

INFERENCES WITHOUT NECESSARY CONCLUSION AND EASY TO DETECT

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Abstract: Aristotle indicates four types of inferences whose conclusion does not necessarily follows its premises. In this paper, I review them from the mental models theory and try to show that, if this last framework is correct, people should note without a lot of effort that their conclusions are not necessary. I mainly base my analysis on the distinction between canonical and noncanonical models provided by Khemlani, Lotstein, Trafton, and Johnson-Laird (2015).

Key Words: Aristotelian logic, inference, mental models, necessary conclusion, quantification.

1. *Introduction*

The mental models theory (e.g., Johnson-Laird 2010; 2012; 2015; Johnson-Laird, Khemlani, & Goodwin 2015; Khemlani, Lotstein, Trafton, & Johnson-Laird 2015; Khemlani, Orenes, & Johnson-Laird 2012; 2014; Orenes & Johnson-Laird 2012; Quelhas, Johnson-Laird, & Juhos 2010; Ragni, Sonntag, & Johnson-Laird 2016) is a contemporary theory on reasoning essentially based on the idea that human beings make inferences considering possibilities sets related to sentences in natural language. Several aspects of Aristotelian logic have been analyzed from the theoretical machinery of this approach (e.g., López-Astorga 2016a; 2016b; 2016c), and the results refer to different ideas. One of them that is very relevant is that, if it is true that the mental models theory (from now on, MMT) really describes and explains human reasoning, certain particular aspects of Aristotle’s logical system can be used as instruments to predict the conclusions that will be deduced in some types of inferences, as well as the inferential mental activities that will be easier or more difficult for people. This is so for two reasons. Firstly, MMT seems to be able to show to what extent the structures in Aristotelian logic require cognitive effort to be understood or applied. Secondly, it appears that certain correspondences between the syllogisms considered correct by Aristotle and the results expected by MMT in those very syllogisms exist.

This paper is intended to continue in this way and to review four inferences that, according to Aristotle, do not have a necessary conclusion. Thus, its aim is to check whether, following MMT, it must be expected that individuals quickly

note that the conclusions of such inferences are not guaranteed by their premises or, on the contrary, that it is very hard to be aware of that fact.

So, this paper can be considered to be a complement of different works with similar goals, but especially of those akin to that of López-Astorga (2016a), in which it is argued that MMT proves that all the moods of the four figures in Aristotelian logic represent schemata that people should apply without difficulties. The challenge here is, nevertheless, to analyze whether or not the situation is as clear in the case of certain incorrect syllogisms as in that of, for example, the moods and figures. To do that, I will begin by commenting on the particular theses of MMT that need to be taken into account in a study of this kind. Then I will describe the exact four syllogisms to which I am referring. Finally, I will indicate what MMT can say with regard to those four inferences.

2. *MMT and quantification*

As pointed out, the literature on MMT is not scarce. However, a very relevant paper about the way it addresses the quantified sentences (i.e., the type of sentences that is used in Aristotelian logic) is evidently that of Khemlani *et al.* (2015). I will primarily base this section on that paper and on the explanation of it given by López-Astorga (2016a: 6ff).

MMT claims that what is important in reasoning and the inferential activity is not the logical form of sentences, but the possibilities that such sentences enable (e.g., Johnson-Laird 2010). In this way, according to the theory, people think about the possibilities of each sentence by means of representations that are iconic in a sense akin to that given by Peirce (1931/1958) to that word (see, e.g., Johnson-Laird 2012). So, following general assumptions such as these ones, Khemlani *et al.* state that, when faced with quantified sentences, people obviously consider the combinations of possibilities that can be assigned to them too. The only problem in this regard is that, as also indicated by the theory for sentences in general, effort is needed to identify all of the possibilities and, for this reason, individuals sometimes only pay attention to the ones that are easier to detect. Thus, the particular expressions used by them in their paper are “canonical models” to refer to the sets of possibilities not requiring a lot of effort to be captured, and “noncanonical models” to mention the other ones. In their Table 1 (Khemlani *et al.* 2015: 2077), they show examples of canonical and noncanonical models for different kinds of quantified sentences. Nevertheless, for my aim here, it is only necessary to mainly take the canonical models of the affirmative and negative universal sentences into account. Starting with the affirmative universal ones, that is, those that can be formally expressed as [A(sp)] (where, obviously, “A” indicates that the sentence is affirmative universal, “s” means “subject”, and “p” represents “predicate”), which claim

