



Measuring airline hub timetable co-ordination and connectivity: definition of a new index and application to a sample of European hubs

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Abstract

In this paper a new index for measuring the timetable co-ordination of an airline hub is proposed, with application to a sample of European hubs. This index is both quite accurate and easy to use, so that it may prove a useful schedule analysis tool for airline managers. In section 1 of this paper, the definition of “wave-system structure” and “ideal wave” are given. In section 2 the problem of measuring hub connectivity and hub timetable co-ordination is discussed. Then, both the so-called “weighted indirect connection number”, which is an index for measuring hub connectivity, and the “connectivity ratio”, which is an index for measuring hub timetable co-ordination, are described, in section 3 and 4 respectively. In section 5, a new index for measuring hub timetable co-ordination is illustrated: the “weighted connectivity ratio”. Some examples of hub timetable co-ordination measures performed with the new index are reported in section 6.

Keywords: Airline hub; Wave-system; Connectivity; Schedule co-ordination; Weighted connectivity ratio.

Introduction

Following deregulation of the airline industry, “hubbing” was soon developed by most of the major companies as a crucial schedule-based product feature (Doganis, 2002).

Federal Express first developed effective hubbing in the aviation industry in the 1970s, using its airport base at Memphis for the carriage of overnight express parcels throughout the United States. Effective hubbing requires that flights from different airports, which are at the “spokes” of a network, arrive at the “hub” airport at approximately the same time. The aircraft then wait on the ground simultaneously, in

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order to facilitate the rapid interchange of passengers and baggage or, in the case of Federal Express, of express parcels. Afterwards, flights depart in quick succession back out along the spokes. The process that involves a wave or bank of arrivals followed shortly afterwards by a wave of departures is described as a “complex” or “wave”.

An airline which operates a “hub-and-spoke” network offers flights between its one or few hub airports and its spoke airports. The airline co-ordinates arrivals and departures at its hub in order to minimize delays for passengers continuing through the hub to final destinations on spokes other than the one on which they originated. This strategy targets passengers travelling between origins and destinations for which traffic volume is not sufficient for conveniently frequent non-stop flights. Passengers departing from any non-hub origin to other destinations in the network generally proceed first to the hub. Similarly, passengers travelling to non-hub destinations generally arrive at these destinations from the hub.

A hub-and-spoke network to be developed requires (Danesi and Lupi, 2005):

- 1) spatial concentration of the network structure,
- 2) temporal co-ordination of the flight schedules at hub airports in waves,
- 3) integration of via-hub sub-services, i.e. the airline has to sell passengers one via-hub fare, from passengers’ point of origin to passengers’ final destinations, and has also to provide automatic baggage transfer at the hub.

European airlines often pride themselves on having operated hub-and-spoke networks long before US airlines. Actually, they only operated spatially concentrated networks not co-ordinated in time, before deregulation. This pattern was a result of pre-deregulation bilateral air service agreement constraints, which polarised flag carrier networks around their main national airports. Therefore, pre-deregulation star-shaped networks of European airlines are not equivalent to hub-and-spoke networks, as long as timetable co-ordination is lacking and a substantial number of services provide transfer only by chance (Burghouwt and De Wit, 2005).

Indeed, the main characteristics of hub-and-spoke services make the definition of “hub” a somewhat contentious one (Dennis, 1994b; Button, 2002; Holloway, 2003). In the US the word “hub” has been traditionally used to define any large airport. In particular, the Federal Aviation Administration defines as a “hub” airport any US airport generating 0.05 per cent or more national passenger boardings, without loading the term with any scheduling or other operational implications¹.

Colloquial and journalistic usage of the term “hub” often refers either to airports that handle large volumes of traffic or to large airline operational bases. Therefore, also in this case the term “hub” has not got any connotation of transfer traffic and does not consider any inbound and outbound flight schedule co-ordination. However, the concept of “hub” simply as a large airport no longer remains very valid (Dennis, 1994b). In recent years, the term “hub” has become more closely associated with an integrated interchange point where one or more specific airlines concentrate traffic and operate waves of flights².

¹ According to FAA, 0.05 - 0.249 per cent of passenger boardings qualifies an airport as a “small hub”, 0.25 – 0.999 per cent as a “medium hub” and one per cent or over as a “large hub”.

² US experience and more recent experience in Europe suggest that for an airport to be effectively a hub it must possess some critical attributes (Doganis, 2002; Bootsma, 1997; Dennis, 1994b): (1) a central geographical location in relation to the markets which the airport is to serve, in order to minimize airline and passenger travel time and costs; (2) ample operational and environmental air-side capacity, which allows the airline to schedule a large number of flights simultaneously, in order to operate its complexes of flights efficiently; (3) ample land-side capacity and suitable configuration of the terminal, which allows

1. Wave-system structures and ideal waves

Temporal co-ordination of an airline hub-and-spoke network refers to the hub timetable organisation, which is based on a certain wave-system structure. According to the definition proposed by Bootsma (1997) and by Burghouwt and De Wit (2005), a wave-system structure of an airline hub is specified, given:

- 1) the number of flight waves,
- 2) the time interval between the same points of two consecutive waves, i.e. the so-called “hub-repeat cycle” (Dennis, 1994b),
- 3) the short-haul and medium-haul aircraft stabling locations.

The short-haul and medium-haul aircraft stabling locations refer to the locations where the airline continental fleet stays overnight. Generally speaking, an airline which operates a hub-and-spoke network can stable aircraft at the hub, at the spokes or, in the so-called “dual stabling” case, both at the hub and at the spokes³.

Let N be the number of waves of a wave-system structure, H the hub-repeat cycle and S a dummy variable, such that: $S = 0$, if airline continental fleet is stabled at the hub; $S = 1$, if airline continental fleet is stabled at the spokes; $S = 2$, in case of dual stabling. Hence, the triple (N, H, S) identifies an airline wave-system structure univocally. The three factors that define a wave-system structure are interrelated. For instance, H will tend to decrease, if N increases, and S affects N too.

An example of airline wave-system structure is represented by the triple $(2,6,0)$: this is a 2-wave structure based on a 6-hour hub-repeat cycle, with continental fleet stabling at the hub and one wave which results broken overnight. On the other hand, considering the structure $(2,6,1)$, i.e. switching the continental fleet from stabling at the hub to stabling at the spokes, the broken wave becomes a complete one. In order to increase the number of waves, the hub-repeat cycle has to decrease and wave-system structures such as $(3,5,0)$ and $(4,4,1)$ can be obtained. A hub-repeat cycle of about 4 hours is the maximum that can be reached by 4-wave structures. For adding other waves, more advanced wave-system structures are usually developed, by using a combination of stabling at the hub as well as at the spokes ($S = 2$). In particular, wave-system structures such as $(5,3,2)$ and $(7,2,2)$ are characterised by a value of the hub-repeat cycle which is shorter than the continental fleet round-trip times (intra-European round trip times are typically clustered around five hours) and this forces the aircraft to operate only a limited number of waves within the wave-system structure⁴.

the airline to process a large number of transfer passengers and baggage very rapidly; (4) satisfactory average weather conditions, in order to prevent the reliability of airline hubbing operation to be frequently jeopardized by fog, snow or thunderstorms; (5) ideally, strong local demand from/to the hub, which helps to support a wide range of airline services from the airport; (6) obviously, an airline willing and prepared to operate hubbing at the airport, scheduling flights in complexes.

