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Istituto Universitario di Architettura di Venezia

DOTTORATO DI RICERCA IN INGEGNERIA DEI TRASPORTI
XIII CICLO

**MODELLI E METODI DI
PROGRAMMAZIONE LINEARE BILIVELLO
PER IL TRASPORTO MERCI**

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XIII CICLO

**BILEVEL LINEAR PROGRAMMING
MODELS AND ALGORITHMS
FOR FREIGHT TRANSPORTATION**

Dottorando:

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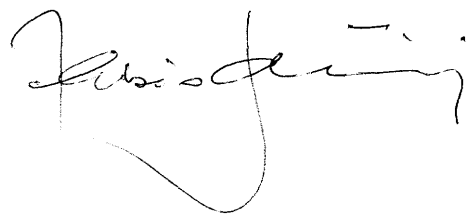
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*A Claudia e
alla mia famiglia*

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Summary

Freight transportation is generally a very complex domain where several players, each with its own set of objectives, act and operate at various decisional levels. There are different players in the field. The *shippers* who decide how much of each commodity to move from every origin to every destination and the means by which the goods will be moved. The *carriers* who respond to this transportation demands and route freight over the actual transportation network under their control. Finally, the *government* defined as the set of international, national and local authorities involved in any way with freight transportation via regulation and the provision of transportation infrastructure. In this work, we consider the case where only one shipper determines the demand for transportation over a network. However, he cannot decide flow levels on arcs in a fully independent way due to the presence of a second agent controlling some links of the network and optimizing her own objective function. This situation is modelled as a game between two players P and Q acting on the same network G . Player P fixes the flows on the arcs of G in such a way that their divergence at some given nodes (sources and sinks) is equal to prescribed values. Such divergences may represent demand and availability levels for some commodity. On the other hand, player Q decides the values of the maximum capacities of some arcs of the network. Both players are interested in the fact that the connectivity between the sources and the sinks in the network is respected, i.e., they both want that the goods can reach their destination. However, they have different objectives. Player P aims at minimizing the transportation costs, whereas player Q aims at maximizing her profit (or, in general, her utility) that is proportional to the flow

passing through the arcs under her control. Note that, in general, the profit of player Q is not assumed to be equal to the cost of player P for the same arc. Such game between players P and Q is modelled as a minimum cost flow problem for player P , where the arc costs are given and the player Q decides the arc capacities.

The modelling of the games under investigation are mainly based upon three different research lines. First, the players understand the freight transportation system as a system where the actors involved do not act simultaneously and they explicitly take into account the sequential nature of the interactions among them. Second, they play a (hierarchical) game over a flow network which causes severe limitations and constraints to their action sets. Finally, the games exhibit linear characteristics and can be solved using bilevel linear programming. All these issues have already been discussed in the scientific literature, even though in different separate contexts. The merging of three mentioned approaches in only one single framework is a major contribution of our modelling perspective. Furthermore, bilevel programming is rich of theoretical results and numerical algorithms, but is scarce in actual applications. From this point of view, the present work might be considered as an interesting addition to the field.

Bilevel noncooperative games in which one player (called the *leader*) declares his strategy first and enforces it on the other players (called the *followers*) who react (rationally) to the leader's decision are referred to as *Stackelberg games*. Since the payoff functions and all the constraints in our Stackelberg games may be expressed in a linear form, these games will be formalized as bilevel linear programming problems (BLPPs). In general, bilevel programming problems are difficult to solve because of their inherent non-convexity and non-differentiability. To face their NP-hard nature, we identify some properties of the game solutions which allow us to define a heuristic algorithm restricting its (local) search on the set of the Nash equilibrium points. The optimal solution of any BLPP lies on a vertex of the leader's inducible region. Relying on this result, we develop an algorithm which allows to move from a starting point of the shipper's inducible region to another point in the shipper's

inducible region always providing a better solution for him. When no further better points may be attained, the algorithm stops. Unfortunately, only a local optimum is identified. The rationale behind the algorithm stems from the consideration that the optimal solution for our BLPP is also a Nash equilibrium point. In particular, the algorithm moves from a Nash equilibrium point to another better Nash equilibrium point of the BLPP under study.

This framework may describe, as an example, the situation where restrictions are imposed by some alpine country on the number of trucks allowed to cross it by road each year. A different context involving the presence of a second agent on the shipper's network occurred when the International Transporters' Association (UND) of Turkey had to face when the war in the Balkans started. This situation motivates our investigation on hierarchical noncooperative network games. The road freight traffic from Turkey to Central and Western Europe and viceversa suffered major disruptions because of the war in Balkans during the nineties. UND is the shipper controlling the quasi-totality of this traffic thus assuming the role of player P . He had to cope with an "adverse entity" able to modify the available capacity on some specific links his vehicles had to pass through. The region involved in the conflict may be represented as a connected subnetwork disconnecting the origin and the destinations of the road transportation network since alternative road routes are not easily affordable. Other possibilities, like the seaborne links now operating, did not exist at that time. Hence the whole freight traffic was performed using a single mode of transport. The models developed in this work allow the shipper to perform a worst-case analysis at the strategic level for this situation assuming that player Q wishes to maximize the costs he has to afford when going through the region under her control. In fact, it is meaningless to talk about the utility or the profit the war may seek to maximize. However, it becomes a sensible modelling when the utility of player Q is strictly related to the costs afforded by player P on this portion of the network. If player Q is maximizing her utility which corresponds to player P 's costs, automatically she plays to maximize player P 's costs. Hence the model represents

a worst-case analysis for player P .

A simple graph composed of 99 nodes and 181 arcs is presented. Player P controls a subnetwork composed of 100 arcs. The others 81 links representing the connections within the Balkans and Eastern Europe form a connected subnetwork. Only the main road links have been considered (motorways or highways). The capacities are calculated taking into account the total number of transit permits available for each country. This figure is annually fixed in bilateral Joint Committee Meetings. Player P 's costs are the average generalized costs derived as a function of lengths and transfer times in the physical links. Player Q does not have profits or losses for the flows passing through the P zone and it is also assumed that the profits she earns for each unit of flow going through the arcs under her control are equal to the costs afforded by the shipper when traversing these arcs. All the relevant data required to calculate these figures are collected in the UND Annual Sector Report 1997-98 (1999). The heuristic algorithm has been tested on this network and its results have been compared with the outcome obtained by using an exact enumeration procedure. Since it turns out that the percentage error of the heuristic algorithm is equal to 0,3%, we may claim that its performances are certainly highly satisfactory, at least in this specific example.

Different extensions of the models and the algorithm developed may be easily envisaged both from the theoretical and the application side. These advances would provide either faster local or global search algorithms either more complete models representing in deeper detail the actual system and the interactions among the actors involved. Hence a decision support system for the shipper's decision making process at the strategic level can be built and effectively used by freight transportation practitioners.

Esposizione riassuntiva

Ogni sistema di trasporto delle merci si presenta generalmente molto articolato e complesso: in particolare l'esistenza di numerosi soggetti che, a diverso livello e con diversi obiettivi, sono tenuti ad operare decisioni rappresenta un elemento che influisce in maniera spesso importante sull'assetto del sistema stesso. Il lavoro prende in esame un sistema di trasporto merci con due attori, denominati P e Q, che attraverso le rispettive decisioni determinano l'assetto dei flussi sulla rete. Il soggetto P, in particolare, è incaricato di soddisfare una data domanda di trasporto (ad esempio espressa mediante una matrice O/D data) e può decidere come ripartire i flussi su una rete multimodale della quale percepisce i tratti fondamentali. Al momento della sua decisione, P conosce il costo generalizzato degli archi della rete e cerca di minimizzare il costo totale del trasporto. Inoltre il giocatore P deve rispettare le decisioni del giocatore Q. Il giocatore Q, che controlla una porzione della rete che connette le origini alle destinazioni di P, invece conosce il profitto unitario che deriva dal transito veicolare sui suoi archi e cerca di massimizzare il proprio profitto complessivo. Nel far questo può modificare la capacità degli archi della sua sottorete, ma anch'egli deve comunque soddisfare la condizione di bilanciamento ai nodi e deve rispettare le decisioni di P.

Quale primo elemento di originalità del presente lavoro può essere considerato il tentativo di condensare in un unico approccio alcuni elementi presenti singolarmente in filoni diversi. Infatti, tra i modelli della letteratura che intendono rappresentare esplicitamente le dinamiche decisionali interattoriali si ricordano i modelli multiattoriali sequenziali, i giochi su rete e la programmazione lineare bilivello i quali

formano il quadro di riferimento in cui la presente lavoro si inserisce.

Il quadro attoriale appena delineato offre l'opportunità di affrontare una serie di problemi diversi, nel campo dell'affidabilità della rete, a seconda dell'ordine con il quale i due giocatori decidono. Infatti il caso in cui la decisione di P preceda quella di Q può essere significativo, per P, al fine di valutare la peggiore situazione che potrebbe presentarsi per effetto di Q una volta stabilito l'assetto dei flussi sulla propria rete. È questo un tipico esempio della cosiddetta "worst case analysis". Viceversa, se gioca prima Q, P riesce a determinare il migliore assetto dei propri flussi nel rispetto di vincoli imposti da Q su una parte della rete interposta tra la sua origine e la destinazione. Si pensi ad esempio alla problematica dell'attraversamento di Paesi, quali Austria e Svizzera, che impongono severe limitazioni per i veicoli pesanti.

Il problema descritto viene formulato come un gioco su rete nel quale i due giocatori, P e Q, non cooperano tra loro. Si ottiene così una formulazione di programmazione lineare bilivello (BLP) dove il giocatore che gioca per primo è il *leader*, mentre l'altro assume il ruolo di *follower*. Ricordando che i problemi BLP sono NP-hard, è stato sviluppato ed implementato un algoritmo euristico di ricerca della soluzione ottima. Sfruttando però l'osservazione che, nel particolare caso in questione, la soluzione ottima del problema BLP è anche un punto di equilibrio di Nash, l'algoritmo restringe la sua ricerca nell'insieme dei punti di equilibrio di Nash. Da un punto di equilibrio di Nash si passa ad un altro corrispondente ad una soluzione "migliore" per il leader fino a quando l'algoritmo non si ferma. Purtroppo però non si è sempre in grado di determinare un ottimo globale, ma solamente un ottimo locale individuando così, nel caso sia P a giocare per primo, un limite superiore alla soluzione ottima.

Lo studio di tale modello è stato motivato dalla volontà di rappresentare, con riferimento al sistema del trasporto merci su gomma tra la Turchia e l'Europa Occidentale, la situazione che si è venuta a creare nella regione dei Balcani a causa dei recenti eventi bellici. Tra le due regioni, annualmente, si registra un traffico

dell'ordine delle centinaia di migliaia di veicoli commerciali. Per ragioni di semplicità, si è fatto riferimento alla sola componente verso l'Europa, fermo restando che la direzione opposta potrebbe essere analizzata in maniera del tutto analoga. Nell'esempio affrontato, la domanda di trasporto delle merci, che viene misurata in numero di veicoli all'anno, e che si sposta con origini diverse nel Sud-Est asiatico e destinazioni pure diverse nell'Europa Occidentale, è stata concentrata in due sole polarità (1 origine e 1 destinazione). Nel sistema appena descritto l'Associazione Industriali della nazione di origine (UND) svolge il ruolo di decisore centrale ed è stata assimilata al giocatore P di cui sopra. In breve, nota la domanda da trasportare, l'UND decide la distribuzione delle merci tra vari percorsi sulla rete che collega l'origine (Turchia) alla destinazione (Europa occidentale). Conosce pure il costo generalizzato degli archi di tale rete e opera le proprie decisioni con l'obiettivo di rendere minimo il costo del trasporto. L'evento bellico ha causato, come riflesso su detto sistema, una decisa modifica alla capacità degli archi di una porzione della rete stradale iniziale, che garantiva la connessione tra origine e destinazione. Alcuni archi sono stati eliminati (la rispettiva capacità posta pari a zero), altri hanno subito una netta riduzione della capacità, o un significativo aumento del costo generalizzato. La guerra quindi ha assunto un comportamento analogo a quello del giocatore Q. In questo caso però, non ha significato parlare di un'utilità che la guerra cerca di massimizzare secondo quanto esposto in precedenza, a meno che non si proceda ad assimilare l'utilità del giocatore Q con i costi di P: se Q gioca per massimizzare la propria utilità e quest'ultima corrisponde ai costi di P, automaticamente Q gioca per massimizzare i costi di P e il modello acquista proprio il significato di una analisi del caso peggiore per P.

La rete considerata è stata semplificata in accordo con il livello di dettaglio delle informazioni di cui dispone P ed è formata da 99 nodi e 181 archi, di cui solamente 100 sotto il controllo di P. Gli altri 81 archi, concentrati nella regione dei Balcani, sono sotto il controllo di Q e costituiscono una sottorete connessa, che disconnette l'origine dalla destinazione. La capacità degli archi è stata determinata

in accordo con il numero dei permessi di transito annui che ogni Stato concede ai veicoli turchi. Tale numero viene annualmente definito, mediante contrattazione tra le parti, in accordi bilaterali. In questa fase non si è tenuto conto delle differenti tipologie di permessi. In accordo con alcune necessarie ipotesi semplificative, la rete stessa è aciclica. I valori del costo per veicolo percepito da parte di P per transitare sugli archi della sottorete propria od altrui rispettivamente, sono stati determinati come funzione del costo monetario, della lunghezza fisica dell'arco e del tempo di percorrenza, tenendo in considerazione le varie voci che concorrono alla formazione del costo unitario (per veicolo-chilometro) di produzione di un servizio di trasporto sull'arco preso in esame. Per quanto riguarda i termini che compaiono nella funzione obiettivo di Q, si suppone che la guerra non tragga beneficio alcuno dal transito dei flussi veicolari sulla rete di P, mentre il profitto di Q è stato posto pari al costo sostenuto da P cambiato di segno come descritto in precedenza. Ai fini di valutare le prestazioni dell'algoritmo, il medesimo problema, viste le sue contenute dimensioni, è stato risolto anche con un algoritmo esatto, cioè in grado di determinare l'ottimo globale. Il risultato dell'algoritmo proposto si discosta di solo lo 0,3% dal risultato ottenuto con una procedura di branch and bound. L'esempio applicativo ha consentito di comprendere le potenzialità dell'approccio proposto e nell'ottica di un suo utilizzo concreto ha fornito delle utili indicazioni su possibili sviluppi da intraprendere legati sia all'algoritmo, sia al modello sia al caso di studio.

In conclusione, il lavoro presenta un modello per la definizione dell'assetto del sistema di trasporto delle merci, con la trattazione esplicita delle dinamiche decisionali interattoriali. In particolare si prendono in considerazione due soggetti, che operano scelte in sequenza gerarchica, uno dei quali agisce per minimizzare i costi totali del trasporto e l'altro cerca invece di massimizzare il proprio profitto che dipende dal volume di traffico lungo gli archi sotto il suo controllo. Si propone una formulazione di programmazione lineare bilivello, per risolvere un gioco infinito statico non cooperativo con insiemi di vincoli accoppiati. Sono descritte le condizioni di esistenza e alcune proprietà dei punti di equilibrio di Nash, dalle quali discende un

algoritmo di ricerca di un ottimo locale. Viene infine discussa un'applicazione del modello al caso del trasporto merci dalla Turchia all'Europa. Alcuni futuri sviluppi sono possibili. Essi portano alla progressiva eliminazione delle assunzioni semplificative che sono state adottate allo stato attuale nella formulazione del modello. In particolare si tratta della configurazione della rete, della struttura e delle proprietà dell'algoritmo (oggi trova solamente un ottimo locale). Inoltre si intende procedere con il perfezionamento del caso di studio.

Chapter 1

The transport system

Freight transportation is generally a very complex domain where several players, each with its own set of objectives, act and operate at various decisional levels. Since it has to adapt to rapidly changing political, social and economic conditions and trends, accurate and efficient methods and tools are required to assist and enhance the analysis of planning and decision-making process (Crainic and Laporte, 1997). There are different players in the field. The *shippers* who decide how much of each commodity to move from every origin to every destination and the means by which the goods will be moved. The *carriers* who respond to this transportation demands and route freight over the actual transportation network under their control. Finally, the *government* defined as the set of international, national and local authorities involved in any way with freight transportation via regulation and the provision of transportation infrastructure (Harker and Friesz, 1986). Because of the complexity of transportation systems, the different decisions and management policies affecting their components are generally classified in three main planning levels. First, strategic planning determines general development policies and shape the operating strategies of the system. International shippers, consulting firms, transportation authorities are committed in this type of activity. Second, tactical planning aims to ensure, over a medium term horizon, an efficient and rational allocation of existing resources in order to improve the performance of the whole system.

Finally, operational planning is performed by local management in a highly dynamic environment where the time factor plays an important role and detailed representations of vehicles, facilities and activities are essential (Crainic and Laporte, 1997).

