

On $\theta_{(\mathcal{I}, \mathcal{J})}$ -continuous functions

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ABSTRACT. *In this paper we investigate some properties of $\theta_{(\mathcal{I}, \mathcal{J})}$ -continuous functions in ideal topological spaces. Moreover the relationships with other related functions are discussed.*

Keywords: ideal topological space, θ -continuous, weakly \mathcal{J} -continuous, strongly θ -continuous, $\theta_{(\mathcal{I}, \mathcal{J})}$ -continuous

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1. Introduction

The concept of ideals in topological spaces is treated in the classic text by Kuratowski [11] and Vaidyanathaswamy [17]. Janković and Hamlett [9] investigated further properties of ideal spaces. An ideal \mathcal{I} on a topological space (X, τ) is a non-empty collection of subsets of X which satisfies the following properties: (1) $A \in \mathcal{I}$ and $B \subseteq A$ implies $B \in \mathcal{I}$; (2) $A \in \mathcal{I}$ and $B \in \mathcal{I}$ implies $A \cup B \in \mathcal{I}$. An ideal topological space (or an ideal space) is a topological space (X, τ) with an ideal \mathcal{I} on X and is denoted by (X, τ, \mathcal{I}) . For a subset $A \subseteq X$, $A^*(\mathcal{I}, \tau) = \{x \in X : A \cap U \notin \mathcal{I} \text{ for every } U \in \tau(X, x)\}$ is called the local function of A with respect to \mathcal{I} and τ [11]. We simply write A^* in case there is no chance for confusion. A Kuratowski closure operator $Cl^*(\cdot)$ for a topology $\tau^*(\mathcal{I}, \tau)$ called the $*$ -topology, finer than τ , is defined by $Cl^*(A) = A \cup A^*$ [17]. The notion of θ -continuity [6] in topological spaces is widely known and investigated. Recently, Yüksel et al. [19] have introduced the notion of $\theta_{(\mathcal{I}, \mathcal{J})}$ -continuous functions between ideal topological spaces. In the present paper, we obtain several characterizations and many properties of $\theta_{(\mathcal{I}, \mathcal{J})}$ -continuous functions.

2. Preliminaries

Let (X, τ) be a topological space with no separation axioms assumed. If $A \subseteq X$, $Cl(A)$ and $Int(A)$ will denote the closure and interior of A in (X, τ) , respectively.

In 1968, Veličko [18] introduced the class of θ -open sets. A set A is said to be θ -open [18] if every point of A has an open neighborhood whose closure is contained in A . The θ -interior [18] of A in X is the union of all θ -open subsets

of A and is denoted by $Int_\theta(A)$. Naturally, the complement of a θ -open set is said to be θ -closed. Equivalently $Cl_\theta(A) = \{x \in X : Cl(U) \cap A \neq \emptyset, U \in \tau \text{ and } x \in U\}$ and a set A is θ -closed if and only if $A = Cl_\theta(A)$. Note that all θ -open sets form a topology on X , coarser than τ , denoted by τ_θ and that a space (X, τ) is regular if and only if $\tau = \tau_\theta$. Note also that the θ -closure of a given set need not be a θ -closed set.

Let (X, τ, \mathcal{I}) be an ideal topological space and $A \subseteq X$. A point x of X is called a $\theta_{\mathcal{I}}$ -cluster point of A if $Cl^*(U) \cap A \neq \emptyset$ for every open set U of X containing x . The set of all $\theta_{\mathcal{I}}$ -cluster points of A is called the $\theta_{\mathcal{I}}$ -closure of A and is denoted by $Cl_{\theta_{\mathcal{I}}}(A)$. A is said to be $\theta_{\mathcal{I}}$ -closed if $Cl_{\theta_{\mathcal{I}}}(A) = A$. The complement of a $\theta_{\mathcal{I}}$ -closed set is called a $\theta_{\mathcal{I}}$ -open set.

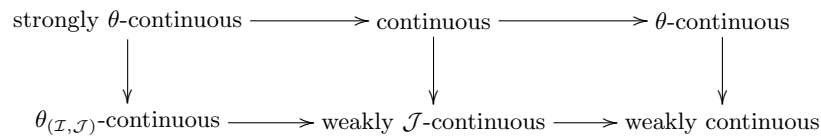
DEFINITION 2.1. *Let (X, τ, \mathcal{I}) be an ideal topological space. A point x of X is called a $\theta_{\mathcal{I}}$ -interior point of A if there exists an open set U containing x such that $Cl^*(U) \subseteq A$. The set of all $\theta_{\mathcal{I}}$ -interior points of A is called the $\theta_{\mathcal{I}}$ -interior of A and is denoted by $Int_{\theta_{\mathcal{I}}}(A)$.*

REMARK 2.2. *For a set A of X , $Int_{\theta_{\mathcal{I}}}(X - A) = X - Cl_{\theta_{\mathcal{I}}}(A)$ so that A is $\theta_{\mathcal{I}}$ -open if and only if $A = Int_{\theta_{\mathcal{I}}}(A)$.*

DEFINITION 2.3. *A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be θ -continuous [6] (resp. strongly θ -continuous [14], weakly continuous [13]) if for each $x \in X$ and each open set V in Y containing $f(x)$, there exists an open set U containing x such that $f(Cl(U)) \subseteq Cl(V)$ (resp. $f(Cl(U)) \subseteq V, f(U) \subseteq Cl(V)$).*

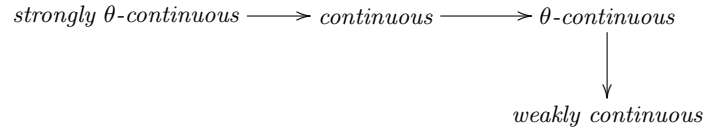
DEFINITION 2.4. *A function $f : (X, \tau, \mathcal{I}) \rightarrow (Y, \sigma, \mathcal{J})$ is said to be weakly \mathcal{J} -continuous [1] (resp. $\theta_{(\mathcal{I}, \mathcal{J})}$ -continuous [19]) if for each $x \in X$ and each open set V in Y containing $f(x)$, there exists an open set U containing x such that $f(U) \subseteq Cl^*(V)$ (resp. $f(Cl^*(U)) \subseteq Cl^*(V)$).*