³ About the stabling location of short and medium-haul aircraft, it should be noted that airlines have traditionally located as many aircraft as possible at their home base, during the nighttime period. This minimises lodging expenses for crew and facilitates servicing and maintenance. However, this arrangement is not ideal in hub operations, because it leads to one complex of connections being broken overnight (the last bank of arrivals in the late evening is followed only the day after, typically in the early morning, by one bank of departures).

⁴ For instance, some destinations may be linked to the hub with flights that depart from one wave, skip the next wave and return to the hub in the second next wave (Bootsma, 1997).

Focusing on the structure of a single wave of flights within an airline hub wave-system, an “ideal wave” or “basic wave” of flights (Bootsma, 1997; Burghouwt and De Wit, 2005) can be defined as a complex of incoming and outgoing flights, structured such that each incoming flight creates a bookable connection with every outgoing flight.

In an ideal wave of flights, the arrival wave, the transfer period and the departure wave can be identified. The arrival wave corresponds to the time-window in which the arriving flights are scheduled to arrive at the hub. The transfer period, which allows passengers and baggage to transfer from the arriving flights to the departing flights, corresponds to the time-window in which no flight is scheduled to arrive or depart to/from the hub. Finally, the departure wave corresponds to the time-window in which departing flights are scheduled to depart from the hub.

Minimum connect times for continental and intercontinental flights, maximum acceptable connect times and the maximum number of flights, which can be scheduled per time period, are the elements that define an ideal wave of flights.

First, connections have to meet the minimum connect time, i.e. the minimum time interval that must elapse between a scheduled arrival and a scheduled departure for the two services to be bookable as a connection (Dennis, 1994b). Minimum connect time is constrained by the minimum transfer time required to passengers and baggage to be transferred at the hub and by the minimum time to turnaround the aircraft. A range of different minimum connect times may be in operation at hub airports, depending on the airport, airline, type of passenger and route. However, minimum connect times for European hubs are typically clustered around 45 minutes.

Secondly, a trade-off exists between the maximum acceptable connect time for passengers and the maximum number of flights that can be scheduled in a time period. Since no airport has unlimited capacity, extra-connections can only be scheduled at the edges of an existing bank and this involves long connection times, which may not be acceptable for transfer passengers. Bootsma (1997) has defined standard maximum connect times for different types of flights and different connection quality thresholds. On the other hand, Dennis (2001) argues that the optimal size of a European hub complex is likely to be around 50 aircraft.

It should be noted that an ideal wave of flights is not very likely to be scheduled by airline managers: for instance, slot shortage, very strong local traffic, need to match competitors’ frequencies or profitable opportunities to raise aircraft utilisation may induce hub-and-spoke carriers to operate flights unconnected to any hub complex. Generally speaking, actual waves are quite far from being ideal: they do not show sharp outlines and sometimes they are difficult to identify. A useful analysis tool for detecting hub waves within an airline hub schedule has been proposed by Bootsma (1997).

2. Measuring hub timetable co-ordination and connectivity

The most relevant purpose of any hub wave-system is to maximise connectivity. Hub connectivity refers to the number and the quality of indirect flights available to passengers via an airline hub (Bootsma, 1997). Hub connectivity depends on:

- 1) the number of markets linked to the hub with direct services,

- 2) service frequencies,
- 3) times of arrival and departure of the flights scheduled at the hub.

Large hub airports have a major advantage, because connectivity tends to increase in proportion to the square of the number of flight movements. Nevertheless, smaller airline hubs can try to compensate for this, by offering a higher level of timetable co-ordination, which does not depend on the size of hub operations (Rietveld and Brons, 2001).

Hub timetable co-ordination can be defined as the action and the effect of organising a hub schedule according to an ordered pattern, so that connectivity can be enhanced without increasing the number of flights. Indeed, concentrating flights in complexes is the common approach adopted by airline managers for implementing hub timetable co-ordination.

A quantitative estimate of hub timetable co-ordination can be obtained by calculating the ratio between the actual value of connectivity registered at the hub and the value of connectivity that would be observed, if flights to/from the hub were scheduled following a fixed reference pattern (such as, for instance, a random uniform distribution of flights). In other words, hub timetable co-ordination can be measured by measuring the degree to which it contributes to improve connectivity.

Hub connectivity can be evaluated through different methodologies, according to the ultimate goal of the measure. Generally, an accurate analysis of both the total number and the quality of hub connections is needed. An example of this kind of approach for measuring hub connectivity is the methodology proposed by Burghouwt and De Wit (2005). On the other hand, Doganis and Dennis (Doganis and Dennis, 1989; Dennis, 1994b and 2001; Doganis, 2002) adopt a less detailed and more straightforward approach for measuring hub connectivity, because they need a connectivity measure only for calculating the so-called “connectivity ratio”, which actually is a hub timetable co-ordination measure.

Both the connectivity measure suggested by Burghouwt and De Wit, i.e. the so-called “weighted indirect connection number”, and the “connectivity ratio” suggested by Doganis and Dennis are illustrated in this paper. Furthermore, a new hub temporal co-ordination measure is proposed in section 5.

3. “Weighted indirect connection number”

Burghouwt and De Wit (2005) suggest to measure airline hub connectivity using an approach that combines the methodologies proposed by Veldhuis (1997) and Bootsma (1997). In order to measure hub connectivity, Burghouwt and De Wit investigate both the quantity and the quality, in terms of passenger attractivity, of indirect flight connections.

Generally speaking, the attractivity of any hub connection depends on several factors (Burghouwt and De Wit, 2005; Veldhuis, 1997; Bootsma, 1997). First, the attractivity of connections declines, with increasing hub transfer time. Secondly, the attractivity of connections declines, with increasing backtracking and in-flight time compared to the direct flight alternative. Flight departure and arrival times, service frequency and

aircraft type also affect connection passenger attractivity. Moreover, in order to evaluate the attractivity of any connection, the attractivity of the other competitive direct and indirect links available to passengers should be evaluated as well. Thus, evaluating passenger attractivity of hub connections is quite a difficult issue, which has to be simplified, in order to get a practical index for measuring hub connectivity.

For the evaluation of hub connectivity, Burghouwt and De Wit (2005) suggest to determine the quality levels of connections, focusing on the role which is played by the perceived transfer time and in-flight time compared to the direct flight option. The authors do not simply distinguish between viable and non viable connections: they propose to calculate the so-called “weighted indirect connection number”, WNX , as a global measure of airline hub connectivity. Considering on-line same-day airline hub connections only,

$$WNX = \sum_k WI_k . \tag{3.1}$$

WI_k indicates a “weighted indirect connection”, being $k = (i, j)$, $i = 1, \dots, n_a$ and $j = 1, \dots, n_d$:

$$\left\{ \begin{array}{l}
 WI_k = \frac{1 \left(1 - \frac{IFT_k - DFT_k}{DFT_k} \right) + 2.4 \left(1 - \frac{TT_k}{MACT_k} \right)}{3.4} = \\
 = \frac{2 - \frac{IFT_k}{DFT_k} + 2.4 \left(1 - \frac{TT_k}{MACT_k} \right)}{3.4}, \quad \text{if } \begin{cases} MCT_k \leq TT_k \leq MACT_k \\ 1 < \frac{IFT_k}{DFT_k} \leq 1.40 \end{cases} ; \\
 WI_k = 0, \quad \text{otherwise.}
 \end{array} \right. \tag{3.2}$$

$i = 1, \dots, n_a$ is any flight arriving at the hub during the time period T . $j = 1, \dots, n_d$ is any flight departing from the hub during the time period T . $t_{a,i}$ is the arrival time of flight i , $t_{d,j}$ is the departure time of flight j , $TT_k = t_{d,j} - t_{a,i}$ is the transfer time scheduled between the arrival of flight i and the departure of flight j , $MACT_k$ is the maximum acceptable connect time between flight i and flight j and MCT_k is the minimum connect time between flight i and flight j , which depends on the connection type as well as on the airport facilities themselves. IFT_k is the indirect in-flight time corresponding to connection $k = (i, j)$, i.e. the sum of the in-flight times of i and j , and DFT_k is the estimated in-flight time for covering the great circle distance between the origin of flight i and the destination of flight j .

