically layered oceanic structural model, i.e. its eigenvalues and eigenfunctions, following the same approach that we have used for tsunami and seismic waves, but extended to frequencies up to 100Hz. We start from a structural model with a thickness of the water layer equal to 130 m (that could be representative of the Adriatic basin), shown in fig. 3.2.1, described by the variations with the depth of the density $\rho$, the P-wave velocity, $v_p$, and the S-wave velocity, $v_s$. Thus, one has first to compute the dispersion relationship $c = c(\omega)$ for the modes of the structure, obtaining phase and the group velocities versus frequency (up to 100Hz), as shown in fig. 3.2.2.
Figure 3.2.1: Example of an oceanic structure (named ’test’) adopted for parametric test in time-domain.
Figure 3.2.2: Eigenvalues of structure 'test': phase velocity (top) and group velocity (bottom).

This analysis allows to recognize the modes who could transport energy in the liquid layer from those that are penetrating in the solid layers: the concentration of the dispersion branches at velocities around 1.5km/s represent the frequency-mode bands trapped in water, while lower and higher values correspond to waves propagating in the solid layers, sedimentary and upper crustal, respectively. The fundamental mode represent mainly an interface (Scholte) wave travelling with a velocity that is approximately the 80% of the lowest S-wave velocity. This kind of analysis can be refined studying of the eigenfunctions of the structure for different frequency bands: this allows to understand important information about the modal features of the structure, and their possible excitation by a source. In particular in fig. 3.2.3 and one can see the radial and the vertical displacements of the fundamental mode in $10^{-1}$-$100Hz$ frequency range: its behavior is typical of an interface wave, in particular of a Scholte wave: it assumes the maximum values in the proximity of the liquid-solid interface and then decays fast away from it.
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Figure 3.2.3: Radial (left) and vertical (right) eigenfunctions of displacement for the fundamental mode of structure 'test', computed from 10 to 100Hz.

In the next figures one can see the behaviour of the higher modes: in fig. 3.2.4 are plotted the radial and vertical eigenfunctions for modes from the first to the 5-th higher, and in fig. 3.2.5 are the eigenfunctions from the 6-th to 15-th higher mode; in both figures frequency range is 40 – 100Hz. From the figures one can argue that, among these selected sets of modes, the higher ones are affected by the presence of the liquid layer more than the lower ones. This will be confirmed in the next section.
Figure 3.2.4: Radial (left) and vertical (right) eigenfunctions of displacement from first to 5-th higher mode of the structure 'test', computed from 40 to 100 Hz.

Figure 3.2.5: Radial (left) and vertical (right) eigenfunctions of displacement from 6-th to 15-th higher mode of structure 'test', computed from 40 to 100 Hz.
3.3 Parametric studies in time domain

Once that the spectral characteristics of a structural model are computed, the seismic signals excited by a given source can be synthetized. We consider an instantaneous explosive source: the related code has been modified following the works of Harkrider (1964) and Takeuchi and Saito (1970). A first test (not shown) was devoted to check the isotropic radiation of this kind of excitation mechanism: the results are those expected and the computed signals do not depend on the source-receiver angle.

A set of parametric tests has then been performed to study the influence of epicentral distance, source and receiver depth. In particular:

- epicentral distance ranging from 15\(km\) to 30\(km\) with a step of 1\(km\), keeping both receiver and source depth at 40\(m\) in the liquid. The results of this test are shown in fig. 3.3.1 for both the radial and the vertical components of displacements.

- source depth ranging from 10\(m\) in the liquid layer to 30\(m\) in the solid layer, keeping the receiver depth fixed at 40\(m\) in the water and the source-receiver distance equal to 15\(km\). The results of this test are shown in fig. 3.3.2 for both the radial and the vertical components of displacements.

- receiver depth ranging from 10\(m\) in the liquid layer to 30\(m\) in the solid layer, keeping the source depth fixed at 40\(m\) in the water and the source-receiver distance equal to 15\(km\). The results of this test are shown in fig. 3.3.3 for both the radial and the vertical components of displacements.
Figure 3.3.1: Parametric test for the signals, radial (left) and vertical (right) components, computed using the structure 'test', with source and receiver placed at a depth of 40m in the water, varying the epicentral distance from 15km (top) to 30km (bottom), with a step of 1km.
Figure 3.3.2: Parametric test for the signals, radial (left) and vertical (right) components, computed using the structure ’test’, epicentral distance fixed to 15km, receiver placed at a depth of 40m under the sea-bottom, varying the source depth from 10m in the water (top) to 30m under the sea-bottom, with a step of 10m.
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Figure 3.3.3: Parametric test for the signals, radial (left) and vertical (right) components, computed using the structure 'test', epicentral distance fixed to 15km, source placed at a depth of 40m in the water, varying the receiver depth from 10m in the water (top) to 30m under sea-bottom, with a step of 10m.