By the above definitions, we have the following diagram and none of these implications is reversible



REMARK 2.5. *In [1, Example 2.1], it is shown that not every weakly continuous function is weakly \mathcal{J} -continuous.*

REMARK 2.6. *The following strict implications are well-known:*



EXAMPLE 2.7. *Let $X = \{1, 2, 3, 4\}$, $\tau = \{X, \phi, \{1, 2, 3\}, \{3\}, \{3, 4\}\}$ with $\mathcal{I} = \{\phi, \{1\}, \{2\}, \{1, 2\}\}$ and $Y = \{a, b, c, d\}$, $\sigma = \{Y, \phi, \{a, b\}, \{b\}, \{d\}, \{b, d\}, \{a, b, d\}, \{b, c, d\}\}$ with $\mathcal{J} = \{\phi\}$. We define a function $f : (X, \tau, \mathcal{I}) \rightarrow (Y, \sigma, \mathcal{J})$ as $f = \{(1, a), (2, b), (3, c), (4, d)\}$. Then f is weakly \mathcal{J} -continuous but not $\theta_{(\mathcal{I}, \mathcal{J})}$ -continuous. In [12, Example 10], it is shown that f is weakly \mathcal{J} -continuous. We show that $f : (X, \tau, \mathcal{I}) \rightarrow (Y, \sigma, \mathcal{J})$ is not $\theta_{(\mathcal{I}, \mathcal{J})}$ -continuous. Let $1 \in X$ and $V = \{a, b\} \in \sigma$ such that $f(1) = a \in V \in \sigma$. But, for every open set $U \subseteq X$ such that $1 \in U$, where $U = \{1, 2, 3\}$ or $U = X$, $Cl^*(U) = X$. Then $f(Cl^*(U)) = Y \not\subseteq Cl^*(V) = \{a, b, c\}$. Therefore, $f : (X, \tau, \mathcal{I}) \rightarrow (Y, \sigma, \mathcal{J})$ is not $\theta_{(\mathcal{I}, \mathcal{J})}$ -continuous.*

EXAMPLE 2.8. *Let $X = \{a, b, c\}$, $\tau = \{X, \phi, \{b, c\}\}$ with $\mathcal{I} = \{\phi, \{a\}\}$ and $Y = \{b, c\}$, $\sigma = \{Y, \phi, \{c\}\}$ with $\mathcal{J} = \{\phi, \{b\}\}$. We define a function $f : (X, \tau, \mathcal{I}) \rightarrow (Y, \sigma, \mathcal{J})$ as $f = \{(a, b), (b, c), (c, b)\}$. Then f is $\theta_{(\mathcal{I}, \mathcal{J})}$ -continuous but not continuous.*

1. *Let $a \in X$ and $V = Y \in \sigma$ such that $f(a) = b \in V$, then there exists an open set $U = X \in \tau$ containing a such that $f(Cl^*(U)) \subseteq Cl^*(V) = Y$.*
2. *Let $b \in X$ and $V = \{c\}$ or $V = Y$ such that $f(b) = c \in V$, then there exists an open set $U = \{b, c\}$ or $U = X$ containing b such that $f(Cl^*(U)) \subseteq Cl^*(V) = Y$.*
3. *Let $c \in X$ and $V = Y$ such that $f(c) = b \in V$, then there exists an open set $U = \{b, c\}$ or $U = X$ containing c such that $f(Cl^*(U)) \subseteq Cl^*(V) = Y$.*

By (1), (2) and (3) f is $\theta_{(\mathcal{I}, \mathcal{J})}$ -continuous. On the other hand, let $b \in X$ and $V = \{c\} \in \sigma$ such that $f(b) = c \in V \in \sigma$. But, for every open set $U \subseteq X$ such that $b \in U$, where $U = \{b, c\}$ or $U = X$. Then $f(U) = Y \not\subseteq V = \{c\}$. Therefore, $f : (X, \tau, \mathcal{I}) \rightarrow (Y, \sigma, \mathcal{J})$ is not continuous.

The following lemma is useful in the sequel:

LEMMA 2.9 ([9]). *Let (X, τ, \mathcal{I}) be an ideal topological space and A, B subsets of X . Then the following properties hold:*

1. *If $A \subseteq B$, then $A^* \subseteq B^*$.*

2. $A^* = Cl(A^*) \subseteq Cl(A)$.
3. $(A^*)^* \subseteq A^*$.
4. $(A \cup B)^* = A^* \cup B^*$.

3. Characterizations of $\theta_{(\mathcal{I}, \mathcal{J})}$ -continuous functions

In this section, we obtain several characterizations of $\theta_{(\mathcal{I}, \mathcal{J})}$ -continuous functions in ideal topological spaces.

THEOREM 3.1. *For a function $f : (X, \tau, \mathcal{I}) \rightarrow (Y, \sigma, \mathcal{J})$, the following properties are equivalent:*

1. f is $\theta_{(\mathcal{I}, \mathcal{J})}$ -continuous;
2. $Cl_{\theta_{\mathcal{I}}}(f^{-1}(B)) \subseteq f^{-1}(Cl_{\theta_{\mathcal{J}}}(B))$ for every subset B of Y ;
3. $f(Cl_{\theta_{\mathcal{I}}}(A)) \subseteq Cl_{\theta_{\mathcal{J}}}(f(A))$ for every subset A of X .

Proof. (1) \Rightarrow (2): Let B be any subset of Y . Suppose that $x \notin f^{-1}(Cl_{\theta_{\mathcal{J}}}(B))$. Then $f(x) \notin Cl_{\theta_{\mathcal{J}}}(B)$ and there exists an open set V containing $f(x)$ such that $Cl^*(V) \cap B = \phi$. Since f is $\theta_{(\mathcal{I}, \mathcal{J})}$ -continuous, there exists an open set U containing x such that $f(Cl^*(U)) \subseteq Cl^*(V)$. Therefore, we have $f(Cl^*(U)) \cap B = \phi$ and $Cl^*(U) \cap f^{-1}(B) = \phi$. This shows that $x \notin Cl_{\theta_{\mathcal{I}}}(f^{-1}(B))$. Thus, we obtain $Cl_{\theta_{\mathcal{I}}}(f^{-1}(B)) \subseteq f^{-1}(Cl_{\theta_{\mathcal{J}}}(B))$.