The number of direct flights to/from the hub, the minimum connect times and the quality of connections with respect to the maximum acceptable connect times determine the weighted indirect connection number WNX for an airline hub. The quality of a generic connection depends on the hub transfer time and on the indirect in-flight time compared to the direct in-flight time. In particular, WI_k is a linear function of both indirect in-flight time and transfer time. It is assumed by the authors that passengers value transfer time 2.4 times as long as in-flight time.

It can be noted that $WI_k < 1$, because: (1) $TT_k \geq MCT_k > 0$; (2) $IFT_k > DFT_k$, due to the extra landing and take-off operations required by the via-hub travel option compared to the direct flight option, even without any circuitry involved. With regard to maximum available connect times, the authors suggest to respect the thresholds proposed by Bootsma (1997) for connections with a poor quality level: $MACT_k = 180'$, in case of connections between two continental flights, $MACT_k = 300'$, in case of connections between one continental flight and one intercontinental flight and $MACT_k = 720'$, in case of connections between two intercontinental flights.

According to Burghouwt and De Wit (2005): if $WNX \geq 2500$, hub connectivity can be considered “high”; if $500 \leq WNX < 2500$, hub connectivity can be considered “medium”; if $WNX < 500$, hub connectivity can be considered “low”.

4. “Connectivity ratio”

In order to measure airline hub temporal co-ordination, Doganis and Dennis propose the so-called “connectivity ratio”, an index that shows the degree to which airline hub connections are more than just purely random (Doganis and Dennis, 1989; Dennis, 1994b and 2001; Doganis, 2002). According to the definition given by Doganis and Dennis, a viable connection is any flight departure, which respects both the minimum connect time MCT and the maximum acceptable connect time $MACT$ after each arriving flight at the hub. Doganis and Dennis suggest $MCT = 45'$ and $MACT = 90'$ as the thresholds to be respected by a couple of one arriving flight and one departing flight for creating a viable connection: these values represent more severe quality standards compared to those proposed by Bootsma (1997).

Let $i = 1, \dots, n_a$ be any flight arriving at the airline hub during the time period T , $j = 1, \dots, n_d$ any flight departing from the hub during the time period T , $t_{a,i}$ the arrival time of flight i , $t_{d,j}$ the departure time of flight j and $TT_k = t_{d,j} - t_{a,i}$, $k = (i, j)$, the transfer time scheduled between flight i and flight j . Furthermore, let m_{ij} be a dummy variable such that: $m_{ij} = 1$, if $MCT = 45' \leq t_{d,j} - t_{a,i} \leq MACT = 90'$; $m_{ij} = 0$, otherwise. Thus, the connectivity ratio proposed by Doganis and Dennis can be written as:

$$CR = \frac{N_c}{N_r} \quad (4.1)$$

being

$$N_c = \sum_i \sum_j m_{ij} \quad (4.2)$$

the total number of viable connections offered at the airline hub during the time period T and

$$N_r = n_a \frac{n_d}{T} (MACT - MCT) = n_a n_d \frac{MACT - MCT}{T} \quad (4.3)$$

the approximate number of viable connections that would be expected to occur in case of a purely random (uniform) arrival and departure timetable across T . Typically, it is assumed that T is equal to one airline operational day, i.e. $T = 15 \div 18$ h.

Indicating with

$$n_c = \frac{N_c}{n_a} \quad (4.4)$$

the average number of viable connections per flight arrival⁵ and with

$$n_r = \frac{N_r}{n_a} \quad (4.5)$$

the average number of viable connections per flight arrival in case of a purely random flight timetable across T , the connectivity ratio can also be written as:

$$CR = \frac{n_c}{n_r} = \frac{n_c}{n_d \frac{MACT - MCT}{T}} \quad (4.6)$$

The main advantage of the connectivity ratio is that it is very easy to be calculated. On the other hand, a major disadvantage is that all viable connections are considered

⁵ It should be noted that the average number of viable connections per flight arrival, n_c , is not a suitable index for measuring hub temporal co-ordination, due to the quadratic nature of market coverage, which can be obtained through hub-and-spoke operations. Every direct link to/from the hub results in a multiplicity of via-hub connections. Thus, airlines offering more direct flights from an airport will generally show a larger value of n_c .

equal. Doganis and Dennis distinguish only between viable and non viable connections and the problem of evaluating the quality of hub connections is not investigated. A connection is simply considered as an indirect link within the airline network that may exist or may not, as it meets or does not meet the hub minimum connect time and maximum acceptable connect time constraints. Connection de-routing and competing paths are not taken into account.

Dennis (1994b) argues that *CR* should ideally be in the range of 2 to 3 for optimal hub temporal co-ordination; connectivity ratios of 1 or less indicate random or even counterproductive hub schedule co-ordination.

5. A new index for measuring hub timetable co-ordination

The connectivity ratio is very useful for evaluating airline hub timetable co-ordination. A major advantage is that the connectivity ratio is very straightforward. On the other hand, this index is easy to be used mainly because it evaluates hub connectivity by means of a very simple procedure, which appears to be a bit rough for some applications (Burghouwt and De Wit, 2005). In particular:

- 1) connections are classified only between viable and non viable connections, without any investigation about the relative quality of viable connections, which are considered equivalent;
- 2) viable connections are defined only looking at temporal constraints, i.e. hub minimum connect time and maximum acceptable connect time, and no spatial or other factor is considered;
- 3) both minimum connect time and maximum acceptable connect time are considered to be hub constants, and it is not taken into account that they depend on the particular connection type, i.e. continental, intercontinental or between one continental and one intercontinental flight.

The weighted indirect connection number (Burghouwt and De Wit, 2005), which specifically aims to evaluate hub connectivity, does suggest some modifications that could be applied to improve the connectivity ratio. A new index for measuring hub temporal co-ordination is proposed and illustrated in this paragraph. This new index is named “weighted connectivity ratio” and it tries to maintain the straightforward structure of the connectivity ratio, while introducing a more accurate procedure to evaluate hub connectivity.

Let $i = 1, \dots, n_a$ be any flight arriving at the airline hub during the time period T , $j = 1, \dots, n_d$ any flight departing from the hub during the time period T , $t_{a,i}$ the arrival time of flight i , $t_{d,j}$ the departure time of flight j and $TT_k = t_{d,j} - t_{a,i}$, $k = (i, j)$, the transfer time scheduled between flight i and flight j . Furthermore, let $n_{a,cont}$ be the number of continental flights arriving at the hub and $n_{d,cont}$ the number of continental flights departing from the hub during the time period T . Similarly, let $n_{a,inc}$ and $n_{d,inc}$ be the number of arriving and departing intercontinental flights.