that “all of the *s* are *p*”, it can be said that Khemlani *et al.* (2015) give an example of canonical model similar to the following:

s & *p*
s & *p*
s & *p*

Each of these three possibilities describes an element. In this case, the three elements have the same characteristics (all of them have both the property “*s*” and the property “*p*”), which is consistent with what a formula such as [*A*(*sp*)] means. Of course, further effort can be made and people can come to other noncanonical models by noting, for example, that an element without the property “*s*” but with the property “*p*” ($\neg s \ \& \ p$, where “ \neg ” is negation) or an element without the property “*s*” and without the property “*p*” ($\neg s \ \& \ \neg p$) are also compatible with the asseveration that “all of the *s* are *p*”. Nonetheless, according to MMT, only a model such as that indicated, i.e., a model in which all the elements have both properties, can be quickly and easily detected given an affirmative universal sentence.

As said, the other type of quantified sentence that is interesting in this paper is the one of the negative universal sentences, that is, of those that can be formally expressed as [*E*(*sp*)] (where “*E*” means that the sentence is negative universal), which claim that “none of the *s* is *p*”. In this case, the Khemlani *et al.*’s example is akin to this one:

s & $\neg p$
s & $\neg p$
 $\neg s \ \& \ p$

Now, the first and the second elements have only the property “*s*”, and not the property “*p*”. On the other hand, the third element does not have the property “*s*”, but it does have the property “*p*”. It is obvious that cognitive effort can reveal other possibilities here as well. However, as mentioned, the canonical models are the most relevant for my goals. This is so because, as also indicated, what I wish to check is whether or not people can note without effort that certain inferences rejected by Aristotle, indeed, are not correct. In this way, if just the canonical models described are enough to show that such inferences are not valid, it will be possible to state that it is not difficult for people to be aware of this last fact, at least if MMT is assumed. Furthermore, as also pointed out, the models that Khemlani *et al.* (2015) assign in Table 1 to other types of quantified inferences are not important here either, since, as accounted for below, the four inferences that will be reviewed have the same two premises, one of them being

an affirmative universal sentence and the other one being a negative universal one.

3. Four inferences with conclusions that are not necessary

The four inferences are exactly in *Ἀναλυτικὰ Πρότερα* (*Analytica Priora*) 26a2-9, and have been studied in different works too (e.g., Johnson 2004; Lukasiewicz 1957). Following the Lukasiewicz's (1957: 21) idea that Aristotelian logic was 'implicational', Johnson (2004: 255) proposes this general structure for them:

$$A(mp) \rightarrow [E(sm) \rightarrow x]$$

Where, evidently, “ \rightarrow ” denotes implication and “m” refers to the middle term.

Thus, the difference between them is the content of [x], which can be one of these formulae:

I(sp) (where “I” means that the sentence is affirmative particular, i.e., that it expresses that “some of the s are p”)

O(sp) (where “O” indicates that the sentence is negative particular, i.e., that it expresses that “some of the s are not p”)

A(sp)

E(sp)

So, the four inferences are formally as follows:

$$A(mp) \rightarrow [E(sm) \rightarrow I(sp)]$$

$$A(mp) \rightarrow [E(sm) \rightarrow O(sp)]$$

$$A(mp) \rightarrow [E(sm) \rightarrow A(sp)]$$

$$A(mp) \rightarrow [E(sm) \rightarrow E(sp)]$$

As said, according to Aristotle, the conclusions, that is, the content of [x] or the consequents of the consequents in the previous formulae, do not necessarily follow from the premises (i.e., from [A(mp)] and [E(sm)]) in these four cases. As explained by Johnson (2004), they are also rejected in the system proposed by Lukasiewicz (1957). Nevertheless, what is interesting for this paper is

whether or not, following MMT, it is easy for human beings without logical background to see that. Because we know which the canonical models of sentences such as [A(sp)] and [E(sp)] are in that theory, it should not be difficult to clarify this point. I try to do it in the next section.

