A NONLINEAR DYNAMICS PERSPECTIVE ON SOME ASPECTS OF TOWING OPERATIONS RELEVANT TO SAFETY AND ENERGY EFFICIENCY

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ABSTRACT

Scheduled and emergency towing operations are carried out daily, both at sea and in restricted waters, as a means of transporting goods, aiding ships during berthing and casting-off or rescuing units in distress. Even in mild weather conditions, such activities can pose serious threats to the safety of the personnel and units involved and to the environment. This study investigates the dynamics of a tug-tow system, where towing is carried out by means of a single elastic towing line. A simplified model is used, in which the tug is modelled as a point-particle with a prescribed motion, while the dynamics of the towed object is modelled as a 3-DOF (surge/sway/yaw) nonlinear dynamical system. Equilibria of the system are analysed through a bifurcation analysis technique, supplemented by time-domain simulations, if/when necessary. An example application is reported for the case of towing of a barge-like hull, showing the effects of several parameters: ship speed, towline length, water depth and wind. The performed calculations confirm that, depending on the configuration parameters, multiple stable/unstable steady-state towing positions may exist. It is also clarified in this study that, in the absence of stable equilibria, the potentially dangerous oscillating phenomenon called “fishtailing” takes place.

Keywords: bifurcation analysis; fishtailing; large drift angles manoeuvring; multiple equilibria; nonlinear dynamics; numerical continuation; shallow water; ship safety; ship towing; wind.

RESUMEN

Maniobras de remolque, programadas y de emergencia, se llevan a cabo diariamente, tanto en mar abierto como en aguas restringidas, con el fin de transportar mercancías, asistir a buques durante el atraque y zarpe o rescatar unidades en situación de emergencia. Incluso en condiciones climáticas leves, estas actividades pueden suponer serias amenazas a la seguridad del personal y las unidades involucradas y al medio ambiente. Este estudio investiga las dinámicas de un sistema de remolque-empuje, donde el remolque se lleva a cabo por medio de una sola línea elástica de remolque. Se ha utilizado un modelo simplificado, en el cual el empujador se modela como una partícula puntual con un movimiento preestablecido, mientras que las dinámicas del objeto remolcado se modelan como un sistema dinámico no lineal a 3 grados de libertad (avance/deriva/guiñada). Los equilibrios del sistema son analizados por medio de una técnica de análisis de bifurcación, añadiendo simulaciones en dominio del tiempo cuando, en caso, sean necesarias. Se presenta un ejemplo de aplicación para el caso del remolque de una carena similar a una barcaza, mostrando los efectos de varios parámetros: velocidad del buque, longitud de la línea de remolque, profundidad del canal y viento. Los cálculos realizados confirman que, dependiendo de la configuración de los parámetros, pueden existir múltiples posiciones, estables e inestables, de estado estacionario del remolque. En el estudio también se aclara que, en ausencia de equilibrios estables, aparece el potencialmente peligroso fenómeno oscilatorio denominado “fishtailing”.

Palabras clave: análisis de bifurcación; fishtailing; maniobra a grandes ángulos de deriva; equilibrios múltiples; dinámica no lineal; continuación numérica; seguridad del buque; remolque del buque; bajas profundidades; efectos del viento.
INTRODUCTION

Towing operations have been carried out for centuries as a relatively cheap means of transporting considerable quantities of goods. There are indications of trade routes relying on barges towed by horses (or sometimes by slaves) along rivers and canals in ancient Mesopotamia as far back as the 3rd Millennium BC, as waterborne transport had proved to be about 170 times more efficient than the average donkey caravan [1]. Canal barges, towed by draft animals on adjacent towpaths, contended with the railway in the early days of the Industrial Revolution, but were ultimately outperformed by the higher speed, lower costs and much greater flexibility of trains.

Nowadays, ship towing serves multiple purposes: on one hand, it is still widely employed as a means of transporting freight both at sea and in internal waters, as to do so helps relieve highway heavy goods vehicles (HGV) traffic or reach areas poorly, if not hardly at all, served by local infrastructures [2]; on the other hand, towing operations are standard procedures for most units during berthing and unberthing in port. All such activities fall into a category that often goes under the name of “planned point-to-point towing” [3] due to the fact that they can be planned in advance, so that many environmental factors (wind, sea-state, tide, currents) can be examined and provisions can be made to reduce the potential risks generated by them. Towing is, however, also needed in emergency situations in order to rescue units in distress due to grounding, propulsion or steering failure, collision or any other reason [4]; in such cases, timing becomes a crucial issue in order to avoid loss of life, further damage to the disabled ship, oil spillages and environmental damage, therefore rescue operations often are carried without the possibility of a detailed planning. Over 40 such operations were carried out in the coastal waters of the UK [5] in the 2002-2007 period, including high-profile ones such as the rescuing of MSC Napoli. Following the growing attention to pollution-related issues due to oil spillages in the maritime environment from wrecked ships, a research project called “SAFETOW” was funded by the EU in order to provide masters of vessels with tools to help them control their ships, should they become disabled and to provide masters of salvage and escort tugs with enough knowledge to enable them to take decisions with the best available information regarding the consequences of their actions [6].

