RECENT ADVANCES IN INVERSE SYSTEMS OF SPACES (*)

by SIBE MARDEŠIĆ (in Zagreb)(**)

SOMMARIO. - *La nozione di sistemi inversi è un elemento importante nel definire e studiare gli spazi, specialmente gli spazi compatti. In particolare, si tenta di rappresentare gli spazi come limite di sistemi inversi di poliedri riducendo così il problema allo studio di tali sistemi. Quest'articolo illustra alcuni recenti progressi in quest'area. In particolare, si discutono sistemi di gauge, risoluzioni e sistemi approssimati. I sistemi di gauge sono sistemi inversi dove ogni membro è dotato di un ricoprimento che determina una nozione di prossimità. Le risoluzioni si possono vedere come sistemi inversi dotati di buone proprietà con possibili applicazioni agli spazi non compatti. Infine, i sistemi approssimati condividono tutte le buone proprietà dei sistemi inversi usuali oltre ad una maggiore flessibilità poichè la condizione funtoriale sulle mappe di bordo è sostituita da una richiesta più debole: le mappe* $p_{aa'}p_{a'a''}$ *e* $p_{aa''}$, $a \leq a' \leq a''$ *possono differire di una quantità adeguatamente controllata.*

SUMMARY. - *The notion of inverse system is an important tool in defining and studying spaces, especially compact spaces. In particular, one tries to represent spaces as limits of inverse systems of polyhedra and thus reduce their study to the study of such systems. This paper surveys some recent developments in the area. In particular, gauged systems, resolutions and approximate systems are discussed. Gauged systems are inverse systems, where each member is endowed with a covering, which determines nearness. Resolutions can be viewed as inverse systems with particularly good properties, which makes the application to noncompact spaces possible. Approximate systems share all good properties with usual inverse systems, but are more flexible, because the functorial condition on the bonding mappings is replaced by the weaker requirement, that the mappings* $p_{aa'}p_{a'a''}$ *and* $p_{aa''}$, $a \leq a' \leq a''$, *may differ by a properly controlled amount.*

Key words: Inverse system, approximate inverse system, inverse limit, resolution, approximate resolution, dimension, shape theory.
AMS (MOS) Subj.Class.: 54B35, 54F45, 54C56

(*) Pervenuto in Redazione il 28 dicembre 1993.

(**) Indirizzo dell'Autore: Department of Mathematics, University of Zagreb, Bijenička cesta 30, 41.000 Zagreb (Croazia).

1. Introduction.

1.1. The notion of an inverse system of spaces (groups) and its limit developed gradually, beginning with the work of P. S. Aleksandrov [2], [3], L. Vietoris [67] and S. Lefschetz [19], followed by the work of Čech [10], L. S. Pontryagin [56], A. G. Kurosh [18], N. Steenrod [59] and H. Freudenthal [13]. In this survey, we use the standard definition, first given in [19], [20]. Also see [14], [4], [15], [37].

1.2. An *inverse system* of spaces $\boldsymbol{X} = (X_a, p_{aa'}, A)$ consists of a directed preordered set $(A, \leq)$, called the *index set*, of spaces X_a, for $a \in A$, called the *members* of the system, and of mappings $p_{aa'} : X_{a'} \to X_a$, for $a \leq a' \leq a''$, called the *bonding mappings*, which must satisfy the following conditions: p_{aa} is the identity mapping 1_{X_a} on X_a and

(I) $$p_{aa'}p_{a'a''} = p_{aa''}, \text{ for } a \leq a' \leq a''.$$

If $A = \mathbb{N}$, one speaks of an *inverse sequence* of spaces.

The *limit* $\boldsymbol{p} : X \to \boldsymbol{X}$ of $\boldsymbol{X}$ consists of a space $X = \lim \boldsymbol{X}$ and of a collection of mappings $p_a : X \to X_a$, $a \in A$, called the *projections*. X is the subspace of the direct product $\prod_{a\in A} X_a$, formed by all the points $x = (x_a)$, $x_a \in X_a$, satisfying $p_{aa'}(x_{a'}) = x_a$, for $a \leq a'$. The projections p_a are the restrictions of the natural projections $\prod_{a\in A} X_a \to X_a$. They satisfy the condition

(L) $$p_{aa'}p_{a'} = p_a, \text{ for } a \leq a'.$$

It is readily seen that whenever another collection of mappings $p'_a : X' \to X_a$ satisfies condition (L), then there exists a unique mapping $f : X' \to X$ such that $p_a f = p'_a$, for all $a \in A$. Consequently, (L) is the universal property, which determines the limit.

1.3. One also considers mappings between inverse systems. A *level-preserving mapping* $\boldsymbol{f} = (f_a) : \boldsymbol{X} \to \boldsymbol{Y}$ between systems $\boldsymbol{X} = (X_a, p_{aa'}, A)$ and $\boldsymbol{Y} = (Y_a, q_{aa'}, A)$, indexed by the same index set A, consists of a collection of mappings $f_a : X_a \to Y_a, a \in A$, such that $f_a p_{aa'} = q_{aa'} f_{a'}$, for $a \leq a'$. Clearly, $\boldsymbol{f}$ determines a unique mapping $f : X \to Y$ between the limits, such that $f_a p_a = q_a f$, for $a \in A$. This mapping is denoted by $f = \lim \boldsymbol{f}$.

If we have another level-preserving mapping $\boldsymbol{g} : \boldsymbol{Y} \to \boldsymbol{Z}$, given by mappings $g_a : Y_a \to Z_a, a \in A$, then the composition $\boldsymbol{gf} : \boldsymbol{X} \to \boldsymbol{Z}$ is defined as the collection of the mappings $g_a f_a : X_a \to Z_a$, $a \in A$. The identity mapping $\mathbf{1}_{\boldsymbol{X}}$ is given by the identity mappings $1_{X_a} : X_a \to X_a$. In this way, for a fixed A, one obtains a category denoted by $\boldsymbol{Top}^A$. Moreover, $\lim \mathbf{1}_{\boldsymbol{X}} = 1_X$ and $\lim \boldsymbol{gf} = \lim \boldsymbol{g} \lim \boldsymbol{f}$, so that $\lim : \boldsymbol{Top}^A \to \boldsymbol{Top}$ is a functor.