4. *Would it be hard for people to reject the inferences?*

As I will show now, from the MMT perspective, the answer to this question is clearly negative. And this is so because, to understand that the conclusions do not follow from the common premises, it is only necessary to take the canonical models of such premises into account. Given that the first one is [A(mp)], its canonical model can be as follows:

[I]: $m \ \& \ p$

[II]: $m \ \& \ p$

[III]: $m \ \& \ p$

Likewise, the one of the second premise can be this:

[IV]: $s \ \& \ \neg m$

[V]: $s \ \& \ \neg m$

[VI]: $\neg s \ \& \ m$

Hence we have six possibilities, [I], [II], [III], [IV], [V], [VI]. Accordingly, what must be done then is to check whether or not what is expressed by the four conclusions matches the description of the domain or universe given by such possibilities. In this way, it can be said that the first conclusion [I(sp)] would follow the premises if at least one element with both the property “s” and the property “p” were found. But we cannot do that because in [I], [II], and [III], while the elements have the property “p”, it is not known whether or not they have the property “s” as well. There is no information about that and, even though it is possible that all of them have the property “s”, it is also possible that none of them has it. Something similar can be said about [IV] and [V]. They represent elements with “s”, but we do not know whether or not they have “p” too. Furthermore, in [VI] the element does not have the property “s” and, therefore, it is not useful to confirm the conclusion. Thus, given the premises, [I(sp)] is not necessarily true.

The situation is even clearer in the case of the second conclusion. Indeed, [O(sp)] would be a valid conclusion if there were at least a case of “s” without “p”. Nevertheless, again, it is not possible to know for sure whether or not there is such a case. [IV] and [V] are the only elements that, undoubtedly, have the property “s”, but it is unknown whether or not they have the property “p”.

Maybe they do not have it but it is also possible that they do. On the other hand, [I], [II], and [III] have “p”, and [VI] does not have “s”.

As far as the third conclusion [A(sp)] is concerned, it is necessary that all the elements with “s” have “p” as well. In [I], [II], and [III], we do not know whether or not they have “s”. In [IV] and [V], what is unknown is whether or not they have “p”. And, in [VI], it is absolutely clear that “s” is there.

Finally, the case of the fourth conclusion is obvious too. We could accept [E(sp)] if all of the “s” were “¬p”. Nonetheless, [I], [II], and [III] cannot be considered because they have “p”. [VI] cannot be taken into account either because it does not have ‘s’. And, in [IV] and [V], we do not know what the value of “p” is. It can be false but it can also be true.

So, it is evident that it can be noted that the four conclusions do not follow the premises by only paying attention to the canonical models of the latter. This, from MMT point of view, means that checking the defect of the inferences should be a very easy, trivial, and simple task for most of the human beings. Accordingly, it appears that, as claimed, for example, by López-Astorga (2016a; 2016c) with regard to other aspects of Aristotelian logic, this last system can play any kind of role in current cognitive science.

5. Conclusions

True, as said in papers such as that of López-Astorga (2016a), if MMT is a correct framework, certain elements and aspects of Aristotle’s logic have the potential to predict the conclusions that individuals can come to given certain inferential structures. Based on this, it can be thought even that Aristotelian logic (if not in entirety, at least some of its parts) describes the real way people reason. And this is so because, if it is possible to notice just by resorting to canonical models whether or not the conclusion follows in the inferences of Aristotelian logic, it is evident that such inferences correspond to ways people usually reason in everyday life.

This is an idea that is also supported by papers such as those of López-Astorga. However, it is clear that, to be absolutely accepted, as also indicated in works such as the aforementioned ones, empirical evidence is needed. There is no doubt that many experimental results confirm the general assumptions of MMT. However, continuing with the basic approach of those papers, it can be said that empirical studies intended to check arguments such as those presented here could be useful in at least two ways. On the one hand, they would enable a better interpretation and understanding of Aristotelian logic, since they could reveal its actual potential and value. On the other hand, they would also allow checking whether or not certain predictions of MMT (for example, those related to the kind of inferences reviewed in this paper) are correct. Furthermore, the basic design that such studies could have is obvious: keeping the formal

structure of the inferences used by Aristotle, new created versions of them with thematic content would be presented to groups of participants, the task for them being just to indicate whether or not, given the premises, the conclusion must be admitted.

Accordingly, it seems that the line of research opened by the works using MMT to analyze particular aspects of Aristotelian logic deserves to be developed to a greater extent. If, for example, the moods of the four figures correspond to structures that allow deriving correct conclusions easily and the inferences analyzed here match structures with clear incorrect conclusions, it is evident that such structures can be used as research instruments in different contexts. Thus, as argued, the benefits to be found are real both in the study of the logic provided by Aristotle (it could be checked whether or not his schemata are consistent with the actual cognitive human behavior) and in the review of the validity of MMT (it could be checked whether or not people resolve reasoning tasks only requiring canonical models without difficulties). And, of course, all this independently of the purely philosophical, psychological or linguistic interest that studies of this kind can also have.

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