Towing operations can be necessary in many different environments, namely internal waters (such as rivers, lakes, canals), coastal waters, open sea and port. Each of these conditions is characterised by a number of potentially hazardous aspects: strong currents, shallow waters, wind and various obstacles are typical of rivers; canals are often narrow and congested; coastal waters are rich in reefs or sandbanks; ports can be overcrowded; the open ocean offers no close shelter in case of a sudden and unexpected weather change. Therefore, regardless of the possibility of planning, even in mild weather conditions towing operations always pose serious threats to the safety of the personnel and units involved and to the environment. As an example of the dangers posed by coastal waters, one could look at the accident occurred to the tug Flying Phantom on 19 December 2007 [7], that led to its capsizing and sinking (with the loss of three lives) after grounding and subsequent girding in a thick fog while assisting Red Jasmine during her transit in the Firth of Clyde [7] or to the loss of several marine engines (worth in excess of 20 million Euros) that were being transported by barge from Trieste to Venice in September 2010 [8][9]. As far as river towing accidents are concerned, on 19 July 2009 tug North Arm Venture capsized in an area of the Sechelt Inlet (British Columbia, Canada) known as the “Sechelt Rapids” after being overtaken in a strong current by the barge it was towing [10]. Finally, anchor-handling vessel Bourbon Dolphin was lost in the North Sea (together with the lives of 8 out of 15 crewmembers) on 12 April 2007 during offshore operations which, though not strictly related to towing, are dynamically similar to it [11].

Scope of this study is to provide an analysis of some aspects of the dynamics of ship towing, since this is dominated by rich and interesting nonlinear effects. For instance, while anyone in charge of
towing operations would desire an ideal, stable, in-line towing, as this condition minimises the resistance (and, consequently, the towing line tension) and is generally less prone to create hazardous situations (unless of course there is a loss of propulsion on the tug, which can cause the tow to “rear-end” it [12]), this can be impossible to achieve for some towed objects. Depending, indeed, on a number of parameters (particularly the position of the towline connection point on the tow [13][14]), stable lateral towing positions may exist, which yield to much greater resistance, as the tow moves with a non-zero drift angle. However, it will be shown that, in some configurations of parameters, stable equilibria may not exist at all. An interesting outcome of this study is that such conditions are directly linked with the inception of a potentially dangerous phenomenon that goes under the name of “fishtailing”. This phenomenon is known to occur not only in case of towing [15], but also in case of moored objects subjected to current (see e.g. [16]); a difference exists, however, between these two situations, as relative velocities in towing may attain higher values, while in offshore operations the current speed is limited by nature. When fishtailing occurs, the towed object becomes subject to a transversal oscillation about the position of the tug. Such a situation can be particularly hazardous, as the alternation of peak tensions and periods of slack line can lead, at best, to quick wear and tear of the line itself and, in a worst-case scenario, to its failure or to the inception of excessive heeling moments on the units involved in the towing operation.

Past studies regarding the dynamics of ship towing have mostly made use of nonlinear derivative-based models [13][14][17], which have the advantage of allowing the application of relatively straightforward analytical procedures in order to study the behaviour of the system. Such methods do, however, have the severe drawback of being applicable only as long as the drift angle remains relatively small, a condition that is not always satisfied during towing. This issue is due to an intrinsic limitation of polynomial models, which invariably only yield good approximations in the neighbourhood of the point about which they are determined. Besides, when some nonlinear derivatives need to be made dimensional, they are to be divided by a reference speed (usually the advance speed), which can, at times, become very small or even zero as in the case of pure drift motion, thus originating a singularity in such conditions. Therefore, the use of a manoeuvring model capable of successfully predicting hydrodynamic forces at large drift angles becomes necessary when studying the dynamics of ship towing. In this paper, the model developed by [18] [19] will be employed to simulate the dynamics of the tow.