This general framework holds at each planning level but has to be appropriately adapted to the transportation modality the players are considering for transporting goods. In particular, when focusing the attention on a road freight transportation system, the decisions of any player heavily rely on the status of the transport network he normally operates on. Unforeseen events may suddenly occur, producing relevant deviations of the actual behavior of the vehicle fleet from the expected one. Unexpected situations encompass either abnormal (or exceptional) events such as natural or man-made disasters, huge accidents, large-scale maintenance works, or normal (or regular) events such as usual variations in the traffic demand and road capacity like congestion during peak period (see, e.g., Bell, 1999, and Iida, 1999). This is why an analysis of the network reliability should support the firm decision maker not only at the operational level, but at the tactical and strategic levels, as well. Connectivity reliability, i.e., the probability that there exists at least one path without disruption or unacceptable delay to a given destination, and travel time (or performance) reliability, i.e., the probability that traffic can reach a given destination within a stated time, are the principal issues in network reliability (again, see Bell, 1999, and Iida, 1999).

In this work, connectivity is granted, i.e., it is supposed that it is always possible to reach the desired destinations. Instead, attention is focused on performance reliability which is expressed in terms of costs to be afforded by the shipper (needless to say, such costs can also model delays). We consider the case that the demand for transportation over the network is determined by only one shipper. However, he cannot decide flow levels on arcs in a fully independent way due to the presence of a second agent controlling some links of the network and optimizing her own objective function. This framework may describe, as an example, the situation where restrictions are imposed by some alpine country on the number of trucks allowed to cross it

by road each year. On one side, the aim is to reduce air pollution. On the other side, the country makes a profit from each vehicle traversing it due to e.g., toll collection or use of restaurants, hotels and petrol stations. A different application involving the presence of a second agent on the shipper's network is described in deeper detail in Chapter 6. In that case, the shipper is the International Transporters' Association (UND) of Turkey requiring to manage approximately 100.000 freight trucks per year in the European road network. Due to the Balkan war in the nineties, we consider the other player as an "adverse entity" (i.e., the war) introducing major limitations on the link capacities.

1.1 The two players

We model this kind of situations as a game between two players P and Q acting on the same network G . Player P fixes the flows on the arcs of G in such a way that their divergence at some given nodes (sources and sinks) is equal to prescribed values. Such divergences may represent demand and availability levels for some commodity. On the other hand, player Q decides the values of the maximum capacities of some arcs of the network. As previously stated, both players are interested in the fact that the connectivity between the sources and the sinks in the network is respected, i.e., they both want that the goods can reach their destination. However, they have different objectives. Player P aims at minimizing the transportation costs, whereas player Q aims at maximizing her profit (or, in general, her utility) that is proportional to the flow passing through the arcs under her control. Note that, in general, the profit of player Q is not assumed to be equal to the cost of player P for the same arc. Such game between players P and Q is modelled as a minimum cost flow problem for player P , where the arc costs are given and the player Q decides the arc capacities. The game may be generalized assuming that player Q controls both arc costs and capacities. However, such a possibility is out of the scope of this work.

1.2 Main assumptions

The problem we face is the assignment of a known and fixed demand of freight on a specific network aiming at minimizing the total transport cost on this network. Freight flows on each arc may be measured in terms of number of vehicles or amount of tons passing through it in a given period of time. We operate at the strategic level where the time horizon under consideration is not restricted to a period comparable to the trip time. Hence we may neglect the individual behavior of the carriers. In addition we assume, as usual in freight transportation, that the units of flow do not have any autonomous decisional capability. As a consequence, the models we develop cannot be considered User Equilibrium or Network Loading Assignment Models fulfilling the condition expressed by the First Wardrop's Principle. Instead, we deal with and extend System Optimum Assignment Models leading to a solution which complies to the Second Wardrop's Principle (see, e.g., Cascetta, 1998). Further assumptions are also taken into account thus strictly defining the applicability environment of the present work. In the following subsections we introduce and justify them.

1.2.1 Costs

At the strategic level any shipper is generally not interested in the precise identification of the operational details each carrier or each vehicle has to examine when performing its journey. In fact, he considers only an aggregated representation of the network goods have to pass through and makes the assignment of the total freight flow to each arc of the entire network just once each large period of time (e.g., month or year). Hence some phenomena, like for instance congestion effects, which might have short-time influence on road freight flow can be neglected, especially when observing shipments non restricted to an urban environment. In this context, the generalized costs associated to each arc may be correctly represented only considering their average values. Of course, if deviations from these mean val-

ues are relevant also at the strategic level, a stochastic model has to be introduced. In this work we assume that the shipper's view fulfills the conditions allowing to use a deterministic model where the (average) generalized costs do not depend on the flow.

1.2.2 Q disconnects sources and sinks

In a real transportation system it is highly unlikely that the shipper does not have any possibility but passing through the zone controlled by the adverse player Q . However, some exceptions may exist. As an example, consider the case of Finland whose two-thirds of overseas freight traffic goes by ship (source: The Ministry of Transport and Communications of Finland). All its vehicles transporting freight to/from Europe heavily relies on sea transportation because of the high costs in terms of time and network unreliability that the alternative road or railways travels would require. A similar example involving a large amount of freight transport using links which are not fully controlled by the shipper occurs when large islands far apart from the continent, e.g., Sardinia, are considered. In both cases, sea and weather conditions may affect the traffic. Hence under specific circumstances, the sea is the adverse entity a considerable amount of traffic must go through and whose arc capacity may significantly vary, at least when observing short periods of time.

The road freight traffic from/to Turkey to/from Western Europe faced to some extent similar problems when the war in the Balkans unexpectedly erupted in the early nineties. In fact, the capacity of some of the road links was going to be modified for a substantial long period of time and no viable road or railways alternatives existed.

Similar issues also arise when the shipper acts on a network where the capacity of (part of) its links is fixed by administrative or environmental regulations like, e.g., the number of road permits for heavy goods vehicle traffic in different European countries. This is the perspective any international shipper operating across the

Alps has to face. In fact, he does not have any choice, but traversing countries where the maximum number of trips per year is imposed, either by severe restrictions (road permits) either by the capacity of alternative train links. As in the previous case, also in this context any variation of the road arc capacities holds for a significant period of time since they comply either to national and international legislations either to bilateral or multilateral agreements.

This work explicitly addresses the situation where all the flow must go through the region controlled by the adverse entity. In practice, player Q arcs form a sub-network disconnecting shipper's sources and sinks. The whole shipper's network will be referred to as a *disconnected* network. Of course, when the adverse player too heavily affects shipper's operations, he may decide, if possible, to modify the network topology, e.g., by adding new links, or to divert part of his freight traffic to other transport modes. When this decision can be taken, the shipper has the possibility to choose among two or more parallel and alternatives different modes of transport connecting his origins with its destinations. As an example, consider the case in which the freight traffic between the same O/D pair can be made either by road or by railways or, as in the freight traffic case study which motivated this work (see Chapter 6), by sea. When the two modes are independent in the sense that freight leaves origin and arrives at the destination on the same transport mode and without any intermodal operation, the whole shipper's transport system may be split and each mode may be considered separately. In this context, our work focuses its attention on a single mode, namely on the relationships between the shipper and an adverse player acting on that specific shipper's monomodal network. A study of the conditions requiring these actions and an investigation of their consequences on the shipper's transport system at the strategic, tactical and operational levels are certainly very interesting and actual issues. However, they go beyond the scope of this work and may be considered as further extensions of it.

1.2.3 Acyclic network

The units of flow are not allowed to cycle, i.e., from any given node they cannot pass again through the same node. This assumption eventually rules out (see, e.g., Ahuja, Magnati and Orlin, 1993) the possibility that the units might go in both directions between any pair of consecutive nodes: only one direction among them is allowed. In a transportation context, this hypothesis may be considered unusual and unnatural because of the possibility to reach a given destination using different paths which might cross some specific links in opposite directions. However, this assumption is made for computational purposes, only. As we shall see in Chapter 5, the property identifying an equilibrium point for the games under study relies on the solution of a Minimum and Maximum Cost Flow Problem for player P and Q , respectively. It is well known (see, e.g., Ahuja, Magnati and Orlin, 1993) that solving a maximum cost flow problem with positive arc costs is a NP-Hard problem if the graph is not acyclic. Thus by dropping this assumption computational difficulties would arise without adding any value or further information to the theoretical results we obtained. Of course, if the model of the transportation system under consideration necessarily requires the presence of directed cycles, the appropriate algorithms should be used and the theorems and properties in Chapter 5 accordingly modified.

1.2.4 Sequential decisions

A further important issue in this work concerns the instants of time the two players take their decisions. When assigning the flow on his network, the shipper either has to react to a previous move of the adverse player either he knows how she would react to his move and he tries to prevent her behavior aiming at minimizing the inconveniences he may suffer. In this framework, simultaneous actions are not considered because in our transportation environment they are meaningless. In fact, at the strategic level the shipper either prevents or reacts to a steady situation. The usual decision making process may be depicted as follows: first, the shipper tries to

prevent the hostile attitude of his adverse player making a flow assignment on the network under concern. He assumes to be perfectly informed about her behavior. If, for any reason, the evil entity changes her mind thus affecting the actual assignment, the shipper tries to do his best to react to this new situation. In practice, we assume that the system is in a steady state which lasts significantly more than the decision making process of both players generating it.

1.3 Case study

The models developed in this work partially motivate considering some specific aspect of road freight traffic among Turkey and Western Europe. We briefly sketch it to further clarify the context we are dealing with. In Chapter 6, some further details on this system are introduced.

Every year approximately 100.000 trucks belonging to the International Transporters' Association (UND) of Turkey leave the country and spread through the whole Europe. They reach their destinations by road through the Balkans, Central and Western or Northern Europe. In this context, UND may be considered as player P . It is on his concern to determine the traffic flow to be assigned to each leg of the network connecting the origin to the destinations. The minimization of the overall costs to be afforded is his main goal. In the early nineties, the sudden war in the former Yugoslavia caused major disruptions in the service. Then, the shipper had to cope with an "adverse entity" able to modify the available capacity on some specific links. The region involved in the conflict may be represented as a connected subnetwork disconnecting the origin and the destinations of the road transportation network since alternative road routes are not affordable. In our example, we assume that the shipper performs a worst-case analysis at the strategic level assuming that player Q wishes to maximize the costs he has to afford when going through the region under her control.

1.4 Structure of the work

This work is organized as follows. In the next Chapter 2, to better understand and properly define the models later developed the necessary background on Game Theory is introduced in Section 2.1 and the characterization of the games we face is precisely stated in Section 2.2. Chapter 3 presents a literature review on the three relevant approaches the model developed in this work is based upon. After a brief overview on the main freight transportation models in Section 3.1, models considering some particular games over a network are presented in Section 3.2, and finally in Section 3.3 a few papers dealing with bilevel linear programming applications on (freight) transportation are mentioned. The games under study are mathematically formalized in the first section of Chapter 4. In addition, a simple explanatory network is also introduced as an example to help the reader in understanding the main concepts and definitions. In Section 4.2, the representation of both games as bilevel linear programming problems (BLPPs) is presented together with some undesired situations to be avoided in Section 4.3. Hence in Section 4.4 we provide the basic definitions and properties for a BLPP in the general case and in Section 4.5 we extend these concepts to the games under consideration. In the next Chapter, Section 5.1 presents the definition of Nash equilibrium for such games and proves that the optimal solutions for a BLPP are also Nash equilibria. Following some theoretical background in Section 5.2, in Section 5.3 relationships of a Nash equilibrium point with the Minimum Cost Flow Problem are investigated. Relying on these results, a heuristic algorithm identifying an upper bound for the optimal solution of the BLPP is presented in Section 5.4. In Chapter 6, the freight traffic system motivating this work is presented in more detail. Relying on some of its features, a simple network is used as a test for the heuristic algorithm previously developed and comparisons with the results obtained from an exact procedure are performed. In Chapter 7 further possible research issues are presented and in the last Chapter some conclusions are eventually drawn.

Chapter 2

Network game

As described in the previous Section 1.1, the two entities acting on the same network mutually influence each other, or, according to some widespread terminology, *play a game*. To better understand and properly define the models developed in the following sections and chapters, in Section 2.1 we introduce the necessary background on Game Theory relying on the two well-known reference books (Osborne and Rubinstein, 1997) and (Başar and Olsder, 1999). In the following Section 2.2, the characterization of the games we face is precisely stated.

2.1 Game theory

Definition 1 *A game may be defined as a description of strategic interaction that include the constraints on the actions that the players can take. The individual making a decision may also be referred to as a player or a person of the game.*

Games are distinguished in different ways:

- **Noncooperative and Cooperative Games:** A game is called Noncooperative if each person involved pursues his own interests which are partly conflicting with others'. When the players share common interests, they cooperate in identifying a possible solution.

- **Strategic (or normal) and Extensive Games:** A strategic game is a model of a situation in which each player chooses his plan of action once and for all, and all players' decisions are made simultaneously (that is, when choosing a plan of action each player is not informed of the plan of action chosen by any other player). By contrast, the model of a extensive game specifies the possible orders of events; each player can consider his plan of action not only at the beginning of the game but also whenever he has to make a decision.
- **Games with Perfect and Imperfect Information:** In the former case, the participants are fully informed about each others' moves, while in the latter case the may be imperfectly informed.
- **Finite and Infinite Games:** In the former case, all the players have at their disposal only a finite number of alternative to choose from, while in the latter case at least one player has infinitely many moves.
- **Symmetrical and Hierarchical Games:** In the former case, no single player dominates the decision process, while in the latter case one of the players has the ability to enforce his strategy on the other player(s). For such decision problems, a hierarchical equilibrium solution concept is introduced.

In symmetrical games, an equilibrium solution of a game is reached when no one of the players can improve his outcome without degrading the performance of the other players. In particular, when the solution of a noncooperative game is such that one player cannot improve his outcome by altering his decision unilaterally, this is a Nash equilibrium solution. However, this solution in general is not Pareto-optimal. In fact, if the players cooperate they could mutually benefit of a better equilibrium solution.

An interesting relationship between the Nash equilibrium concept and the transportation field is given by the property stating that the Nash equilibrium converges to the Wardrop equilibrium when the number of users becomes large. In particular,

Nash equilibrium is strictly related with User Equilibrium expressed by the First Wardrop's Principle which is often used in the context of road traffic. It has been shown in (Haurie and Marcotte, 1985) that the Wardrop Equilibrium is the unique limit of any sequence of Nash equilibria obtained for a sequence of games in which the number of users is finite and tends to infinity, even in those games where the Nash equilibrium is not unique (Altman, Başar and Srikant, 1999).

2.2 Characteristics of the game

In this section we characterized the game we deal with. We first show that the presence of flow balancing constraints limits the action space of both players. In the following we precisely define the type of game according to the classification presented in Section 2.1. Finally, we introduce two different possible Stackelberg games the two actors can play in the framework we established.

2.2.1 A two-player constrained game

In this work we face a particular game because the two players are not allowed to act independently one to other, i.e., the constraints of each player may depend on the strategy of the other player (Rosen, 1965). In fact, both players have also to take into account the topology and the characteristics of the network they act on. Figure 2.1 may help to understand the context. The shipper (player P) has to ship a given amount of freight from node A to nodes H and I aiming at minimizing the cost of the transport. Unfortunately, he has to cope with an adverse entity (player Q) who is entitled to fix the capacity of the dashed line arcs aiming at maximizing her utility due to the passage of the units of flow through the region she controls. As it is explained in Section 1.2.2, we assume that all flow from the origins to the destinations must go through the Q region. In this case, it is easy to prove that player Q action is no longer a decision on arc capacities but reduces to a flow assignment on her region. Since connectivity is granted, the same amount of flow

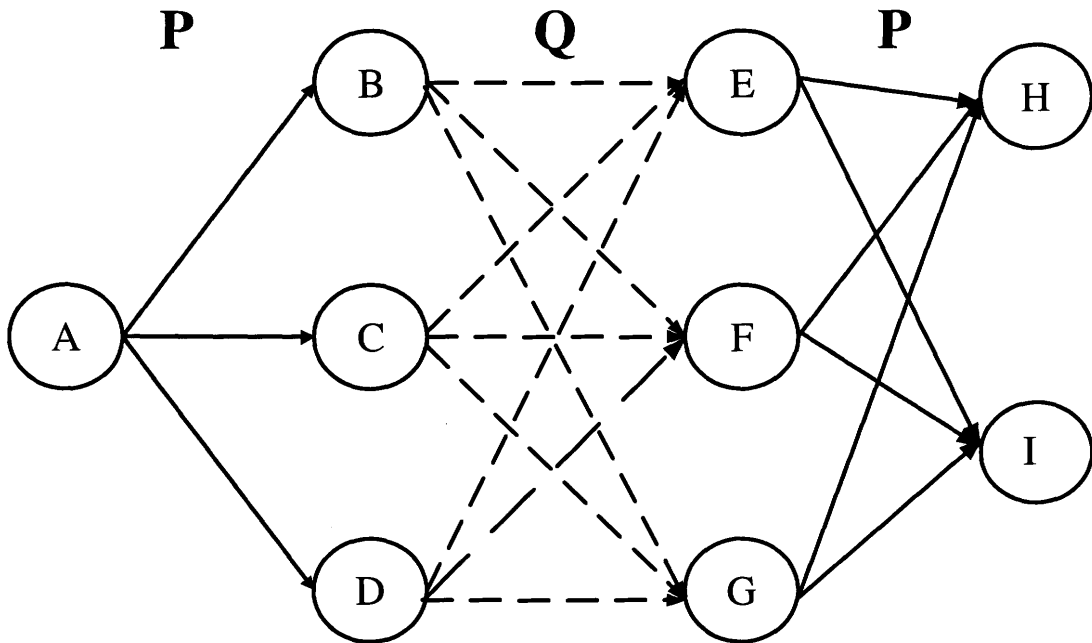


Figure 2.1: Players P and Q acting on the same network

entering each intermediate node (i.e., B , C , D , E , F and G) also leaves it, i.e., flow balancing constraints hold. For the arcs entering and leaving each of these nodes do not belong to the same actor (we will call them *frontier node*, see Section 4.2), the two players cannot freely assign the flows on the arcs they control: only assignments fulfilling flow balancing constraints on each intermediate (and thus, frontier) node are allowed.

