34th Danubia-Adria Symposium on Advances in Experimental Mechanics
University of Trieste, Italy, 2017

AN INNOVATIVE MECHANICAL TESTING METHOD FOR MEASURING YOUNG'S MODULUS OF A THIN FLEXIBLE MATERIAL (OWN-WEIGHT CANTILEVER METHOD)

Atsumi OHTSUKI[1]

[1] Meijo University, Faculty of Science and Technology, Department of Mechanical Engineering, 1-501 Shiogamaguchi, Tenpaku-ku, Nagoya, 468-8502 Japan. E-mail: ohtsuki@meijo-u.ac.jp

1. Introduction

This paper describes a new Young's modulus measuring method (*Own-Weight Cantilever Method).* By applying a nonlinear large deformation theory, analytical solutions are derived by using Bessel functions. Using this method, Young's modulus of a thin and long flexible material can be easily obtained by just measuring the horizontal displacement or the vertical displacement at the free end of the cantilever.

Besides the Own-Weight Cantilever Method studied here, the *Axial Compression Method* [1], [2], the *Cantilever Method* [3], the *Circular Ring Method* [4], [5] for a thin flexible material.

2. Theory

Although conventional testing methods (e.g. three- or four-point bending test and so) are based on the small deformation theory are very simple, they have several disadvantages (e.g., a stress concentration, a gripping problem of specimen). From this point of view, a new testing method (*Own-Weight Cantilever Method*) is derived considering large deformation behaviors of a test specimen.

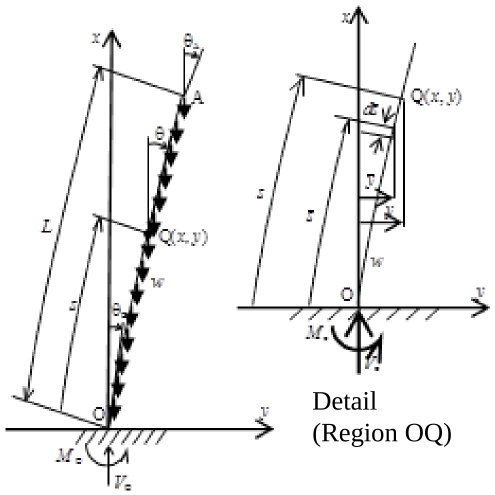

Fig.1 Large deformations of vertical cantilever subjected to own-weight.

2.1 Basic equations

A typical illustration of deformations is given in Fig.1 for a thin flexible cantilever (length L) subjected to the own-weight w (distributed load per unit length) with a supporting angle θ_0. The horizontal displacement is denoted by y, the vertical displacement by x, and θ is the deflection angle. Furthermore, the arc length is denoted by s, the radius of curvature by R and the bending moment by M. The relationships between R, M, s, x, y and θ are given by:

$$1/R = -d\theta/ds = M/(EI), dy = ds\cdot\sin\theta, dx = ds\cdot\cos\theta \quad (1)$$

where E, I are Young's modulus, the second moment of area, respectively.

Finally, the nonlinear equation is derived from Eqs.(1) in the form of :

$$EI\left(d^2\theta/ds^2\right) + w(L-s)\sin\theta = 0\,. \qquad (2)$$

Introducing the following nondimensional variables

$$\xi = \frac{x}{L}, \eta = \frac{y}{L}, \zeta = \frac{s}{L}, \gamma = \frac{wL^3}{EI}, \beta = \frac{ML}{EI}\,. \qquad (3)$$

and transforming the variables ($s\rightarrow\zeta$), equation (2) reduces to Eq.(7).

$$d^2\theta/d\zeta^2 + \gamma(1-\zeta)\sin\theta\,. \qquad (4)$$

Assuming the following relationships [Eq.(5)] in Eq.(4), equation (6) is obtained.

$$\psi = \theta - (\theta_A + \theta_O)/2\,. \qquad (5)$$

$$\left[-\{(\theta_A - \theta_O)/2\} \leqq \psi \leqq \{(\theta_A - \theta_O)/2\}\right]$$

$$d^2\psi/d\zeta^2 + \gamma(1-\zeta)\begin{bmatrix}\cos\{(\theta_A+\theta_O)/2\}\cdot\sin\psi \\ +\sin\{(\theta_A+\theta_O)/2\}\cdot\cos\psi\end{bmatrix} = 0\,. \quad (6)$$

where

$$\sin\psi = \sum_{n=0}^{\infty} 2\cdot\sin\left(\frac{n\cdot\pi}{2}\right)\cdot J_n\left(\frac{\theta_A-\theta_O}{2}\right)\cdot T_n\left(\frac{2\psi}{\theta_A-\theta_O}\right). \qquad (7)$$

$$\cos\psi = J_0\left(\frac{\theta_A-\theta_O}{2}\right) + \sum_{n=1}^{\infty} 2\cdot\cos\left(\frac{n\cdot\pi}{2}\right)\cdot J_n\left(\frac{\theta_A-\theta_O}{2}\right)\cdot T_n\left(\frac{2\psi}{\theta_A-\theta_O}\right) \qquad (8)$$

The functions $T_n(x)$, $J_n(x)$ in Eqs.(7), (8) are Chebyshev polyniminal and Bessel function, respectively.