With reference to river navigation, fluvial ship towing may become particularly relevant in areas characterised by poorly developed infrastructures, as it may be the only method of transporting goods (pusher-barge systems are also widely employed, although such configurations need specifically engineered vessels and fittings, while towing can be carried out much more easily); furthermore, towing is the standard procedure adopted in the emergency recovery of disabled vessels or moving special-purpose vessels from place to place. Additional effects, such as those due to a steady wind or to a limited underkeel clearance, become relevant in such an environment and will be studied in this paper, as they may significantly alter the behaviour of the towed objects and possibly lead to hazardous situations.

The paper is organised as follows. Firstly, the characteristics of the used mathematical model are presented. Then a description is given of the technique of analysis which will be used to carry out the analysis of system’s equilibria. Some application examples will then provided, and the paper will close with some final remarks.

MATHEMATICAL MODEL

This study investigates the dynamics of a tug-tow system, where towing is carried out by means of a single elastic towing line (more complex arrangements will not be studied in this paper, although
they are sometimes adopted [3][20]). Similarly to [13][14], a simplified model is used, in which the tug is modelled as a point-particle with a prescribed motion, while the dynamics of the towed object is simulated through a 3-DOF (surge/sway/yaw) MMG-type manoeuvring model. Of the three reference systems employed, one is Earth-fixed ($\Omega \xi \eta \zeta$), another one is tow-fixed ($O \chi \psi \zeta$), the origin $O$ not necessarily being coincident with the ship’s centre of gravity, and the third is tug-fixed ($Q \sigma \tau \zeta$); the sign conventions adopted are illustrated in Figure 1. In Figure 1 the yaw/heading angle is indicated, as usual, as $\psi$.

\begin{figure}[h]
\centering
\includegraphics[width=\linewidth]{reference-systems.png}
\caption{Reference systems.}
\end{figure}

As a result of the previously mentioned modelling assumptions, only the dynamics of the tow will be considered in this paper; the overall dynamics of the two interacting systems, however important and interesting, has not been examined in order to keep the model relatively simple and in consideration of the fact that even the behaviour of the sole towed object is already quite interesting and rich.

The motion of the towed object is assumed to be governed by the following standard 3-DOF planar manoeuvring model referenced to the point $O$ (see Figure 1):

\begin{equation}
\begin{aligned}
& m \cdot (\ddot{u} - v \cdot r - x_G \cdot r^2) = X_M + X_W + \mathbf{T} \cdot \hat{i} \\
& m \cdot (\ddot{v} + u \cdot r + x_G \cdot \dot{r}) = Y_M + Y_W + \mathbf{T} \cdot \hat{j} \\
& I_{zz} \cdot \dot{r} + m \cdot x_G \cdot (\ddot{v} + u \cdot \dot{r}) = N_M + N_W + (\mathbf{\bar{x}}_P \wedge \mathbf{T}) \cdot \dot{k}
\end{aligned}
\end{equation}

where, given the origin $O$, $m$ [kg] is the ship mass, $u$ [m/s] and $v$ [m/s] are the longitudinal and transversal velocities of the tow, $r$ [rad/s] is the yaw rate, $x_G$ [m] is longitudinal position of the centre of gravity with respect to $O$ (for symmetry $y_G$ is assumed to be zero, with a centre of gravity on the centreplane of the tow), and $I_{zz}$ [kg·m²] is the yaw moment of inertia with respect to point $O$. Dots in the equations represent differentiation with respect to time. Coming to the right hand side of (1), and following the idea of the MMG modelling [21] of separating external actions according to their cause, the tug is assumed to be subject to manoeuvring (hydrodynamic) force/moment (subscript “M”), wind force/moment (subscript “W”) and the effect of the towline. The towline force is represented by the force vector $\mathbf{T}$ [N] which is assumed to be applied at the towing point $\mathbf{\bar{x}}_P$ (ship fixed reference system). Vectors $\hat{i}$, $\hat{j}$ and $\dot{k}$ are unit vectors of the three ship-fixed reference axes $x$, $y$ and $z$. Note that, in (1), a dot ($\cdot$) represents the dot product, while the caret symbol ($\wedge$) indicates the standard cross product.

As previously mentioned, the technique that is most widely used for the determination of the manoeuvring forces is the nonlinear derivative-based model. Besides the uncertainty related to the determination of the nonlinear derivatives, for which few reliable regression formulae are available,
the main drawback of the derivative-based approach is its inability to properly reproduce the hydrodynamic actions on a ship when its lateral speed becomes comparable (or greater than) its advance speed (i.e.: when the small drift-angle hypothesis ceases to be valid due to the fact that the drift angles becomes very large).