By exploiting this characteristic, we identify some properties of the game solutions which allow us to define a heuristic algorithm restricting its (local) search on the set of the Nash equilibrium points, as it is explained in Section 4.5 and in the following Chapter 5.

2.2.2 Game specification

Since players P and Q have conflicting objectives, in this work our attention is only focused on noncooperative games. In addition, we also assume that each player

has perfect information about the other player behavior. Furthermore, both players have to choose a feasible flow in the network under their own control satisfying flow balancing and capacity constraints, then they have infinitely many alternatives to consider. Due to the presence of flow balancing constraints, the game under study may be referred to as a *static noncooperative infinite game with coupled constraint sets* (Başar and Olsder, 1999).

2.2.3 Stackelberg games

Unfortunately, no clear interpretations of a Nash or a saddle-point equilibrium of the strategic form of these games still exist (Başar and Olsder, 1999). For this and other (see Section 1.2.4) reasons, the simultaneous game is explicitly not addressed in this paper. Conversely, we face the situation wherein one of the players has the ability to enforce his/her decision on the other player which reacts independently and rationally (Osborne and Rubinstein, 1997). Hence, only the hierarchical frameworks (i.e., player P plays first or player Q plays first) are considered. As a consequence, we deal with games in their extensive form. In particular, games in which at least one player is allowed to act more than once and with possibly different information sets at each level of play, are known as *multi-act games*. In fact, since the shipper takes his strategic decisions aiming to prevent or to react to the other actor behavior, two different two-player infinite multi-act games can be considered:

Game 1 *Player P plays first. If P fixes the flows all over the network, then no decision is left to player Q . In fact, Q may only adjust the arc capacities according to the flows imposed by P . Such a trivial possibility is not considered any more in the rest of the paper. A more realistic situation is that P plays first, initially deciding only on the flows over the arcs not controlled by Q and possibly leaving unbalanced divergences in some nodes. Then, Q fixes the capacities of the arcs under her control with the only constraints that it must be possible to balance the flows left unbalanced. Finally, P plays again deciding on the flows in the arcs controlled by Q .*

Game 2 *Player Q plays first. This situation occurs, as an example, when a local traffic authority imposes the maximum flow of trucks that the roads under her control may bear. Given the capacities fixed by Q, then player P may decide his flows.*

Game 1 is a three-stage decision process. However, in Section 4.2 we prove that the hypothesis about connectivity allows to remove the third level and leads to a two-level program. Differently, Game 2 is clearly already a two-stage process. Hence in this work we focus the attention on bilevel games only. In particular, we deal with bilevel noncooperative games in which one player (called the *leader*) declares his strategy first and enforces it on the other players (called the *followers*) who react (rationally) to the leader's decision. Such games are referred to as *Stackelberg games*.

In Section 4.2, we show that the payoff functions and all the constraints in both Stackelberg games may be expressed in a linear form. Hence these games will be formalized as bilevel linear programming problems (see, e.g, Bard, 1998).

Chapter 3

Literature review

The modelling of the two games under investigation and their characterization as a bilevel linear programming problems are mainly based upon three different research lines. First, the players understand the freight transportation system as a system where the actors involved do not act simultaneously and they explicitly take into account the sequential nature of the interactions among them. Second, they play a (hierarchical) game over a flow network which causes severe limitations and constraints to their action sets. Moreover, one of the two players cannot be considered as a “rational” person maximizing her utility. Instead, she acts as an adverse player seeking to maximize inconveniences to her opposite player. Finally, the games exhibit linear characteristics and can be solved using bilevel linear programming.

All these issues have already been discussed in the scientific literature, even though in different separate contexts. The merging of three mentioned approaches in only one single framework is a major contribution of our modelling perspective. Furthermore, bilevel programming is rich of theoretical results and numerical algorithms, but is scarce in actual applications. From this point of view, the present work might be considered as an interesting addition to the field. In this chapter, a broad overview of the main available results on freight models and on the other two research lines with specific focus on their application to freight transportation is provided.

3.1 Freight transportation models

In the past and recent years, passengers mobility, with particular emphasis on the development of modal choice and assignment models, has certainly been more widely investigated than freight transportation. This discrepancy might have been caused, on one side, by the greater complexity of the freight mobility system with respect to the passengers one. On the other side, the collection of reliable and detailed data is a highly difficult task (Camus et al., 1998). Nevertheless, “freight transportation is one of today’s most important activities, not only as measured by the yardstick of its own share of a nation’s gross national product (GNP), but also by the increasing influence that transportation and distribution of goods have on the performance of virtually all other economic sectors” (Crainic and Laporte, 1997). To suitably face the new theoretical challenges and effectively meet the operator requirements, in the recent years more attention is paid to the freight transportation system by the academic community and practitioners. For instance, new International Journals (e.g., *Transportation Research, Part E: Logistics*), Special Issues of scientific Journals, Workshops (e.g., *Odysseus: Logistics and Freight Transportation*, to be held triennially) and Conference streams completely devoted on logistics and freight transportation problems are increasing every year. This interest together with the availability of more performing computers leads to the development of more powerful algorithms thus providing more realistic models both in terms of complexity and size. In this section, we briefly describe the principal methodological approaches for modelling freight transportation systems that have been useful in developing this work.

3.1.1 Equilibrium models

They represent the modal choice and assignment of a freight transportation system in an aggregate way: interactions among the different decision makers are not explicitly taken into account. Instead, flow (tons or trucks) on the multimodal network are

represented considering the demand expressed in terms of the O/D matrices. The representation of the infrastructural network is performed with particular care: the aim is to differentiate by means of appropriate cost functions either the actual network arcs from those representing intermodal exchanges, either the arcs related to the different nodes of transport. A seminal model following the described approach is the Harvard - Brookings model (Kresge and Roberts, 1971). Later contributions are the Freight Network Equilibrium Model (Harker and Friesz, 1986) which introduces the congestion phenomenon and the Multimode Multiproduct Network Assignment Model (Guélat, Florian and Crainic, 1990) which allows to obtain a description of the multimodal transport system able to support decisions at the strategic level. These network models provide a clear description of the system but, as a drawback, they lack in accuracy with respect to the decisional process occurring in the system. In addition, they do not consider some typical aspects of a freight transportation system as the back-hauling phenomenon and the limitations on the availability of vehicles. However, they provide an analytical approach for representing the transportation costs of the network.

3.1.2 Sequential models

Due to the widespread fragmentation in both the demand and supply sides, different actors are generally involved in the freight transportation system. The sequential models explicitly describe the decision making process for both sides. Unlike passengers mobility, these decisions have usually more influence on the actual behavior of a freight transportation system rather than the determination of the minimum generalized transportation cost only. The operators involved may be usually classified in four families: Producers and Consumers (transportation demand), Public Authorities (Local entities, Port Authorities, Governments), Shippers and Carriers. Each of them deals with problems which are mutually interconnected at the different hierarchical levels, i.e., at the strategic, tactical, and operational level. As a rough

approximation, the main features of the decision making process may be adequately represented by considering the role of shippers and carriers only. In fact, producers, consumers and Public Authorities affect the system in a less dynamic way and their influences may be taken into account by means of constraints imposed on the shippers and the carriers (Camus et al., 1998). The transportation demand usually can be adequately expressed in terms of O/D matrices, whereas public authorities' activity mainly affects the design of the infrastructure supply and/or the link cost functions of the multimodal network. On the other hand, shippers and carriers may act, at different levels, on several decision making variables, and influence the final structure of the system. Relationships among all the actors involved in the transport of goods are conceptually described, e.g., in the framework proposed by Harker and Friesz, 1986. However, the other existing models generally take into account only the decision making process between shippers and carriers.

Among the most interesting models, the approach proposed by Friesz, Gottfried and Morlok (1986) assumes a merely sequential structure to describe the hierarchical decision process and proposes a nonlinear optimization model to represent it. At the beginning, shippers, on the basis of aggregate information about the network and the transport demand in terms of production and needs, decide origins and destinations of the movements, transport modes and, if any, nodes of intermodal exchange aiming at minimizing their own total generalized cost. In the following, a correspondence between the nodes of the aggregate (shipper's) and the detailed (carrier's) networks is established. Then each carrier receives from the shippers fixed amounts of demand and optimizes its own transport subsystem in order to minimize the generalized cost on this subnetwork. In particular, carriers may decide on the path to be followed on the detailed network and on other operative issues as, e.g., fleet and crew rostering and scheduling. At the end of this two-stage decision process, arc and path flows and costs are obtained. Criticism is raised to this model because of the lack of interaction between the two hierarchical levels, namely, by no means carriers' choices are allowed to influence shippers' decisions. In particular,

shippers are not able to evaluate the actual cost in the different arcs because it depends on the load allocated by the lower level decision maker. Thus this is not an equilibrium model allowing shippers and carriers to take simultaneous decisions.

3.1.3 Mathematical programming models

Mathematical programming models and algorithms have proved highly suitable for the solution of road freight transportation problems at the strategic, tactical and operational levels. They use linear, integer or mixed integer programming methodologies both in a static and dynamic context on a deterministic or stochastic network. When the exact solution can not be obtained in due time because of the computational *NP*-hardness of the problems, approximate or heuristic algorithms are also developed and implemented. Restricting the attention to the strategic level only, goods producing firms are mainly concerned on the distribution network design problem whereas transportation or logistics firms focus their attention on the transportation network design problem (Speranza, 1999). In the former case, the decision maker has to decide on the node characteristics (i.e., plant facilities, central or regional warehouses, intersshipment points), on their amount, location and capacity, on the distribution routes, and on the transportation modality for each possible link. Cost minimization while satisfying a given level of service for customers is his main objective. In the latter case, the aim is to choose links in a network, along with capacities, eventually, in order to enable goods to flow between origin and destination at the lowest possible system cost, i.e., the total fixed cost of selecting the links plus the total variables cost of using the network (Crainic and Laporte, 1997).

A wide range of different problems on these topics exists according to the constraints the system under study is subject to and the objectives of the decision maker. More complex challenges arise when integrating the strategic and the tactical levels, e.g., simultaneously considering the just above described aspects with the collection and/or distribution routing and scheduling issues.

3.2 Network Game models

When a network with non-cooperating agents acting on it is considered, the game theoretic approach may be relevant. Indeed, the search for Nash or Stackelberg equilibria has been employed in, e.g., flow control, routing and virtual-path bandwidth allocation in modern networking (Libman and Orda, 1999). In this context, models dealing with very simple topologies (i.e., a common source and a common destination node interconnected by a number of parallel links) and considering either a single follower and a multi follower Stackelberg game have been presented (Korilis, Lazar and Orda, 1997). They propose a method for architecting noncooperative equilibria in the run time phase, i.e., during the actual operation of the network. This approach is based in the observation that, apart from the flow generated by the self-optimizing users, typically, there is also some network flow that is controlled by a central entity, also called “the manager”. As an example, we might consider the traffic generated by signalling and/or control mechanisms, as well as traffic of users that belong to virtual network. The manager attempts to optimize the system performance, through the control of its portion of flow. This framework is generally not suitable for transportation networks because in this latter case the user controls just an infinitesimally small portion of the network flow (i.e., a car on the road), whereas we are concerned with users controlling non negligible portions of flow (Orda, Rom and Shimkin, 1993). However, it provides complementary aspects with the freight transportation network we propose in this work. On one side, it considers a simpler network topology (i.e., only parallel arcs) than we do. On the other side, it assumes that the leader and follower may control portions of the flow in a given arc, whereas we always imagine that in each arc all the flow is fully controlled by only one player. As a further research, it would be interesting to investigate more thoroughly the links between these two approaches.

The game theoretic approach is explicitly used in a transport network by Bell (1999) and Bell (2000). In these papers, the author envisages a two-player, non-

cooperative, zero sum game between a network user seeking to a path to minimize the expected trip cost on one hand and an “an evil entity” imposing link costs on the user so as to maximize the expected trip cost on the other. The user guesses what link costs will be imposed and the evil entity guesses which path will be chosen. At the Nash mixed strategy equilibrium, the user is unable to reduce the expected trip cost by changing its path choice probabilities while the evil entity is unable to increase the expected trip cost by changing the scenario probabilities, without cooperating. At the equilibrium, this game offers a useful measure of network reliability defined as the expected trip costs that are acceptable even when users are extremely pessimistic about the state of the network. These papers differ from our approach because they consider a simultaneous game with mixed strategies. In addition, the network user seeks to solve a shortest path problem, whereas we face a minimum cost flow problem. However, they introduce the concept of an “evil entity” operating on a network to be dealt with by the user. This provides the theoretical framework for performing a worst-case analysis as a basis for a cautious approach to network design.

Even though the game theoretic approach is not explicitly addressed, another similar network game to be considered has been introduced by Lozovanu and Trubin, 1994. They present a different two-player game over a network. In their model, a partition of the network nodes in two classes is considered. The player controlling the first (second) class chooses the arcs in order to maximize (minimize) the sum of their costs. The game consists of a sequence of moves that incrementally build the solution path. Each player moves (i.e., includes new arcs in the path under construction) if and only if the arc to be chosen to reach destination belongs to his/her own class. It follows that at each node it is possible to identify the maximin or the minimax path from a given origin, according to the class the node belongs to. In contrast to this incremental approach, in our system each player assigns flows at a given time or, respectively, capacities to all the arcs under his/her own control. Note that the game is not necessarily simultaneous in the sense that the two players

may or may not make their move at the same time. Each actor decides at the same time for all his/her arcs, but each move is taken, in general, at a different time.

3.3 Bilevel models

In general, bilevel programming problems are difficult to solve because of their inherent non-convexity and non-differentiability (Florian and Chen, 1995). Due to their NP-hard nature, practitioners are committed in developing new exact or heuristic algorithms able to identify a (acceptable) solution in not too long computation time. In spite of the substantial development of research in this area, there is still scope for improvement. Bard (1998) provides a recent and updated summary of the theoretical results and introduces the most widespread solution techniques for the linear, non linear and general case, respectively. This book is used as a reference throughout this work whenever bilevel programming issues are concerned.

Among the different applications, some transport studies have been performed using bilevel models. In particular, they mainly focus on passengers' mobility with emphasis on the road transport sector. The early papers deal with highway network system design (see, e.g., Ben-Ayed, Boyce and Blair, 1988) also considering congestion effects (Marcotte, 1986). Traffic control models, like, e.g., traffic signal setting, optimal road capacity improvement, estimation of the origin-destination matrices from traffic counts, ramp metering in freeway-arterial corridor and optimization of road tolls have also been developed using bilevel programming technique (Yang and Bell, 2001). Recent advances in this field rely on non-traditional formulations of static and dynamic equilibrium network design and on the development of algorithms for urban traffic optimization models (again, Yang and Bell, 2001).

Even though bilevel programming enables the representation of competition between players, hardly ever freight transportation modelling relies on this technique. A relevant exception to this trend is the paper written by Brotcorne et al., 2000. They develop a bilevel programming formulation for freight transportation focusing

the attention to a tariff-setting problem involving two decision makers acting non-cooperatively and in a sequential way. The leader consists in one among a group of competing carriers and the follower is a shipper. At the upper level, a given carrier maximizes its revenues by setting optimal tariff on the subset of arcs under its control. On one side, the reaction from its competitors is neglected, on the other side, the reaction of the shipper company to its price schedule is explicitly taken into account. A shipper company (the follower) ships a prescribed amount of goods from origins to its customers at minimum cost. The lower level problem provides the flow repartition solving a standard transshipment problem where the tariffs are added to the initial arc costs. Supply at the origin nodes and demand from the customers are both assumed to be known and fixed. Hence the shipper minimizes its transportation costs, given the tariff schedule set by the leader.

Chapter 4

Mathematical formulation

Games 1 and 2 are mathematically formalized in the first sections of this chapter. In addition, a simple explanatory network is also introduced as an example to help the reader in understanding the main concepts and definitions. In Section 4.2, we claim that both games may be represented as bilevel linear programming problems (BLPPs). Hence in Section 4.4 we provide the basic definitions and properties for a BLPP in the general case and in Section 4.5 we extend these concepts to the games under consideration.