1.4. A more general situation occurs if $\boldsymbol{X}$ and $\boldsymbol{Y}$ are indexed by different sets, say, if $\boldsymbol{X} = (X_a, p_{aa'}, A)$ and $\boldsymbol{Y} = (Y_b, q_{bb'}, B)$. Then one defines an *order-preserving mapping* $\boldsymbol{f} = (f, f_b) : \boldsymbol{X} \to \boldsymbol{Y}$, which consists of an increasing function $f : B \to A$ and of a collection of mappings $f_b : X_{f(b)} \to Y_b$, $b \in B$, such that, whenever $b \leq b'$, one has $f_b p_{f(b)f(b')} = q_{bb'} f_{b'}$. The composition of (f, f_b) with (g, g_c) is defined by $(fg, g_c f_{g(c)})$. In this way one obtains a category *inv*-$\boldsymbol{Top}$. There is a unique mapping $f = \lim \boldsymbol{f} : X \to Y$, which satisfies the condition

$$\text{(LM)} \qquad f_b p_{f(b)} = q_b f, \ b \in B,$$

and $\lim :$ *inv*-$\boldsymbol{Top} \to \boldsymbol{Top}$ is a functor.

1.5. The next level of generality is attained by allowing $f : B \to A$ to be an arbitrary function. Then $\boldsymbol{f} = (f, f_b) : \boldsymbol{X} \to \boldsymbol{Y}$ is called a *mapping of systems* provided, for $b \leq b'$, there is an $a \geq f(b), f(b')$, such that the following condition holds:

$$\text{(M)} \qquad f_b p_{f(b)a} = q_{bb'} f_{b'} p_{f(b')a}.$$

In this case too, $\boldsymbol{f}$ induces a unique limit mapping $f = \lim \boldsymbol{f} : X \to Y$ satisfying condition (LM).

Composition of mappings of systems is defined as in 1.4. Furthermore, one defines an equivalence relation $\sim$ between mappings of systems $\boldsymbol{f}, \boldsymbol{f}' : \boldsymbol{X} \to \boldsymbol{Y}$, putting $\boldsymbol{f} \sim \boldsymbol{f}'$ provided, for each $b \in B$, there is an $a \geq f(b), f'(b)$, such that

$$\text{(E)} \qquad f_b p_{f(b)a} = f'_b p_{f'(b)a}.$$

It is easy to see that $\boldsymbol{f} \sim \boldsymbol{f}'$ and $\boldsymbol{g} \sim \boldsymbol{g}'$ implies $\boldsymbol{gf} \sim \boldsymbol{g}'\boldsymbol{f}'$, so that composition of equivalence classes is well-defined by $[\boldsymbol{g}][\boldsymbol{f}] = [\boldsymbol{gf}]$. Moreover, $\boldsymbol{f} \sim \boldsymbol{f}'$ implies $\lim \boldsymbol{f} = \lim \boldsymbol{f}'$ and one puts $\lim[\boldsymbol{f}] = \lim \boldsymbol{f}$. In this way one obtains a category *pro*-$\boldsymbol{Top}$ and a functor $\lim :$ *pro*-$\boldsymbol{Top} \to \boldsymbol{Top}$ (see e.g., [37]).

1.6. By a *polyhedron* P we understand the geometric realization $|K|$ of a simplicial complex K, endowed with the $CW-$topology. It is well-known that polyhedra are paracompact and therefore, topologically complete spaces (see, e.g., [37]). If K is a finite complex, $P = |K|$ is a metric compact space. In view of the combinatorial nature of simplicial complexes, polyhedra can be considered as relatively simple objects. Therefore, it is natural to try to express more general spaces as limits of inverse systems of polyhedra and express mappings between spaces as limits of mappings between inverse systems of polyhedra. This idea is present ever since the introduction of inverse systems.

1.7. We now state some results concerning polyhedral expansions of spaces and mappings, which explain why the method of inverse systems can be useful in studying spaces.

THEOREM 1. *The limit X of an inverse system (sequence) $\boldsymbol{X}$ of compact polyhedra is a compact Hausdorff (compact metric) space. The limit X of an inverse system of arbitrary polyhedra is a topologically complete space.* (see, e.g., [37]).

THEOREM 2. *Every compact Hausdorff (compact metric) space X is the limit of an inverse system (sequence) $\boldsymbol{X}$ of compact polyhedra. Every topologically complete space X is the limit of an inverse system $\boldsymbol{X}$ of polyhedra* (see [14], [1], [5], [53], [37]).

THEOREM 3. *Every mapping $f : X \to Y$ between compact Hausdorff (compact metric) spaces is the limit of a mapping of systems (sequences) $\boldsymbol{f} : \boldsymbol{X} \to \boldsymbol{Y}$ consisting of compact polyhedra. Every mapping $f : X \to Y$ between topologically complete spaces is the limit of a mapping $\boldsymbol{f} : \boldsymbol{X} \to \boldsymbol{Y}$ between systems of polyhedra* ([26], [16]).

We now state three natural questions to which one would like to have a positive answer.

QUESTION 1. *Let $f : X \to Y$ be a mapping and let $\boldsymbol{p} = (p_a) : X \to \boldsymbol{X}$ and $\boldsymbol{q} = (q_b) : Y \to \boldsymbol{Y}$ be limits of polyhedral systems. Is there a mapping of systems $\boldsymbol{f} : \boldsymbol{X} \to \boldsymbol{Y}$ such that $f = \lim \boldsymbol{f}$?*

QUESTION 2. *Let $\boldsymbol{X} = (X_a, p_{aa'}, A)$ and $\boldsymbol{X}' = (X'_a, p'_{aa'}, A')$ be polyhedral systems having the same limit X. Are the Čech homology groups $\check{H}_n(\boldsymbol{X}) = \lim(\check{H}_n(X_a), (p_{aa'})_*, A)$ and $\check{H}_n(\boldsymbol{X}') = \lim(\check{H}_n(X'_a), (p'_{aa'})_*, A')$ isomorphic?*

QUESTION 3. *Let X be a compact Hausdorff space with covering dimension* $\dim X \leq n$. *Is X the limit of an inverse system of compact polyhedra* $\boldsymbol{X} = (X_a, p_{aa'}, A)$ *with* $\dim X_a \leq n$.

Unfortunately, using the classical notions of inverse system and limit, all three questions have a negative answer (see [32]).

This situation recently led to modifications of the classical concept of an inverse system and its limit and three new notions emerged: *gauged systems* (see §2), *resolutions* (see §3) and *approximate systems* (see §4). The main purpose of this paper is to describe these notions and outline the main results obtained up to now. In particular, using the new notions, all three of the above questions have a positive answer.