As far as the standard manoeuvres need to be simulated, such a condition is not normally achieved in normal operations because the drift angle remains sufficiently small, whereas in case of non-standard manoeuvres, similar to those carried out during dynamic positioning, crabbing, berthing and unberthing, fishing activities and in certain phases of towing, the drift angle becomes larger, and the derivative-based model becomes, therefore, unsuitable for studying such operations. In order to overcome this difficulty, in this work, a model capable of reproducing the manoeuvring forces at high-drift angles was used, as developed in [18][19]. A comparison between the non-dimensional forces and moments obtained by using the model developed by [18] and a standard non-linear derivative-based (1st and 3rd order) model is shown in Figure 2. The example uses, as sample vessel, the barge described in [17] (this is actually the barge which, apart from scaling, will be used in the example application later in this paper). In the calculation of transversal force and yaw moment, dimensionless derivatives given in [17] were made dimensional using the total ship speed $U = \sqrt{u^2 + v^2}$. Figure 2 also shows the effect of a reduced water depth (actually a quite shallow water condition) on the force/moment as predicted by the two methodologies. Shallow water effects have been introduced in the model on the basis of Ankudinov’s formulae [22] and taking into account the indications found in [23]. It can be noticed that, for this particular case, the lateral force given by the derivative-based model agrees with the large-drift-angles model in case of shallow water, while clear differences are visible in case of deep water. Regarding the yaw moment, the derivative-based model is not able to reproduce the behaviour of the moment indicated by the large-drift-angles model.

![Figure 2](image)

As far as wind effects are concerned, the model used in [24] was adopted. Data from [24] gives the possibility of evaluating the forces and moments acting on the ship when the relative velocity, the angle of attack and the shape of the superstructures are known. Note that, although aerodynamic effects are indicated here as “wind effects”, such aerodynamics effects also arise in “still air”. Accordingly, the modelling used in the simulations, takes into account the relative velocity between the vessel and the air, and then aerodynamic coefficients are applied taking into account the velocity of the apparent (relative) wind. Finally, the towline tension $T$ is defined as a function of the elongation of the towline and of its breaking load. It is, therefore, dependent on the distance
between the towline connection points on the tug and on the tow ($\ell$) and on the diameter of the rope. The towing line chosen for the applications in this paper is an unused high-tenacity polyester 12-strand roundline [25], whose load-extension diagram is shown in Figure 3.

The diameter selected is $d = 36\, mm$, equivalent to a guaranteed minimum breaking force of $273\, kN$ [25]. The choice was based on some considerations regarding the typical bollard pull of a small river-tug, which was assumed to be equal to $BP = 73.5\, kN$ for the calculations which will follow, and on the IMO recommendations concerning the minimum recommended breaking load of towing lines, which is set as three times the tugboat’s bollard pull force in case this is less than $392\, kN$ [26]. The possibility of towing line failure (line tension exceeding the breaking load) was, however, not considered. This because the main purpose of this paper is an analysis of the nonlinear dynamic behaviour of tow, rather than a strict examination of the practical operational problems connected with such activities, which is left as a further task. As a result, the nominal elongation/tension graph (Figure 3) has been extended above the breaking load using a pure quadratic function of the elongation. In this example, The “New 12-Strand” rope (Figure 3) was selected as a reference. Accordingly, the resulting model for the towline tension vector applied in the point $P$ becomes:

$$
\begin{align*}
\vec{T} &= \vec{0} \iff \ell \leq L_W \text{ (slack towing line)} \\
\vec{T} &= f\left(\frac{\ell - L_W}{L_W}\right) \cdot BL \cdot \hat{\chi} \iff L_W < l \leq L_{BL} \text{ (taut towing line – physical region)} \\
\vec{T} &= k \cdot \left(\frac{\ell - L_W}{L_W}\right) \cdot BL \cdot \hat{\chi} \iff \ell > L_{BL} \text{ (notional extrapolation region)}
\end{align*}
$$

(2)

where $\ell$ is the instantaneous distance between the towline end points, $L_W$ is the length of the unstretched towline, $BL$ is the minimum breaking load found in [25], $L_{BL}$ is the length of the towline at the nominal breaking load, $f((\ell - L_W)/L_W)$ is the empirical load vs. extension relationship defined by Figure 3, $k = 17.23$ for this specific case and $\hat{\chi}$ is the unit vector (i.e. the versor) that defines the direction of the towline force acting on the tow (as indicated in Figure 1).