4.1 Notation and definitions

Given a digraph $G = (N, A)$, where N is the set of nodes and A is the set of arcs, each node $i \in N$ is either a *source node* or a *sink node* or simply a *transshipment node*. A flow divergence vector b is given, with $b_i > 0$ if i is a source node, $b_i < 0$ if i is a sink node, and $b_i = 0$ if i is a transshipment node. The arcset A in G is partitioned in two subsets, $A = A^P \cup A^Q$, with B^P and B^Q the corresponding incidence matrices. The cardinalities of sets A^P and A^Q are defined as p and q , respectively. Each arc $a \in A$ has an upper bound \bar{u}_a on the maximum flow that can pass through it. All the lower bounds are assumed equal to zero.

Player P decides the values of the flows through the arcs in A^P necessary to

satisfy the demand expressed by the divergence vector b . Player Q decides the values of the capacities of the arcs in A^Q , i.e., she may reduce the upper bound \bar{u}_e of any arc e down to a lower value u_a . Define $x \in \mathbb{R}^p$ the vector of the flows of the arcs in A^P and $y \in \mathbb{R}^q$ the vector of the flows of the arcs in A^Q . The two players have, possibly different, objectives $\phi^P = c^{PP}x + c^{PQ}y$ and $\phi^Q = c^{QP}x + c^{QQ}y$, respectively. We assume that all the components of ϕ^P and ϕ^Q are constant and non negative.

A simple example (see Fig. 4.1) may help to better understand the network and the notation just introduced.

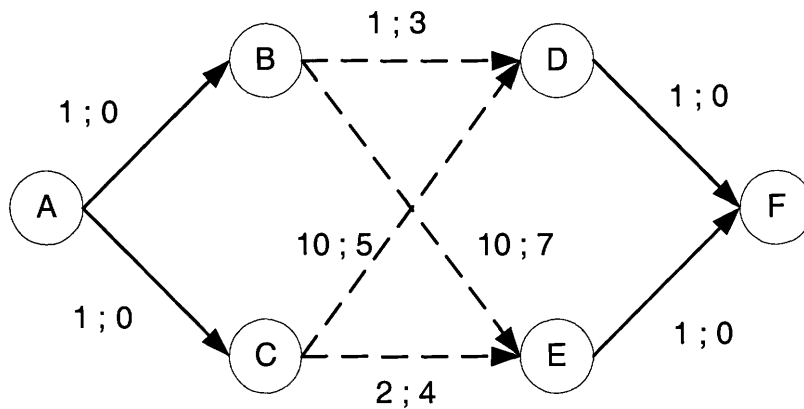


Figure 4.1: Players P and Q network

Player P controls the solid line arcs and player Q the dashed line ones. Thus the arcset $A = A^P \cup A^Q$ is composed of

$$A^P = \{AB, AC, DF, EF\} \quad \text{and} \quad A^Q = \{BD, BE, CD, CE\}.$$

In Tab. 4.1 the costs and the capacities associated to each arc $a \in A$ are shown:

a	c^{PP}	c^{PQ}	c^{QP}	c^{QQ}	u_a
AB	1	-	0	-	1
AC	1	-	0	-	1
BD	-	1	-	3	1
BE	-	10	-	5	1
CD	-	10	-	7	1
CE	-	2	-	4	1
DF	1	-	0	-	2
EF	1	-	0	-	2

Table 4.1: Arc costs and capacities

We finally assume that 2 units of flow leave source A and have to reach destination F , i.e., $b_A = 2, b_B = b_C = b_D = b_E = 0, b_F = -2$.

4.2 Game formulations

According to the above notation, Game 1 claims that given two players, P and Q , and a network $G = (N, A)$, the first player P decides the values of the flows x , then player Q decides the values of the capacities u^Q , finally player P decides the values of the flows y . We may observe that both the payoff functions and all the constraints, i.e., flow balancing, capacity and nonnegative constraints, are linear. Hence this is a *linear* three-stage game which may be formalized as follows:

$$\min_x \phi^P = c^{PP}x + c^{PQ}y \quad (1a)$$

$$B^P x = b^P \quad (1b)$$

$$\text{subject to } \max_{u^Q} \phi^Q = c^{QP}x + c^{QQ}y \quad (1c)$$

$$\min_y c^{PQ}y \quad (1d)$$

$$B^Q y = b^Q \quad (1e)$$

$$B^{PQ}x + B^{QP}y = 0 \quad (1f)$$

$$0 \leq x \leq \bar{u}^P \quad (1g)$$

$$0 \leq y \leq u^Q \leq \bar{u}^Q \quad (1h)$$

where (1a) is the objective of player P , (1c) is the objective of player Q , (1d) is the second move of player P and conditions (1b), (1e) and (1f) are the flow balancing constraints for nodes in which only arcs in A^P are incident, nodes in which only arcs in A^Q are incident, and nodes in which arcs both in A^P and A^Q are incident, respectively. In the following, these last nodes are referred to as *frontier* nodes. Denote $\hat{N} \subset N$ the set of the frontier nodes. As an example, referring again to Fig. 4.1, $\hat{N} = \{B, C, D, E\}$. We also define B^{PQ} as the incidence matrix for frontier nodes corresponding to arcs in A^P and B^{QP} as the incidence matrix for the frontier nodes corresponding to arcs in A^Q . Note that there is no loss of generality in assuming that no sink or source can be a frontier node.

Theorem 1 *When network $G^Q = (N, A^Q)$ disconnects network $G^P = (N, A^P)$, letting player Q choose the capacities u^Q corresponds to make her define the flows y over the subnetwork $G^Q = (N, A^Q)$, provided that the flow balancing constraints with flows x are met.*

Proof: Each arc in G^Q has a maximum capacity \bar{u}^Q . Player Q would lower these capacities to let pass as much flow as possible in the more profitable arcs and as less flow as possible in the less profitable arcs. In this way, she fixes the exact amount of flow going through each arc, i.e., she assigns the flow on her network. Hence player P does not have any further room to decides on the flows y , i.e., equation (1d) may be removed. On the contrary, this equation still holds when some flow can reach destination by overcoming G^Q . In this case, once player Q has fixed the capacities on her arcs, player P is allowed to assign to G^Q less flow than the available capacity by sending some flow over the arcs not passing through it. \square

Under the above hypotheses, Game 1 simplifies: first P decides flows x , then Q decides flows y . Model (1) describing Game 1 may be restated as the following bilevel linear programming problem, or linear Stackelberg game (see, e.g., Bard, 1998), since player P cannot do anything different from giving up the control of flows y to player Q :

$$\min_x \phi^P = c^{PP}x + c^{PQ}y \quad (2a)$$

$$\text{subject to } \max_y \phi^Q = c^{QP}x + c^{QQ}y \quad (2b)$$

$$B^P x = b^P \quad (2c)$$

$$B^Q y = b^Q \quad (2d)$$

$$B^{PQ}x + B^{QP}y = 0 \quad (2e)$$

$$0 \leq x \leq \bar{u}^P \quad (2f)$$

$$0 \leq y \leq \bar{u}^Q. \quad (2g)$$

Similarly, Game 2 where, given two players P and Q , and a network $G = (N, A)$, player Q decides first, may be formally stated by means of BLPP (2) with the only difference that the role of the two objective functions (2a) and (2b) are exchanged.

By means of the example previously introduced in Fig. 4.1, we present the three different possibilities that the shipper (i.e., player P) may face when deciding to move two units of flow from source A to destination F and solving BLPP (2).

- **Player P plays first (Game 1).** In this case, player P has only three choices for allocating the two units of flow (see Tab. 4.2). Player Q reacts to each of them in such a way to maximize her profit. Observe that when P considers the second alternative, Q may react in two ways: $y_{BD} = 1, y_{BE} = 0, y_{CD} = 0, y_{CE} = 1$ yielding $\phi^Q = 7$ or $y_{BD} = 0, y_{BE} = 1, y_{CD} = 1, y_{CE} = 0$ yielding $\phi^Q = 12$. Obviously, she will choose the latter one. Since this behavior is known by player P , he will finally choose the third alternative that leads to a minimization of his costs, i.e., $x_{AB} = 1, x_{AC} = 1, x_{DF} = 2, y_{EF} = 0$ and $\phi^P = 15$ (Fig. 4.2a).

Player P choices				Player Q consequent choice				$\max \phi^Q$	ϕ^P
AB	AC	DF	EF	BD	BE	CD	CE		
1	1	0	2	0	1	0	1	11	16
1	1	1	1	0	1	1	0	12	24
1	1	2	0	1	0	1	0	8	15

Table 4.2: Player P plays first

- **Player Q plays first (Game 2).** In this case, player Q has four different possible moves and player P may react to each of them in only a single way (see, Tab. 4.3). It is easy to show that the third alternative is the optimal solution in this game (Fig. 4.2b) and $\phi^P = 24$.

Player Q choices				Player P consequent choice				$\min \phi^P$	ϕ^Q
BD	BE	CD	CE	AB	AC	DF	EF		
1	0	1	0	1	1	2	0	15	8
1	0	0	1	1	1	1	1	7	7
0	1	1	0	1	1	1	1	24	12
0	1	0	1	1	1	0	2	16	11

Table 4.3: Player Q plays first

- **Player Q does not exist.** This situation is presented only to compare the two previous games with the most favorable case the shipper may afford: he fully controls all the arcs of the graph. Thus he has simply to solve a Minimum Flow Cost Problem over the entire network (Fig. 4.2c). The result turns out to be $\phi^P = 7$ which is the smallest value among the three possible alternatives, as it should have been expected.

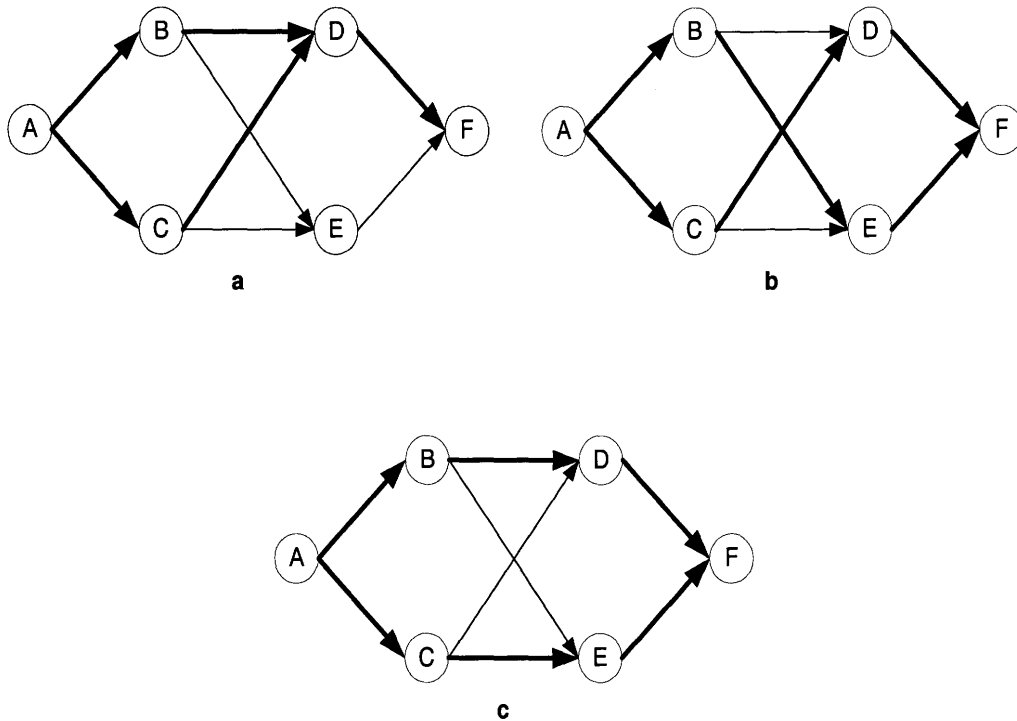


Figure 4.2: The three alternatives the shipper may face

However, in the remainder of this work we always consider Game 1 under the hypotheses defining BLPP (2).

Comparison with another network game

To further clarify the games under study, we compare player P 's results when he moves first with the assignment obtained when the players moves in accordance with the game proposed by Lozovanu and Trubin (1994) which has already been introduced in Section 3.2. Also in this latter case, player P aims at minimizing the cost of the transport and player Q aims at maximizing her utility. Conversely to our approach, each player does not act on all the arcs under his/her control at the same time. Starting from the origin, player P assigns the flow on the first part of his network, then he waits for player Q 's choice on her subgraph and, finally, he makes

the assignment on his remaining arcs thus reaching destination. Always relying on the simple network introduced in Fig. 4.1, it is easy to show that the outcome of this game is depicted in Fig. 4.2b. Even though he moves first, player P gets a worse payoff (than the result shown in Fig. 4.2a representing Game 1) because he is not able to completely prevent the other player's reaction as he normally does in Game 1.

4.3 Undesired situations

Undesired situations may occur and have to be avoided.

1. Player Q does not have to force circulations in the flow. In fact, player P would never play against player Q if he knew that she could force his vehicles to indefinitely loop in the region under her control. To prevent this situation, it is assumed that either G^Q is cycle-free or that in (2) the appropriate constraints preventing flow circulation, possibly exponential in number, are added. In this latter case, the algorithm we later present may still be used, but the solution of the maximum cost flow problem in one of its steps would become a NP-hard problem.
2. When multiple optima for player Q leading to different values of $C^{PQ}y$ exist, player P may not achieve his optimal solution even though the feasible region of BLPP (2) is not empty. To overcome this problem, observe that once player Q has reached optimality, she is no longer in competition with player P . Hence, we assume a benevolent (cooperative) attitude of player Q with respect to player P . Then, the two players may collaborate and player P may encourage player Q to choose a desirable solution for him. This framework allows player P to always identify an optimal solution, as described in Bialas and Karwan, 1984. As an example, consider Fig. 4.3 which is a slight modification of the network introduced in Fig. 4.1. In this network, player Q always earns

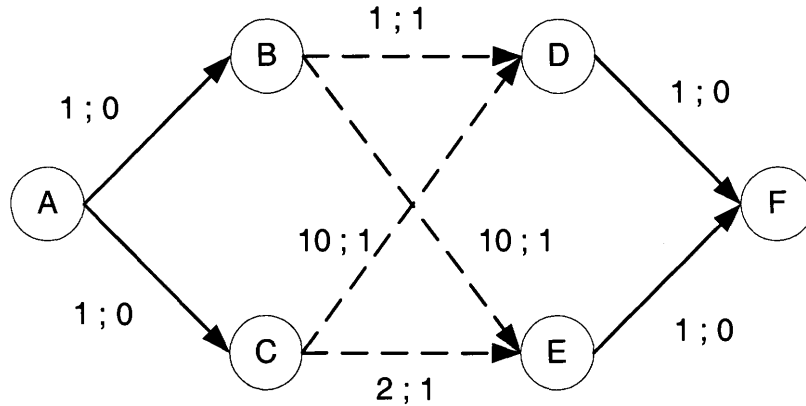


Figure 4.3: Player Q has multiple optima

4. Since she would choose the most favorable alternative to player P , i.e., $y_{BD} = 1, y_{BE} = 0, y_{CD} = 0, y_{CE} = 1$, he would play $x_{AB} = 1, x_{AC} = 1, x_{DF} = 1, x_{EF} = 1$ leading to $\phi^P = 7$.

4.4 Bilevel linear programming

In the previous section, we claim that the game under investigation is formalized as a bilevel linear programming problem (BLPP). To solve it, some basic background has to be introduced. Relying on (Bard, 1998), it is first given for the general case. Then its adaptation to BLPP (2) is provided in Section 4.5.

A general BLPP for continuous variable can be written as follows:

$$\begin{aligned}
 \min_{x \in X} F(x, y) &= c_1 x + d_1 y \\
 \text{subject to} & \quad A_1 x + B_1 y \leq b_1 \\
 & \quad \min_{y \in Y} f(x, y) = c_2 x + d_2 y \\
 & \quad \text{subject to } A_2 x + B_2 y \leq b_2
 \end{aligned}$$

where $x \in X \subset R^n$, $y \in Y \subset R^m$, $F : X \times Y \rightarrow R$, and $f : X \times Y \rightarrow R$.

In addition, $c_1, c_2 \in R^n$, $d_1, d_2 \in R^m$, $b_1 \in R^p$, $b_2 \in R^q$, $A_1 \in R^{p \times n}$, $B_1 \in R^{p \times m}$, $A_2 \in R^{q \times n}$, $B_2 \in R^{q \times m}$. Once the leader selects an x , the first term in the follower's objective function becomes a constant and can be removed from the problem. Hence y can be viewed as a function of x , i.e., $y = y(x)$.

