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Corrigendum to "Some remarks on substitution and composition operators"

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Abstract. *We correct an error in Part (c) of Proposition 3.3 in our paper "Some remarks on substitution and composition operators"* [Rend. Istit. Mat. Univ. Trieste **53** (2021), Art. No. 6, pp. 1–25].

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Part (c) of Proposition 3.3 in [1] is not correct as stated. The operator $S_\varphi : Lip \to Lip$ is certainly not an isometry in *Lip* for $\varphi(t) \equiv 1 - t$ which becomes immediate after computing S_φ at the identity function. Therefore, this option in the statement of Proposition 3.3 (c) must be erased. Instead, its correct formulation is as follows.

Proposition 3.3. *Let* $\varphi : [0,1] \to [0,1]$ *be Lipschitz continuous. With* S_φ *given by (1.1) the following is true.*

(a) The function $\varphi : [0,1] \to [0,1]$ *is injective if the operator* $S_\varphi : Lip \to Lip$ *is surjective.*

(b) The operator $S_\varphi : Lip \to Lip$ *is injective if and only if the corresponding function* $\varphi : [0,1] \to [0,1]$ *is surjective.*

(c) The operator $S_\varphi : Lip \to Lip$ *is an isometry if and only if* $\varphi(t) \equiv t$.

Proof. The proofs for (a) and (b) remain unchanged. For the proof of (c) first note that $\varphi(t) \equiv t$ generates the identity operator which clearly is an isometry in *Lip*.

To prove the "only if" part of (c), suppose that S_φ is an isometry, hence injective. From (b) it follows then that φ is surjective.

Now, S_φ being an isometry in *Lip* implies $\varphi(0) + lip(\varphi) = 1$ and hence

$$|\varphi(s) - \varphi(t)| \leq \big(1 - \varphi(0)\big)|s - t| \qquad (0 \leq s, t \leq 1). \tag{5}$$

This yields on the one hand that $\varphi(0) < 1$, because otherwise φ would be constant, contradicting its surjectivity. On the other hand, $\varphi(0) = 0$. Otherwise, we would find $s, t \in (0, 1]$ such that $\varphi(s) = 0$ and $\varphi(t) = 1$. But then $1 = |\varphi(s) - \varphi(t)| \leq \big(1 - \varphi(0)\big)|s - t| < 1$, a contradiction.

It now follows from (5) that $0 \leq \varphi(t) \leq t$ for $0 \leq t \leq 1$, and the surjectivity of φ implies that $\varphi(1) = 1$. Now, if $\varphi(\tau) < \tau$ for some $\tau \in (0,1)$, we would obtain from (5)

$$1 \geq \frac{\varphi(1) - \varphi(\tau)}{1 - \tau} > \frac{1 - \tau}{1 - \tau} = 1,$$

a contradiction. Consequently, $\varphi(t) = t$ for all $t \in [0,1]$. □

The error, however, only propagates to a remark following the above critical proposition and to Table 2 in which the false statement is cited. The correct equivalence in Table 2 should therefore be "S_φ isometry $\Leftrightarrow$ $\varphi(t) \equiv t$" in the space *Lip*. The error does not affect the rest of the paper.

References

[1] J. Appell, B. López, S. Reinwand, K. Schöller: *Some remarks on substitution and composition operators*, Rend. Istit. Mat. Univ. Trieste **53** (2021), Art. No. 6, pp. 1–25.

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