2. Gauged Systems.

2.1. The only way to avoid the difficulties related to Question 1 is to allow as mappings of systems $\boldsymbol{f}$ also collections of mappings f_b, which need not satisfy equality (M). Instead, the two sides of (M) can differ by an amount controlled in such a manner that $\boldsymbol{f}$ still induces a limit mapping. For metric compacta and inverse sequences, one finds this idea in [51]. However, it was T. Watanabe [70], who introduced and developed in full generality the needed notion of a *gauged inverse system*, i.e., an inverse system $\mathcal{X} = (X_a, \mathcal{U}_a, p_{aa'}, A)$, where each term X_a is endowed with a normal covering $\mathcal{U}_a$, called the *mesh* at a. The introduction of meshes enables one to measure discrepancy from commutativity in the mappings forming $\boldsymbol{f}$.

2.2. The essential condition to which meshes are subject is condition

$$\text{(A3)} \qquad (\forall a \in A)(\forall \mathcal{U} \in Cov(X_a))(\exists a' \geq a)(\forall a'' \geq a') \quad \mathcal{U}_{a''} \prec p_{aa''}^{-1}(\mathcal{U});$$

where $Cov(Z)$ denotes the set of all normal coverings of Z.

In this paper we always denote systems with boldface characters and gauged systems with script characters.

2.3. Now one can define an *approximate mapping* $\boldsymbol{f} : \mathcal{X} \to \mathcal{Y}$ from one gauged system $\mathcal{X}$ to another one $\mathcal{Y} = (Y_b, \mathcal{V}_b, q_{bb'}, B)$ as a collection consisting of a function $f : B \to A$ and of mappings $f_b : X_{f(b)} \to Y_b$, $b \in B$, such that the following condition is satisfied

(AM) $$(\forall b \leq b')(\exists a \geq f(b), f(b'))(\forall a' \geq a)$$
$$(q_{bb'} f_{b'} p_{f(b')a'}, f_b p_{f(b)a'}) \prec \mathrm{st}\mathcal{V}_b,$$

where $(\phi, \psi) \prec \mathcal{U}$ means that the mappings ϕ, ψ are $\mathcal{U}-$near and st$\mathcal{V}$ denotes the star of the covering $\mathcal{V}$. (Note that in the above definitions the meshes of $\mathcal{X}$ have not been used so that it also makes sense to speak of an approximate mapping $\boldsymbol{f} : \boldsymbol{X} \to \mathcal{Y}$ from an inverse system $\boldsymbol{X}$ to a gauged system $\mathcal{Y}$.)

2.4. Theorem 4. *If systems $\mathcal{X}$ and $\mathcal{Y}$ consist of topologically complete spaces (e.g., polyhedra) and $\boldsymbol{f} : \mathcal{X} \to \mathcal{Y}$ is an approximate mapping, then there exists a unique mapping $f : X \to Y$, called the* limit *of $\boldsymbol{f}$, and characterized by the following condition*

(LAM) $$(\forall b \in B)(\forall \mathcal{V} \in Cov(Y_b))(\exists b' \geq b)(\forall b'' \geq b')$$
$$(q_{bb''} f_{b''} p_{f(b'')}, q_b f) \prec \mathcal{V}$$

(see [70], [45]).

One also defines equivalence of approximate mappings, $\boldsymbol{f} \sim \boldsymbol{f}'$, by requiring that, for each $b \in B$, there is an $a \geq f(b), f'(b)$, such that

(AEM) $$(f_b p_{f(b)a'}, f'_b p_{f'(b)a'}) \prec \mathcal{V}_b, \text{ for } a' \geq a.$$

One proves that the limit depends only on the equivalence class $[\boldsymbol{f}]$ of $\boldsymbol{f}$ ([70], [45]).

2.5. If $\boldsymbol{f} : \mathcal{X} \to \mathcal{Y}$ and $\boldsymbol{g} : \mathcal{Y} \to \mathcal{Z}$ are approximate mappings, it is natural to attempt to define their composition $\boldsymbol{h} = \boldsymbol{g}\boldsymbol{f}$ as in the case of mappings of systems, i.e., by putting $\boldsymbol{h} = (h, h_c)$, where $h = fg$ and $h_c = g_c f_{g(c)}$. Unfortunately, $\boldsymbol{h}$ does not satisfy condition (AM), i.e., is not an approximate mapping $\boldsymbol{h} : \mathcal{X} \to \mathcal{Z}$. Nevertheless, T. Watanabe [70] succeeded in defining composition of equivalence classes of approximate mappings between systems over cofinite index sets. He proved that for given $\boldsymbol{f}$ and $\boldsymbol{g}$, there exists a $\boldsymbol{g}' : \mathcal{Y} \to \mathcal{Z}$ such that $\boldsymbol{g}' \sim \boldsymbol{g}$ and $\boldsymbol{g}'\boldsymbol{f}$ is an approximate mapping $\mathcal{X} \to \mathcal{Z}$. Furthermore, $\boldsymbol{g}'$ is *uniform*, i.e., $\mathcal{V}_{g(c)} \prec g_c^{-1}(\mathcal{W}_c)$, for each $c \in C$. Finally, for any other such $\boldsymbol{g}''$, one has $\boldsymbol{g}'\boldsymbol{f} \sim \boldsymbol{g}''\boldsymbol{f}$, which makes possible the definition of $[\boldsymbol{g}][\boldsymbol{f}]$. Watanabe also proved that in this way one obtains a category, which he named *Appro-**Top***.

If one restricts *Appro-**Top*** to cofinite systems of topologically complete spaces, one obtains a subcategory *Appro-**CTop***. Watanabe showed that lim : *Appro-**CTop*** $\to$ ***CTop*** is a functor to the category ***CTop*** of topologically complete spaces and mappings.

2.6. There exist inverse systems $\boldsymbol{X}$, whose terms cannot be endowed with *admissible meshes*, i.e., meshes which will turn $\boldsymbol{X}$ into a gauged system $\mathcal{X}$. Indeed, for systems $\boldsymbol{X}$ consisting of compact Hausdorff spaces X_a and surjective bonding maps $p_{aa'}$, the following is a necessary condition in order that $\boldsymbol{X}$ admits meshes [41]:

$$\text{(C)} \qquad (\forall a \in A)\ \operatorname{card}(A) \geq \operatorname{weight}(X_a).$$

A generalization to topological spaces is given by the following theorem [41].