(2) \Rightarrow (1): Let $x \in X$ and V be an open set of Y containing $f(x)$. Then we have $Cl^*(V) \cap (Y - Cl^*(V)) = \phi$ and $f(x) \notin Cl_{\theta_{\mathcal{J}}}(Y - Cl^*(V))$. Therefore, $x \notin f^{-1}(Cl_{\theta_{\mathcal{J}}}(Y - Cl^*(V)))$ and by (2) we have $x \notin Cl_{\theta_{\mathcal{I}}}(f^{-1}(Y - Cl^*(V)))$. There exists an open set U containing x such that $Cl^*(U) \cap f^{-1}(Y - Cl^*(V)) = \phi$ and hence $f(Cl^*(U)) \subseteq Cl^*(V)$. Therefore, f is $\theta_{(\mathcal{I}, \mathcal{J})}$ -continuous.

(2) \Rightarrow (3): Let A be any subset of X . Then we have $Cl_{\theta_{\mathcal{I}}}(A) \subseteq Cl_{\theta_{\mathcal{I}}}(f^{-1}(f(A))) \subseteq f^{-1}(Cl_{\theta_{\mathcal{J}}}(f(A)))$ and hence $f(Cl_{\theta_{\mathcal{I}}}(A)) \subseteq Cl_{\theta_{\mathcal{J}}}(f(A))$.

(3) \Rightarrow (2): Let B be a subset of Y . We have $f(Cl_{\theta_{\mathcal{I}}}(f^{-1}(B))) \subseteq Cl_{\theta_{\mathcal{J}}}(f(f^{-1}(B))) \subseteq Cl_{\theta_{\mathcal{J}}}(B)$ and hence $Cl_{\theta_{\mathcal{I}}}(f^{-1}(B)) \subseteq f^{-1}(Cl_{\theta_{\mathcal{J}}}(B))$. \square

DEFINITION 3.2 ([1]). *An ideal topological space (X, τ, \mathcal{I}) is called an $F\mathcal{I}^*$ -space if $Cl(U) \subseteq U^*$ for every open set U of X .*

DEFINITION 3.3 ([3]). *Let (X, τ, \mathcal{I}) be an ideal topological space. \mathcal{I} is said to be codense if $\tau \cap \mathcal{I} = \phi$.*

REMARK 3.4. *In [12], Kuyucu et al. showed the following properties:*

1. *an ideal topological space (X, τ, \mathcal{I}) is an $F\mathcal{I}^*$ -space if and only if \mathcal{I} is codense,*

2. if (X, τ, \mathcal{I}) is an $F\mathcal{I}^*$ -space, then $V^* = Cl^*(V) = Cl(V)$ for every open set V of X .

THEOREM 3.5. For a function $f : (X, \tau, \mathcal{I}) \rightarrow (Y, \sigma, \mathcal{J})$, the following implications: (1) \Leftrightarrow (2) \Rightarrow (3) \Leftrightarrow (4) hold. Moreover, the implication (4) \Rightarrow (1) holds if (Y, σ, \mathcal{J}) is an $F\mathcal{J}^*$ -space.

1. f is $\theta_{(\mathcal{I}, \mathcal{J})}$ -continuous;
2. $f^{-1}(V) \subseteq Int_{\theta_{\mathcal{I}}}(f^{-1}(Cl^*(V)))$ for every open set V of Y ;
3. $Cl_{\theta_{\mathcal{I}}}(f^{-1}(V)) \subseteq f^{-1}(Cl(V))$ for every open set V of Y ;
4. For each $x \in X$ and each open set V of Y containing $f(x)$, there exists an open set U of X containing x such that $f(Cl^*(U)) \subseteq Cl(V)$.

Proof. (1) \Rightarrow (2): Suppose that V is any open set of Y and $x \in f^{-1}(V)$. Then $f(x) \in V$ and there exists an open set U containing x such that $f(Cl^*(U)) \subseteq Cl^*(V)$. Therefore, $x \in U \subseteq Cl^*(U) \subseteq f^{-1}(Cl^*(V))$. This shows that $x \in Int_{\theta_{\mathcal{I}}}(f^{-1}(Cl^*(V)))$. Therefore, we obtain $f^{-1}(V) \subseteq Int_{\theta_{\mathcal{I}}}(f^{-1}(Cl^*(V)))$. (2) \Rightarrow (1): Let $x \in X$ and $V \in \sigma$ containing $f(x)$. Then, by (2) $f^{-1}(V) \subseteq Int_{\theta_{\mathcal{I}}}(f^{-1}(Cl^*(V)))$. Since $x \in f^{-1}(V)$, there exists an open set U containing x such that $Cl^*(U) \subseteq f^{-1}(Cl^*(V))$. Therefore, $f(Cl^*(U)) \subseteq Cl^*(V)$ and hence f is $\theta_{(\mathcal{I}, \mathcal{J})}$ -continuous.

(2) \Rightarrow (3): Suppose that V is any open set of Y and $x \notin f^{-1}(Cl(V))$. Then $f(x) \notin Cl(V)$ and there exists an open set W containing $f(x)$ such that $W \cap V = \phi$; hence $Cl^*(W) \cap V \subseteq Cl(W) \cap V = \phi$. Therefore, we have $f^{-1}(Cl^*(W)) \cap f^{-1}(V) = \phi$. Since $x \in f^{-1}(W)$, by (2) $x \in Int_{\theta_{\mathcal{I}}}(f^{-1}(Cl^*(W)))$. There exists an open set U containing x such that $Cl^*(U) \subseteq f^{-1}(Cl^*(W))$. Thus we have $Cl^*(U) \cap f^{-1}(V) = \phi$ and hence $x \notin Cl_{\theta_{\mathcal{I}}}(f^{-1}(V))$. This shows that $Cl_{\theta_{\mathcal{I}}}(f^{-1}(V)) \subseteq f^{-1}(Cl(V))$.