Now, considering on-line same-day airline hub connections only, let MCT_k be the minimum connect time between i and j , $MACT_k$ be the maximum acceptable connect time for passengers having a viable connection between flight i and j and let define “intermediate connect time” (ICT_k) an intermediate threshold for taking into account the different quality levels, in terms of passenger attractivity, of “rapid connections” ($MCT_k \leq TT_k \leq ICT_k$) compared to the other viable but less desirable connections (“slow connections”, $ICT_k < TT_k \leq MACT_k$). In Tab. 1, typical values of MCT_k are listed and possible values of ICT_k and $MACT_k$ are suggested for both continental and intercontinental connections.

Table 1: Values of minimum, intermediate and maximum acceptable connect times proposed for the calculation of the weighted connectivity ratio⁶.

<i>CONNECT TIMES (minutes)</i>		<i>MCT_k</i>	<i>ICT_k</i>	<i>MACT_k</i>
CONNECTION TYPE	Continental - Continental	45	90	120
	Continental - Intercontinental	60	120	180
	Intercontinental - Intercontinental	60	120	180

Recalling and adjusting the definition of “connectivity matrix” given by Ivy (1993), the “temporal connectivity matrix” can be defined as the matrix \underline{TCM} , with n_a rows and n_d columns, such that, for the generic element τ_{ij} , $i = 1, \dots, n_a$, $j = 1, \dots, n_d$, the following holds:

$$\begin{cases} \tau_{ij} = 1 & \text{if } MCT_k \leq t_{d,j} - t_{a,i} \leq ICT_k \\ \tau_{ij} = 0.5 & \text{if } ICT_k < t_{d,j} - t_{a,i} \leq MACT_k \\ \tau_{ij} = 0 & \text{otherwise} \end{cases} \quad (5.1)$$

Similarly, the “spatial connectivity matrix” can be defined as the matrix \underline{SCM} , with n_a rows and n_d columns, such that, for the generic element δ_{ij} , $i = 1, \dots, n_a$, $j = 1, \dots, n_d$, the following holds:

⁶ In Tab. 1, minimum connect times correspond to typical values for European hubs, but they actually depend both on the connection type and on the particular airport facilities. Note also that the values of maximum acceptable and intermediate connect times represent more severe level-of-service standards compared to the thresholds proposed by Bootsma (1997); in particular, note that the same thresholds have been chosen for connections between two intercontinental flights and for connections between one continental flight and one intercontinental flight.

$$\begin{cases} \delta_{ij} = 1 & \text{if } DR_k \leq 1.20 \\ \delta_{ij} = 0.5 & \text{if } 1.20 < DR_k \leq 1.50 \\ \delta_{ij} = 0 & \text{otherwise} \end{cases} \quad (5.2)$$

where

$$DR_k = \frac{ID_k}{DD_k} \quad (5.3)$$

is the so-called “de-routing index” ($DR_k \geq 1$), with DD_k the great circle distance between the point of origin of flight i and the destination of flight j and ID_k the sum of the great circle distances corresponding to flights i and j .

Furthermore, the “weighted connectivity matrix” can be defined as the matrix \underline{WCM} , with n_a rows and n_d columns, such that the generic element w_{ij} , $i = 1, \dots, n_a$, $j = 1, \dots, n_d$, corresponds to the so-called “weighted connection”

$$w_{ij} = \tau_{ij} \delta_{ij} \quad (5.4)$$

Now, the “weighted connectivity ratio” can be defined as:

$$WCR = \frac{WN_c}{WN_r} \quad (5.5)$$

where

$$WN_c = \sum_i \sum_j w_{ij} = \sum_i \sum_j \tau_{ij} \delta_{ij} \quad (5.6)$$

is the number of weighted connections offered at the airline hub during the time period T and

$$WN_r = \left[n_{a,cont} \frac{n_{d,cont}}{T} \left(ICT_1 - MCT_1 + \frac{MACT_1 - ICT_1}{2} \right) + \right.$$

$$\begin{aligned}
 & + n_{a,cont} \frac{n_{d,inc}}{T} \left(ICT_2 - MCT_2 + \frac{MACT_2 - ICT_2}{2} \right) + \\
 & + n_{a,inc} \frac{n_{d,cont}}{T} \left(ICT_2 - MCT_2 + \frac{MACT_2 - ICT_2}{2} \right) + \\
 & + n_{a,inc} \frac{n_{d,inc}}{T} \left(ICT_3 - MCT_3 + \frac{MACT_3 - ICT_3}{2} \right) \left] \frac{\sum_i \sum_j \delta_{ij}}{n_a n_d} = \\
 & = \frac{\sum_i \sum_j \delta_{ij}}{n_a n_d} \left[n_{a,cont} n_{d,cont} \frac{MACT_1 + ICT_1 - 2MCT_1}{2T} + \right. \\
 & + \left(n_{a,cont} n_{d,inc} + n_{a,inc} n_{d,cont} \right) \frac{MACT_2 + ICT_2 - 2MCT_2}{2T} + \\
 & \left. + n_{a,inc} n_{d,inc} \frac{MACT_3 + ICT_3 - 2MCT_3}{2T} \right] = WN_r \tag{5.7}
 \end{aligned}$$

the approximate number of weighted connections that would be expected to occur in case of a purely random (uniform) arrival and departure timetable across T (for both continental and intercontinental flights)⁷. For practical applications, it is typically assumed that T is one airline operational day, i.e. $T = 15 \div 18$ h.

The weighted connectivity ratio shows if the viable weighted connections are more than purely random. Thus, the thresholds suggested by Dennis (1994b) indeed remain valid: ideally, WCR should be in the range of 2 to 3 for optimal hub temporal co-ordination, whereas connectivity ratios of 1 or less indicate random or even counterproductive hub schedule co-ordination.

In contrast to the connectivity ratio proposed by Doganis and Dennis, the weighted connectivity ratio classifies viable connections in different quality levels, according to their spatial as well as temporal characteristics. Indeed, the definition of two quality levels for both spatial and temporal attributes allows any weighted connection to vary between a set of three values other than zero:

$$\begin{cases} w_{ij} = 1 & \text{if } \delta_{ij} = \tau_{ij} = 1 \\ w_{ij} = 0.5 & \text{if } \delta_{ij} + \tau_{ij} = 1.5 \\ w_{ij} = 0.25 & \text{if } \delta_{ij} = \tau_{ij} = 0.5 \\ w_{ij} = 0 & \text{otherwise} \end{cases} \tag{5.8}$$

⁷ In formula (5.7), continental, intercontinental and continental-intercontinental connections have to be distinguished, as they refer to different minimum, intermediate and maximum connect times. Furthermore, rapid and slow connections have to be considered separately, because they lead to different values of τ_{ij} . Finally, the arithmetic mean of the elements of SCM has to be taken into account, in order to compute WN_r .

with $i = 1, \dots, n_a$, $j = 1, \dots, n_d$. Moreover, different connect time thresholds can be considered, with respect to the different connection types that may occur (continental, continental - intercontinental, intercontinental) and to the particular hub facilities. Indeed, the hub connectivity evaluation procedure, which leads to the computation of WCR , is quite precise and WN_c could be considered itself an acceptable hub connectivity measure.