For a BLPP the following definitions are introduced:

1. Constraint region of the BLPP:

$$S = \{(x, y) : x \in X, y \in Y, A_1x + B_1y \leq b_1, A_2x + B_2y \leq b_2\}$$

2. Feasible set for the follower for each fixed $x \in X$:

$$S(x) = \{y \in Y : B_2y \leq b_2 - A_2x\}$$

3. Projection of S onto the leader's decision space:

$$S(X) = \{x \in X : \exists y \in Y, A_1x + B_1y \leq b_1, A_2x + B_2y \leq b_2\}$$

4. Follower's rational reaction set for $x \in S(X)$:

$$P(x) = \{y \in Y : y \in \arg \min[f(x, \hat{y}) : \hat{y} \in S(x)]\}$$

5. Inducible region:

$$IR = \{(x, y) : (x, y) \in S, y \in P(x)\}$$

To ensure that the BLPP is well posed it is common to assume that S is nonempty and compact, and that for decisions taken by the leader, the follower has some room to respond, i.e., $P(x) \neq \emptyset$. The rational reaction set $P(x)$ defines the response while the inducible region IR represents the set over which the leader may optimize. Thus in terms of the above notation, the BLPP can be written as:

$$\min \{F(x, y) : (x, y) \in IR\} .$$

Unlike linear programming, the identification of the optimal solution for a BLPP is generally a very hard task. More formally, it has been proved that the BLPP is strongly NP-hard. Thus the need for heuristics or approximate algorithms for finding a solution of good quality in acceptable time. However, some properties of the optimal solution of a BLPP are known:

Property 1 *Unless the follower's terms in F and f are collinear (i.e., $d_1 = \alpha d_2, \alpha > 0$) there is no assurance that the optimal solution to the BLPP will be Pareto optimal.*

Property 2 *The inducible region can be written equivalently as a piecewise linear equality constraint comprised of supporting hyperplanes of S .*

Property 3 *An optimal solution to BLPP occurs at a vertex of IR .*

Property 4 *An optimal solution to BLPP occurs at a vertex of S .*

The meaning of the above definitions and properties are geometrically clarified and explained considering the following BLPP:

$$\begin{aligned}
 \max_x \quad & -x + \frac{9}{2}y \\
 s. t. \quad & \min_y \\
 & x + y \geq 2 \\
 & -x + y \geq 1 \\
 & x + 2y \leq 14 \\
 & 2x - y \leq 8 \\
 & x, y \geq 0
 \end{aligned}$$

Fig. 4.4 depicts this BLPP. The constraint region S is the light gray region. For a given point x_1 in the projection set $S(X)$, the feasible set $S(x_1)$ and the reaction set $P(x_1)$ are given. The thick solid line represents the inducible region IR . In this specific case Property 1 holds. In fact, the optimal solution P^* is dominated by all

the other points in the region delimited by the hyperplanes $y = 4$, $-x + (9/2)y = 12$ and $-x + y = 1$. Finally, it is easy to see that Properties 2 - 4 are fulfilled.

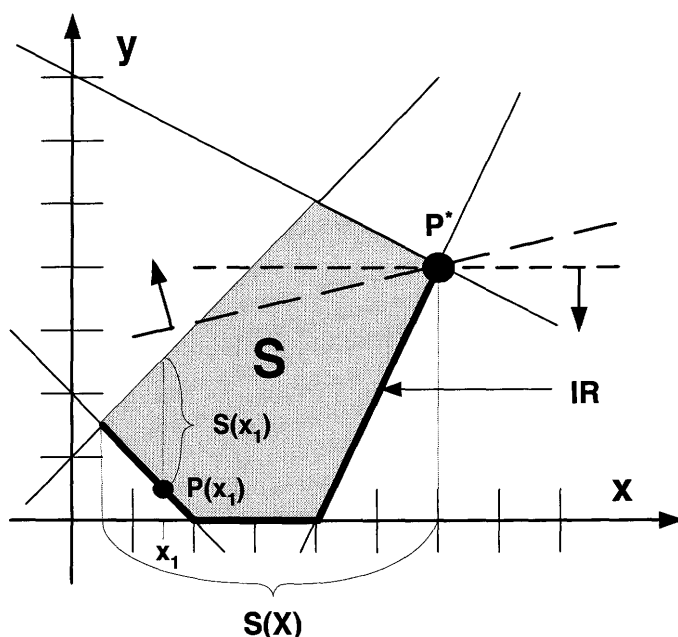


Figure 4.4: Bilevel linear programming problem

4.5 Inducible regions

According to Property 3, the optimal solution lies on a vertex of the inducible region. This is the leader's inducible region, of course. However, if we assume to exchange the role of the players, a follower's inducible region may be similarly defined. As it will be shown in the next Chapter 5, for solving BLPP (2) the inducible regions of both players are required. In this section, we define them and we show that due to the presence of the frontier nodes or, otherwise stated, due to the fact that the subnetwork G^Q disconnects sources and destinations, the inducible region may be easily determined by solving a Minimum and a Maximum Cost Flow Problem, respectively.

Definition 2 Given a network $G = (N, A)$ where constraints (2c) - (2g) hold, define as frontier flow vector any vector $\kappa \in \mathfrak{R}^{|\hat{N}|}$ such that $B^{PQ}x = \kappa$ and $B^{QP}y = -\kappa$.

Note that the component κ_i of vector κ is nothing else than the flow passing from the subnetwork G^P to the subnetwork G^Q through the frontier node i . For this reason the vectors κ are referred to as *frontier flow vectors*. As an example, in Fig. 4.2a the frontier vector is $\kappa = (-1, -1, 2, 0)$ and in both Fig. 4.2b and 4.2c is $\kappa = (-1, -1, 1, 1)$.

Definition 3 For any frontier flow vector κ , define

- $X^k = \{x : B^P x = b^P, B^{PQ}x = \kappa, 0 \leq x \leq \bar{u}^P\},$
- $Y^k = \{y : B^Q y = b^Q, B^{QP}y = -\kappa, 0 \leq y \leq \bar{u}^Q\}.$

According to these definitions, the following theorem trivially holds:

Theorem 2 If a point (x, y) is feasible for constraints (2c) - (2g), then it exists a frontier flow vector κ such that $x \in X^k$ and $y \in Y^k$. Conversely, if a frontier flow vector κ exists such that both X^k and Y^k are non empty, then any point (x, y) where $x \in X^k$ and $y \in Y^k$ is a feasible solution for BLPP (2).

By extending the definitions introduced in Section 4.4 and closely paralleling the arguments in Section 4.3 in Başar and Olsder (1999), it is now possible to define the leader's and follower's rational reaction sets and inducible regions.

Definition 4 Given BLPP (2), let exist a frontier flow vector κ such that both X^k and Y^k are non empty. Since the minimum of $\phi^P(x, y)$ with respect to $x \in X^k$ is attained for each $y \in Y^k$, the set $R^P(y) \subset X^k$ defined by

$$R^P(y) = \{\xi \in X^k : \phi^P(\xi, y) \leq \phi^P(x, y) \forall x \in X^k\}$$

is called the optimal response or rational reaction set of player P. The set

$$IR^Q = \{(x, y) : x \in R^P(y), y \in Y^k\}$$

is called the inducible region of player Q . Likewise, $R^Q(x) = \{\zeta \in Y^k : \phi^Q(x, \zeta) \geq \phi^Q(x, y) \forall y \in Y^k\}$ and $IR^P = \{(x, y) : x \in X^k, y \in R^Q(x)\}$ are, respectively, the rational reaction set of player Q and the inducible region of player P .

According to this definition, in BLPP (2), being the flows κ fixed, $R^P(y)$ is determined solving a Minimum Cost Flow Problem. Similarly, $R^Q(x)$ is the solution of a Maximum Cost Flow Problem. Hence the following theorem holds:

Theorem 3 $R^P(y)$ and $R^Q(x)$ depend only on the frontier flow vector κ , i.e., $R^Q(x) = f^Q(\kappa)$, $\forall x \in X^k$, and $R^P(y) = f^P(\kappa)$, $\forall y \in Y^k$, where f^P and f^Q solve a minimum and maximum cost flow problem, respectively.

Chapter 5

The optimal solution

The optimal solution of any BLPP lies on a vertex of the leader's inducible region. Since in this work we always refer to Game 1 where player P plays first, the optimal solution of BLPP (2) lies on a vertex of the shipper's inducible region. In Section 4.5, we state that for any feasible frontier flow κ , the shipper's and follower's rational reaction sets are easily determined by solving a Maximum and Minimum Cost Flow Problem, respectively. Relying on this result, we now develop an algorithm which allows to move from a starting point of the shipper's inducible region to another point in the shipper's inducible region always providing a better solution for him. When no further "better" points may be attained, the algorithm stops. Unfortunately, only a local optimum is identified.

The rationale behind the algorithm stems from the consideration that the optimal solution for BLPP (2) is also a Nash equilibrium point for BLPP (2). In particular, the algorithm moves from a Nash equilibrium point for BLPP (2) to another "better" Nash equilibrium point. Section 5.1 presents the definition of Nash equilibrium for BLPP (2) and proves that the optimal solutions are Nash equilibria. The identification of such equilibria relies on the Negative Cycle Optimality Conditions for solving a Minimum Cost Flow Problem. Section 5.2, entirely based on Ahuja, Magnati and Orlin (1993) provides the relevant definitions and theorems for introducing such conditions in the general case. Section 5.3 extends these defini-

tions and theorems to the two-players network under study and links them to the Nash equilibrium point of BLPP (2). Finally, Section 5.4 introduces the local search algorithm.

5.1 Equilibrium points

In this section, we introduce the concept of Nash equilibrium point for BLPP (2) and we prove that any optimal solution for BLPP (2) is also a Nash equilibrium point.

Given a leader's choice x , the follower's rational reaction set represents the best value(s) that can be attained by her for that specific move x . Likewise, given a follower's choice y , the leader's rational reaction set represents the best value(s) that can be attained by him for that specific move y . It follows that none of the players can improve his (her) outcome without degrading the performance of the other player if and only if their choice (x, y) lies on both reaction sets, i.e., we may state the following definition (see Başar and Olsder, 1999):

Definition 5 *A feasible point (x, y) which belongs to both the inducible regions IR^P and IR^Q is called a Nash Equilibrium Point for BLPP (2), i.e., the Nash Equilibria Set (NES) is formally defined as:*

$$NES = IR^P \cap IR^Q = \{(x, y) : x \in R^P(y), y \in R^Q(x)\}.$$

As a counterexample, consider the frontier vector $\kappa = (1, 1, -1, -1)$. The vector $x = \{x_{AB} = 1, x_{AC} = 1, x_{DF} = 1, x_{EF} = 1\}$ and $y = \{y_{BD} = 1, y_{BE} = 0, y_{CD} = 0, y_{CE} = 1\}$ is a feasible solution and belongs to IR^P , but y is not a point of player Q rational reaction. This is clearly not a Nash equilibrium because player Q can improve her outcome (moving to her rational reaction point $y = \{y_{BD} = 0, y_{BE} = 1, y_{CD} = 1, y_{CE} = 0\}$) without degrading the shipper's choice.

Theorem 4 *A feasible point (x^κ, y^κ) where $x^\kappa = \arg \min_{x \in X^k} c^{PP}x$ and $y^\kappa = \arg \max_{y \in Y^k} c^{QQ}y$ is a Nash equilibrium point.*

Proof: Just check that $x^\kappa \in R^P(y)$ and $y^\kappa \in R^Q(x)$. □

Hence from Theorem 3 and given a feasible frontier vector κ , a Nash equilibrium point is obtained by solving a minimum and a maximum cost flow problem, respectively.

We may now characterize the optimal solution of BLPP (2). First, we prove the existence of the optimal solution. Then, we show that it is also a Nash equilibrium point.

Theorem 5 *If BLPP (2) has feasible solutions, it always admits an optimal solution.*

Proof: If conditions (2c) - (2g) define a nonempty set, this is a compact set and $R^Q(x) \neq \emptyset$. If $R^Q(x)$ is a singleton, BLPP (2) is well posed and admits optimal solution (cf., e.g., Bard, 1998). If $R^Q(x)$ is not single-valued and holding the cooperative attitude of player Q assumption, the unique optimal solution for the player P may be achieved by means of incentive schemes as described in Bialas and Karwan (1984). □

Theorem 6 *An optimal solution for BLPP (2) is a Nash equilibrium point.*

Proof: Let (x^*, y^*) be an optimal solution for BLPP (2) and κ^* the corresponding frontier flow vector. On one side, since player P plays first, (x^*, y^*) certainly belongs to IR^Q . On the other side, we prove that (x^*, y^*) belongs also to IR^P , i.e., $\phi^P(x^*, y^*) \leq \phi^P(x, y^*)$ for any feasible x given y^* (i.e., $\forall x \in X^{k^*}$). By contradiction, let $x \in X^{k^*}$ be such that $\phi^P(x, y^*) < \phi^P(x^*, y^*)$. It follows that $c^{PP}x < c^{PP}x^*$. Since (x^*, y^*) is an optimal solution of BLPP (2),

$$c^{PP}x^* + c^{PQ}y^* = c^{PP}x^* + c^{PQ}R^Q(x^*) \leq c^{PP}x + c^{PQ}R^Q(x).$$

Hence $R^Q(x^*) < R^Q(x)$. But Theorem 3 contradicts this result. □

A Nash equilibrium point is always referred to as the solution of a simultaneous game (see, e.g., Osborne and Rubinstein, 1997). Conversely, a Stackelberg equilibrium point is the solution of a sequential game. Theorem 6 states that the optimal

solution of the BLPP (2) (a sequential game) may also be obtained as an outcome of a simultaneous game (which is a completely different situation, of course). Thus we could conclude that it would be sufficient to play a simultaneous game to solve BLPP (2), as well. This would be a very attractive result because the simultaneous game may be solved simply by means of minimum and maximum cost flow problems, as proved earlier. Unfortunately, this is not true. In fact, when a simultaneous game is played, many different Nash equilibria may exist. But a-priori we do not have any evidence that both players would meet exactly at the Nash equilibrium which is also the Stackelberg one. And, if they reach a Nash point that is not a Stackelberg one they do not have any reason to move apart from there. Hence the importance of Theorem 6: BLPPs are NP-hard problems, but this theorem allows to restrict the search for the optimal solution in the Nash Equilibria Set (*NES*) only. The algorithm introduced in the last section of this chapter performs a local search in this *NES*. Sections 5.2 and 5.3 provide the theoretical framework required to correctly define this algorithm.

5.2 Residual network

This section introduces the Negative Cycle Optimality Conditions for solving a Minimum Cost Flow Problem (MCFP) in the general case.

Let $G = (N, A)$ be a directed network with a cost c_{ij} and a capacity u_{ij} associated with every arc $(i, j) \in A$. We associate with each node $i \in N$ a number $b(i)$ which indicates its supply or demand depending on whether $b(i) > 0$ or $b(i) < 0$. The minimum cost flow problem can be stated as follows:

$$\begin{aligned} \min z(x) &= \sum_{(i,j) \in A} c_{ij} x_{ij} \\ \text{subject to} & \sum_{\{j:(i,j) \in A\}} x_{ij} - \sum_{\{j:(j,i) \in A\}} x_{ji} = b(i) \quad \forall i \in N \\ & 0 \leq x_{ij} \leq u_{ij} \quad \forall (i, j) \in A \end{aligned}$$

We further make the following assumptions:

1. All data (cost, supply/demand, and capacity) are integral;
2. The network is directed;
3. The supplies/demands at the nodes satisfy the condition $\sum_{i \in N} b(i) = 0$ and the minimum cost flow problem has a feasible solution;
4. All arc costs are nonnegative.

Definition 6 *The residual network $G(x)$ corresponding to a flow x is defined as follows. We replace each arc $(i, j) \in A$ by two arcs (i, j) and (j, i) . The arc (i, j) has cost c_{ij} and the residual capacity $r_{ij} = u_{ij} - x_{ij}$, and the arc (j, i) has cost $c_{ji} = -c_{ij}$ and residual capacity $r_{ji} = x_{ij}$. The residual network consists only of arcs with positive residual capacity.*

Definition 7 *A cycle W (not necessarily directed) in G is called an augmenting cycle with respect to the flow x if by augmenting a positive amount of flow $f(W)$ around the cycle, the flow remains feasible.*

In terms of residual networks, each augmenting cycle W with respect to a flow x corresponds to a directed cycle W in $G(x)$, and vice versa.

Theorem 7 Augmenting Cycle Theorem. *Let x and x° be any two feasible solutions of a network flow problem. Then x equals x° plus the flow on at most m directed cycles in $G(x^\circ)$. Furthermore, the cost of x equals the cost of x° plus the cost of flow on these augmenting cycles.*

Theorem 8 Negative Cycle Optimality Conditions. *A feasible solution x^* is an optimal solution of the minimum cost flow problem if and only if it satisfies the negative cycle optimality conditions: namely, the residual network $G(x^*)$ contains no negative cost (directed) cycle.*

Proof: Suppose that x is a feasible flow and that $G(x)$ contains a negative cycle. Then x cannot be an optimal flow, since by augmenting positive flow along the cycle

we can improve the objective function value. Therefore, if x^* is an optimal flow, then $G(x^*)$ cannot contain a negative cycle. Now suppose that x^* is a feasible flow and that $G(x^*)$ contains no negative cycle. Let x° be an optimal flow and $x^\circ \neq x^*$. The augmenting cycle property shows that we can decompose the difference vector $x^\circ - x^*$ into at most m augmenting cycles with respect to the flow x^* and the sum of the costs of flows on these cycles equals $cx^\circ - cx^* \geq 0$, or $cx^\circ \geq cx^*$. Moreover, since x° is an optimal flow, $cx^\circ \leq cx^*$. Thus $cx^\circ = cx^*$, and x^* is also an optimal flow. This arguments shows that if $G(x^*)$ contains no negative cycle, then x^* must be optimal, and this conclusion completes the proof of the theorem. \square

For further details and examples, the interested reader may directly refer to Ahuja, Magnanti and Orlin, 1993.

