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OPEN PROBLEMS

WHY OPEN TEXTURE?

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Waismann (1945) introduces the notion of open texture or conceptual porosity: according to Waismann, "most of our empirical concepts are not delimited in all possible directions"; our definitions are, by contrast, "always corrigible or emendable". The notion of open texture bears significance for several contemporary debates. In this short note we outline an open problem concerning open texture, mainly in its connections with logical and mathematical concepts.

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Let's start with a warm-up example. Imagine that Tibbles looks and behaves just likes an ordinary cat. However, Tibbles has some peculiar property that distinguish it from ordinary cats; for example, imagine that Tibbles can grow to an abnormally giant size. Should we say in this case *(i)* that we have just learnt something new about cats, i.e. that some of of them are much bigger than normal, or *(ii)* that Tibbles was not a cat in the first place?[1]

Waismann (1945) contends that none of these two answers would be correct; he argues, by contrasts, that this example reveals the *open texture* of our empirical concepts. In Section 1. we introduce Waismann's notion of open texture and discuss its relevance for traditional and contemporary debates in analytic philosophy; we then address an open problem concerning the notion of open texture, mainly in its connections with logical and mathematical concepts.

1. Open Texture

According to Waismann, "most of our empirical concepts are not delimited in all possible directions" (1945, p. 122); our definitions are, by contrast, "*always* corrigible or emendable" (p. 123).

We can think of a concept as being associated with a procedure which enables us to answer a given class of questions, and, in particular, questions whether that concept applies to a given object. However, according to Waismann, empirical concepts are not defined in such a way that their definitions settle all possible questions about the application of those concepts. By contrast, there will always be some possible cases that our definitions leave undecided. For example, the definition of *cat* leaves it open whether a giant cat is still a cat.

More precisely, let *a* be *definitely an F* iff the following two conditions are met:[2]

[1] Cf. Waismann (1945, pp. 121-2); cf. also Wittgenstein (1953, § 80) and Blackburn (1994, p. 370).

[2] We borrow the notion of *being definitely an F* from Shapiro (2013, p. 309); Shapiro in turn borrows the notion from McGee, V., & McLaughlin, B. (1994). "Distinctions Without a Difference", *Southern Journal of Philosophy* 33: 2013-51.

(i) the thoughts and practices of speakers of the language determine conditions of application for F, and

(ii) the facts about *a* determine that those conditions are met.

Let the sentence 'F(a)' be *unsettled* if a is not definitely an F and a is not definitely a non-F. Finally, let a be a *borderline case* of a concept F if 'F(a)' is unsettled.
We can therefore formulate a preliminary definition of open texture;[3] this definition covers most of Waismann's examples:

> **Open Texture (OT)**: A concept F displays OT iff there might be some a such that it it would be unsettled whether F(*a*) – i.e., *a* is a borderline case of F.

Waismann argued that the fact that (most of) our empirical concepts display open texture bears great philosophical significance. More precisely, the notion of open texture figures prominently in Waismann's discussion of the two dogmas of empiricism as famously stated by Quine (1951), namely

> *I – Reductionism*: "each meaningful statement is equivalent to some logical construct upon terms which refer to immediate experience;"
>
> *II – Analytic/Synthetic Distinction*: "[we can distinguish] between truths which are *analytic*, or grounded in meanings independently of matters of fact and truths which are *synthetic*, or grounded in fact."

As regards reductionism, Waismann (1945) claims that the OT of empirical concepts implies that reductionism is false, since one could never exclude all possible cases in which it would be unsettled, for example, whether there is a cat next door. As regards the analytic/synthetic distinction, Waismann argues that the OT of empirical concepts blurs the distinction between analytic and synthetic truths: in many cases there will be no fact of the matter as to whether a new discovery counts as a change in meaning or a change in view (Waismann, 1949, 1950, 1951a, 1951b, 1952a, 1952b).

[3]Note that Waismann applies the notion of OT to linguistic expressions, while others (e.g. Shapiro (2013) and Tanswell (2018) apply it to concepts; in this note we will stick to the formulation in terms of concepts for the sake of convenience.