(3) \Rightarrow (4): Suppose that $x \in X$ and V is any open set of Y containing $f(x)$. Then $V \cap (Y - Cl(V)) = \phi$ and $f(x) \notin Cl(Y - Cl(V))$. Therefore $x \notin f^{-1}(Cl(Y - Cl(V)))$ and by (3) $x \notin Cl_{\theta_{\mathcal{I}}}(f^{-1}(Y - Cl(V)))$. There exists an open set U containing x such that $Cl^*(U) \cap f^{-1}(Y - Cl(V)) = \phi$. Therefore, we obtain $f(Cl^*(U)) \subseteq Cl(V)$.

(4) \Rightarrow (3): Let V be any open set of Y . Suppose that $x \notin f^{-1}(Cl(V))$. Then $f(x) \notin Cl(V)$ and there exists an open set W containing $f(x)$ such that $W \cap V = \phi$. By (4), there exists an open set U containing x such that $f(Cl^*(U)) \subseteq Cl(W)$. Since $V \in \sigma$, $Cl(W) \cap V = \phi$ and $f(Cl^*(U)) \cap V \subseteq Cl(W) \cap V = \phi$. Therefore, $Cl^*(U) \cap f^{-1}(V) = \phi$ and hence $x \notin Cl_{\theta_{\mathcal{I}}}(f^{-1}(V))$. This shows that $Cl_{\theta_{\mathcal{I}}}(f^{-1}(V)) \subseteq f^{-1}(Cl(V))$.

(4) \Rightarrow (1): Since (Y, σ, \mathcal{J}) is an $F\mathcal{J}^*$ -space, $Cl(V) \subseteq Cl^*(V)$ for every open set V of Y and hence f is $\theta_{(\mathcal{I}, \mathcal{J})}$ -continuous. \square

PROPOSITION 3.6. *A function $f : (X, \tau, \mathcal{I}) \rightarrow (Y, \sigma, \mathcal{J})$ from an $F\mathcal{I}^*$ -space to an $F\mathcal{J}^*$ -space is $\theta_{(\mathcal{I}, \mathcal{J})}$ -continuous if and only if it is θ -continuous.*

Proof. This follows from the Remark 3.4. \square

4. Some properties of $\theta_{(\mathcal{I}, \mathcal{J})}$ -continuous functions

DEFINITION 4.1. *An ideal topological space (X, τ, \mathcal{I}) is said to be $\theta_{\mathcal{I}}\text{-}T_2$ (resp. $*$ -Urysohn) if for each distinct points $x, y \in X$, there exist two $\theta_{\mathcal{I}}$ -open (resp. open) sets $U, V \in X$ containing x and y , respectively, such that $U \cap V = \phi$ (resp. $Cl^*(U) \cap Cl^*(V) = \phi$).*

THEOREM 4.2. *If $f, g : (X, \tau, \mathcal{I}) \rightarrow (Y, \sigma, \mathcal{J})$ are $\theta_{(\mathcal{I}, \mathcal{J})}$ -continuous functions and (Y, σ, \mathcal{J}) is $*$ -Urysohn, then $A = \{x \in X : f(x) = g(x)\}$ is a $\theta_{\mathcal{I}}$ -closed set of (X, τ, \mathcal{I}) .*

Proof. We prove that $X - A$ is a $\theta_{\mathcal{I}}$ -open set. Let $x \in X - A$. Then $f(x) \neq g(x)$. Since Y is $*$ -Urysohn, there exist open sets V_1 and V_2 containing $f(x)$ and $g(x)$, respectively, such that $Cl^*(V_1) \cap Cl^*(V_2) = \phi$. Since f and g are $\theta_{(\mathcal{I}, \mathcal{J})}$ -continuous, there exists an open set U containing x such that $f(Cl^*(U)) \subseteq Cl^*(V_1)$ and $g(Cl^*(U)) \subseteq Cl^*(V_2)$. Hence we obtain that $Cl^*(U) \subseteq f^{-1}(Cl^*(V_1))$ and $Cl^*(U) \subseteq g^{-1}(Cl^*(V_2))$. From here we have $Cl^*(U) \subseteq f^{-1}(Cl^*(V_1)) \cap g^{-1}(Cl^*(V_2))$. Moreover $f^{-1}(Cl^*(V_1)) \cap g^{-1}(Cl^*(V_2)) \subseteq X - A$. This shows that $X - A$ is $\theta_{\mathcal{I}}$ -open. \square

DEFINITION 4.3. *An ideal topological space (X, τ, \mathcal{I}) is said to be $*$ -regular if for each closed set F and each point $x \in X - F$, there exist an open set V and an $*$ -open set $U \in \tau^*$ such that $x \in V$, $F \subseteq U$ and $U \cap V = \phi$.*

EXAMPLE 4.4. *Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{a, b\}\}$ and $\mathcal{I} = \mathcal{P}(X)$, then (X, τ, \mathcal{I}) is an $*$ -regular space which is not regular.*

LEMMA 4.5 ([1]). 1. *A function $f : (X, \tau) \rightarrow (Y, \sigma, \mathcal{J})$ is weakly \mathcal{J} -continuous if and only if for each open set V , $f^{-1}(V) \subseteq Int(f^{-1}(Cl^*(V)))$.*

2. *If an ideal space (Y, σ, \mathcal{I}) is an $F\mathcal{J}^*$ -space and a function $f : (X, \tau, \mathcal{I}) \rightarrow (Y, \sigma, \mathcal{I})$ is weakly \mathcal{J} -continuous, then $Cl^*(f^{-1}(G)) \subseteq f^{-1}(Cl^*(G))$ for every open set G in Y .*

The equivalence of (1) and (2) in the following theorem is suggested by the referee.

THEOREM 4.6. *Let (Y, σ, \mathcal{J}) be an $F\mathcal{J}^*$ -space. For a function $f : (X, \tau) \rightarrow (Y, \sigma, \mathcal{J})$, the following properties are equivalent:*

1. f is weakly \mathcal{J} -continuous;
2. $Cl(f^{-1}(V)) \subseteq f^{-1}(Cl^*(V))$ for every open set V of Y ;
3. f is weakly continuous.