5.3 Relationships with MCFP

According to Theorems 3 and 4, the optimal solutions of BLPP (2) can be characterized in terms of Nash equilibrium points. In this section, it is shown the tight relationship of a Nash equilibrium point with the solution of a particular MCFP. Let us introduce the following definition:

Definition 8 Define $G_r(x, y) = (N, A_r)$ as the residual network of G with respect to the flows x, y , where the arcset is $A_r = \{(i, j) : (i, j) \vee (j, i) \in A\}$. The arc capacities and costs are defined according to the following Table 5.1:

	$(i, j) \in A^P$	$(j, i) \in A^P$	$(i, j) \in A^Q$	$(j, i) \in A^Q$
Capacity	$\bar{u}_{ij} - x_{ij}$	x_{ij}	$\bar{u}_{ij} - y_{ij}$	y_{ij}
Costs	c_{ij}^{PP}, c_{ij}^{QP}	$-c_{ij}^{PP}, -c_{ij}^{QP}$	c_{ij}^{QQ}, c_{ij}^{PQ}	$-c_{ij}^{QQ}, -c_{ij}^{PQ}$

Table 5.1: Arc capacities and costs of the residual network $G_r(x, y)$

Trivially, $G_r(x, y)$ represents the network of the residual capacities that can be still used on the network G given the flows x, y . Accordingly, define $G_r^P(x)$ and $G_r^Q(y)$

the residual subnetworks corresponding to subnetworks G^P and G^Q , respectively.

As an example, consider the network in Figure 5.1. Figure 5.1.a) shows a network G and a feasible flow on it. Let the black arcs of G belong to A^P and the grey arcs to A^Q . Each arc $(i, j) \in A^P$ is characterized by a couple of values $(c_{ij}^{PP}, \bar{u}_{ij})$, indicated near the arc, and by the value c_{ij}^{QP} that is assumed equal to c_{ij}^{PP} , e.g., the arc (s, A) has $c_{sA}^{PP} = c_{sA}^{QP} = 1$ and $\bar{u}_{ij} = 2$. Similarly, each arc $(i, j) \in A^Q$ is characterized by a couple of values $(c_{ij}^{PQ}, \bar{u}_{ij})$ and $c_{ij}^{QQ} = c_{ij}^{PQ}$. Let node s be the origin and t be the destination with $b_s^P = 2$ and $b_t^P = -2$, respectively. Finally, let the thick arcs in G indicate arcflow different from zero. In particular, $x_{sA} = x_{sB} = x_{Ct} = x_{Dt} = 1$ and $y_{AC} = y_{BD} = 1$. Figure 5.1.b) shows the residual network $G_r(x, y)$ induced to the above described feasible flow. The values $(c_{ij}^{PP}, \bar{u}_{ij})$ or $(c_{ij}^{PQ}, \bar{u}_{ij})$ are again represented close to the arcs.

The following theorem allows to practically identify a Nash equilibrium point.

Theorem 9 *A point (x, y) is a Nash equilibrium point for BLPP (2) if and only if the residual subnetworks $G_r^P(x)$ and $G_r^Q(y)$ contain no negative, with respect to the values c^{PP} , and positive, with respect to the values c^{QQ} , cost cycles, respectively.*

Proof. Let us prove first the “only if” part by contradiction. Let (x, y) be a Nash equilibrium point. Then consider the frontier flow vector κ associated to it and assume that a residual $G_r^P(x)$ contains a negative cost cycle. Then, according to the Negative Cycle Optimal Conditions Theorem for the MCFP (see Section 5.2), player P can define a new flow x' , which imposes the same frontier vector κ , such that $c^{PP}x' < c^{PP}x$. Being κ constant, the feasibility of (x, y) implies the feasibility of (x', y) . Since $c^{PP}x' + c^{PQ}y < c^{PP}x + c^{PQ}y$, then (x, y) is not a Nash equilibrium point. Symmetrical arguments hold for player Q if $G_r^Q(y)$ contains a positive cost cycle.

Consider, now, the “if” part. Let (x, y) be given and since $G_r^P(x)$ and $G_r^Q(y)$ contain, respectively, no negative and positive cost cycles, we prove that (x, y) is a Nash equilibrium point. Consider player P (symmetrical arguments hold for player

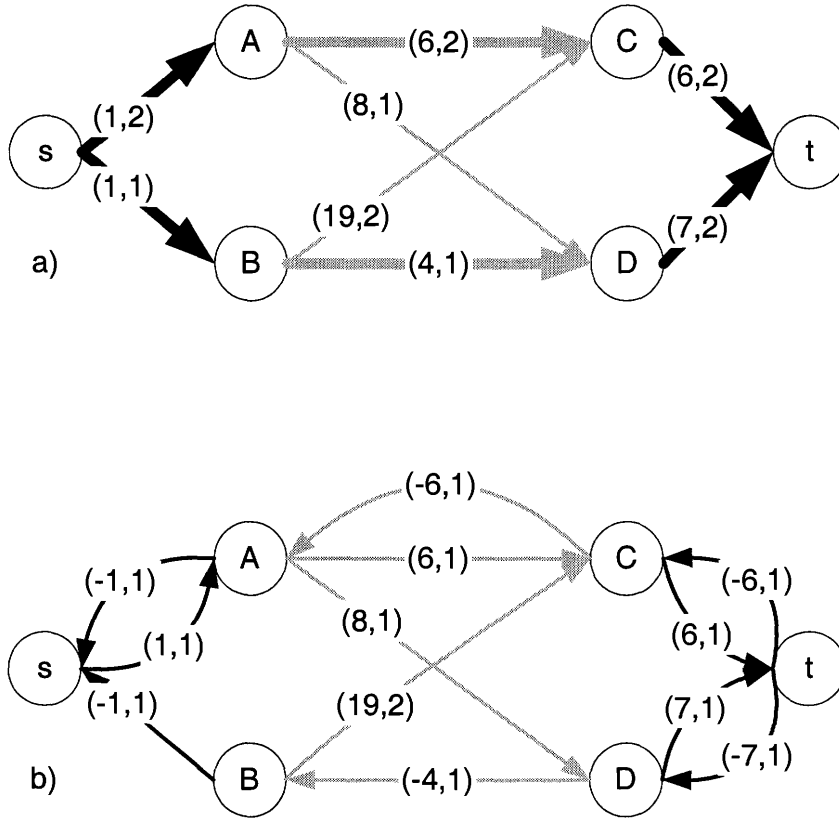


Figure 5.1: a residual network

Q). Since flows y are fixed, and hence the value of the frontier flows κ , Negative Cycle Optimal Conditions Theorem for the MCFP guarantees that P cannot define a different feasible solution x' such that $c^{PP}x' < c^{PP}x$. Being the cost component $c^{PQ}y$ fixed by hypothesis, the previous argument proves that (x, y) is a Nash equilibrium point. \square

As an example, consider again the network in Figure 5.1. Figure 5.1.a) shows a feasible flow on it, minimal for player P , but which is not a Nash equilibrium point. Actually, the residual subnetwork $G_r^Q(y)$ contains a positive cost cycle, as shown by the thick arcs in Figure 5.2.a). The cost of the cycle is 17 and the maximum flow that can circulate in it values one. The thick arcs in Figure 5.2.b) show the new

feasible flow (x, y') that is also a Nash equilibrium point. Such a flow is obtained summing the arcflow defined by the positive cycle to the original flow (x, y) . In (x, y') all the non null arcflows are equal to one.

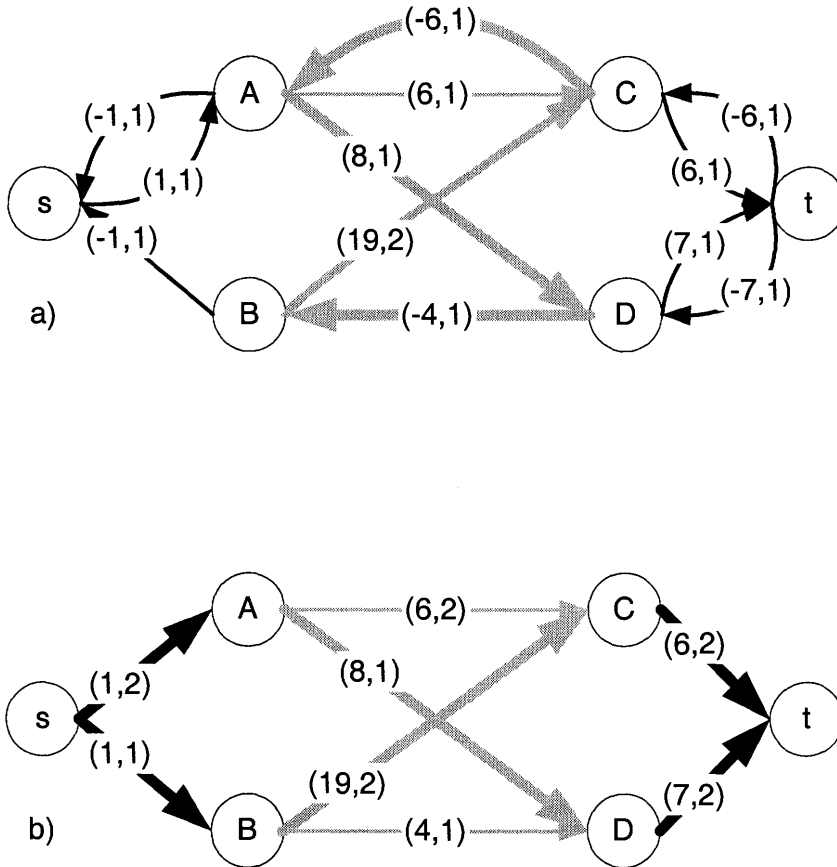


Figure 5.2: A positive cycle and an equilibrium flow

The following definitions and results, given a Nash equilibrium point, allow to determine a new one associated to a different frontier flow vector, when it exists.

Definition 9 Define $\hat{G}(x, y) = (\hat{N}, \hat{A})$ as the network of the residual paths of G with respect to the flows (x, y) , where the set of arcs \hat{A} includes a pair of directed arcs $(i, j)^P, (j, i)^Q$ for each ordered pair of nodes $i, j \in \hat{N}$ such that there exists a directed path from i to j in $G_r^P(x, y)$ and a directed path from j to i in $G_r^Q(x, y)$.

Each arc $(i, j)^P$ has associated a cost $\hat{c}_{(i,j)^P}^P$ corresponding to the cost of the minimal path from i to j on the residual network $G_r^P(x)$. Besides, it has associated a cost $\hat{c}_{(i,j)^P}^Q$ equal to the cost of the previously identified path when evaluated with costs c^{QP} . Accordingly, each arc $(j, i)^Q$ has a cost $\hat{c}_{(j,i)^Q}^Q$ corresponding to the cost of the maximal path from j to i on the residual network $G_r^Q(y)$ and it has also associated a cost $\hat{c}_{(j,i)^Q}^P$ equal to the cost of the previously identified path when evaluated with costs c^{PQ} . Each arc has also a capacity \hat{u} equal to the minimum capacity of the arcs of the associated path.

Definition 10 Given $\hat{G}(x, y) = (\hat{N}, \hat{A})$, define as an alternating cycle a (directed) cycle C made of an arc $(i, j)^P$ and an arc $(j, i)^Q$. Finally, define also δ_C the capacity of an alternating cycle C as the minimum capacity of the arcs defining C .

Figure 5.3.a) shows the network of the residual path $\hat{G}(x, y')$ induced by the flow in Figure 5.2.b). In particular, the thick arcs highlight a feasible flow (x_c, y_c) of value 1 in an alternating cycle with negative cost -14 with respect to ϕ^P . Here and in the following, with an abuse of notation, (x_c, y_c) indicate both a feasible flow in the alternating cycle C of $\hat{G}(x, y')$ and the flow of equal value on the arcs of G defining the two paths which compose C .

Theorem 10 Given a Nash equilibrium point (x, y) , assume that the associated network of the residual paths $\hat{G}(x, y)$ has an alternating cycle C , with $\delta_C > 0$. Then, any point $(x + x_c, y + y_c)$, where (x_c, y_c) is a feasible flow for C , is a new Nash equilibrium point.

Proof: Any point $(x + x_c, y + y_c)$ is trivially feasible by construction. Let us now prove that it is a Nash equilibrium point by contradiction. Assume that $(x + x_c, y + y_c)$ is not a Nash equilibrium point. Then, by Theorem 9, there should exist either at least a negative cycle in $G_r^P(x + x_c)$ or at least a positive cycle in $G_r^Q(y + y_c)$. However, the existence of a negative cycle in $G_r^P(x + x_c)$ contradicts the minimality of the path associated to the arcs in C . Symmetrical arguments hold for a positive cycle in

$G_r^Q(y + y_c)$. Actually, since (x, y) is a Nash equilibrium point, there are no negative cycles in $G_r^P(x)$. Networks $G_r^P(x)$ and $G_r^P(x + x_c)$ differ only for a path $p(j, i)$ from a frontier node j to another frontier node i . Then any negative cycle must include at least a part of such a path. Such an argument implies the contradiction that there exists a path from i to j on $G_r^P(x)$ which is less expensive than the one associated to $(i, j)^P$. \square

The thick arcs in Figure 5.3.b) show the new feasible flow $(x + x_c, y' + y_c)$ that is also a Nash equilibrium point. Such a flow is obtained summing the flow (x, y') in Figure 5.2.b) with the one defined by the alternating cycle highlighted in Figure 5.3.a). In $(x + x_c, y' + y_c)$ all the non null arcflows are equal to one except the one on arc (D, t) which is equal to 2.

5.4 A heuristic algorithm

In this section, an upper bound to the optimal solution to BLPP (2) is proposed when the subnetwork G^P is disconnected. In this context, a modification of the basic *Cycle-Cancelling* algorithm for MCFP problems based on Negative Cycle Optimal Conditions Theorem for the MCFP is introduced. Our algorithm, as the original one for MCFP, turns out to be pseudopolynomial, but it can be reduced to be polynomial by appropriate scaling of the network parameters. This reduction is identical to the corresponding one used for the algorithms of the MCFP, then it is not reported in this work. The interested reader may refer again to Ahuja, Magnanti and Orlin, 1993.

First the concept of a network without negative P cycles is stated.

Definition 11 *A residual network $G_r(x, y)$ is defined as negative P cycle free if none of the following conditions is true:*

(i) *there is a positive cycle in $G_r^Q(y)$;*

(ii) *there is a negative cycle in $G_r^P(x)$;*

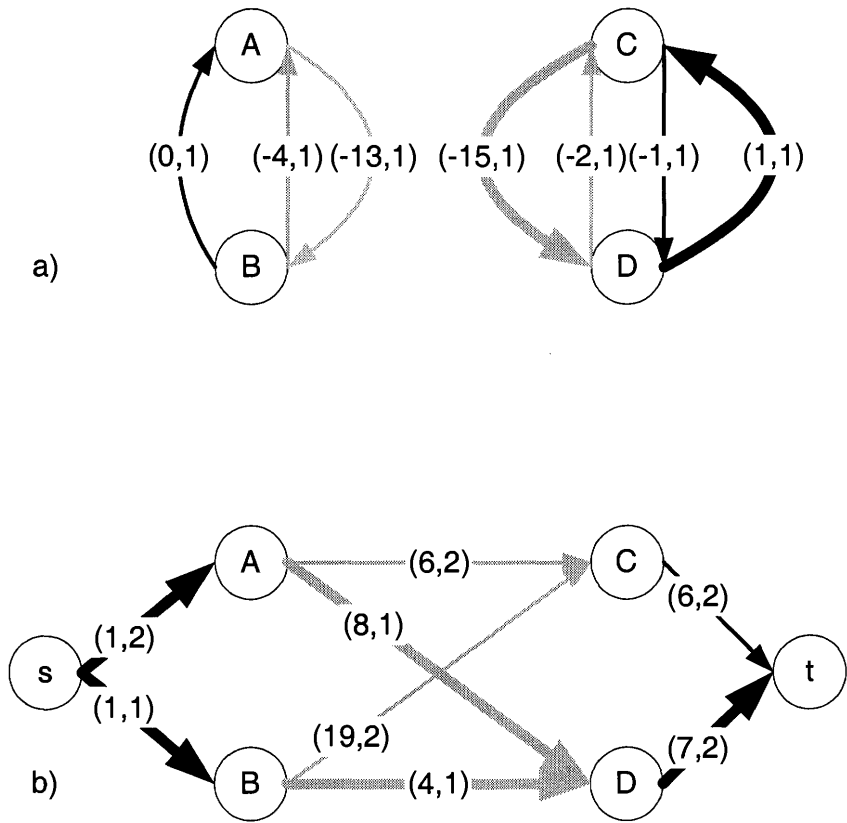


Figure 5.3: An alternating cycle and the induced equilibrium flow

(iii) there is an alternating cycle in $\hat{G}(x, y)$ such that the sum of the costs ϕ^P of the arcs of the cycle is negative.