OT bears connections with many contemporary debates as well, in particular *(a)* in the philosophy of legal language, *(b)* in the philosophy of information and artificial intelligence, *(c)* in the philosophy of biology, *(d)* in the philosophy of physics, and *(e)* in the philosophy of logic and mathematics. More specifically, Hart (1961) claims that legal rules displays OT in Waismann's sense (for discussion cf. Bix, 1991). Hart's claims have been implemented using AI techniques by Stranieri, Zeleznikow, Gawler, e Lewis (1999) and Costantini e Lanzarone (1995). As regards biology, the notion of OT could be relevant to the understanding of the concepts of heredity and extended heredity (cf. Bonduriansky e Day, 2018). Moreover, OT is relevant to the philosophy of science in general, and in particular to the debate on quantum mechanics (see 2.3 below). Finally, in connection with the philosophy of logic and mathematics, Waismann's notion of OT has ramifications for the debates on vagueness (Shapiro, 2013), conceptual change (Wilson, 2006), explication (Tanswell, 2018), and *conceptual engineering* (Cappelen, 2018).[4]
This note addresses an open problem concerning the notion of OT by focusing on logical and mathematical concepts.[5]

2. Why Open Texture?

The question that we will tackle in this note is *why* concepts display OT in Waismann's sense. Note that in order for OT to the have the philosophical significance which Waismann ascribes to it, one would need to argue that our (empirical) concepts inevitably display open texture to some extent. We will now consider five possible routes to (such strong version of) OT; the options in 2.1, 2.2, and 2.3 traces back to Waismann, while the ones in 2.4 and 2.5 are original.

2.1 Open texture all the way down

The first route to open texture is suggested by Waismann, who links the OT of empirical concepts to the "*essential incompleteness* of an empirical description" Waismann (1945, p. 43-4).

[4] The notion of OT bears significance also for Waismann's aesthetics; we are grateful to Antonia Soulez for having pointed our attention to this (see Narboux e Soulez, 2008).

[5] As we point out below (cp. 2.4) Waismann claimed that logical and mathematical concepts do *not* display OT (contrary to what Shapiro 2013 and Tanswell, 2018) have recently argued); those concepts are therefore, in a sense, the "hard cases" in the debate on OT: if one could show that logico-mathematical concepts display OT, it is likely that many other kinds of concepts would be OT as well; see 2.5 below.

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With 'incompleteness' Waismann means that, no matter how far we can go, an empirical description can always be enriched by adding more information:

> If I had to describe the right hand of mine which I am now holding up, I may say different things of it: I may state its size, its shape, its colour, its tissue, the chemical compound of its bones, its cells, and perhaps add some more particulars; but however far I go, I shall never reach a point where my description will be completed: logically speaking, it is always possible to extend the description by adding some detail or other. (ibid.)

This has a direct bearing on OT. If an empirical description could be completed, then that description would also be apt to provide necessary and sufficient conditions for the application of the concept, and, therefore, a complete definition of that concept. However, since those cannot be completed, any such description will not provide a complete definition, and so new and unforeseen possibilities might always arise such that the definition leaves it open whether the concept should apply or not.
Let's introduce the symbol '$\rightleftharpoons$' to indicate the relation between a concept and its incomplete definition, as for instance in:

$$F(x) \ \rightleftharpoons \ \phi(x)$$

Note that $\rightleftharpoons$ cannot be understood in terms of a material (bi)conditional. If so, then the definition would allow us to decide all possible cases; Waismann points out, by contrast, that any definition of an empirical concept will always leave some case undecided (i.e., there might be a such that a satisfy $\phi(x)$, while, at the same time, it is unsettled whether a is in fact an F).
The question is now why empirical descriptions are incomplete. An option is that OT might go, so to say, all the way down; this option is tabled below:

Open Texture
⇑
incomplete definition
⇑
incomplete description
⇑
Open Texture

On this first route, empirical concepts would display OT because their definitions involve concepts that display OT temselves. There is an obvious problem of circularity with this account – however, Waismann suggests two more options.