6. Examples of application of the new index

Fig. 1 and Fig. 2 illustrate the Alitalia hub timetable structures, which correspond to the Alitalia Winter 2004/2005 schedule (schedule “AZW05”).

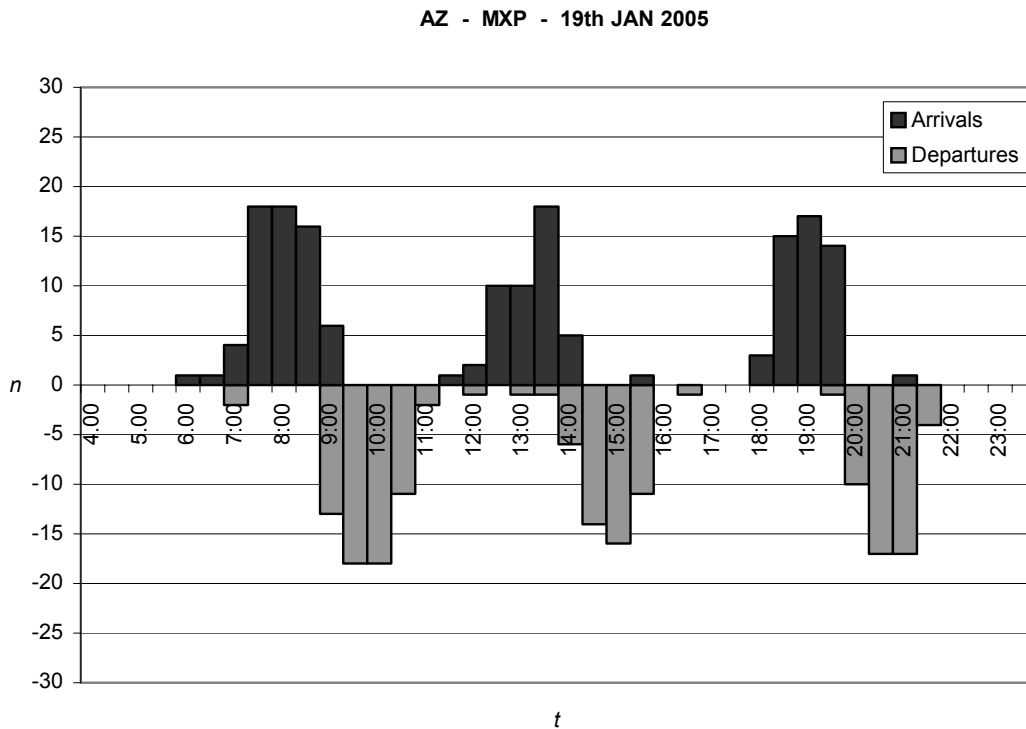


Figure 1: Schedule structure of Alitalia hub in Milan Malpensa (MXP), on Wednesday 19th January 2005.

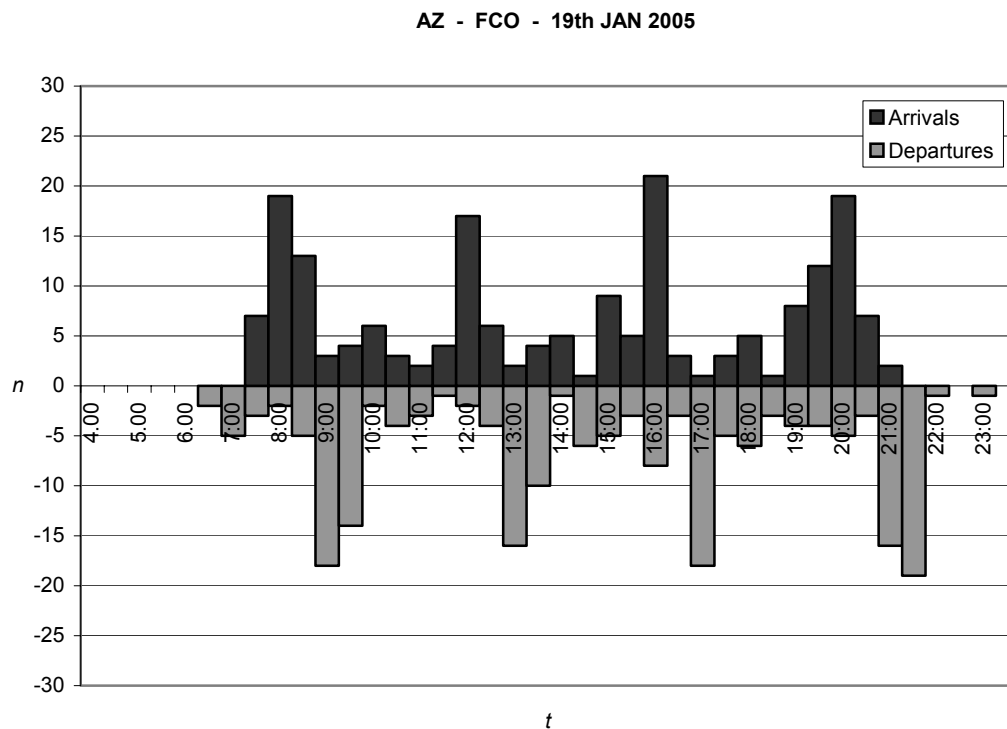


Figure 2: Schedule structure of Alitalia hub in Rome Fiumicino (FCO), on Wednesday 19th January 2005.

Schedule AZW05 requires 127 narrow-body aircraft and 19 wide-body aircraft to serve a wide range of continental as well as intercontinental destinations. As a result of schedule AZW05, the wave-system structure of Alitalia hub in Milan Malpensa (MXP) is characterised by three waves with centres at 8:55 a.m., 1:50 p.m. and 8:00 p.m. Since the continental fleet is stabled at the spokes and the average hub-repeat cycle is equal to 5h30', the wave-system structure of Alitalia hub in Milan Malpensa can be described by the triple $(3, 5 \frac{1}{2}, 1)$. On the other hand, with schedule AZW05, the wave-system structure of Alitalia hub in Rome Fiumicino (FCO) is characterised by four waves with centres at about 9:00 a.m., 1:00 p.m., 5:00 p.m., 9:00 p.m. As the continental fleet is stabled at the spokes and the average hub-repeat cycle is equal to four hours, the wave-system structure of Alitalia hub in Rome Fiumicino can be described by the triple $(4, 4, 1)$.

Fig. 3, Fig. 4, Fig. 5 and Fig. 6 illustrate the Winter 2004/2005 schedule structures of some major European airline hubs, namely: KLM hub in Amsterdam (Fig. 3), British Airways hub in London Heathrow (Fig. 4), Iberia hub in Madrid Barajas (Fig. 5), Air France hub in Paris Charles de Gaulle (Fig. 6). It can be observed that the schedule structures of KLM hub in Amsterdam and of Air France hub in Paris Charles de Gaulle, as well as the schedule structures of Alitalia hubs, exhibit quite clear wave-system patterns. In particular, it can be noted that KLM hub in AMS is based on a 4-wave-system structure, whereas Air France hub in CDG is based on a 6-wave-system structure. On the other hand, no flight wave can be easily identified within the schedule structures of British Airways in LHR and of Iberia in MAD. In any case, a computerised analysis tool, like that proposed by Bootsma (1997), is needed for an accurate detection of the number and the characteristics of hub waves within an airline hub schedule.