Other tests have then been performed to analyse the different modal contributions with different source and receiver depths configurations and the figures from 3.3.4 to 3.3.8 shows the results. For every figure the first row represents the sum of the first 21 modes while other ones represent the contribution of the single modes; the source-receiver distance is fixed to 15km and the different source-receiver configurations are:

- both source and receiver at a depth at 40m in the water. The results are shown in fig.3.3.4
- receiver on the sea-bottom and source at a depth of 40m in the water. The results are shown in fig. 3.3.5
- source on the sea-bottom and receiver at a depth of 40m in the water. The results are shown in fig. 3.3.6
- both source and receiver on the sea-bottom. The results are shown in fig. 3.3.7
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- both source and receiver at a depth of 30\(m\) in the solid layer. The results are shown in fig. 3.3.8

Figure 3.3.4: Parametric test for the signals, radial (left) and vertical (right) components, computed using the structure 'test', epicentral distance fixed to 15\(km\), source and receiver placed at a depth of 40\(m\) in the water, on the modal summation. The first row represents the signal computed with the first 21 modes, the others represent the contribution of the single modes.
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Figure 3.3.5: Parametric test for the signals, radial (left) and vertical (right) components, computed using the structure ‘test’, epicentral distance fixed to 15 km, receiver placed at the sea-bottom, and source placed at a depth of 40 m in the water, on the modal summation. The first row represents the signal computed with the first 21 modes, the others represent the contribution of the single modes.
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Figure 3.3.6: Parametric test for the signals, radial (left) and vertical (right) components, computed using the structure ‘test’, epicentral distance fixed to 15km, source placed at the sea-bottom, and receiver placed at a depth of 40m in the water, on the modal. The first row represents the signal computed with the first 21 modes, the others represent the contribution of the single modes.
Figure 3.3.7: Parametric test for the signals, radial (left) and vertical (right) components, computed using the structure ‘test’, epicentral distance fixed to 15 km, source and receiver placed at the sea-bottom, on the modal summation. The first row represents the signal computed with the first 21 modes, the others represent the contribution of the single modes.
The main conclusion that can be drawn is that the source and receiver configuration, respect to the water layer, plays a fundamental role: the amplitude of the prominent arrivals depends on the different mode contributions, e.g. travelling at the velocity of sound (P) waves in the water, at the average S-wave velocity of the upper sediments or at about 80 S-wave velocity of the sediments (fundamental mode).

In the shallow water, the physical properties of the sediments and the presence of low velocity channels in the water layer can have a relevant effect on the seismo-acoustic wavefield generation and propagation. To analyze this effect, we decided to consider other two structural models: one without low velocity sediments (i.e. with lowest S-wave velocity higher than 1.5 km/s), and one without low velocity sediments but with a low velocity channel in the water layer, are shown in fig. 3.3.9. From now on we call these structures ‘test2’ and ‘test3’ respectively.

Figure 3.3.8: Parametric test for the signals, radial (left) and vertical (right) components, computed using the structure ‘test’, epicentral distance fixed to 15 km, source and receiver placed at a depth of 30 m under the sea-bottom, on the modal summation. The first row represents the signal computed with the first 21 modes, the others represent the contribution of the single modes.
Figure 3.3.9: Example of the oceanic structures with homogenous liquid layer (named 'test2', on the left) and with a low velocity channel in the liquid layer (named 'test3', on the right).

To highlight the effect of the low velocity sedimentary layers we perform a comparison between the modal characteristics of structure 'test' (in fig. 3.2.1) and structure 'test2', using the same sets of eigenfunctions of figs.: 3.2.3, 3.2.4 and 3.2.5: fundamental mode in the frequency range of $10^{-1}100\text{Hz}$; from first to 5-th higher mode, in the frequency range of $40^{-1}100\text{Hz}$ and from the 6-th to the 15-th higher mode in the frequency range of $40^{-1}100\text{Hz}$. The results are plotted in figs. 3.3.10, 3.3.11 and 3.3.12: the behaviour the fundamental mode is the same, both in shape and amplitude, while the higher modes considered here are mainly “acoustic” ones.
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Figure 3.3.10: Radial (left) and vertical (right) eigenfunctions of displacement for fundamental of structure 'test2', computed from 10 to 100 Hz.