2.2 Ceteris paribus Conditions

Waismann claims also that the definition of an empirical concept can be seen as a sort of *ceteris paribus* condition - i.e. such that if the antecedent of the conditional obtain then, *other things being equal*, the consequent obtains as well. This option gets its bite by adding that the notion of *ceteris paribus* is itself open-ended, that is, it is never possible to fully specify which those background conditions are:

> the vague supposition that 'a normal situation subsists', that 'no disturbing factors are present' or in whatever way we may hint at the possibility of intervention of some unforeseen conditions. The relation between L and s, then, when exactly stated, is this: Given such-and-such laws L_l, L_2 .. L_m, given such-and-such initial and boundary conditions c_l, c_2 .. c_n and no other disturbing factors being present, so-and-so will happen. And here it must be stressed that behind the words italicized a presupposition is concealed which cannot be split up into clear, separate statements.[6]

The question is, however, why *ceteris paribus* itself would be open-ended. If that is due to the fact that the concepts that might be involved in any specification of the background conditions would display OT themselves, then this second route would collapse on the first one.

2.3 Metaphysical and epistemic indeterminacy

A different answer to our question is inspired by Waissman's considerations on quantum mechanics (QM). As the conceptual revolution imposed by quantum mechanics poses several challenges to scientific realism, he argues that our very concept of reality might be systematically ambiguous (Waismann, 1945, p. 141). If one endorses Waismann's conclusion, then the OT of a given empirical concept might be seen as depending on the indeterministic features of the objects falling under that concept, as probabilistically described by quantum mechanics.

[6] Waismann (1945, p. 130).

Unlike classical objects, the objects described by quantum mechanics seem to violate the classical requirement of value definiteness - according to which we must be able to associate definite values to the properties of a given object at any time (*e.g.* also prior to detection).[7] According to this picture, one could say that the descriptions associated with our empirical concepts would be essentially incomplete *because of* the indeterminacy of the physical reality those concepts apply to. The metaphysical indeterminacy would thus ground OT - and a form of epistemic indeterminacy would obviously follow the metaphysical one, as we will now mention.

However, we think that a similar answer to our starting problem (basically: Why OT?) would be hardly satisfactory. First of all, note that this argument for OT would rely on the specific assumption of metaphysical indeterminacy. However, it would seem reasonable to favour positions, if any, that does not require specific metaphysical assumptions - and especially *heavy* metaphysical assumptions.

Waismann presents also an epistemic version of the same argument. In this case, what matters for OT is not that quantum mechanics offers the correct description of the world (in some relevant sense), but only that it is possible (read: conceivable) that the picture delivered by quantum mechanics is correct; the mere possibility of quantum mechanics would entail that it is (at least epistemically) possible that new and unforeseen cases might emerge.

However, even if this argument was sound, it seems to us that it would concern only the OT of specific empirical concepts - i.e. those for the extension of which a quantum description of reality is valid. The extension of such a position to all empirical concepts, and then to concepts of a different nature (such as logical or mathematical ones), would remain problematic and without metaphysical foundations.

2.4 Conceptual engineering

In these final sections we will examine two more routes to open texture. First, one might claim that, even if OT might in principle be eliminated, concepts which display OT are *better*, in some sense to be specified, that concepts that do not. Second, the *logical* concepts that are employed in definitions might display OT themselves. We will consider these two options in their turn.

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[7]The objects in question being particles, or systems of particles. For a useful discussion of the metaphysical commitment involved by this thesis, see Calosi e Wilson (2019).

First of all, we can distinguish between two flavours of open texture.[8] The first one is Waismann (1945)'s. Waismann argues that even if it was settled for any individual in the standard domain whether the concepts applies to it (e.g., whether that individual is a cat), still a new case might always emerge *outside* the intended domain of application of that concept such that it would be unsettled whether the concept would apply to that case (e.g., a giant cat). We will refer to OT in this first sense as *openness*:

> **(OT_1)** A concept or term displays open texture iff there are possible objects falling outside of the standard domain of application for which there is no fact of the matter as to whether they fall under the concept of not.[9]

The second characterization is due to Shapiro (2013). According to Shapiro, what is really open is the possibility for competent speakers to go either ways with respect to borderline cases *within* the intended domain of the concept. For him, "unsettled entails open": if it is unsettled whether *a* is an F, then competent speakers can faultlessly go either way, and assert either 'F*a*' or its negation (even though they cannot assert both in the same context; more on this in section 3). We will refer to this second sense of OT as *porosity*:

> **(OT_2)** A concept or term displays open texture iff there are cases for which a competent, rational agent may acceptably assert either that the concept applies or that it disapplies.[10]

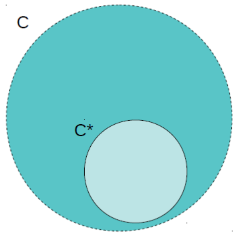
This two senses of open texture, openness and porosity, give raise to two distinct possibilities. On the one hand, *porous* concepts can be *sharpened* by settling some, or all, borderline cases within their domain of application:

> **Sharpening**: Let C be open-textured$_2$, and let a and b be two borderline cases for C; a sharper concept C^* can be introduced, such that *a* is determinately a C*, and b is determinately non-C*.

[8]The formulations of OT_1 and OT_2 are taken from Tanswell (2018, p. 3).

[9]Cf. Tanswell (2018, p. 3).

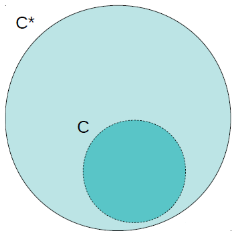
[10]Cf. Tanswell (2018, p. 3-4)



As an example of sharpening, consider Shapiro's discussion about the notion of *computability*: Shapiro maintains that, as many others mathematical notions, the intuitive, pre-formal notion of computability is used long before being sharpened and replaced by the rigorously defined notion of *recursiveness* (Shapiro, 2007, p. 428-30).

On the other hand, an *open* concept can be *expanded* as to apply to new domains:

> **Expansion**: Let C be open-textured$_1$, and let $a_1, \ldots a_n$ be objects outside the original domain of application of C such that it is unsettled whether those objects are C; an expanded concept C^* can be introduced, such that $a_1, \ldots a_n$ are determinately C*.



An example of expansion concerns the concept of *number*, which extended both mathematically - from integers to rationals, reals, and complex numbers by generalization – and historically - from positive integers to rationals, negatives, irrationals, imaginary numbers by a process of gradual inclusion. We will now examine these two possibilities in their turn.

2.4.1 Porosity and Vagueness

In this section we will clarify the relations between open texture and vagueness; we will compare Shapiro's and Waismann's characterizations of open texture along the way.

Waismann articulates the relation between OT and vagueness as follows:

> *Vagueness* should be distinguished from open texture. A word which is actually used in a fluctuating way (such as 'heap' or 'pink') is said to be vague; a term like 'gold', though its actual use may not be vague, is non-exhaustive or of an open texture in that we can never fill up all the possible gaps through which a doubt may seep in. Open texture, then, is something like *possibility of vagueness*.[11]

Waismann submits, moreover, that "vagueness can be remedied by giving more accurate rules, open texture cannot" (Waismann, 1945, p. 123): even if one can sharpen the predicate 'bald' by settling all the borderline cases in the domain of cases which are available at a given moment or in a given set of circumstances, there would still be the possibility that a new case might emerge such that it would be unsettled whether it should count as a case of baldness.[12]

OT is by contrast at the heart of Shapiro's account of vagueness. As we saw, Shapiro submits that competent speakers can go either ways with respect to borderline cases "without offending against the meanings of the terms, the non-linguistic facts, and the like" (Shapiro, 2013, p. 310) .

However, openness does not entail that competent speakers might go either ways *in any possible circumstance*.[13] By contrast, the use of vague predicates would be further constrained by the conversational context those speakers are in. Shapiro gives the following example. Imagine that 2000 cards are lined up in a row. The first card is determinately green, and the last card is determinately yellow; however, two adjacent cards cannot be distinguished in normal lightening conditions. Suppose that a group of speakers is asked to decide, for each card, whether that card is green or yellow, and then move to the next one. By assumption, at least some of those cards are neither determinately green nor determinately yellow; speakers could in principle go either ways with respects to those cards. However, they should also be consistent with their previous verdicts - e.g., if they say that card 1025 is green, they will also

[11] Waismann (1945, p. 123).