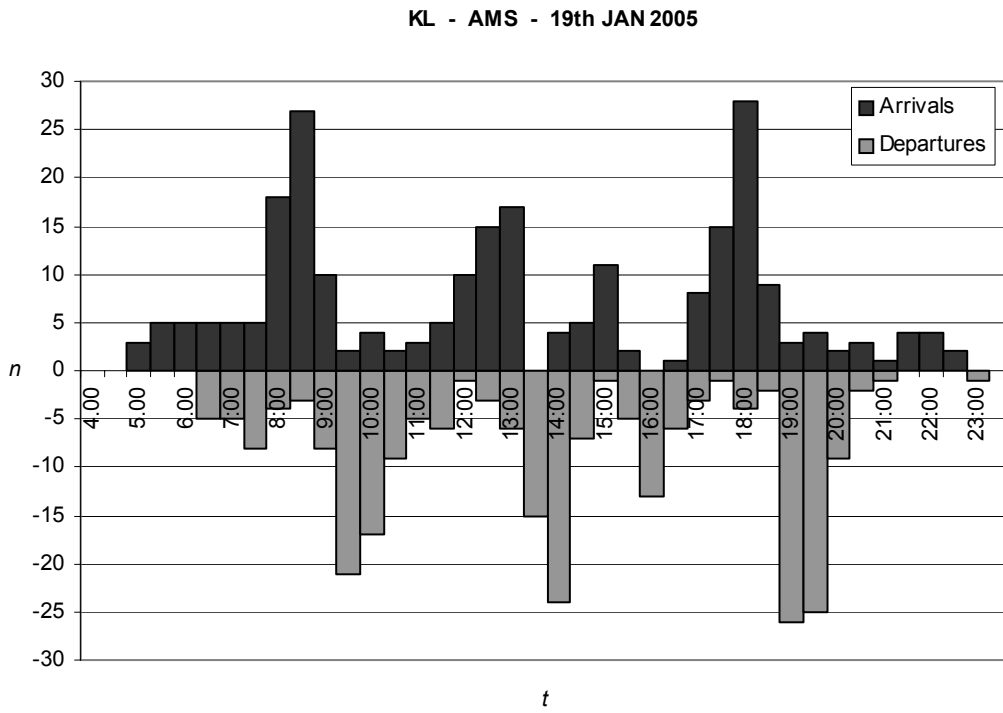


Figure 3: Schedule structure of KLM hub in Amsterdam (AMS), on Wednesday 19th January 2005 (source OAG data).

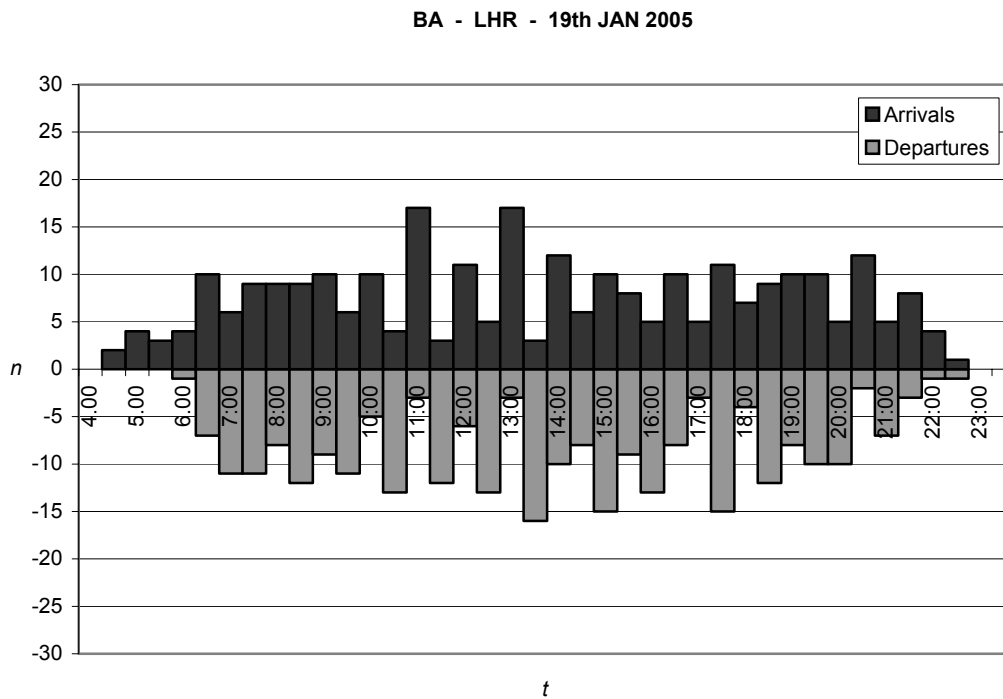


Figure 4: Schedule structure of British Airways hub in London Heathrow (LHR), on Wednesday 19th January 2005 (source OAG data).

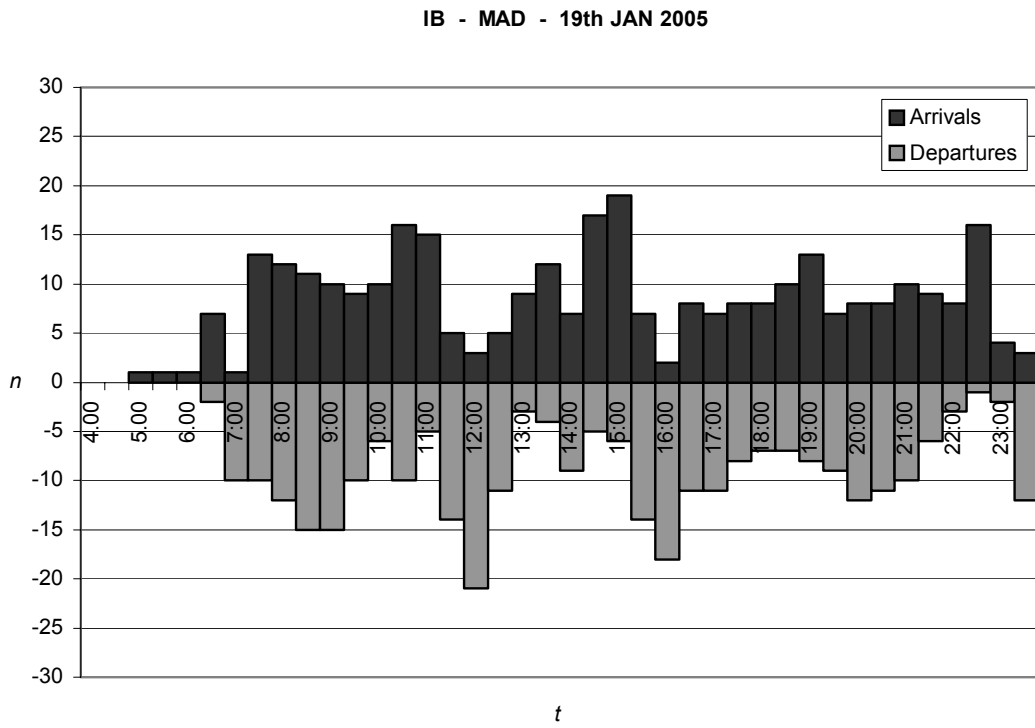


Figure 5: Schedule structure of Iberia hub in Madrid Barajas (MAD), on Wednesday 19th January 2005 (source OAG data).