Figure 3.3.11: Radial (left) and vertical (right) eigenfunctions of displacement from first to 5-th higher mode of the structure 'test2', computed from 40 to 100 Hz.
With these tests one can also identify and isolate the waveguide effects of the low velocity channel. This problem is often studied by the ray theory (classicaly, a complementary approach to modal method): the critical refraction mechanism, trapping the rays in the channel, is a common feature in ray propagation models. From fig. 3.3.13 one can see that the first higher mode samples the low velocity channel in the liquid layer and this is confirmed by its eigenfunctions plotted in fig. 3.3.14.
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Figure 3.3.13: Eigenvalues of structure 'test2': phase velocity (top) and group velocity (bottom).

The importance of the presence of this low velocity channel for acoustic wave transmission in water is shown in fig. 3.3.15 where the radial and the vertical displacements for the first higher mode of 'test2', without the low-velocity channel. The differences are evident: in this case the energy of the mode, especially for vertical component, is concentrated in the solid layers while in the presence of low velocity channel is concentrated in the water: this may have important effects on the signals.
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Figure 3.3.14: Radial (left) and vertical (right) eigenfunctions of displacement for the first higher mode of structure ‘test3’, computed from 10 to 100Hz.

Figure 3.3.15: Radial (left) and vertical (right) eigenfunctions of displacement for the first higher mode of structure ‘test2’, computed from 10 to 100Hz.

In figs. 3.3.16 and 3.3.17 the two component of the stress $\sigma$, the normal one, and $\tau$, the shear one, are plotted. They show the same behavior of the displacement components, and the shear
stress is zero in the liquid layers. All these plots show the frequency frequency from 30Hz to 100Hz.

Figure 3.3.16: Displacements and stresses for the structure ‘test3’, with the low velocity channel.
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Figure 3.3.17: Displacements and stresses for structure ‘test2’, without the low velocity channel.

3.4 Frequency time analysis

The considerations about the modal characteristics of the different structural models studied in the previous sections can be confirmed, and highlighted, considering their excitation and propagation for different source-site configurations. We adopt an instantaneous explosive source, and a epicenter-receiver distance of 15 km, to synthetize the signals in time domain, that are analysed with the FTAN method to reveal the different dispersed nature of the generated wavefield, with and without the low velocity sediments, and with and without a low velocity channel in the water layer. The results obtained by the FTAN are in the following figures; as mentioned in section 2.3.3 this approach permits to isolate the more energetic arrivals concentrated around the dispersion curves of the different excited modes, represented by light coloured zones, and
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the analysis can be enriched by using the spectrogram representation.

The different (model, source depth, receiver depth, modes) configurations that have been considered are (source and receiver depth are expressed in m from the free surface):

1. 'test', 40, 40, all
2. 'test2', 40, 40, all
3. 'test3', 30, 40, all
4. 'test3', 70, 40, all
5. 'test2', 30, 40, all
6. 'test2', 70, 40, all
7. 'test3', 30, 40, fundamental
8. 'test3', 70, 40, fundamental
9. 'test2', 30, 40, fundamental
10. 'test2', 70, 40, fundamental
11. 'test3', 30, 40, first overtone
12. 'test3', 70, 40, first overtone
13. 'test2', 30, 40, first overtone
14. 'test2', 70, 40, first overtone
15. 'test3', 40, 30, all
16. 'test3', 40, 70, all
17. 'test2', 40, 30, all
18. 'test2', 40, 70, all

In figs. 3.4.1 and 3.4.2 (configurations 1 and 2) we show an example of FTAN in which, recognizing its dispersion curve on the FTAN map, we have isolated the contributions of fundamental mode in the signal and in the amplitude spectra. Having set both source and receiver in the water layer, this result confirms the fact that this mode, being a Scholte wave, does not transport energy in liquid. Thus, we can confirm that the evident differences between the two signals (visible also on the two spectrograms shown in fig. 3.4.3) are due to the higher modes.
Considering the other configurations some results are evident: the first is the great difference in the amplitude values, independently from the source position, between the results obtained using structure 'Test3' with low-velocity channel (figs. 3.4.4 and 3.4.5 and relative spectrograms shown in fig. 3.4.6) and those using the 'test2' (figs.3.4.7 and 3.4.8). The difference can be noted also in their amplitude spectra.

Source position may affects as well the signals: the signals computed for structure 'test2', without the low velocity channel, for the different source depths are very similar in shape with a just small difference in amplitude due to different depth excitation of the smooth eigenfunction (see figs. 3.3.15 and 3.3.17).