**TECHNIQUE OF ANALYSIS**

The model (1), which describes the motion of the tow, is a nonlinear dynamical model. As such, it can be characterised, in principle, by rich and interesting dynamical phenomena. Herein, we concentrate our attention on the determination of equilibrium positions and on some partial assessment of the already mentioned phenomenon of “fishtailing”. Since the model is nonlinear, we
can expect multiple equilibrium conditions, with different stability properties, to exist for the same set of parameters. This was shown in the past, on an analytical basis, in [13]. It is therefore important to have a picture, to be as large as possible, of the behaviour of the system.

The model reported in equation (1) is of course suitable for direct time-domain simulations. These, however, can be time-consuming, and therefore, in certain cases, impractical. Moreover, brute force numerical simulation only provide with a limited set of data. Aspects such as the simultaneous existence of stable and unstable solutions for a certain condition cannot be determined via time-domain simulations. In this study, we have decided to address the model (1) by means of a bifurcation analysis of the equilibria, supplemented by some time-domain simulations intended for the analysis of the system, in particular, under the inception of fishtailing. If the mathematical model is sufficiently simple and fully analytical, equilibria can be analytically determined and characterised in terms of their stability as done, in the past, in [13]. However, in the case under investigation, this approach is impossible, as neither the manoeuvring model [19], nor the wind model [24] are fully analytical. As a result, a fully numerical bifurcation analysis has been applied. A similar approach, although not applied to towing, but to ship manoeuvring in wind, can be found in [27][28]. The approach can be summarised as follows.

Considering a generic bifurcation parameter $\lambda$ (for instance the longitudinal position of the towing point on the tow, the wind speed, ...), equilibria of the dynamical system are sought by using transformed state variables making reference to the tug-fixed reference system:

$$\begin{pmatrix} \dot{\sigma} \\ \dot{t} \\ \dot{r} \end{pmatrix} = \begin{pmatrix} u \cdot \cos \psi - v \cdot \sin \psi - U_{tug} \\ u \cdot \sin \psi + v \cdot \cos \psi \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

(3)

In equation (3) the variables $\sigma$ and $\tau$ are the coordinates of the tow as defined in Figure 1. The resulting equilibrium problem is basically a system of three nonlinear equations, in three unknowns $(\sigma_{eq}, \tau_{eq}, \psi_{eq})$ and with one parameter ($\lambda$), which can be written, in general, as:

$$\begin{cases} F(y, \lambda) = 0 \\ y = (\sigma, \tau, \psi)^T \Rightarrow y_{eq} = (\sigma_{eq}, \tau_{eq}, \psi_{eq})^T \end{cases}$$

(4)

The system of equilibrium equations (4) is given from the original dynamical system (1) combined with the transformation of variables (3). Given a particular value of the parameter $\lambda$, solutions of (4) can be determined by means of any zero-search method, after providing a suitable initial guess. In case of multiple coexisting solutions, i.e. multiple solutions of (4), (some of) these solutions can be determined by making use of different initial guesses. In this work, as a first step, a series of equilibria have been determined as solutions of (4) by using a set of initial guesses, for different values of the bifurcation parameter $\lambda$. Making use of this first set of the equilibrium points, a numerical continuation approach has been applied using a predictor-corrector algorithm based on the secant method [29]. The idea of the method is to trace branches of equilibrium solutions using a simple and efficient algorithm, where the guess value for each step $j \rightarrow j + 1$ of the continuation is obtained using information on the last two known solutions $(y_{j-1}, \lambda_{j-1})$ and $(y_j, \lambda_j)$ as shown in Figure 4.
The new guess, i.e. the predictor point \( \hat{y}_{j+1}, \hat{\lambda}_{j+1} \) is determined along the line identified by \( (y_{j-1}, \lambda_{j-1}) \) and \( (y_j, \lambda_j) \) by imposing a (Euclidean-norm) distance \( \delta s \) from the previous solution \( (y_j, \lambda_j) \). The actual solution \( (y_{j+1}, \lambda_{j+1}) \) is then sought starting from the predictor point \( (\hat{y}_{j+1}, \hat{\lambda}_{j+1}) \) using the zero-search procedure to solve (4). It is however to be underlined that, in order to allow the procedure to deal with turning points of the solution branch, in the corrector step, the bifurcation parameter \( \lambda \) is left free as an additional unknown. Since this makes the system (4) underdetermined, a new equation is introduced which is intended to fix the (Euclidean-norm) distance from the previous solution \( (y_j, \lambda_j) \). This approach implicitly assumes the introduction of a parameterization along the solution branches which is based on the curvilinear abscissa (arclength, or, better, in this case, pseudo-arclength) \( s \), i.e. \( (y(s), \lambda(s)) \). The additional equation takes, in principle, the following form:

\[
\sum_{i=1}^{n} \left[ (y - y_j^i)^2 + (\lambda - \lambda_j)^2 - (\delta s)^2 \right] = 0
\]  

(5)

where the subscript \( i \) indicates the i-th component of the solution vector. In addition to the above, the implemented method also embeds the following characteristics:

- A normalization of the state variables, of the bifurcation parameter and of the equations by using appropriate normalization factors, in order to improve the numerical accuracy;
- The introduction of a penalty function technique, in order to avoid the corrector step to go “backward”.