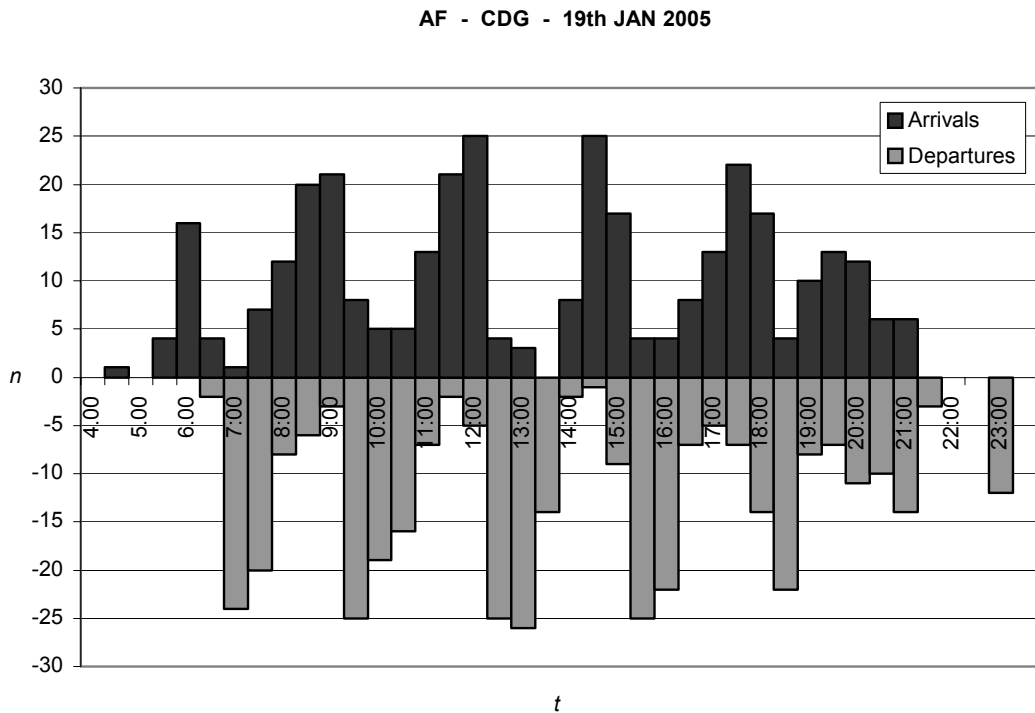


Figure 6: Schedule structure of Air France hub in Paris Charles de Gaulle (CDG), on Wednesday 19th January 2005 (source OAG data).

Tab. 2 reports the results of the analysis, which has been performed for comparing the temporal co-ordination and connectivity levels of the selected European hubs, on Wednesday 19th January 2005. According to the definition given in section 5, the weighted number of connections (WN_c) and the weighted connectivity ratio (WCR) have been calculated, in order to estimate hub connectivity and hub temporal co-ordination respectively. The results of the analysis demonstrate how even relatively small airline hubs can offer a huge number of weighted connections, through a satisfactory timetable co-ordination level, that is by achieving a high value of the weighted connectivity ratio. For example, KLM hub in Amsterdam offers 4526 weighted connections with 247 daily arriving flights, whereas Iberia hub in Madrid Barajas offers 3957 weighted connections with a far higher number of daily arriving flights (320). This can be easily explained, by observing that the weighted connectivity ratio is equal to 1.32 for Iberia at Madrid Barajas and to 1.75 for KLM at Amsterdam.

Alitalia hub in Milan Malpensa shows a very high degree of temporal co-ordination (2.48), which can be considered “ideal”, according to the classification proposed by Dennis (section 4 and 5). This confirms the impression that could be derived comparing the representation of the waves of Milan Malpensa (Fig. 1) with the timetable structures of the other hubs. Indeed, Alitalia is able to offer as many as 2942 weighted connections, by scheduling only 163 daily arriving flights at Milan Malpensa hub. Alitalia hub in Rome Fiumicino exhibits a quite high degree of temporal co-ordination and connectivity as well: with 200 daily arriving flights and a weighted connectivity ratio equal to 1.53, almost 2000 daily weighted connections are available to passengers.

Table 2: Daily number of arriving flights (n_a), number of weighted connections (WN_c) and weighted connectivity ratio (WCR) for selected European hubs on Wednesday 19th January 2005⁸ (source OAG data).

<i>AIRLINE (CODE)</i>	<i>AIRPORT (CODE)</i>	na	WNc	WCR
Air France (AF)	Paris (CDG)	380	7285	1.42
Iberia (IB)	Madrid (MAD)	320	3967	1.32
British Airways (BA)	London H. (LHR)	280	3788	1.23
KLM (KL)	Amsterdam (AMS)	247	4526	1.75
Alitalia (AZ)	Rome (FCO)	203	1983	1.53
Alitalia (AZ)	Milan (MXP)	163	2942	2.48

In Tab. 3, the estimates of hub temporal co-ordination obtained using the weighted connectivity ratio are compared to the values obtained using the connectivity ratio by Doganis and Dennis. Some significant though not radical differences can be observed, analysing the results produced by the two indices. This seems to confirm that the connectivity ratio is a valid tool for measuring hub timetable co-ordination but it also indicates that the weighted connectivity ratio can lead to considerably more accurate estimates, based on a more detailed connectivity analysis.

⁸ Note that: $T = 18 h$ for CDG, LHR and MAD; $T = 16 h$ for AMS; $T = 15 h$ for FCO and MXP.

Table 3: Weighted connectivity ratio (WCR) and connectivity ratio (CR) for selected European hubs on Wednesday 19th January 2005⁹ (source OAG data).

<i>AIRLINE (CODE)</i>	<i>AIRPORT (CODE)</i>	WCR	CR
Alitalia (AZ)	Milan (MXP)	2.48	2.23
KLM (KL)	Amsterdam (AMS)	1.75	1.87
Alitalia (AZ)	Rome (FCO)	1.53	1.60
Air France (AF)	Paris (CDG)	1.42	1.70
Iberia (IB)	Madrid (MAD)	1.32	1.23
British Airways (BA)	London H. (LHR)	1.23	1.16