Proof. (1) \Rightarrow (2): Let V be any open set of Y . Suppose that $x \notin f^{-1}(Cl^*(V))$. Then $f(x) \notin Cl^*(V)$. Since (Y, σ, \mathcal{J}) is an $F\mathcal{J}^*$ -space, $f(x) \notin Cl(V)$ and there exists $W \in \sigma$ containing $f(x)$ such that $W \cap V = \phi$, hence $Cl^*(W) \cap V = Cl(W) \cap V = \phi$. Since f is weakly \mathcal{J} -continuous, there exists $U \in \tau$ containing x such that $f(U) \subseteq Cl^*(W)$. Therefore, we have $f(U) \cap V = \phi$ and $U \cap f^{-1}(V) = \phi$. Since $U \in \tau$, $U \cap Cl(f^{-1}(V)) = \phi$ and hence $x \notin Cl(f^{-1}(V))$. Therefore, we obtain $Cl(f^{-1}(V)) \subseteq f^{-1}(Cl^*(V))$.

(2) \Rightarrow (3): Let V be any open set of Y . Since (Y, σ, \mathcal{J}) is an $F\mathcal{J}^*$ -space, by (2) we have $Cl(f^{-1}(V)) \subseteq f^{-1}(Cl(V))$. It follows from [16, Theorem 7] that f is weakly continuous.

(3) \Rightarrow (1): Let f be weakly continuous. By [13, Theorem 1]

$$f^{-1}(V) \subseteq Int(f^{-1}(Cl(V)))$$

for every open set V of Y . Since (Y, σ, \mathcal{J}) is an $F\mathcal{J}^*$ -space, $Cl(V) = Cl^*(V)$ and we have $f^{-1}(V) \subseteq Int(f^{-1}(Cl^*(V)))$. Therefore, by Lemma 4.5 (1) f is weakly \mathcal{J} -continuous. \square

DEFINITION 4.7 ([5]). An ideal space (X, τ, \mathcal{I}) is said to be $*$ -extremally disconnected if the $*$ -closure of every open subset of X is open.

LEMMA 4.8. An ideal topological space (X, τ, \mathcal{I}) is $*$ -regular if and only if for each open set U containing x there exists an open set V such that $x \in V \subseteq Cl^*(V) \subseteq U$.

PROPOSITION 4.9. Let (X, τ, \mathcal{I}) be an $*$ -regular space. Then $f : (X, \tau, \mathcal{I}) \rightarrow (Y, \sigma, \mathcal{J})$ is $\theta_{(\mathcal{I}, \mathcal{J})}$ -continuous if and only if it is weakly \mathcal{J} -continuous.

Proof. Every $\theta_{(\mathcal{I}, \mathcal{J})}$ -continuous function is weakly \mathcal{J} -continuous. Suppose that f is weakly \mathcal{J} -continuous. Let $x \in X$ and V be any open set of Y containing $f(x)$. Then, there exists an open set U containing x such that $f(U) \subseteq Cl^*(V)$. Since X is $*$ -regular, by Lemma 4.8 there exists an open set W such that $x \in W \subseteq Cl^*(W) \subseteq U$. Therefore, we obtain $f(Cl^*(W)) \subseteq Cl^*(V)$. This shows that f is $\theta_{(\mathcal{I}, \mathcal{J})}$ -continuous. \square

THEOREM 4.10. Let an ideal space (Y, σ, \mathcal{J}) be an $F\mathcal{J}^*$ -space and $*$ -extremally disconnected. Then $f : (X, \tau, \mathcal{I}) \rightarrow (Y, \sigma, \mathcal{J})$ is $\theta_{(\mathcal{I}, \mathcal{J})}$ -continuous if and only if it is weakly \mathcal{J} -continuous.

Proof. It is clear that every $\theta_{(\mathcal{I}, \mathcal{J})}$ -continuous function is weakly \mathcal{J} -continuous. Conversely, suppose that f is weakly \mathcal{J} -continuous. Let $x \in X$ and V be an open set of Y containing $f(x)$. Then by Lemma 4.5 (1), $x \in f^{-1}(V) \subseteq \text{Int}(f^{-1}(Cl^*(V)))$. Let $U = \text{Int}(f^{-1}(Cl^*(V)))$. Since (Y, σ, \mathcal{I}) is an $F\mathcal{J}^*$ -space and $*$ -extremally disconnected, by using Lemma 4.5 (2) $f(Cl^*(U)) = f(Cl^*(\text{Int}(f^{-1}(Cl^*(V)))) \subseteq f(Cl^*(f^{-1}(Cl^*(V)))) \subseteq f(f^{-1}(Cl^*(Cl^*(V)))) \subseteq Cl^*(V)$. Hence f is $\theta_{(\mathcal{I}, \mathcal{J})}$ -continuous. \square

COROLLARY 4.11. *Let an ideal space (Y, σ, \mathcal{J}) be an $F\mathcal{J}^*$ -space and $*$ -extremally disconnected. For a function $f : (X, \tau, \mathcal{I}) \rightarrow (Y, \sigma, \mathcal{J})$, the following properties are equivalent:*

1. f is $\theta_{(\mathcal{I}, \mathcal{J})}$ -continuous;
2. f is weakly \mathcal{J} -continuous;
3. $f^{-1}(V) \subseteq \text{Int}(f^{-1}(V^*))$ for every open set V in Y ;
4. $f^{-1}(V) \subseteq \text{Int}(f^{-1}(Cl(V)))$ for every open set V of Y ;
5. f is weakly continuous.

Proof. By Theorem 4.10, we have the equivalence of (1) and (2). The equivalences of (2), (3) and (4) follow from Lemma 4.5 (1) and Remark 3.4. The equivalence of (4) and (5) is shown in [13, Theorem 1]. \square

A subset A of an ideal space (X, τ, \mathcal{I}) is said to be pre- \mathcal{I} -open [4] if $A \subseteq \text{Int}(Cl^*(A))$. A function $f : (X, \tau, \mathcal{I}) \rightarrow (Y, \sigma, \mathcal{J})$ is said to be pre- \mathcal{I} -continuous [4] if the inverse image of every open set of Y is pre- \mathcal{I} -open in X .