THEOREM 5. *Let $\boldsymbol{X}$ be an inverse system of topological spaces X_a, whose Lindelöf numbers $l(X_a)$ satisfy the inequality*

$$\text{(D)} \qquad 2^{l(X_a)} \leq \operatorname{card}(A),$$

and let each $p_{aa'}(X_{a'})$ be dense in X_a. If $\boldsymbol{X}$ admits meshes, then the following condition holds:

$$\text{(C)}^* \qquad (\forall a \in A)\ \operatorname{card}(A) \geq \operatorname{cw}(X_a),$$

where $\operatorname{cw}(Z) \geq \aleph_0$ *denotes the* covering weight *of Z, i.e., the least cardinal of a basis of normal coverings of Z.*

2.7. Conversely, condition (C)* is sufficient for the existence of admissible meshes on an arbitrary cofinite inverse system $\boldsymbol{X}$. In particular, cofinite inverse systems of metric compacta over unbounded index sets admit meshes, because $\operatorname{cw}(X_a)$ equals the weight $\operatorname{w}(X_a) = \aleph_0$, and therefore, (C)* is satisfied. Moreover, with every inverse system $\boldsymbol{X} = (X_a, p_{aa'}, A)$ one can associate an inverse system $\boldsymbol{X}^* = (X_{a^*}, p_{a^*a^{*\prime}}, A^*)$ and an increasing surjective function $s : A^* \to A$, such that A^* is cofinite and antisymmetric, $X^*_{a^*} = X_{s(a^*)}, p^*_{a^*a^{*\prime}} = p_{s(a^*)s(a^{*\prime})}$, and $\boldsymbol{X}^*$ has property (C)*, so that it can be endowed with admissible meshes. Moreover, if $\boldsymbol{p} = (p_a) : X \to \boldsymbol{X}$ is the limit of $\boldsymbol{X}$, then $\boldsymbol{q} = (q_{a^*}) : X \to \boldsymbol{X}^*$ is the limit of $\boldsymbol{X}^*$, where $q_{a^*} = p_{s(a^*)}$ (Theorem (3.7) of [70]; also see [31] and [64]).

2.8. There is a natural candidate for the definition of approximate mappings $\boldsymbol{f} : \boldsymbol{X} \to \boldsymbol{Y}$ between nongauged inverse systems. One defines it using the same data as before. However, condition (AM) is replaced by the requirement that, for each $b \in B$ and each $\mathcal{V} \in Cov(Y_b)$, there exists $b_0 \geq b$ such that, for all $b', b'' \geq b_0$, there exists $a \geq f(b'), f(b'')$ having the property

$$\text{(AM)}^* \qquad (q_{bb'} f_{b'} p_{f(b')a'}, q_{bb''} f_{b''} p_{f(b'')a'}) \prec \mathcal{V}, \text{ for all } a' \geq a.$$

There is also a natural candidate for the definition of equivalence of approximate mappings. One puts $\boldsymbol{f} \sim \boldsymbol{f}'$, provided, for each $b \in B$ and each normal covering $\mathcal{V}$ of Y_b, there exists a $b_0 \geq b$ having the property that, for each $b' \geq b_0$, there exists an $a \geq f(b'), f'(b')$ such that

(AEM)* $$(q_{bb'} f_{b'} p_{f(b')a'}, q_{bb'} f'_{b'} p_{f'(b')a'}) \prec \mathcal{V}, \text{ for } a' \geq a.$$

If systems $\boldsymbol{X}, \boldsymbol{Y}$ do admit meshes, i.e., are obtainable from gauged systems $\mathcal{X}$ and $\mathcal{Y}$ by forgetting their meshes, then there is a bijection between the set of equivalence classes of approximate mappings $\boldsymbol{f} : \boldsymbol{X} \to \boldsymbol{Y}$, in the sense just defined, and equivalence classes of approximate mappings $\boldsymbol{f} : \mathcal{X} \to \mathcal{Y}$, as defined in 2.4. However, S. Mardešić and N. Uglešić [42] have recently produced an example showing that in general, it is impossible to define a composition of equivalence classes of approximate mappings (in the above sense) yielding a category of nongauged inverse systems on which lim would be a functor. This shows that meshes are indeed indispensable.

3. Resolutions

3.1. One can answer affirmatively Question 1, for compact spaces X, Y and cofinite systems of compact polyhedra X_a, Y_b, provided one endows the systems with admissible meshes and uses approximate mappings $\boldsymbol{f}$. Unfortunately, the analogous result for gauged polyhedral systems with noncompact limits does not hold. Furthermore, it is well-known that, for an inverse system of compact polyhedra $\boldsymbol{X} = (X_a, p_{aa'}, A)$ with limit $X = \lim \boldsymbol{X}$, the Čech homology group $\check{H}_p(X)$ is the limit of the induced inverse system of groups $H_p(\boldsymbol{X}) = (H_p(X_a), p_{aa'*}, A)$ (see e.g., [37]). However, this assertion fails if one omits the compactness assumption. This is a very special case of the well-known phenomenon in shape theory of noncompact spaces, that one cannot replace spaces X by arbitrary polyhedral inverse systems $\boldsymbol{X}$ with $X = \lim \boldsymbol{X}$. These and other difficulties with limits of noncompact spaces were the reasons for introducing the notion of resolution, which can be viewed as a well-behaved type of inverse system. This notion was introduced and developed in several papers by P. Bacon [5], K. Morita [52], [53], [54] and S. Mardešić [25], [26].

3.2. According to [26] (also see [37]), a *resolution of a space* X consists of an inverse system $\boldsymbol{X}$ and of a collection $\boldsymbol{p} : X \to \boldsymbol{X}$ of mappings $p_a : X \to X_a$, $a \in A$, satisfying condition (L). In addition one requires that, for any polyhedron P and any $\mathcal{V} \in Cov(P)$, the following two conditions be satisfied:

$$\text{(R1)*} \qquad (\forall f : X \to P)(\exists a \in A)(\forall a' \geq a)(\exists g : X_{a'} \to P)$$
$$(gp_{a'}, f) \prec \mathcal{V};$$

$$\text{(R2)*} \qquad (\exists \mathcal{V}' \in Cov(P))(\forall a \in A)(\forall g, g' : X_a \to P)$$
$$(gp_a, g'p_a) \leq \mathcal{V}' \Longrightarrow (\exists a' \geq a)(\forall a'' \geq a')(gp_{aa''}, g'p_{aa''}) \prec \mathcal{V}.$$

An inverse system $\boldsymbol{X}$ is said to be a *resolution* provided there exist a topologically complete space X and a resolution $\boldsymbol{p} : X \to \boldsymbol{X}$ of X.

A *resolution of a mapping* $f : X \to Y$ consists of two resolutions of spaces $\boldsymbol{p} : X \to \boldsymbol{X}$ and $\boldsymbol{q} : Y \to \boldsymbol{Y}$ and of a mapping of systems $\boldsymbol{f} : \boldsymbol{X} \to \boldsymbol{Y}$, which satisfies condition (LM).