Once the equilibrium solutions have been determined, for the analysis to be complete it is necessary to establish their nature. For this purpose, the Jacobian matrix of the dynamical system (1) is numerically determined at each equilibrium solution along the branch. The analysis of the eigenvalues of the Jacobian allows, then, to determine whether the obtained solution is stable or unstable. Indeed, the solution is unstable whenever there is at least one eigenvalue of the Jacobian showing a positive real value. The whole procedure is repeated using different starting points, with the intention of covering the various solution branches.
EXAMPLE APPLICATION

In this paper, an example preliminary application was carried out on a barge-like, full block-coefficient towed object. Such flat-bottomed, low-draught vessels are mainly built for river and canal transportation of heavy bulk goods, are usually not self-propelled and therefore need to be towed by appropriately chosen tugboats. Many boats designed for river service are, however, characterised by a barge-like shape; in the present paper, a scaled version of the barge described in [17] is used, with the scale factor being 0.5. Main particulars of the considered barge are reported in Table 1. The used scaling factor leads to a barge with particulars similar to those of the hospital ship described in [2] for operation in the Amazonian region. Since the shape of the superstructures of the hospital ship in [2] resembles that of a parallelepiped very closely, such an approximation was considered when defining the windage areas and choosing the coefficients from [24]. It must, however, be said that the wind coefficients used in this work are based on data from [24] associated with height/breadth ratio of 1.0, while the actual ratio for the considered vessel is about 0.5. A better estimation is therefore necessary, and the obtained results are to be considered only as a preliminary example. Regarding hydrodynamic forces, the non-dimensional derivatives needed in order to calculate some of the coefficients required by the manoeuvring model [19] were found directly in [17].

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Value</th>
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<tbody>
<tr>
<td>L_{pp} [m]</td>
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</tr>
<tr>
<td>B [m]</td>
<td>10.670</td>
</tr>
<tr>
<td>T [m]</td>
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<tr>
<td>D [m]</td>
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</tr>
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<td>C_{h} [-]</td>
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<td>A_{F} [m^{2}]</td>
<td>54.7</td>
</tr>
<tr>
<td>A_{L} [m^{2}]</td>
<td>156</td>
</tr>
</tbody>
</table>

Scope of this work is to show some examples of analysis of the behaviour of the system when some relevant parameters are changed. Such analysis will be reported in the form of so-called bifurcation diagrams. The parameters which have been taken into account are: length of the towline (considering information in [12][30][31]), tug speed (considering information in [32]), longitudinal position \( x_{P} \) of the tow point on the tow (following previous studies in [13][14]), wind (wind can increase along rivers, as shown by [33], and wind speed at night-time, when operations are more difficult and dangerous, can be greater than during daytime [34]). As far as the position of the towline connection point on the tow is concerned, one should notice that towing is rarely carried out with values of \( x_{P}/L_{pp} < 0.4 \); the dynamics of the system is going to be nevertheless analysed for the full range \( 0 \leq x_{P}/L_{pp} \leq 0.5 \), both to provide a complete mathematical examination of the problem and to study the behaviour of possible non-standard or emergency towing configurations.

As a first example, Figure 5 show the effect of the variation of the longitudinal position of the towline connection point on the equilibrium lateral offset and yaw angle respectively. Different towing speeds are considered, as well as two values of the towline length. In Figure 5, solid blue lines represent stable equilibria, while dotted red lines symbolise unstable equilibria. According to Figure 5, the lateral offset of the equilibrium position of the towed object is a function of the length of the unstretched towing line, whereas the latter has no influence on the stable/unstable behaviour of the system or on the yaw angle of the stable equilibria. It is noted that the result obtained in Figure 5 is similar to that obtained by [16]. Yaw angles equal to \( \psi = \pm \pi \) represent a situation in which the tow is subject to a reverse motion; it is to be noticed that the equilibrium conditions \( \psi_{eq} = \pm \pi \) are exactly the same. An example representation of coexisting equilibria is shown in Figure 6, where two lateral stable equilibrium solutions coexists with two unstable inline towing
configurations, i.e. towing ahead and reverse towing. Of course, although all these cases are solutions of the dynamical systems, not all these configurations are actually physically realizable.