THEOREM 4.12. *If $f : (X, \tau, \mathcal{I}) \rightarrow (Y, \sigma, \mathcal{J})$ is a pre- \mathcal{I} -continuous function and $Cl^*(f^{-1}(U)) \subseteq f^{-1}(Cl^*(U))$ for every open set U in Y , then f is $\theta_{(\mathcal{I}, \mathcal{J})}$ -continuous.*

Proof. Let $x \in X$ and U be an open set in Y containing $f(x)$. By hypothesis, $Cl^*(f^{-1}(U)) \subseteq f^{-1}(Cl^*(U))$. Since f is pre- \mathcal{I} -continuous, $f^{-1}(U)$ is pre- \mathcal{I} -open in X and so $f^{-1}(U) \subseteq \text{Int}(Cl^*(f^{-1}(U)))$. Since $x \in f^{-1}(U) \subseteq \text{Int}(Cl^*(f^{-1}(U)))$, there exists an open set V containing x such that $x \in V \subseteq Cl^*(V) \subseteq Cl^*(f^{-1}(U)) \subseteq f^{-1}(Cl^*(U))$ and so $f(Cl^*(V)) \subseteq Cl^*(U)$ which implies that f is $\theta_{(\mathcal{I}, \mathcal{J})}$ -continuous. \square

The following corollary follows from Lemma 4.5 and Theorems 4.6 and 4.12.

COROLLARY 4.13. *Let $f : (X, \tau, \mathcal{I}) \rightarrow (Y, \sigma, \mathcal{J})$ be pre- \mathcal{I} -continuous and (Y, σ, \mathcal{J}) is an $F\mathcal{J}^*$ -space. The following properties are equivalent:*

1. f is $\theta_{(\mathcal{I}, \mathcal{J})}$ -continuous;

2. $Cl^*(f^{-1}(V)) \subseteq f^{-1}(Cl^*(V))$ for every open set V in Y ;
3. $Cl(f^{-1}(V)) \subseteq f^{-1}(Cl^*(V))$ for every open set V in Y ;
4. f is weakly \mathcal{J} -continuous.

5. Preservation theorems

A subset A of a space X is said to be quasi H^* -closed relative to X if for every cover $\{V_\alpha : \alpha \in \Lambda\}$ of A by open sets of X , there exists a finite subset Λ_0 of Λ such that $A \subseteq \cup\{Cl^*(V_\alpha) : \alpha \in \Lambda_0\}$. A space X is said to be quasi H^* -closed if X is quasi H^* -closed relative to X .

THEOREM 5.1. *If $f : (X, \tau, \mathcal{I}) \rightarrow (Y, \sigma, \mathcal{J})$ is $\theta_{(\mathcal{I}, \mathcal{J})}$ -continuous and K is quasi H^* -closed relative to X , then $f(K)$ is quasi H^* -closed relative to Y .*

Proof. Suppose that $f : (X, \tau, \mathcal{I}) \rightarrow (Y, \sigma, \mathcal{J})$ is a $\theta_{(\mathcal{I}, \mathcal{J})}$ -continuous function and K is quasi H^* -closed relative to X . Let $\{V_\alpha : \alpha \in \Lambda\}$ be a cover of $f(K)$ by open sets of Y . For each point $x \in K$, there exists $\alpha(x) \in \Lambda$ such that $f(x) \in V_{\alpha(x)}$. Since f is $\theta_{(\mathcal{I}, \mathcal{J})}$ -continuous, there exists an open set U_x containing x such that $f(Cl^*(U_x)) \subseteq Cl^*(V_{\alpha(x)})$. The family $\{U_x : x \in K\}$ is a cover of K by open sets of X and hence there exists a finite subset K_* of K such that $K \subseteq \cup_{x \in K_*} Cl^*(U_x)$. Therefore, we obtain $f(K) \subseteq \cup_{x \in K_*} Cl^*(V_{\alpha(x)})$. This shows that $f(K)$ is quasi H^* -closed relative to Y . \square

DEFINITION 5.2. *A function $f : (X, \tau, \mathcal{I}) \rightarrow (Y, \sigma, \mathcal{J})$ is said to be $\theta_{(\mathcal{I}, \mathcal{J})}$ -irresolute if for every $\theta_{\mathcal{J}}$ -open set U in Y , $f^{-1}(U)$ is $\theta_{\mathcal{I}}$ -open in X .*

THEOREM 5.3. *Every $\theta_{(\mathcal{I}, \mathcal{J})}$ -continuous function is $\theta_{(\mathcal{I}, \mathcal{J})}$ -irresolute.*

Proof. Let $f : X \rightarrow Y$ be a $\theta_{(\mathcal{I}, \mathcal{J})}$ -continuous function and U be a $\theta_{\mathcal{J}}$ -open set in Y . Let $x \in f^{-1}(U)$. Then, $f(x) \in U$. Since U is $\theta_{\mathcal{J}}$ -open, there exists an open set V in Y such that $f(x) \in V \subseteq Cl^*(V) \subseteq U$. By $\theta_{(\mathcal{I}, \mathcal{J})}$ -continuity of f , there exists an open set W in X containing x such that $f(Cl^*(W)) \subseteq Cl^*(V) \subseteq U$. Thus $x \in W \subseteq Cl^*(W) \subseteq f^{-1}(U)$. Hence $f^{-1}(U)$ is $\theta_{\mathcal{I}}$ -open and hence f is $\theta_{(\mathcal{I}, \mathcal{J})}$ -irresolute. \square

DEFINITION 5.4. (1) *An ideal space (X, τ, \mathcal{I}) is said to be $\theta_{\mathcal{I}}$ -compact if every cover of X by $\theta_{\mathcal{I}}$ -open sets admits a finite subcover.*

(2) *A subset A of an ideal space (X, τ, \mathcal{I}) is said to be $\theta_{\mathcal{I}}$ -compact relative to X if every cover of A by $\theta_{\mathcal{I}}$ -open sets of X admits a finite subcover.*