The non-dimensional horizontal and vertical displacements ξ (=x/L), η (=y/L) at an arbitrary position Q(x,y) are obtained as follows.

$$\xi = \int_0^{\zeta} \cos\left[\left\{\frac{\theta_A - \theta_O}{2} - q\right\}\cdot\left\{1 + \frac{m^2\cdot\delta^3}{2\cdot(4/3)} + \frac{m^4\cdot\delta^6}{2\cdot 4\cdot(8/3)\cdot(10/3)} + \cdots\right\} + q + \frac{\theta_A + \theta_O}{2}\right] d\zeta \quad .(9)$$

$$\eta = \int_0^{\zeta} \sin\left[\left\{\frac{\theta_A - \theta_O}{2} - q\right\}\cdot\left\{1 + \frac{m^2\cdot\delta^3}{2\cdot(4/3)} + \frac{m^4\cdot\delta^6}{2\cdot 4\cdot(8/3)\cdot(10/3)} + \cdots\right\} + q + \frac{\theta_A + \theta_O}{2}\right] d\zeta \quad .(10)$$

where,

$$\delta = \gamma\cdot(1-\zeta) \quad [0 \le \delta \le \gamma]. \quad (11)$$

$$q = -\frac{(\theta_A - \theta_O)\cdot J_0\left(\frac{\theta_A - \theta_O}{2}\right)\cdot \sin\left(\frac{\theta_A + \theta_O}{2}\right)}{4J_1\left(\frac{\theta_A - \theta_O}{2}\right)\cdot \cos\left(\frac{\theta_A + \theta_O}{2}\right)} \quad .(12)$$

$$m^2 = -\frac{16J_1\left(\frac{\theta_A - \theta_O}{2}\right)\cdot \cos\left(\frac{\theta_A + \theta_O}{2}\right)}{9\gamma^2\cdot(\theta_A - \theta_O)}. \quad (13)$$

3. Experimental investigation

In order to assess the applicability of the new method, several experiments were carried out using a thin piano wire (SWP-A, length: L=500.0-1400.0 mm, diameter: d=0.9 mm, distributed load per unit length: w=49.245×10^{-3} N/m) Horizontal and vertical displacements y_A , x_A at the free end are measured by using a grid paper(1 mm spacing).

Young's modulus obtained by applying Method 1 and Method 2 are shown in Figs. 2 and 3, respectively. The measured values of each method remain nearly constant for various lengths L in the range of 500.0 — 1400.0 mm and the standard deviation (S.D.) is small although every method has a little scattered values. As a reference, Young's modulus measured by using conventional three-point bending test based on the small deformation theory is E_0 =205.8 GPa.

In this manner, Young's modulus can be obtained by applying an easy method without a large scale testing machine.

4. Conclusions

The principal conclusions are drawn as follows from the results of theoretical and experimental analyses.

(1) The new method is based on the nonlinear large deformation theory.

(2) A thin material (a piano wire, SWP-A) was tested.

(3) Experimental results clarify that the new method is suitable for measuring Young's modulus of a thin flexible material.

(4) Based on the assessments, the proposed method is applied to thin sheets and thin fiber materials (e.g., steel belts, glass fibers, carbon fibers, optical fibers, etc.).

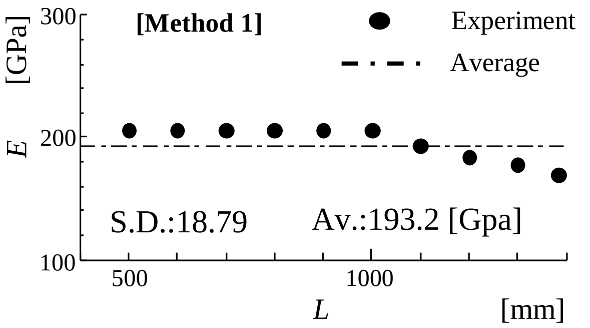

Fig.2 Comparison of Young's moduli for ϕ=0.9 piano wires(SWPA)(: Data x_A).

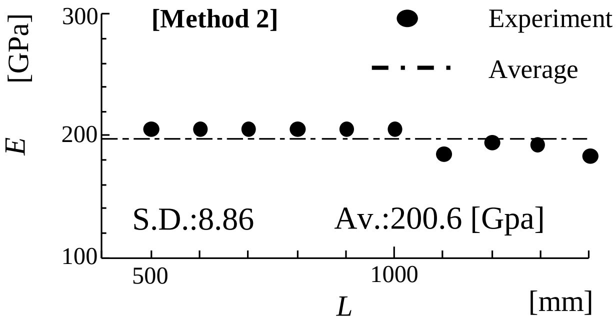

Fig.3 Comparison of Young's moduli for ϕ=0.9 piano wires(SWPA)(: Data y_A).

References

[1] A.Ohtsuki Proc. 4th Inter. Conf. Exp. Mech., Southampton, Vol.3-4, pp 53-58, 2005.

[2] A.Ohtsuki 4th Inter.Conf.Thin-Walled Struct. Loughborough, Vol.1, pp 233-240, 2004.

[3] A.Ohtsuki Proc. 2006 SEM Ann. Conf. Exp. Appl. Mech, St.Louis, 78, pp.78(1)-78(8).

[4] A.Ohtsuki and H. Takada Trans.JSSR No.47, pp 27-31, 2002

[5] A.Ohtsuki Proc.2005 SEM Ann. Conf. Exp. Appl.Mech., Portland, 72, pp 113(1)-113(8).