In the example shown in Figure 5, low values of $x_p/L_{pp}$ (i.e.: towing point close to amidships) originate two stable lateral equilibrium solutions and two unstable in-line solutions (towing ahead, with $\psi_{eq} = 0$, and in reverse, with $\psi_{eq} = \pm \pi$). A towing point very close to amidships causes the barge to be towed with a relatively small lateral offset, but at a high drift angle, which requires a very large towing force. As $x_p/L_{pp}$ increases, so does the lateral offset $\tau$, while the yaw angle $\psi$ reduces up to a point where the stable equilibrium solutions cease to exist and first give way to a triplet of unstable equilibria and, soon after, to a single unstable equilibrium condition independently of the value of $x_p/L_{pp}$. The behaviour of the system when there are no stable equilibria is the inception of a transversal oscillating motion that goes under the name of “fishtailing” for its analogy with the motion of a fish’s fin. An example of this phenomenon is shown in Figure 7 (red parts denote periods of taut towing line, blue parts represent time intervals of slack towing line); in the example shown, a limit cycle is eventually reached, but the oscillation is not transversally symmetrical about the position of the tug. By changing the initial conditions of the simulation appropriately (i.e. changing $\tau(0) \rightarrow -\tau(0)$), another limit cycle is obtained, but this time the oscillation takes place on the opposite side of the tug’s position and is exactly symmetrical with respect to the previous one. Two distinct mirrored not-symmetrical oscillations can therefore be obtained depending on the initial condition chosen for the simulation, and each simulation can be considered to belong to a specific attraction domain. This behaviour is a consequence of the underlying portside-starboard symmetry of the system. It appears from Figure 5 that a variation of the towing speed does not appreciably affect the characteristics of the equilibrium solutions in terms of lateral offset and yaw angle, while it has an obvious repercussion on the towing line tension. For the same case, Figure 8 shows the maximum towline tensions as a function of $x_p/L_{pp}$. Solid and dotted lines represent the tension determined via the bifurcation analysis; blue squares, circles and diamonds denote the maximum tensions that occur in the corresponding time-domain simulations after the transient effects have died out. As expected, the forces determined from the time-domain simulations perfectly coincide with those calculated from the direct search of equilibria as long as a pair of stable solutions exists; beyond the threshold of fishtailing inception, peak tensions increase dramatically as a result of the oscillating motion, especially immediately after the disappearance of the stable equilibria.

Figure 5: Transversal equilibrium position $\tau_{eq}/L_{pp}$ (left) and equilibrium yaw angle $\psi_{eq}$ (right) as a function of $x_p/L_{pp}$.
Figure 6: Example of equilibrium towing configurations.

Figure 7: Time-domain simulations showing fishtailing with two mirroring initial conditions.
Moving now to another example, a critical problem that arises whenever river navigation is involved is due to the effects of shallow water. The lower course of the Amazon River is rather deep: in the Brazilian area it is not infrequent to encounter sections deeper than 100 m [35]; proceeding upstream, however, in the Peruvian regions the water depth can be as high as 40 m but in certain parts also as shallow as 3 m, with significant seasonal oscillations [35]. Simulations with a ratio $h/T = 1.3$ ($h$ being the water depth) have been carried out in order to compare the dynamics of towing in deep water and in shallow water; the results are shown in Figure 9. The obtained results show that, in this example, the effects of shallow water tend to reduce the region of $x_p/L_{pp}$ where stable equilibria exist and to reduce the lateral offset with respect to the corresponding deep water conditions. On the other hand, they also show that yaw angles at the equilibrium positions during towing in shallow water tend to be higher than those in deep water. The corresponding effect on the towline tension is shown in Figure 10. It turns out that maximum steady-state towline tensions can be $3 \div 5$ times greater in shallow water than in deep water, both in case of stable lateral towing and in case of unstable towing that leads to fishtailing. This is not surprising, as the drag coefficients of the manoeuvring model increase when the underkeel clearance reduces [23]. This fact should be borne very well in mind when planning towing operations, since a towing line whose strength is perfectly acceptable during operations in deep water may break due to the increased tension necessary to perform the same activity once the water becomes shallow (Figure 10). In the present analysis, the use of Ankudinov’s corrections for shallow water effect was adopted, as it was considered to be the most convenient; it is, however, possible that such simplified corrections may not reproduce the shallow water effects with high accuracy. The same is true for the corrections applied to the drag coefficient. As a result, shallow water simulations should be considered with care.
Figure 9: Transversal equilibrium position $\tau_{eq}/L_{pp}$ (left) and equilibrium yaw angle $\psi_{eq}$ (right) as a function of $x_p/L_{pp}$.