3.3. The relationship between resolutions and limits is given by the next theorem.

THEOREM 6. *If $\boldsymbol{p} : X \to \boldsymbol{X}$ is a resolution consisting of completely regular spaces (in particular, a polyhedral resolution) and the space X is topologically complete, then $\boldsymbol{p}$ is also a limit of $\boldsymbol{X}$. If all X_a and X are compact Hausdorff spaces, also the converse holds, i.e., if $\boldsymbol{p}$ is a limit, then $\boldsymbol{p}$ is a resolution* [53], [26].

3.4. For resolutions, Theorems 1, 2 and 3 assume the following form.

THEOREM 7. *If a completely regular space X admits a resolution consisting of compact polyhedra, then X is pseudocompact* [23], [30].

THEOREM 8. *Every pseudocompact space X admits a resolution $\boldsymbol{p} : X \to \boldsymbol{X}$, where $\boldsymbol{X}$ consists of compact polyhedra. Every topological space X admits a resolution $\boldsymbol{p} : X \to \boldsymbol{X}$, where $\boldsymbol{X}$ consists of polyhedra.* [23], [26], [30].

THEOREM 9. *Every mapping $f : X \to Y$ between pseudocompact spaces admits a resolution consisting of compact polyhedra and mappings. Every mapping $f : X \to Y$ between topological spaces admits a polyhedral resolution* ([26], [16], [30]).

3.5. A *gauged resolution* $\boldsymbol{p} : X \to \mathcal{X}$ of a space X is a resolution $\boldsymbol{p} : X \to \boldsymbol{X}$, where $\boldsymbol{X}$ is obtained from the gauged system $\mathcal{X}$ by forgetting the meshes. A gauged resolution of a mapping $f : X \to Y$ consists of gauged resolutions $\boldsymbol{p} : X \to \mathcal{X}$ and $\boldsymbol{q} : Y \to \mathcal{Y}$ and of an approximate mapping $\boldsymbol{f} : \mathcal{X} \to \mathcal{Y}$, which satisfies condition (LAM). Both notions were introduced by Watanabe [69], [70].

Clearly, the gauged analogues of Theorems 7, 8 and 9 remain valid. More important is the next result, which gives the desired affirmative answer to Question 1 and is referred to as the *Expansion theorem* [70].

THEOREM 10. *Let $\boldsymbol{p} : X \to \mathcal{X}$ and $\boldsymbol{q} : Y \to \mathcal{Y}$ be gauged resolutions and let $f : X \to Y$ be a mapping. If $\mathcal{Y}$ is cofinite and all Y_b are polyhedra, then there exists an approximate mapping $\boldsymbol{f} : \mathcal{X} \to \mathcal{Y}$, which satisfies condition* (LAM) *and one has an approximate resolution of f.*

It is a consequence of Theorem 10, that lim defines an equivalence of categories between the category *Appro-**Top*** restricted to cofinite polyhedral resolutions and the category of topologically complete spaces and mappings. This shows that gauged polyhedral resolutions are indeed a suitable tool for studying topologically complete spaces and their mappings.

Recently, S. Mardešić and N. Uglešić produced examples showing that the analogue of Theorem 10 for nongauged resolutions (using approximate mappings as defined in 2.8) is no longer true [42]. This demonstrates again the importance of meshes.

3.6. It is well-known that, inverse systems of non-empty compact Hausdorff spaces always have a non-empty limit. However, for systems of non-compact spaces this is no more true, even when one assumes surjectivity of the bonding mappings (see e.g., Exercise 2.5.A.(b) of [15]). It is also well-known that for systems $(X_a, p_{aa'}, A)$ of compact Hausdorff spaces of covering dimension $\dim X_a \leq n$, the limit X has dimension $\dim X \leq n$. Neither this result generalize to non-compact spaces. Indeed, M. G. Charalambous [7], [8] has produced inverse sequences of 0-dimensional Lindelöf spaces having a normal limit X of dimension $\dim X > 0$. These anomalies cannot happen if limits are replaced by resolutions, which is another advantage of resolutions over limits.

3.7. Let *Ho-**Pol*** denote the homotopy category of polyhedra. In shape theory one defines shape morphisms between spaces $F : X \to Y$ as morphisms in the pro-category *pro-Ho-**Top*** between adequate objects associated with X and Y respectively. If $\boldsymbol{p} : X \to \boldsymbol{X} = (X_a, p_{aa'}, A)$ and

$\boldsymbol{q} : Y \to \boldsymbol{Y} = (Y_b, q_{bb'}, B)$ are polyhedral resolutions, then such adequate object are $(X_a, [p_{aa'}], A)$ and $(Y_b, [q_{bb'}], B)$, where $[\phi]$ denotes the homotopy class of the mapping ϕ (see, e.g. [37]).

Polyhedral resolutions (or ANR-resolutions) are fine enough tools to allow the description of morphisms in the strong shape theory and strong homology [21], [22], [28], [29], [6], [12]. Another non-trivial application of resolutions is in the theory of shape fibrations for arbitrary topological spaces [26], [76]. T. Watanabe has successfully used resolutions in the theory of approximate absolute neighborhood retracts, fixed point theory and the Vietoris-Smale type theorems [71, 72, 73].

4. Approximate Systems.

4.1. In view of 3.6. and Theorem 1, it is natural to ask if every compact Hausdorff space X with $\dim X \leq n$ can be expressed as the limit of an inverse system of compact polyhedra X_a with $\dim X_a \leq n$? Surprisingly, a negative answer has been given long ago [55], [24]. This defect of inverse limits was corrected by S. Mardešić and L. R. Rubin [34], who in 1989 introduced, a more flexible type of inverse systems of metric compacta, which need not satisfy condition (I) from 1.2. The members of these approximate systems are endowed with numerical meshes, subject to conditions, which weaken condition (I). The notion was generalized by Mardešić and Watanabe to systems of arbitrary spaces as follows [45].

An *approximate system* is a collection $\mathcal{X} = (X_a, \mathcal{U}_a, p_a, A)$ consisting of the same data as a gauged system, subject to condition (A3) from 2.2, and two additional conditions:

(A1) $\quad (\forall a \leq a' \leq a'')\ (p_{aa'}p_{a'a''}, p_{aa''}) \prec \mathcal{U}_a;$

(A2) $\quad (\forall a \in A)(\forall \mathcal{U} \in Cov(X_a))(\exists a' \geq a)(\forall a_2 \geq a_1 \geq a')$
$\quad (p_{aa_1}p_{a_1a_2}, p_{aa_2}) \prec \mathcal{U}.$

As in the case of usual inverse systems, the *limit* $\boldsymbol{p} : X \to \mathcal{X}$ of $\mathcal{X}$ consists of a space $X = \lim \mathcal{X}$ and of a collection of mappings $p_a : X \to X_a$, $a \in A$, called projections. If all X_a are completely regular spaces, X is the subspace of the direct product $\prod_{a\in A} X_a$, formed by all the points $x = (x_a)$, $x_a \in X_a$, satisfying the condition

(AI) $\quad \lim_{a'\geq a} p_{aa'}(x_{a'}) = x_a$, for each $a \in A$.