For structure 'test3', with the low velocity channel, the situation is different: the peak amplitudes remain more or less the same but the shape of the leading wavefield changes. In particular, when the source is located in the channel (fig.3.4.4) the first leading arrival, at 10.1s, a direct acoustic wave, is strongly attenuated while the main phase is retarded and arrives at about 10.34s, that is the arrival time the we expect for a wave travelling for 15km in the channel, where $V_p = 1.45km/s$.

Considering the spectrum of the structure 'test3' (see fig. 3.3.13) we have argued that the fundamental mode is not affected by the presence of the channel. This is confirmed by tests 7, 8, 9, 10, e which we have computed signals for structure 'test3' and 'test2' for the two different source depths but considering the fundamental mode only. The results in figs. 3.4.9, 3.4.10, 3.4.11 and 3.4.12 show that the fundamental mode propagates in the same way in the two structures.

From the plot of eigenfunctions (figs. 3.3.14 and 3.3.15) we have argued that the first higher mode behaves very differently and it is strongly affected by the presence of channel. The results of figs. 3.4.13, 3.4.14, 3.4.15 and 3.4.16 confirm this: for example fig. 3.4.13 highlights that the main arrival due to a source in the channel (see fig. 3.4.4) is due to the first higher mode as we could expect. This can be argued also considering form their FTAN maps where one can see that the main part of the energy carried by the mode travels at a velocity, corresponding to sound velocity in the channel.

The result obtained from tests 17 and 18, considering the source fixed and varying the receiver depth, confirm that the signals are model-consistent, and that the main effect of the receiver depth variation is on the peak amplitudes and not on the shape of the signals. Thus, the low velocity channel plays an important role in acoustic wave propagation affecting especially the relative amplitudes.
Figure 3.4.1: FTAN results for the signal computed using all modes, with the source at a depth of 30m, the receiver at a depth of 40m, in structure 'test', corresponding to configuration 1.
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Figure 3.4.2: FTAN results for the signal computed using all the modes with the source at 30m, the receiver at 40m, in the structure ‘test2’, corresponding to configuration 2.

Figure 3.4.3: Spectrograms representations of configuration 1 (left) and 2 (right).
Figure 3.4.4: FTAN results for the signal computed using all the modes with the source at 30m, the receiver at 40m, in the structure ‘test3’, corresponding to configuration 3.
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**Figure 3.4.5:** FTAN results for the signal computed using all the modes with the source at 70m, the receiver at 40m, in the structure ‘test3’, corresponding to configuration 4.

**Figure 3.4.6:** Spectrograms representations of configuration 3 (left) and 4 (right)
Figure 3.4.7: FTAN results for the signal computed using all the modes with the source at 30m, the receiver at 40m, in the structure 'test2', corresponding to configuration 5.
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Figure 3.4.8: FTAN results for the signal computed using all the modes with the source at 70m, the receiver at 40m, in the structure 'test2', corresponding to configuration 6.
Figure 3.4.9: FTAN results for the signal computed using the fundamental mode with the source at 30m, the receiver at 40m, in the structure 'test3', corresponding to configuration 7.
Figure 3.4.10: FTAN results for the signal computed using the fundamental mode with the source at 70m, the receiver at 40m, in the structure 'test3', corresponding to configuration 8.
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Figure 3.4.11: FTAN results for the signal computed using the fundamental mode with the source at 30m, the receiver at 40m, in the structure 'test2', corresponding to configuration 9.
Figure 3.4.12: FTAN results for the signal computed using the fundamental mode with the source at 70\(m\), the receiver at 40\(m\), in the structure 'test2', corresponding to configuration 10.
Figure 3.4.13: FTAN results for the signal computed using the first higher mode with the source at 30m, the receiver at 40m, in the structure 'test3', corresponding to configuration 11.
Figure 3.4.14: FTAN results for the signal computed using the first higher mode with the source at 70m, the receiver at 40m, in the structure ‘test3’, corresponding to configuration 12.
Figure 3.4.15: FTAN results for the signal computed using the first higher mode with the source at 30m, the receiver at 40m, in the structure ‘test2’, corresponding to configuration 13.
Figure 3.4.16: FTAN results for the signal computed using the first higher mode with the source at 70m, the receiver at 40m, in the structure 'test2', corresponding to configuration 14.
Figure 3.4.17: FTAN results for the signal computed using all the modes with the source at 40m, the receiver at 30m, in the structure 'test3', corresponding to configuration 15.
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Figure 3.4.18: FTAN results for the signal computed using all the modes with the source at 40m, the receiver at 70m, in the structure ‘test3’, corresponding to configuration 16.
Figure 3.4.19: FTAN results for the signal computed using all the modes with the source at 40m, the receiver at 30m, in the structure 'test2', corresponding to configuration 17.
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Figure 3.4.20: FTAN results for the signal computed using all the modes with the source at 40m, the receiver at 70m, in the structure 'test2', corresponding to configuration 18.
Conclusions