According to this definition, the following result holds:

Theorem 11 For any optimal solution (x, y) to BLPP (2), the corresponding residual network $G_r(x, y)$ is negative P cycle free.

Proof. Since an optimal solution of BLPP (2) is a Nash equilibrium point (Theorem 6), the residual subnetworks $G_r^P(x)$ and $G_r^Q(y)$ contain no negative and positive cost cycles, respectively (Theorem 9). Hence the first two conditions are fulfilled. The third condition is proved by contradiction. If (x, y) is an optimal solution point,

assume that there exists an alternating cycle C in $\hat{G}(x, y)$ such that the sum of the costs c^P of the arc of the cycle is negative. Then Theorem 10 guarantees the existence of a different Nash equilibrium point $(x + x_C, y + y_C)$ with an associated frontier flow κ_C . By construction of C , $c^{PP}(x + x_C) + c^{PQ}(y + y_C) < c^{PP}x + c^{PQ}y$, then the new Nash equilibrium point $(x + x_C, y + y_C)$ is preferred by P and can be obtained by imposing the flow $x + x_C$ which, in turn, imposes κ_C . Hence (x, y) is not an optimal solution to BLPP (2). \square

On the basis of the above Theorem 11, starting from a feasible flow in $G(N, A)$ the following algorithm determines a Nash equilibrium point and moves, if it exists an alternating cycle in $\hat{G}(x, y)$ such that the sum of the costs ϕ^P of the arcs of the cycle is negative, to another Nash equilibrium point improving the value of ϕ^P . Since these are points in player Q rational reaction set $R^Q(x)$, they belong to the inducible region IR^Q identifying an upper bound to the optimal solution of BLPP (2). When no negative P cycles can be determined, the algorithm stops in a local optimum.

ALGORITHM Cycle Cancelling

1. determine an initial feasible flow (x^0, y^0) and set $k = 0$
2. look for negative P cycles C and modify the flow adding to (x^k, y^k) the maximum circulation (x_C, y_C) feasible for C obtaining $(x^{k+1}, y^{k+1}) = (x^k, y^k) + (x_C, y_C)$
3. set $k = k + 1$ and repeat step 2 until no negative P cycles can be determined.

The algorithm terminates in a finite number of steps. The initial feasible flow (x^0, y^0) is detected, as an example, first relaxing the bilevel linear problem by dropping the lower level objective function and solving the corresponding Minimum Cost Flow Problem over the whole network. If ties for player Q choice exist, zero cost cycles with respect to the values c^{QQ} in $G_r^Q(y)$ may be detected. In this case, under

the assumption of a cooperative attitude of player Q , a Nash equilibrium point is reached when in $G_r^Q(y)$ none of the zero cost cycles with respect to c^{QQ} is a negative cost cycle with respect to c^{PQ} . Hence the algorithm has to be slightly modified to encompass this situation.

Chapter 6

The freight traffic

The road freight traffic from Turkey to Central and Western Europe and vice versa suffered major disruptions because of the war in Balkans during the nineties. The International Transporters' Association of Turkey is the shipper controlling the quasi-totality of this traffic thus assuming the role of player P . He had to cope with an "adverse entity" able to modify the available capacity on some specific links his vehicles had to pass through. The region involved in the conflict may be represented as a connected subnetwork disconnecting the origin and the destinations of the road transportation network since alternative road routes are not easily affordable. Other possibilities, like the seaborne links now operating, did not exist at that time. Hence the whole freight traffic was performed using a single mode of transport. The models developed in this work allow the shipper to perform a worst-case analysis at the strategic level for this situation assuming that player Q wishes to maximize the costs he has to afford when going through the region under her control. In fact, it is meaningless to talk about the utility or the profit the war may seek to maximize. However, it becomes a sensible modelling when the utility of player Q is strictly related to the costs afforded by player P on this portion of the network. If player Q is maximizing her utility which corresponds to player P 's costs, automatically she plays to maximize player P 's costs. Hence the model represents a worst-case analysis for player P .

The situation UND had to face when the war in the Balkans started motivates our investigation on network games when handling uncertainty in the analysis of traffic and transportation systems at the strategic level is a major concern. As highlighted in Chapter 3, we propose an approach which extends and merges together results from different contexts. In Chapter 4 and in Chapter 5 we prove original theorems and implement a new heuristic algorithm. However, as described in Chapter 7 where some possible further research lines are introduced, some model refinements and advances are necessary to correctly depict the whole complexity of the shipper's past and present actual behavior. In fact, we do not address all the aspects and details of the actual freight transportation system, but we provide a sound theoretical framework to capture some of its main properties. In this context, this dissertation might be considered as a building block to be later exploited and further expanded when the modelling of the whole complex system is required. To this aim, relying on actual and updated data, we build a simplified graph representing the road network under study, even though some strict assumptions had to be introduced. Hence the results provided in Section 6.3 give an estimate of the economic advantages the shipper may benefit when modelling the worst-case analysis as an adverse player and solving the game with bilevel linear programming.

In the next Section 6.1, the main aspects of the road freight traffic within the existing freight transportation framework are briefly outlined. The following Section 6.2 provides the relevant assumption, information and data used in the graph construction. Finally, in Section 6.3 the Cycle Cancelling algorithm has been applied to the network just introduced and its results have been compared with the outcome obtained by using an exact enumeration procedure. Since it turns out that the percentage error of the heuristic algorithm is equal to $\varepsilon\% < 0,5\%$, we may claim that its performances are certainly highly satisfactory, at least in this specific example.

6.1 Description of the system

The main features of the road freight transportation system of the Republic of Turkey are now introduced. Since the most complete and updated data set backs to 1997, all the data are referred to this specific year and may be found in “Annual Report of UND 1997-1998” published in 1999.

The foreign trade of Turkey as per transport modalities is shown in the following Table 6.1. The export and import flows measured in terms of thousands of tons are represented considering both the turkish and foreign flag carriers.

Modality	Export		Import		Total	%
	TR Flag	Others	TR Flag	Others		
Sea	7.334	14.676	26.438	47.100	95.548	87,2%
Railway	145	6	96	85	332	0,3%
Road	4.791	1.643	3.325	1.811	11.570	10,6%
Airway	67	48	194	140	449	0,4%
Others	61	0	1.604	18	1.683	1,5%
Total	12.398	16.373	31.657	49.156	109.582	100,0%

Table 6.1: Foreign Trade of Turkey as per Transport Modalities in 1997

The last column indicates the incidence of each transport mode out of the total foreign trade and shows that the road transport is the second mode for freight transport accounting for a little more than 10% out of the total. Within this transport modality, the turkish flag carriers handle 8.116.588 thousands of tons representing the 70,1% of the freight transported by road.

In this context, UND (Uluslararası Nakliyeciler Derneği, i.e., International Transporters' Association) is the largest turkish association of the sector. In fact, 87% of transport companies active at international level are UND members. Their number increases each year: from 375 members in 1993 to 623 members in 1997 and 794 in 1998. Since the breakdown of the freight transported by UND members and UND

non members is not available, we approximate by saying that all the road freight traffic is performed by UND members. Hence UND is a shipper which controls 7,4% of the overall freight traffic of Turkey. UND fleet is composed of 23.629 tractors and 28.915 (semi-) trailers, 4.789 trucks and 640 tankers providing a payload capacity of 697.588 thousands of tons. In 1997, UND vehicles globally made 375.181 transports. Each transport can be made in two different ways: either it is completely a road transport, either part of it is performed by road and in the remaining part the vehicle is load on a RO-RO vessel. Some destinations are reached only by road (e.g., Iran and Middle East). By contrast, European countries and the Republics of the former Soviet Union are connected either by road either by RO-RO lines.

Since we are interested in the way the shipper afforded the Balkans war during the nineties, the freight traffic between Turkey and Central and Western Europe is considered. Detailed data are available for the export component only, i.e., from Turkey towards Europe. Obviously, similar analysis may be performed for the freight traffic in the opposite direction. Table 6.2 depicts the breakdown of UND export transports as per aggregate destinations in 1997.

Destination	Export transports
Europe	110.028
Middle East	35.211
Ex Soviet Union	71.532
Total	216.771

Table 6.2: UND Export Transports as per Aggregate Destinations in 1997

According to 1997 figures, approximately 65.000 (i.e., 59%) of these road transports to Europe actually perform its entire travel by road. The remaining 41% loads the vehicles on RO-RO vessels operating on four different lines connecting Turkey to Italy. The total number of trips to be considered is further reduced because ca. 15.000 journeys end before entering or inside the Balkans. Hence the final amount of flow units passing through the region controlled by the adverse entity and reaching

Central and Western Europe is nearly equal to 50.000 transports.

6.2 A simple graph

According to the assumptions made in Section 1.2 and in Chapter 4, the bilevel linear games under study apply to a single origin, single destination and acyclic network disconnected by the region controlled by the adverse entity. At the aggregate strategic level, this framework depicts to some extent the situation UND faces when considering the flow from Turkey and the North West of Europe. This destination far from the Balkan region has also been chosen to make the network not too trivial, i.e., with a significant number of nodes and arcs in both the player P and Q regions. Hence the total traffic taken into account from 50.000 units is further reduced to 28.000 vehicles and an acyclic network which includes a single source (Turkey) and a single sink (near Paris) is considered. As a further assumption, for balancing the size of sub-networks G^P and G^Q in terms of number of arcs and nodes, the adverse region has been enlarged and comprehends countries which were not directly involved in war operations (i.e., Rumania, Bulgaria, Greece, Hungary, Slovakia, Czech Republic and Eastern Austria).

The graph is composed of 99 nodes and 181 arcs. The nodes differentiate in 10 frontiers nodes (Table 6.4), 36 nodes in the subnetwork G^Q (Table 6.3) and 53 nodes in the subnetwork G^P (Table 6.5). Actually, for modelling purposes each node representing a country border gate is split in two parts and a dummy arc connecting these two nodes has been inserted. For sake of clarity, only one of these two nodes is shown in the just mentioned Tables.

Player P controls a subnetwork composed of 100 arcs. The others 81 links representing the connections within the Balkans and Eastern Europe form a connected subnetwork. In Figure 6.1 the network layout is provided. The dark arcs belong to region P and the light arcs are controlled by player Q . Only the main road links have been considered (motorways or highways). The capacities are calculated taking

into account the total number of transit permits available for each country. This figure is annually fixed in bilateral Joint Committee Meetings. Player P 's costs, c^{PP} and c^{PQ} , are the average generalized costs derived as a function of lengths and transfer times in the physical links. Since player Q does not have profits or losses for the flows passing through the P zone, c^{QP} are set equal to zero. It is also assumed that the profits she earns for each unit of flow going through the arcs under her control are equal to the costs afforded by the shipper when traversing these arcs, i.e., $c^{QQ} = -c^{PQ}$. All the relevant data required to calculate these figures are collected in the UND Annual Sector Report 1997-98. In Table 6.6 and Table 6.7, the origin and destination nodes of each arc are identified for both regions. The values of c^{PP} or c^{QQ} (thousands of Italian Lira) and u^P or u^Q (thousands of vehicles) are also provided for each link. Dummy arcs connecting the two nodes representing a national border gate are also shown.

Node	Label	Node	Label	Node	Label
105	GR-YU	125	Praha	198	Szeghed
106	BG-YU	147	Salzburg	199	Sibiu
107	BG-R (Ruse)	187	R-H	207	Beograd
108	BG-R (Siret)	188	SLO-A	230	Botevgrad
109	Nis	190	Skopje	232	Caransebes
110	Bucarest	191	Cacak	233	Craiova
112	Ljubljana	192	Titograd	234	Hateg
113	SLO-H	193	Dubrovnik	235	Timisoara
114	Budapest	194	Sarajevo	236	Sebes
117	H-A	195	Novisad	237	Brasov
118	Wien	196	Zadar	238	Turda
119	H-SK	197	Zagreb	239	Oradea

Table 6.3: Nodes in G^Q

Node	Label	Node	Label
102	TR-GR (Ipsala)	141	I-SLO
103	TR-BG (Kapikule)	143	I-A (Tarvisio)
104	TR-BG (Derekoy)	148	A-D (Salzburg)
124	D-CZ	158	A-D (Passau)
140	Trieste	206	Dresden

Table 6.4: Frontier nodes

Node	Label	Node	Label	Node	Label
1	Turkey	155	Köln	205	Jena
2	France	156	D-B	208	Stuttgart
100	Izmit	157	Bruxelles	231	Portile de Fier
101	Istanbul	158	F-B	240	Bellinzona
136	Genova	159	Amsterdam	241	Bad Ragaz
137	I-F (Ventimiglia)	160	D-NL	242	Ulm
138	Milano	161	Osnabrück	243	Ansbach
139	Verona	173	Narbonne	244	Würzburg
142	Palmanova	174	Oange	245	Frankfurt
144	I-F (Frejus)	175	Lyon	246	Dortmund
145	I-A (Brennero)	176	Dijon	247	Clermont Ferrand
146	A-D (Innsbruck)	177	Bordeaux	248	Brive
149	München	178	Paris	249	Fontainebleau
150	Linz	200	Zürich	250	Orleans
151	A-D (Passau)	201	Poitiers	251	Le Mans
152	Nürnberg	202	Toulouse	252	Rennes
153	Berlin	203	Metz	253	Nantes
154	Hannover	204	Kassel		

Table 6.5: Nodes in G^P

ON	DN	c^{PP}	u^P	ON	DN	c^{PP}	u^P	ON	DN	c^{PP}	u^P	ON	DN	c^{PP}	u^P	ON	DN	c^{PP}	u^P
1	100	81	1000	149	242	806	66	175	176	1337	14	208	203	2194	66	247	250	3738	14
100	101	908	1000	152	243	292	66	175	200	3285	14	208	155	2604	66	248	177	2413	14
101	102	3688	1000	152	243	292	66	175	247	2250	14	208	245	1403	66	248	201	2638	14
101	103	3530	1000	152	205	1368	66	176	249	1604	14	240	241	1348	66	249	178	535	14
101	104	3774	1000	152	204	2488	66	177	201	1757	14	240	200	1208	42	250	178	1027	14
124	152	941	66	153	154	2594	66	178	2	81	1000	241	242	1768	66	250	253	2696	14
136	137	1372	24	154	161	965	66	200	176	3089	14	241	200	632	40	250	251	1725	14
137	1371	627	1000	155	159	1813	66	200	208	1768	66	242	243	1411	66	251	178	1785	14
138	136	1258	24	155	156	941	66	201	250	1486	14	242	208	528	66	252	251	1330	14
138	144	2307	24	156	1561	627	1000	201	253	2238	14	243	208	1097	66	253	251	1598	14
138	240	847	24	157	1581	528	18	202	177	1736	14	243	244	729	66	253	252	946	14
139	145	2116	24	158	178	2119	14	202	248	2663	14	244	204	1097	66	1371	174	2823	14
139	138	1328	24	159	157	1803	7	203	1561	1882	18	244	245	896	66	1411	140	132	24
140	142	440	24	160	1601	627	1000	203	178	2299	14	245	246	1486	66	1461	149	677	66
142	139	2049	24	161	155	986	66	204	154	1507	66	245	155	1188	66	1461	241	1741	40
143	142	827	24	161	160	667	66	204	246	1389	66	246	154	1236	66	1481	149	1170	66
144	1441	627	1000	173	247	5050	14	205	204	1319	66	246	155	535	66	1511	152	1891	66
145	1451	627	1000	173	202	1014	14	205	153	1389	66	247	249	3863	14	1561	157	941	18
146	1461	627	1000	174	173	1645	14	206	153	1118	66	247	248	2325	14	1581	158	627	1000
149	152	1433	66	174	175	1733	14	206	205	1014	66	247	201	3625	14	1601	159	1627	7

Table 6.6: Arcs in G^P

ON	DN	c^{QQ}	u^Q	ON	DN	c^{QQ}	u^Q	ON	DN	c^{QQ}	u^Q	ON	DN	c^{QQ}	u^Q	ON	DN	c^{QQ}	u^Q
102	1021	627	1000	117	1171	627	1000	190	109	1661	39	230	231	3713	6	1031	107	8642	6
103	1031	627	1000	118	150	1583	40	191	192	3922	39	230	106	788	6	1041	108	5209	6
104	1041	627	1000	118	125	2563	40	191	194	2906	39	231	233	1475	42	1051	190	1850	39
105	1051	627	1000	118	143	4053	40	192	193	2859	39	231	232	1350	42	1061	109	1288	39
106	1061	627	1000	119	1191	627	1000	193	194	4825	39	232	234	900	42	1071	110	918	42
107	1071	627	1000	125	206	1875	14	193	196	5906	39	233	110	2036	42	1131	113	627	1000
107	230	4563	6	125	1241	2483	14	194	195	1775	39	233	234	3234	42	1171	118	541	40
108	1081	627	1000	141	1411	627	1000	195	197	3455	39	234	236	500	42	1241	124	627	1000
109	207	2938	39	143	1431	627	1000	196	197	4484	39	234	235	2750	42	1431	147	2231	40
110	237	2125	42	147	148	108	40	196	140	4703	39	235	199	797	42	1441	175	2587	14
110	236	3363	42	148	1481	627	1000	197	140	2339	5	236	199	2700	42	1451	146	1256	40
112	141	783	3	150	147	1249	40	197	112	1688	5	236	238	1734	42	1871	114	3229	39
112	188	718	3	150	151	844	40	198	195	2913	39	237	238	3984	42	1881	143	488	40
113	112	2261	3	151	1511	627	1000	199	198	1313	39	238	239	2275	42				
114	117	1919	39	187	1871	627	1000	199	187	688	42	239	199	1463	42				
114	119	1148	39	188	1881	627	1000	207	195	382	39	1021	105	6529	10				
114	1131	3975	39	190	191	1488	39	207	235	1975	39	1031	106	5152	6				