Figure 10: Maximum absolute towline tensions as a function of $x_p/L_{pp}$ (comparison between shallow-water and deep-water conditions).

As previously mentioned, many areas in the close proximity of the Amazon River and its tributaries tend to show an increased wind speed [33], especially at night [34]. It is therefore instructive to investigate the effects of wind on the dynamics of the system. An example of analysis of effects due to wind is reported in Figure 11. Constant ahead and astern wind, with a wind speed of $10 \, \text{m/s}$ has been considered for this purpose, with a steady-state towing speed of $2 \, \text{m/s}$ and an unstretched towing line length equal to the ship length. The same case, but without wind, is also reported for comparative purposes. Results reported in Figure 11 show that, in this case, towing in head wind causes the region characterised by stable equilibria to reduce its extension, while the opposite happens when towing is carried out in following wind.
FINAL REMARKS

Towing operations can be vital to the economy of a region, as they can prove to be very cost-effective and allow otherwise isolated areas to be reached. From a technical point of view, towing dynamics can be very rich and strongly non-linear and can lead to unexpected behaviours of the towed objects even in mild weather conditions. Typical non-linear phenomena can appear when analysing towing even with a very simplified model that approximates the tug with a point moving at constant speed, as it was done in this study; conditions such as multiple coexisting stable/unstable equilibria can be present, which can cause the towed object to assume a very cost-ineffective and potentially dangerous non-symmetrical steady-state motion characterised by non-zero drift and yaw angles. Such a condition significantly increases the tug pulling force required to carry out the towing operation at the given towing speed. This, in turn, increases fuel consumption and therefore emissions, leading to reduced energy efficiency for the considered operation. Furthermore, the possibility of a complete lack of stable equilibria can lead to a self-induced oscillation, the so-called “fishtailing” motion, which can be very hazardous for all the units and the personnel involved.

Regarding the modelling of the system, towed objects can, at times, operate at high drift angles, which makes the classical derivative-based models ineffective, as they are unable to properly account for such conditions. This requires implementing mathematical models suited for large drift angles. In addition to these models, wind effects also need to be incorporated. In both cases, typically, the modelling of the forces is not fully analytical.

All the above aspects make towing an interesting dynamical problem, which, in general, cannot be tackled in a fully analytical way. Brute-force application of time-domain simulations can only provide a limited insight of the problem, as these are unable to show the presence of unstable solutions. To respond to this situation, we have implemented a model-free bifurcation analysis procedure, which allows to create bifurcation diagrams considering influential parameters, such as the towline connection point on the tow or the wind speed. To the best of the authors’ knowledge, such an approach, though rather common in the field of nonlinear dynamics, appears to be new to the field of ship towing.

For the considered example cases, it was determined that important parameters that can be easily modified by captains or tow planners (such as the towline length or the towing speed) have little or no influence on the nature of the equilibrium solutions (although they do indeed affect important
aspects such as the lateral equilibrium offset and the towing line tension respectively). On the other hand, it was confirmed that the towline connection point on the tow is a very important parameter, as it has a significant influence on the overall equilibrium of the system. Water depth also turned out to be noteworthy, since, in the considered example, its reduction tends to decrease the extension of the region of existence of stable equilibria and causes the towline tension to increase considerably with respect to the corresponding deep water condition; this particular effect should make tow planners very well aware of the importance of a careful bathymetry analysis of the towing route. As far as the fishtailing motion is concerned, greater towline tensions were observed in the regions characterised by the largest oscillations. This corresponds to the transition zone between the range of the parameter where stable equilibria exist, to the range of the parameter where stable equilibria no longer exist. Wind was also shown to have quite remarkable effects on the dynamics of towing, as it reduces or extends the region characterised by the presence of lateral stable equilibria depending on whether towing occurs against the wind itself or downwind.

ACKNOWLEDGMENTS

The present paper was written within the framework of the corresponding author’s PhD programme (PhD scholarship M/1/2-Cycle XXVII funded by the University of Trieste). The authors would like to thank Ing. Pedro Nicolas Mendoza Vassallo for providing useful information regarding the Peruvian Amazon Basin and for the help in the translation of the abstract from English into Spanish.

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