PROPOSITION 5.5. *Every quasi H^* -closed space (X, τ, \mathcal{I}) is $\theta_{\mathcal{I}}$ -compact.*

Proof. More generally, we show that if A is quasi H^* -closed relative to a space X , then A is $\theta_{\mathcal{I}}$ -compact relative to X . Let $A \subseteq \cup\{V_{\alpha} : \alpha \in \Lambda\}$, where each V_{α} is $\theta_{\mathcal{I}}$ -open, and A be quasi H^* -closed relative to X , then for each $x \in A$ there exists an $\alpha(x) \in \Lambda$ with $x \in V_{\alpha(x)}$. Then there exists an open set $U_{\alpha(x)}$ with $x \in U_{\alpha(x)}$ such that $Cl^*(U_{\alpha(x)}) \subseteq V_{\alpha(x)}$. Since $\{U_{\alpha(x)} : x \in A\}$ is a cover of A by open set in X , then there is a finite subset $\{x_1, x_2, \dots, x_n\} \subseteq A$ such that $A \subseteq \cup\{Cl^*(U_{\alpha(x_i)}) : i = 1, 2, \dots, n\} \subseteq \cup\{V_{\alpha(x_i)} : i = 1, 2, \dots, n\}$. Hence A is $\theta_{\mathcal{I}}$ -compact relative to X . \square

THEOREM 5.6. *If $f : (X, \tau, \mathcal{I}) \rightarrow (Y, \sigma, \mathcal{J})$ is a $\theta_{(\mathcal{I}, \mathcal{J})}$ -irresolute surjection and (X, τ, \mathcal{I}) is $\theta_{\mathcal{I}}$ -compact, then Y is $\theta_{\mathcal{J}}$ -compact.*

Proof. Let \mathcal{V} be a $\theta_{\mathcal{J}}$ -open covering of Y . Then, since f is $\theta_{(\mathcal{I}, \mathcal{J})}$ -irresolute, the collection $\mathcal{U} = \{f^{-1}(U) : U \in \mathcal{V}\}$ is a $\theta_{\mathcal{I}}$ -open covering of X . Since X is $\theta_{\mathcal{I}}$ -compact, there exists a finite subcollection $\{f^{-1}(U_i) : i = 1, \dots, n\}$ of \mathcal{U} which covers X . Now since f is onto, $\{U_i : i = 1, \dots, n\}$ is a finite subcollection of \mathcal{V} which covers Y . Hence Y is a $\theta_{\mathcal{J}}$ -compact space. \square

COROLLARY 5.7. *The $\theta_{(\mathcal{I}, \mathcal{J})}$ -continuous surjective image of a $\theta_{\mathcal{I}}$ -compact space is $\theta_{\mathcal{J}}$ -compact.*

DEFINITION 5.8. *An ideal topological space (X, τ, \mathcal{I}) is said to be $*$ -Lindelöf if for every open cover $\{U_{\alpha} : \alpha \in \Lambda\}$ of X there exists a countable subset $\{\alpha_n : n \in \mathbb{N}\} \subseteq \Lambda$ such that $X = \cup_{n \in \mathbb{N}} Cl^*(U_{\alpha_n})$.*

THEOREM 5.9. *Let $f : (X, \tau, \mathcal{I}) \rightarrow (Y, \sigma, \mathcal{J})$ be a $\theta_{(\mathcal{I}, \mathcal{J})}$ -continuous (resp. weakly \mathcal{J} -continuous) surjection. If X is $*$ -Lindelöf (resp. Lindelöf), then Y is $*$ -Lindelöf.*

Proof. Suppose that f is $\theta_{(\mathcal{I}, \mathcal{J})}$ -continuous and X is $*$ -Lindelöf. Let $\{V_{\alpha} : \alpha \in \Lambda\}$ be an open cover of Y . For each $x \in X$, there exists $\alpha(x) \in \Lambda$ such that $f(x) \in V_{\alpha(x)}$. Since f is $\theta_{(\mathcal{I}, \mathcal{J})}$ -continuous, there exists an open set $U_{\alpha(x)}$ of X containing x such that $f(Cl^*(U_{\alpha(x)})) \subseteq V_{\alpha(x)}$. Now $\{U_{\alpha(x)} : x \in X\}$ is an open cover of the $*$ -Lindelöf space X . So there exists a countable subset $\{U_{\alpha(x_n)} : n \in \mathbb{N}\}$ such that $X = \cup_{n \in \mathbb{N}} (Cl^*(U_{\alpha(x_n)}))$. Thus $Y = f(\cup_{n \in \mathbb{N}} (Cl^*(U_{\alpha(x_n)}))) \subseteq \cup_{n \in \mathbb{N}} f(Cl^*(U_{\alpha(x_n)})) \subseteq \cup_{n \in \mathbb{N}} V_{\alpha(x_n)}$. This shows that Y is $*$ -Lindelöf. In case X is Lindelöf the proof is similar. \square

A function $f : (X, \tau, \mathcal{I}) \rightarrow (Y, \sigma, \mathcal{J})$ is said to be $\theta_{(\mathcal{I}, \mathcal{J})}$ -closed if for each $\theta_{\mathcal{I}}$ -closed set F in X , $f(F)$ is $\theta_{\mathcal{J}}$ -closed in Y .

The following characterization of $\theta_{(\mathcal{I}, \mathcal{J})}$ -closed functions will be used in the sequel.

THEOREM 5.10. *A surjective function $f : (X, \tau, \mathcal{I}) \rightarrow (Y, \sigma, \mathcal{J})$ is $\theta_{(\mathcal{I}, \mathcal{J})}$ -closed if and only if for each set $B \subseteq Y$ and for each $\theta_{\mathcal{I}}$ -open set U containing $f^{-1}(B)$, there exists a $\theta_{\mathcal{J}}$ -open set V containing B such that $f^{-1}(V) \subseteq U$.*

Proof. Necessity. Suppose that f is $\theta_{(\mathcal{I}, \mathcal{J})}$ -closed. Since U is $\theta_{\mathcal{I}}$ -open in X , $X - U$ is $\theta_{\mathcal{I}}$ -closed and so $f(X - U)$ is $\theta_{\mathcal{J}}$ -closed in Y . Now, $V = Y - f(X - U)$ is $\theta_{\mathcal{J}}$ -open, $B \subseteq V$ and $f^{-1}(V) = f^{-1}(Y - f(X - U)) = X - f^{-1}(f(X - U)) \subseteq X - (X - U) = U$.