The projections p_a are the restrictions of the natural projections $\prod_{a\in A} X_a \to X_a$. They satisfy the condition

$$\text{(AL)} \qquad (\forall a \in A)(\forall \mathcal{U} \in Cov(X_a))(\exists a' \geq a)(\forall a'' \geq a')$$
$$(p_{aa''}p_{a''}, p_a) \prec \mathcal{U}.$$

This condition is universal and characterizes the limit. If $\mathcal{X}$ is commutative, i.e., if by forgetting the meshes one obtains a usual inverse system $\boldsymbol{X}$, then the limits of $\mathcal{X}$ and $\boldsymbol{X}$ coincide.

Approximate systems of metric compacta with numerical meshes as in [34] always admit coverings $\mathcal{U}_a \in Cov(X_a)$, which make them into gauged approximate systems. Conversely, members of a gauged approximate system of metrizable compacta can always be provided by suitable metrics and numerical meshes to become systems in the sense of [34] (see [36] and [75]).

Approximate resolutions, approximate mappings, as well as equivalence and composition of approximate mappings are defined as for commutative systems, i.e., using the same conditions (R1), (R2), (AM), (AEM) and (LAM). In spite of greater generality, one can prove the analogues of all the results stated before for commutative (usual) systems or resolutions. In particular, one obtains a category ***Apres*** of cofinite approximate resolutions, formed by topologically complete spaces and lim is a functor from this category to the category ***CTop*** of topologically complete spaces. If one restricts ***Apres*** to polyhedral approximate resolutions, lim becomes an equivalence of categories [45].

4.2. Due to greater flexibility of approximate resolutions, one can prove results, whose analogues were not true for commutative systems.

THEOREM 11. *A compact Hausdorff space X has dimension* $\dim X \leq n$ *if and only if it is the limit of an approximate system* $\mathcal{X} = (X_a, \mathcal{U}_a, p_{aa'}, A)$ *of compact polyhedra with* $\dim X_a \leq n$ [34].

THEOREM 12. *A topological space X has dimension* $\dim X \leq n$ *if and only if it admits an approximate resolution* $\boldsymbol{p} : X \to \mathcal{X} = (X_a, \mathcal{U}_a, p_{aa'}, A)$ *consisting of polyhedra with* $\dim X_a \leq n$ [74].

THEOREM 13. *A topological space X is finitistic if and only if it admits an approximate resolution* $\boldsymbol{p} : X \to \mathcal{X} = (X_a, \mathcal{U}_a, p_{aa'}, A)$ *consisting of finite-dimensional polyhedra* [49].

Using some recent results on irreducible mappings [43], Vlasta Matijević [50] proved the following theorem, which can be viewed as a generalization of a classical result of H. Freudenthal [13].

THEOREM 14. *A normal space X with* $\dim X \le n$ *admits an approximate polyhedral resolution* $\boldsymbol{p} = (p_a) : X \to \mathcal{X} = (X_a, \mathcal{U}_a, p_{aa'}, A)$ *such that* $\dim X_a \le n$ *and the mappings* $p_{aa'}, p_a$ *are irreducible.*

4.3. Let $\mathcal{P}$ be a class of polyhedra. A space X is said to be $\mathcal{P} -$ *like* provided, for every normal covering $\mathcal{U}$ of X, there exists a polyhedron $P \in \mathcal{P}$ and a $\mathcal{U}-$mapping $f : X \to P$ such that $f(X)$ is dense on P.

THEOREM 15. *If* $\mathcal{P}$ *is a class of connected locally compact polyhedra, then* X *is* $\mathcal{P}-$*like if and only if there exists an approximate resolution* $\boldsymbol{p} : X \to \mathcal{X} = (X_a, \mathcal{U}_a, p_{aa'}, A)$ *such that all* $X_a \in \mathcal{P}$ *and all* $p_{aa'}$ *are surjective* [38],[33].

4.4. Another result, which holds for approximate systems and fails for commutative systems is given by the following theorem.

THEOREM 16. *If an approximate cofinite resolution* $\boldsymbol{p} = (p_a) : X \to \mathcal{X} = (X_a, \mathcal{U}_a, p_{aa'}, A)$ *of topologically complete spaces satisfies condition* (C)* *of* 2.6, *then it is* stable, *i.e., for each pair* $a \le a'$, *there exists a normal covering* $\mathcal{V}_{aa'}$ *of* X_a *such that, for any choice of mappings* $p'_{aa'} : X_{a'} \to X_a$, *satisfying* $(p_{aa'}, p'_{aa'}) \prec \mathcal{V}_{aa'}$, $\boldsymbol{p} = (p_a) : X \to \mathcal{X}' = (X_a, \mathcal{U}_a, p'_{aa'}, A)$ *is also an approximate resolution. Moreover,* $\mathcal{X}$ *and* $\mathcal{X}'$ *are isomorphic objects of* ***Apres***.

This theorem was first established for systems of metric compacta [39] and it was then generalized to the present form [63]. The proof uses a criterion, which gives necessary and sufficient conditions in order that two objects of ***Apres*** be isomorphic [61], [62].

In 1991 M. Charalambous [9] considered nongauged approximate systems (of uniform spaces) using only condition (A2) of 4.1. This notion was further investigated by Mardešić [31], Uglešić [64] and Matijević [48]. These authors showed that one can indeed develop a satisfactory theory of such systems. However, as we have already pointed out, this approach does not extend to mappings and one cannot build a category like ***Apres*** [42].

4.5. We now give two theorems, whose statements do not involve inverse systems of any kind, but their proofs essentially use the techniques of approximate systems.

THEOREM 17. *A compact Hausdorff space* X *has integral cohomological dimension* $\dim_{\mathbb{Z}} X \leq n,\ n \geq 1$, *if and only if there exist a compact Hausdorff space* Y *with covering dimension* $\dim Y \leq n$ *and weight* $\mathrm{w}(Y) \leq \mathrm{w}(X)$ *and a* $CE-$*mapping* $f : Y \to X$ [35].

In the special case when X is a compact metric space, this is an important theorem of R. D. Edwards and J. J. Walsh [68], which is an essential ingredient in the recent solution of the $CE-$ dimension raising problem [11]. In the nonmetric case, the proof required the construction of Y and it is here that the extra flexibility of approximate systems was crucial. The proof given in [35] does not use the result in the metric case but gives an alternate proof for that result.