In the above scheme we show a summary of wave phenomena typically studied in geophysics: from left to right, their period, frequency, wavelength, phase velocity, penetration depth of penetration and possible fields of application, are shown. In this work, referring to the figure, we can imagine a logical path from low to high frequencies: in particular, the area shaded in red represents the topic of the first chapter, the area shaded in light blue is the topic of the second, while the dark blue one represents the subject of the third and final chapter. The initial goal we set was to demonstrate the versatility of the modal method, originally developed for seismic waves, for different physical phenomena and at different frequency ranges. In the first chapter we have shown the extension of this technique to tsunami, which is treated as a long period mode (about 1000s) due to the presence of gravity as main restoring force. This methodology allows, both on a theoretical and computational basis, several approaches to modeling. We have developed and used the methodology, and the related code, which considers the extension to the case of tsunami of the WKBJ approach for wave propagation in models with both 3D and 2D oceanic heterogeneities. At an operational level, we made another choice considering a stronger approximation: the bathymetry is modeled considering only two structures, associated respectively to the receiver and the source, and a phenomenological parameter that
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describes the gradient. This simplification can appear very strong in some cases, however, as we proved, it allows much faster calculations: with the aim to hazard scenario computations, i.e. in which many of the details of the tsunamigenic source are unknown, a faster calculation may be preferable to greater accuracy. In particular, from the practical point of view the latter approximation allows to efficiently calculate: a) tsunami signals using an extended source model and b) tsunami hazard scenarios for wide target areas, that are more realistic than those obtained using a laterally homogeneous bathymetry. These methodologies have been applied in different geographical contexts: the western Mediterranean Sea, the Tohoku earthquake and tsunami of March 2011, the Adriatic Sea and Vietnam coasts. The first two cases have been studied also to validate the related codes, and, in particular as regards the Tohoku tsunami, given the abundance of recorded data, we made comparison tests with real data: considering the extended source modeling with 2D bathymetry, full and simplified, the results obtained are very encouraging. In the other two cases tsunami hazard scenarios were calculated for the target areas, obtaining in both cases comparable results with available catalogues and literature. In particular for the Adriatic Sea, have also been calculated signals considering a realistic bathymetry, with heterogeneity in all directions, using the extension of the WKBJ method to tsunami. The WKBJ method has as a condition of validity, both for the seismic waves and for tsunami, that the heterogeneity treated are "smooth", i.e. that the physical quantities characterizing the medium vary little for distances comparable to a wavelength. If this condition is not met, like boundaries between ocean and continent or deep faults, the formalism to be used must consider the coupling coefficient at the interface between two structures, essentially a full scalar product between the relative stress-displacement vectors. During this work it has been formalized extension of the above method also to 2D sequences of structures with solid-liquid and liquid-solid vertical interfaces: this topic was discussed in the second chapter. A first test of these new codes was done using structural models, oceanic and continental, related to Iberian coast within the framework of the international project MARSH ("Mitigation of Seismic Risk through Integrated Scenarios of Crustal Deformation, Earthquake Hazard, Earthquake Damage and Losses, and Real Time Ground-shaking at Regional and Urban Scales ") for the estimation of seismic hazard from submarine sources. In addition to this more application-oriented purpose, we think that the detailed study of these liquid-solid coupling mechanisms may also contribute to the understanding of some wave phenomena whose origin is not yet clear (e.g. the generation of T-waves). It has also been shown an example of these calculations up to a maximum frequency of 10Hz (more sensitive to smaller-scale lateral heterogeneities), that have been studied by means of a very powerful analysis tool, that has been used extensively in chapter three: the FTAN (frequency-time analysis). Going to higher frequencies, up to 100Hz, the software package, originally developed for seismic waves, has been tested and, after the necessary improvements, used to study the propagation of acoustic waves in oceanic models. The purpose
of these tests, a subject of the third and final chapter of this work, is to understand the sensitivity of the acoustic waves to the characteristics of the structure in which they propagate, such as the presence of low velocity sedimentary layers or a low velocity channel in the liquid. We have carried out an analysis of the spectral characteristics of the different structures, studying their eigenvalues and eigenfunctions, and, considering an explosive source model, we carried out a series of parametric tests in the time domain, in which we evaluated the effect of different source-receiver configurations, and of various modal contributions, on the calculated signals. Finally, to further refine this analysis, we have made use of FTAN, thanks to which we have identified more precisely the influence of these aspects on the propagation of seismo-acoustic waves, showing the validity and the polyhedric nature of the modal approach.
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