Sufficiency. Let A be a $\theta_{\mathcal{I}}$ -closed set in X . To prove that $f(A)$ is $\theta_{\mathcal{J}}$ -closed, we shall show that $Y - f(A)$ is $\theta_{\mathcal{J}}$ -open. Let $y \in Y - f(A)$. Then $f^{-1}(y) \cap f^{-1}(f(A)) = \emptyset$ and so $f^{-1}(y) \subseteq X - f^{-1}(f(A)) \subseteq X - A$. By hypothesis there exists a $\theta_{\mathcal{J}}$ -open set V containing y such that $f^{-1}(V) \subseteq X - A$. So $A \subseteq X - f^{-1}(V)$ and hence $f(A) \subseteq f(X - f^{-1}(V)) = Y - V$. Thus $V \subseteq Y - f(A)$ and so the set $Y - f(A)$ being the union of $\theta_{\mathcal{J}}$ -open sets is $\theta_{\mathcal{J}}$ -open. \square

THEOREM 5.11. *Let $f : (X, \tau, \mathcal{I}) \rightarrow (Y, \sigma, \mathcal{J})$ be a $\theta_{(\mathcal{I}, \mathcal{J})}$ -closed surjection such that for each $y \in Y$, $f^{-1}(y)$ is $\theta_{\mathcal{I}}$ -compact relative to X . If Y is $\theta_{\mathcal{J}}$ -compact, then X is $\theta_{\mathcal{I}}$ -compact.*

Proof. Let $\mathcal{U} = \{U_{\alpha} : \alpha \in \Lambda\}$ be a $\theta_{\mathcal{I}}$ -open covering of X . Since for each $y \in Y$, $f^{-1}(y)$ is $\theta_{\mathcal{I}}$ -compact relative to X , we can choose a finite subset Λ_y of Λ such that $\{U_{\beta} : \beta \in \Lambda_y\}$ is a covering of $f^{-1}(y)$. Now, by Theorem 5.10, there exists a $\theta_{\mathcal{J}}$ -open set V_y containing y such that $f^{-1}(V_y) \subseteq \cup\{U_{\beta} : \beta \in \Lambda_y\}$. The collection $\mathcal{V} = \{V_y : y \in Y\}$ is a $\theta_{\mathcal{J}}$ -open covering of Y . In view of $\theta_{\mathcal{J}}$ -compactness of Y there exists a finite subcollection $\{V_{y_1}, \dots, V_{y_n}\}$ of \mathcal{V} which covers Y . Then the finite subcollection $\{U_{\beta} : \beta \in \Lambda_{y_i}, i = 1, \dots, n\}$ of \mathcal{U} covers X . Hence X is a $\theta_{\mathcal{I}}$ -compact space. \square

Let (X, τ) be a space with an ideal \mathcal{I} on X and $D \subseteq X$. Then $\mathcal{I}_D = \{D \cap A : A \in \mathcal{I}\}$ is obviously an ideal on D .

THEOREM 5.12. *Let $f : (X, \tau, \mathcal{I}) \rightarrow (Y, \sigma, \mathcal{J})$ be a function, D be a dense subset in the topological space (Y, σ^*) and $f(X) \subseteq D$. Then the following properties are equivalent:*

1. $f : (X, \tau, \mathcal{I}) \rightarrow (Y, \sigma, \mathcal{J})$ is $\theta_{(\mathcal{I}, \mathcal{J})}$ -continuous;
2. $f : (X, \tau, \mathcal{I}) \rightarrow (D, \sigma_D, \mathcal{J}_D)$ is $\theta_{(\mathcal{I}, \mathcal{J}_D)}$ -continuous.

Proof. (1) \Rightarrow (2): Let $x \in X$ and W be any open set of D containing $f(x)$, that is $f(x) \in W \in \sigma_D$. Then exists a $V \in \sigma$ such that $W = D \cap V$. Since $f : (X, \tau, \mathcal{I}) \rightarrow (Y, \sigma, \mathcal{J})$ is $\theta_{(\mathcal{I}, \mathcal{J})}$ -continuous and $f(x) \in V \in \sigma$, there exists $U \in \tau$ such that $x \in U$ and $f(Cl^*(U)) \subseteq Cl^*(V)$. If D is a dense subset in the topological space (Y, σ^*) , then D is a dense subset in the topological space

(Y, σ) since $Cl^*(D) \subseteq Cl(D)$. Since $\sigma \subseteq \sigma^*$, $V \in \sigma^*$. So, $Cl^*(D \cap V) = Cl^*(V)$ since D is dense. Thus $f(Cl^*(U)) \subseteq Cl^*(V) \cap f(X) \subseteq Cl^*(D \cap V) \cap D \subseteq Cl^*(V) \cap D$. Since $W = D \cap V$, $Cl_D^*(W) = Cl^*(V) \cap D$ by [7, Lemma 4] $f(Cl^*(U)) \subseteq Cl_D^*(W)$. Hence we obtain that $f : (X, \tau, \mathcal{I}) \rightarrow (D, \sigma_D, \mathcal{J}_D)$ is $\theta_{(\mathcal{I}, \mathcal{J}_D)}$ -continuous.

(2) \Rightarrow (1): Let $x \in X$ and V be any open set Y containing $f(x)$. Since $f(x) \in D \cap V$ and $D \cap V \in \sigma_D$, by (2) there exists $U \in \tau$ containing x such that $f(Cl^*(U)) \subseteq Cl_D^*(D \cap V) = Cl^*(D \cap V) \cap D \subseteq Cl^*(V)$. This shows that f is $\theta_{(\mathcal{I}, \mathcal{J})}$ -continuous. \square

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