THEOREM 18. *For a locally connected Hausdorff continuum* X *the hyperspaces* 2^X *of nonempty closed subsets of* X *and* $C(X)$ *of subcontinua of* X *have the fixed point property* [58].

Until now, this was known only for metrizable X.

4.6. Finally a word of warning. If $\mathcal{X} = (X_n, \mathcal{U}_n, p_{nn'}, \mathbb{N})$ is an approximate inverse sequence of metric compacta, it is natural to consider the usual inverse sequence $\boldsymbol{X}' = (X_n, p'_{nn'}, \mathbb{N})$, where $p'_{n,n+1} = p_{n,n+1}$ and $p'_{nn'} = p_{n,n+1} \ldots p_{n'-1,n'}$, for $n+1 < n'$. It can well happen that the limit X of $\mathcal{X}$ and the limit X' of $\boldsymbol{X}'$ are not homeomorhic [60], [65]. However, according to [9], there exists always a cofinal subset $M \subseteq \mathbb{N}$ such that the same construction applied to the subsequence $(X_m, \mathcal{U}_m, p_{mm'}, M)$ yields an inverse sequence whose limit is homeomorphic to X.

References

[1] ALDER M.D., *Inverse limits of simplicial complexes*, Compositio Math. **29** (1974), 1-7.

[2] ALEKSANDROFF P., *Une définition des nombres de Betti pour un ensemble fermé quelconque*, C. R. Acad. Sci. Paris **184** (1927), 317-319.

[3] ALEKSANDROFF P., *Untersuchungen über Gestalt und Lage abgeschlossener*

Mengen beliebiger Dimension, Ann. of Math. **30** (1929), 101-187.

[4] ALEKSANDROV P.S. and PASYNKOV B.A., *Introduction to dimension theory*, (Russian), Nauka, Moscow, 1973.

[5] BACON P., *Continuous functors, General Topology and Appl.* **5** (1975), 321-331.

[6] CATHEY F.W. and SEGAL J., *Strong shape theory and resolutions*, Topology and Appl. **15** (1983), 119-130.

[7] CHARALAMBOUS M.G., *An example concerning inverse limit sequences of normal spaces*, Proc. Amer. Math. Soc. **78** (1980), 605-608.

[8] CHARALAMBOUS M.G., *The dimension of inverse limit and N-com- pact spaces*, Proc. Amer. Math. Soc. **85** (1982), 648-652.

[9] CHARALAMBOUS M.G., *Approximate inverse systems of uniform spaces and an application of inverse systems*, Comment. Math. Univ. Carolinae **32** (1991), 551-565.

[10] ČECH E., *Théorie générale de l'homologie dans un espace quelconque*, Fund. Math. **19** (1932), 149-183.

[11] DRANISHNIKOV A.N., *On a problem of P. S. Aleksandrov* (Russian), Mat. Sb. **135** (177) (1988), 4, 551-557.

[12] DYDAK J. and NOWAK S., *Strong shape for topological spaces*, Trans. Amer. Math. Soc. **323** (1991), 765-796.

[13] FREUDENTHAL H., *Entwicklungen von Räumen und ihren Gruppen*, Compositio Math. **4** (1937), 145-234.

[14] EILENBERG S. and STEENROD N.E., *Foundations of algebraic topology*, Princeton Univ. Press, Princeton, 1952.

[15] ENGELKING R., *General topology*, Polish Scientific Publishers, Warszawa, 1977.

[16] HAXHIBEQIRI Q., *Shape fibrations for compact Hausdorff spaces*, Publ. Inst. Math. Beograd **31** (45) (1982), 33-49.

[17] KOYAMA A., MARDEŠIĆ S. and WATANABE T., *Spaces which admit AR-resolutions*, Proc. Amer. Math. Soc. **102** (1988), 749-752.

[18] KUROSCH A., *Kombinatorischer Aufbau der bikompakten topologischen Räume*, Compositio Math. **2** (1935), 471-476.

[19] LEFSCHETZ S., *On compact spaces*, Ann. of Math. **32** (1931), 521-538.

[20] LEFSCHETZ S., *Algebraic topology*, Amer. Math. Soc. Colloquium Publ. **27**, New York, 1942.

[21] LISICA JU. T. and MARDEŠIĆ S., *Steenrod-Sitnikov homology for arbitrary spaces*, Bull. Amer. Math. Soc. **9** (1983), 207-210.

[22] LISICA JU. T. and MARDEŠIĆ S., *Coherent prohomotopy and strong shape theory*, Glasnik Mat. **19** (1984), 335-399.

[23] LONČAR I., *Some results on resolutions of spaces*, Rad Jugoslav. Akad. Znan. Umjetn. Matem. Znan. **428** (**6**) (1987), 37-49.

[24] MARDEŠIĆ S., *On covering dimension and inverse limits of compact spaces*, Illinois J. Math. **4** (1960), 278-291.

[25] MARDEŠIĆ S., *Inverse limits and resolutions*, in *Shape theory and geometric topology*, Proceedings, Dubrovnik 1981, Lecture Notes in Math. 870, Springer, Berlin 1981, 239-252.

[26] MARDEŠIĆ S., *Approximate polyhedra, resolutions of maps and shape fibrations*, Fund. Math. **114** (1981), 53-78.

[27] MARDEŠIĆ S., *Approximate inverse systems and approximate resolutions*, Rendiconti Circolo Mat. Palermo, Suppl., Ser.II, **24** (1990), 145-157.

[28] MARDEŠIĆ S., *Resolutions of spaces are strong expansions*, Publ. Inst. Math. Beograd **49** (63) (1991), 179 - 188.

[29] MARDEŠIĆ S., *Strong expansions and strong shape theory*, Topology and its Appl. **38** (1991), 275-291.

[30] MARDEŠIĆ S., *Strong shape of the Stone-Čech compactification*, Commentationes Math. Univ. Carolinae **33**, No. 3 (1992), 533-539.

[31] MARDEŠIĆ S., *On approximate inverse systems and resolutions*, Fund. Math. **142** (1993), 241-255.

[32] MARDEŠIĆ S., *Approximating spaces by polyhedra*, in *Proceedings, 2nd Gauss Symposium*, München 1993, Vol. A. Mathematics and Theoretical Physics (to appear).

[33] MARDEŠIĆ S. and MATIJEVIĆ V., *$\mathcal{P}-$like spaces are limits of approximate $\mathcal{P}-$resolutions*, Topology and its Appl. **45** (1992), 189-202.

[34] MARDEŠIĆ S. and RUBIN L.R., *Approximate inverse systems of compacta and covering dimension*, Pacific J. Math. **138** (1989), 129-144.