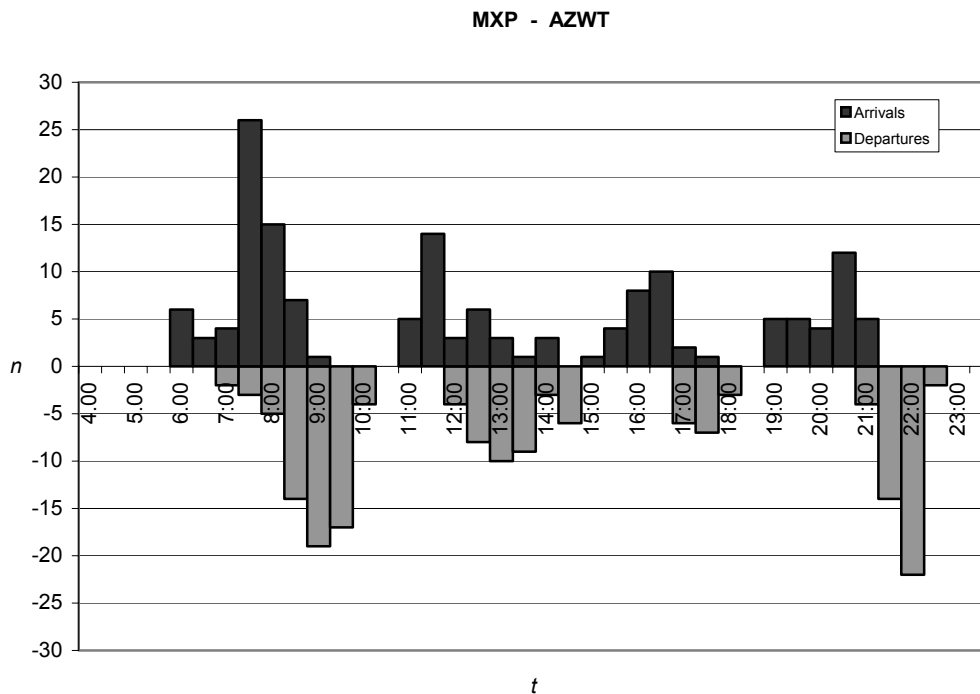


Figure 7: Schedule structure of Alitalia hub in Milan Malpensa, with test-schedule AZWT being operated.

In particular, the weighted connectivity ratio may prove very useful in order to evaluate hub temporal co-ordination of different schedule configurations of the same airline hub (Danesi, 2006). For example, a case study can be discussed, which regards the comparison of the temporal co-ordination levels of two different schedules of Alitalia hub in Milan Malpensa: the first one (Fig. 1) corresponds to the Alitalia Winter 2004/2005 schedule (schedule “AZW05”); the second one (Fig. 7) corresponds to the implementation of the test-schedule named “AZWT”.

Schedule AZWT serves the same range of destinations and requires the same number and type of aircraft of AZW05, but it is based on a 4-wave-system structure (with

⁹ Note that: $T = 18 h$ for CDG, LHR and MAD; $T = 16 h$ for AMS; $T = 15 h$ for FCO and MXP.

centres at about 9:00 a.m., 1:00 p.m., 5:00 p.m., and 9:00 p.m.) at Milan Malpensa hub. Since the continental fleet is stabled at the spokes and the hub-repeat cycle is equal to four hours, the wave-system structure of Alitalia hub in Milan Malpensa, which corresponds to schedule AZWT, can be described by the triple (4, 4, 1).

Tab. 4 presents the results of the analysis of Milan Malpensa hub timetable co-ordination, with reference to schedules AZW05 and AZWT. Schedule AZW05 achieves a *WCR* score equal to 2.48 and a *CR* score equal to 2.23, while schedule AZWT achieves a *WCR* score equal to 1.96 and a *CR* score equal to 2.05. Thus, both *CR* and *WCR* measures indicate that a reduction in the temporal co-ordination level of Alitalia hub in Milan Malpensa would occur, if schedule AZW05 were replaced by schedule AZWT. On the other hand, the difference between *CR* and *WCR* scores are quite significant and this justifies the more detailed connectivity analysis performed by *WCR* compared to *CR* index.

Table 4: Comparative analysis of the temporal co-ordination levels of schedule AZW05 and AZWT for Alitalia hub in Milan Malpensa: score of the connectivity ratio (*CR*) and score of the weighted connectivity ratio (*WCR*), with T = 15h.

Schedule	CR (MXP)	WCR (MXP)
AZW05	2.23	2.48
AZWT	2.05	1.96
<i>Difference</i>	-8.07%	-20,97%

Finally, some general considerations can be added. First, the choice of the wave-system structure is a major determinant of the temporal co-ordination level of an airline hub schedule. In particular, if a certain amount of flight frequencies is re-distributed within a higher number of hub waves, the connectivity and temporal co-ordination level of the hub schedule will tend to diminish. However, the hub temporal co-ordination measures, such as *CR* and *WCR*, do evaluate (actual) schedules and not wave-system structures (that are ideal models). Indeed, both *CR* and *WCR* could be applied to evaluate any type of timetable, even if no wave-system is operated at the airport and connections are purely random.

Second, the weighted connectivity ratio aims to estimate only one of a huge number of parameters to be optimised by airline managers for improving the quality of hub operations. In particular, *WCR* refers to the performance of a hub schedule and not to the performance of an airline network as a whole. Nevertheless, future researches may investigate the problem of evaluating the mutual co-ordination of different hub schedules implemented within a certain multi-hub system.

Conclusions

Temporal co-ordination of an airline hub-and-spoke network refers to the hub timetable organisation, which is based on a certain wave-system structure. The most relevant purpose of a hub wave-system is to maximise connectivity.

Hub connectivity refers to the number and the quality of indirect flights available to passengers via an airline hub. Large hub airports have a major advantage, because connectivity tends to increase in proportion to the square of the number of flight movements. Nevertheless, smaller airline hubs can try to compensate for this, by offering a higher level of timetable co-ordination, which does not depend on the size of hub operations.

Hub timetable co-ordination can be defined as the action and the effect of organising a hub schedule according to an ordered pattern, so that connectivity can be enhanced without increasing the number of flights. Indeed, concentrating flights in complexes is the common approach adopted by airline managers for implementing hub timetable co-ordination.

In order to measure hub connectivity, Burghouwt and De Wit propose to calculate the so-called "weighted indirect connection number". The authors investigate both the quantity and the quality, in terms of passenger attractivity, of indirect flight connections. Generally speaking, the attractivity of any hub connection depends on several factors and Burghouwt and De Wit suggest to focus on the role that is played by the perceived transfer time and in-flight time compared to the direct flight option.

The "connectivity ratio" by Doganis and Dennis is very useful for evaluating airline hub timetable co-ordination, which can be measured by measuring the degree to which it contributes to improve connectivity. A major advantage of the connectivity ratio is that it is very straightforward; on the other hand, this index is easy to be used mainly because it evaluates hub connectivity by means of a very simple procedure, which appears to be a bit rough for some applications.

A new index for measuring hub temporal co-ordination is proposed in this paper. Its name is "weighted connectivity ratio" and it tries to maintain the straightforward structure of the connectivity ratio, while introducing a more accurate procedure to evaluate hub connectivity. In contrast to the connectivity ratio proposed by Doganis and Dennis, the weighted connectivity ratio classifies viable connections in different quality levels, according to their spatial as well as temporal characteristics. Moreover, different maximum and minimum connect time thresholds can be considered, with respect to the different connection types that can occur (continental, continental - intercontinental, intercontinental-continental, intercontinental) and to the particular hub facilities.

The weighted connectivity ratio has been applied for evaluating the temporal co-ordination and connectivity levels of a sample of European hubs and for comparing the temporal co-ordination of two alternative schedule configurations of Alitalia hub in Milan Malpensa. The results of the analysis demonstrate how even relatively small airline hubs can offer a huge number of weighted connections, by achieving a high value of the weighted connectivity ratio. Furthermore, the case study indicates that the weighted connectivity ratio can generate more accurate estimates compared to the connectivity ratio by Doganis and Dennis, especially for evaluating the temporal co-ordination of different schedule configurations of the same airline hub. Thus, the

weighted connectivity ratio is a hub schedule analysis tool that may prove particularly helpful to airline managers.

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