Table 6.7: Arcs in G^Q

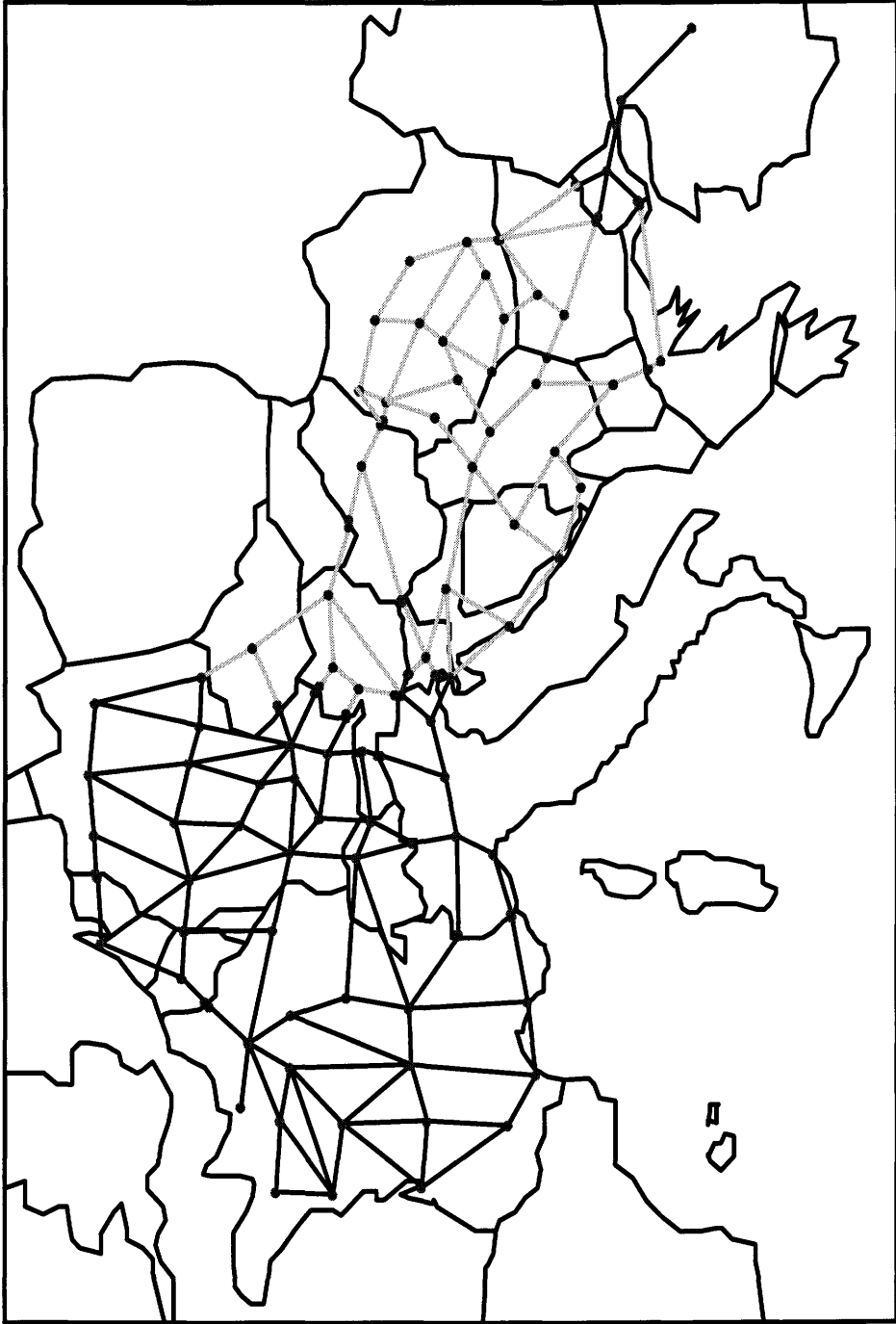


Figure 6.1: Network layout

6.3 Numerical experiments

Since $c^{QQ} = -c^{PQ}$, multiple optima for player Q do not vary the value of ϕ^P . Thus the following steps of the *Cycle Cancelling Algorithm* described in Section 5.4 identify a local optimum:

1. An initial feasible flow (x^0, y^0) is detected first relaxing the bilevel linear problem by dropping the lower level objective function and solving the corresponding Minimum Cost Flow Problem over the whole network. The generalized cost obtained is 1.063.710 MI t l. Then, given the divergences at the frontier nodes, the Maximum Cost Flow Problem is solved for the Q region. At the equilibrium point, the cost to be afforded by the shipper turns out to be equal to 1.385.868 MI t l. Note that, the presence of negative (positive) cycles in $G_r^P(x)$ ($G_r^Q(y)$) is ruled out due to the minimality (maximality) of the solution just obtained for the P (Q) region.
2. A first alternating cycle C_1 is identified. Then, adding to (x^0, y^0) the maximum circulation (x_{C_1}, y_{C_1}) feasible for C_1 , a new equilibrium point $(x^1, y^1) = (x^0, y^0) + (x_{C_1}, y_{C_1})$ is obtained. The new value of P objective function is equal to 1.370.979 MI t l.
3. A second alternating cycle C_2 is identified. Then, adding to (x^1, y^1) the maximum circulation (x_{C_2}, y_{C_2}) feasible for C_2 , a new equilibrium point $(x^2, y^2) = (x^1, y^1) + (x_{C_2}, y_{C_2})$ is obtained. The new value of P objective function is equal to 1.329.003 MI t l.
4. No further alternating cycles exist, then the algorithm stops.

The global optimum value for the considered problem is 1.324.927 MI t l and is obtained with a branch-and-bound procedure. Hence, the percentage error is equal to $\varepsilon\% < 0,5\%$. The first rational solution used to initialize the branch-and-bound procedure was the one defining the equilibrium point at step 1 of the above Cycle

Cancelling Algorithm. The same procedure, after exploring less than ten nodes of the enumeration tree, determines the solution whose value is 1.370.979 Mitl. Finally, after visiting 1.247 nodes the optimal solution is established.

These results prove the benefit of such bilevel linear programming modelling. The generalized cost obtained relaxing the bilevel linear problem by dropping the lower level objective function and solving the corresponding Minimum Cost Flow Problem over the whole network depicts the case where the shipper fully controls all the arcs of the network his flow goes through. Unfortunately, this favorable situation has to be ruled out when an adverse entity imposes her assignment on some given arcs. If the shipper is aware of such hostile attitude, some actions to prevent it has to be taken. In fact, if player Q is freely allowed to reassign the flow in her region without any further moves from player P , his costs would increase by 30%. When the shipper starts solving the bilevel linear programming game by identifying new Nash equilibrium points, the first flow reassignment would lower his extra-costs to a value equal to 28% more than the minimum cost flow assignment. In the following, a second alternating cycle allows player P to detect a new Nash equilibrium point which leads to further reduction of his extra-costs to 24,8% more than the minimum cost flow assignment. Since no further alternating cycles exist, then the algorithm stops. As a consequence of this game, the shipper understands that the perfect knowledge of the behavior of his hostile player allows him to prevent her moves minimizing his losses due to her actions on the subnetwork his units must go through. When the shipper is not able to solve the exact bilevel linear programming problem (e.g., because of the large size of the network), the heuristic algorithm detects in polynomial time a solution which is an upper bound for the global optimum. In the specific example under investigation the heuristic and the exact solutions are very close: only 0,3% far apart.

Chapter 7

Further researches

Different extensions of the models and the algorithm developed may be easily envisaged both from the theoretical and the application side. These advances would provide either faster local or global search algorithms either more complete models representing in deeper detail the actual system and the interactions among the actors involved. Hence a decision support system for the shipper's decision making process at the strategic level can be built and effectively used by freight transportation practitioners. In this Section, we briefly introduce the main lines of development to be possibly followed according to the issue we are more interested in.

7.1 The Algorithm

The heuristic algorithm introduced in Chapter 5 has been applied on a simple network and in Section 6.3 its results have been compared with the outcome of a Branch & Bound procedure. In this specific case heuristics perform very well with a percentage error equal to 0,3% with respect to the global optimum. Of course, this comparison is affordable for small problem instances only, as for the network we propose in Section 6.2. As a further research on this issue, it would be interesting to test the algorithm on larger, more complex and topological different networks to possibly identify its average and worst-case performances. Instances specifically

devoted to be used as test-beds for network algorithms are available on the Internet and could be considered as a reliable starting point for this task.

A further interesting investigation stems from the possibility to integrate our approach with other exact procedures and implement an (enumeration) algorithm able to identify, on the average, in affordable time also the global optima of large instances of the two games. In fact, bilevel linear programming problems are *NP*-hard problems, then the only certain way to attain a global optimum relies on enumeration algorithms. Bard (1998) provides an updated review of the main exact approaches for solving BLP problems. According to our main results, the new enumeration algorithm would restrict the search for the global optimal solution to the set of the Nash equilibrium points. In addition, it would use the local optimum identified in polynomial time by the Cycle Cancelling Algorithm as an upper bound for pruning the Branch & Bound tree of the cost minimization problem.

7.2 The Model

Many developments are possible on this issue. Disregarding the initial main assumptions (see Section 1.2) the investigation of the consequent extended models may be performed.

- **Costs** We first drop the assumption that costs are data already given. It might be more realistic to consider the case where player Q decides not only on the arc capacities but also on the arc costs. In particular, the following situation may be depicted: player Q plays the role of an Authority looking for regulation of player P 's freight traffic. For instance, she may decide to support a specific transport modality and discourage the usage of another one. This behavior can be accomplished by imposing tolls or fares on the links of the penalized mode such that the shipper would suffer higher costs when using it rather than passing through the toll-free arcs. Hence player Q has to decide on which arcs (of the penalized mode) and at which value the tolls should be fixed, provided

that her overall profit should attain at least a minimum level. If we further assume that player Q bears some expenses in the revenue collection phase, she would also minimize the number of her arcs where tolls or fares are imposed. In a simple network composed of a single origin and a single destination, player Q would solve a minimum capacity cut problem. When multi-origins and multi-destinations are considered, the computational complexity increases and the problem becomes NP -hard. In addition, a decision on the arc capacities has also to be taken when the adverse entity still wishes to control the flow in the region under her control. As usual, to minimize the effects of these undesirable higher costs on his transport system, the shipper would play a bilevel linear programming game.

- **Disconnection** A different modelling issue assumes that player Q network does not disconnect origin and destination. In this case, model (2) can no longer be derived from model (1). Hence the remaining of this work when player P plays first no longer holds. The three-stage model has to be faced and conditions for the existence of the optimal solution and methods to identify it have to be investigated. Finally, new algorithms to detect it have to be implemented. This extension would allow the shipper to partially or fully overcome the region controlled by the adverse entity and consider alternative routes which are completely under his control. In addition, different competing transport modalities between the same O/D pairs may also be compared.
- **Competition** At the strategic level, competition is often a major concern in the decision making process of any shipper. As we mentioned in Section 3.2, some results on multi-level noncooperative games where several actors compete for taking portions of the link capacity over a network are available in the communication sector. Unfortunately, these models consider only a very simple network topology: a single origin and a single destination connected by parallel arcs. This basic structure is clearly unfit to depict transportation

environments. Indeed, the extension of this approach to a more general graph topology would provide an useful framework to be further exploited when modelling several actors in competition over the same network.

- **Minor issues** More trivial extensions consider multiple origins and/or multiple destinations. Also the assumption of acyclic network may be easily disregarded, provided that the appropriate algorithm to prevent positive cycles in the subgraph G^Q is integrated.

7.3 The Freight Traffic

The situation UND had to face years ago when the Balkan war suddenly started gave us the stimulus to investigate the actions a shipper can take when he is not able to fully control all the arcs of network his goods go through. From the available data the network in Fig. 6.1 has been built and the Cycle Cancelling Algorithm has been tested on this simple, but not trivial, real graph. However, the actual freight traffic system from Turkey to Europe could be investigated in more detail.

As explained in the previous Chapter 6, all the data we refer to are actual data and particular accuracy has been given to correctly define the average generalized costs and capacities of each arc. Nevertheless some more refinement would be possible provided more complete data sources are available. In this field, similar advances calibrating the generalized cost function of a modal choice model comparing sea, road and rail freight transportation in the Adriatic sea context (with particular emphasis on the Port of Trieste) are currently under study (Dall'Acqua, 2002).

Relying on the different theoretical modelling issues introduced in the previous Section 7.2 to be further developed in the future, we might expand our view to capture several aspects of the whole UND freight traffic system. First, the real multi-origin and multi-destination network could be established. Second, all the different transportation modes used by UND vehicles could be considered simultaneously. In fact, by disregarding the disconnection property of G^Q the actual competition among

road and seaborne paths can be represented. The interest in this line of research is also testified by some recent studies where a macro-economic analysis comparing the road and RO-RO transport in the Mediterranean (including the freight traffic from Turkey to Europe) is performed (Torbianelli, 2000).

The comparison between the results obtained when performing the worst-case analysis and the actual system behavior is another important point to be further developed. However, difficulties arise because the freight traffic is continuously subject to major variations. On the sea side, the number of weekly calls at the Port of Trieste raised in the last year from 11 to 14. It means that the seaborne link increased its capacity by about 30.000 vehicles affecting shipper's decision making process at the strategic level. On the road side, the past and present international political situation has currently strong influence at each hierarchical level. At the strategic level, war operations quickly moved through different neighboring regions (i.e., Bosnia, Kosovo, Macedonia etc.) thus modifying different times the capacity of the same arc. This prevented the adverse entity to attain a steady state. At the lower levels, the highly variable variance of the time spent in each trip to cross the several country border gates has always been a major concern. This situation even worsened after September 11, 2001, attacks. In fact, stricter regulations have been enforced for customs and anti-terrorism inspections. At the time being, we are not in the position to predict whether they would significantly affect (e.g., as a large annual variation of the amount of vehicles through some specific arc) the network when considering its aggregate view. By contrast, the acquisition of a new RO-RO vessel (a shipper's strategic decision) has certainly much more impact on his whole transport system. Because of this high degree of uncertainty and dynamic evolution of the environment wherein UND normally operates, stochastic and dynamic network models might be considered.

Chapter 8

Conclusions

Noncooperative games in their extensive form may be appropriate to support shipper strategic decisions when the whole transportation network is not under his full control. In this context, a game between two players acting on the same road transportation network is considered. The first player aims at minimizing the transportation costs, whereas the second player aims at maximizing her profit (or, in general, her utility) that is proportional to the flow passing through the arcs under her control. This framework may describe, as an example, the situation where restrictions are imposed by some alpine country on the number of trucks allowed to cross it by road each year. A different context involving the presence of a second agent on the shipper's network occurred when the International Transporters' Association (UNDT) of Turkey had to face when the war in the Balkans started. The war is considered as an hostile player or an adverse entity introducing major limitations on the link capacities. The situation motivates our investigation on network games when handling uncertainty in the analysis of traffic and transportation systems at the strategic level is a major concern. As highlighted in Chapter 3, we propose an approach which extends and merges together results from different contexts, namely multi-actor sequential models, network game models and bilevel linear programming models. The games under study are eventually modelled in Chapter 4 as bilevel linear programming formulations. In Chapter 5 we prove that the op-

timal solution of the bilevel linear programming problem under concern is a Nash equilibrium point. Relying on the Negative Cycle Optimality Conditions, original theorems and properties identifying such Nash equilibria have been proved. This results allowed to define the Cycle Cancelling algorithm which is a heuristic algorithm detecting a local optimum only, i.e., an upper bound for the global optimum. In Chapter 6 the Cycle Cancelling algorithm has been tested on a simple network and its results have been compared with the outcome obtained by using an exact enumeration procedure. Since it turns out that the percentage error of the heuristic algorithm is equal to 0,3 %, we may claim that its performances are certainly highly satisfactory, at least in this specific example. Even though some strict assumptions had to be introduced, the just mentioned graph has been built relying on actual and updated data related to road freight traffic from Turkey to Western Europe. Hence it is given an estimate of the economic advantages the shipper may benefit when modelling the worst-case analysis as an adverse player and solving the game with bilevel linear programming. Different extensions of the models and the algorithm developed are described in Chapter 7 both from the theoretical and the application side. These advances would provide either faster local or global search algorithms either more complete models representing in deeper detail the actual system and the interactions among the actors involved. Hence a decision support system for the shipper's decision making process at the strategic level can be built and effectively used by freight transportation practitioners.

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