[35] MARDEŠIĆ S. and RUBIN L.R., *Cell-like mappings and non-metrizable compacta of finite cohomological dimension*, Trans. Amer. Math. Soc. **311** (1989), 53-79.

[36] MARDEŠIĆ S., RUBIN L.R. and UGLEŠIĆ N., *A note on approximate systems of metric compacta*, Topology and its Appl. (to appear).

[37] MARDEŠIĆ S. and SEGAL J., *Shape Theory*, North-Holland, Amsterdam, 1982.

[38] MARDEŠIĆ S. and SEGAL J., *$\mathcal{P}-$like continua and approximate inverse limits*, Mathematica Japonica **33** (1988), 895-908.

[39] MARDEŠIĆ S. and SEGAL J., *Stability of almost commutative inverse systems of compacta*, Topology and its Appl. **31** (1989), 285-299.

[40] MARDEŠIĆ S. and SEGAL J., *Mapping approximate inverse systems of compacta*, Fund. Math. **134** (1990), 73-91.

[41] MARDEŠIĆ S. and UGLEŠIĆ N., *Approximate inverse systems which admit meshes*, Topology and its Appl. (to appear).

[42] MARDEŠIĆ S. and UGLEŠIĆ N., *Morphisms of approximate systems require meshes* (submitted).

[43] MARDEŠIĆ S. and UGLEŠIĆ N., *On irreducible mappings into polyhedra*, Topology and its Appl. (to appear).

[44] MARDEŠIĆ S. and WATANABE T., *Strong homology and dimension*, Topology and its Appl. **29** (1988), 185-205.

[45] MARDEŠIĆ S. and WATANABE T., *Approximate resolutions of spaces and mappings*, Glasnik Mat. **24** (1989), 587-637.

[46] MATIJEVIĆ V., *An approximate resolution of a topological pair*, Rad Hrvat. Akad. Znan. Umjetn., Matem. Znan (to appear).

[47] MATIJEVIĆ V., *An approximate resolution of a quotient space*, Rad Hrvat. Akad. Znan. Umjetn., Matem. Znan. (to appear).

[48] MATIJEVIĆ V., *A note on limits and resolutions for nongauged approximate systems*, Glasnik Mat. (to appear).

[49] MATIJEVIĆ V., *Spaces having approximate resolutions consisting of finite-dimensional polyhedra*, Publ. Math. Debrecen (to appear).

[50] MATIJEVIĆ V., *Irreducible resolutions of finitistic spaces*, Rendiconti dell'Istituto di Matem., Univ. Trieste (to appear).

[51] MIODUSZEWSKI J., *Mappings of inverse limits*, Colloquium Math. **10** (1963), 39-44.

[52] MORITA K., *On shapes of topological spaces*, Fund. Math. **86** (1975), 251-259.

[53] MORITA K., *On expansions of Tychonoff spaces into inverse systems of polyhedra*, Sci. Rep. Tokyo Kyoiku Daigaku **13** (1975), 66-74.

[54] MORITA K., *Resolutions of spaces and proper inverse systems in shape theory*, Fund. Math. **124** (1984), 263-270.

[55] PASYNKOV B., *On polyhedral spectra and dimension of bicompacta, in particular of bicompact groups* (Russian), Dokl. Akad. Nauk SSSR **121** (1958), 45-48.

[56] PONTRYAGIN L., *The theory of topological commutative groups*, Ann. of Math. **35** (1934), 361-388.

[57] SEGAL J., SPIEŻ S. and GÜNTHER B., *Strong shape of uniform spaces*, Topology and its Appl. **49** (1993), 237-249.

[58] SEGAL J. and WATANABE T., *Cosmic approximate limits and fixed points*, Trans. Amer. Math. Soc. **333** (1992), 1-61.

[59] STEENROD N., *Universal homology groups*, Amer. J. Math. **58** (1936), 661-701.

[60] UGLEŠIĆ N., *An example in the theory of approximate systems*, Commentationes Math. Univ. Carolinae **34** (1993), 575-581.

[61] UGLEŠIĆ N., *Isomorphisms in the category of approximate resolutions*, Glasnik Mat. (to appear).

[62] UGLEŠIĆ N., *Domination in the category of approximate resolutions*, Glasnik Mat. (to appear).

[63] UGLEŠIĆ N., *Stability of approximate systems*, Rad Hrvat. Akad. Znan. Umjetn., Matem. Znan. (to appear).

[64] UGLEŠIĆ N., *A simple construction of meshes in approximate systems*, Tsukuba J. Math. (to appear).

[65] UGLEŠIĆ N., *Approximate sequences versus inverse sequences*, Rendiconti dell'Istituto di Matem., Univ. Trieste (to appear).

[66] UGLEŠIĆ N. and MATIJEVIĆ V., *An approximate resolution of the product with a compact factor*, Tsukuba J. Math. **16** (1992), 75-84.

[67] VIETORIS L., *Über den höheren Zusammenhang kompakter Räume und eine Klasse von zusammenhangstreuen Abbildungen*, Math. Ann. **97** (1927), 454-472.

[68] WALSH J.J., *Dimension, cohomological dimension and cell-like mappings*, in: *Shape theory and geometric topology*, Proceedings, Dubrovnik 1981, Lecture Notes in Math. **870**, Springer, Berlin 1981, 105-118.

[69] WATANABE T., *Approximative expansions of maps into inverse systems*, in *Geometric and algebraic topology*, Banach Center Publ. **18** (1986), Polish Sci. Publ. Warsaw, 363-370.

[70] WATANABE T., *Approximative shape I*, Tsukuba J. Math. **11** (1987), 17-59.

[71] WATANABE T., *Approximative shape II - Generalized ANRs*, Tsukuba J. Math. **11** (1987), 303-339.

[72] WATANABE T., *Approximative shape III - Fixed point theorems*, Tsukuba J. Math. **12** (1988), 1-41.

[73] WATANABE T., *Approximative shape IV - UV^n-maps and the Vietoris-Smale theorem*, Tsukuba J. Math. **12** (1988), 273-319.

[74] WATANABE T., *Approximate resolutions and covering dimension*, Topology Appl. **38** (1991), 147-154.

[75] WATANABE T., *Numerical meshes and covering meshes of approximate inverse systems of compacta*, Proc. Amer. Math. Soc. (to appear).

[76] YAGASAKI T., *Fiber shape theory, shape fibrations and movability of maps*, in: *Geometric topology and shape theory*, Proceeding, Dubrovnik 1986, Lecture Notes in Math. 1283, Springer, Berlin 1986, 240-252.