[12] Cp. Tanswell (2018, p. 5): "the term 'bald' will still be open-textured . . . because of the possibility of other hard cases not previously considered, say a two-headed person – maybe where one head has no hair and the other has some – how do we apply a term like 'bald' then?".

[13] As Shapiro (2013, p. 310) remarks, this would make "the use of vague predicates considerably less useful".

say that card 1026 is green, since, by assumption, they cannot really tell the two cards apart. If, for some reason, they are willing to say that card 1026 is yellow, then the speakers should (at least tacitly) retract their verdict about card 1025, and so on.

The second difference concerns the *extension* of OT. Waismann submits that, while "most, though not all, empirical concepts" are open-textured, formal concepts, and in particular *mathematical* ones, do *not* display OT. In fact, he employs mathematical concepts as contrast cases in order to illustrate the OT of empirical concepts.

Shapiro (2006, 2013, and Tanswell, 2018) claim, by contrast, that (informal) mathematical concepts can display open texture in Waismann's sense.

In particular, Shapiro draws attention to the role of *formalization* in counteracting open texture drawing on the famous discussion of the definition of the concept of 'polyhedron' in Lakatos (1976). Roughly, while Euler's theorem seemed to offer a good criterion to distinguish standard polyhedra, several counterexamples arise (e.g. twin tetrahedrons with one edge in common): the question is to settle if these new objects are pathological objects, or new polyhedra. The Lakatosian dialogue ends when a set-theoretic definition of polyhedron is found that generalises the original, pre-formal concept, extending its application to new objects.

One might wonder, however, whether there is a real difference; here is Waismann:

> "Cardinal number", "integer", "rational number" will be called sharply defined concepts, for each of them is defined by a calculus. However, what are we to understand by a number (in general)? The best answer that can be given is to declare that the above formations as well as all those which are somehow similar to these fall under the concept "number"; wherein we deliberately say nothing about the kind of similarity. Let us put in this way: the individual number concepts (cardinal number, integer, etc.) form a *family*, whose terms have a family similarity. ...We will describe this by saying that the word "number" does not designate a concept, but a "family of concepts". [14]

In the next section we will focus on the way in which this last picture has been eleborated by Mark Wilson.

[14] Waismann (1951c, p. 235-7).

2.4.2 Openness and Mathematics

In his book *Wandering Significance* (Wilson, 2006), Mark Wilson proposes a non-classical account for the definition of physical concepts. Wilson criticizes what he takes to be the traditional philosophical view of concepts[15] and aims at offering an alternative account by presenting many examples drawn from physics and mathematics. According to Wilson, the complexity of concepts can be pictured by thinking of several networks of complementary areas of significance, called "facades" (Wilson, 2006, p. 7) and "patches" (Wilson, 2006, p. 41) respectively.

To explain his conception of the complex conceptual networks we deal with, Wilson uses the metaphor of the atlas (Wilson, 2006, pp. 289-94). In an atlas, each map represents the geography of a portion of Earth by maximizing specific features and overlooking others. In order to have a full understanding of the geography of the Earth, we need to combine the information supplied in several maps, in a way depending on our practical goals:

> The overlapping set of the maps included in an atlas represent the inspirational prototype for my *facades*, for an atlas represents an evocative way to visualize the ways in which various blocks of a usage need to fit together in order to cover a subject matter effectively.[16]

This picture is meant to account for the "multi-valuedness" of our concepts (Wilson, 2006, p.456). Consider Wilson's discussion of the meaning of the predicate "hard" (Wilson, 2006, Chapter 6.ix). The hardness property of a material is characterized by the relevant measurement method (e.g. the Brinell test for metals, the Rockwell test for plastics), which depends on practical issues such as the type of material, the conditions under which the test is carried out, and so on. Depending on the measurement method used, the hardness is matched to a better defined physical property (e.g. deformation resistance for metals, plasticity for caoutchouc). The predicate "hard" is thus multi-valued for Wilson, in that:

> The end result is that our employment of "hardness" silently distributes itself into a patchwork of sheets, locally distinguished by a certain vein

[15] Russell's *The Problems of Philosophy* (Russell, 1912) is taken as representative of what Wilson calls the "classical glue". (Wilson, 2006, Chapter 3)

[16] Wilson (2006, p. 292).

> of probing (scratching, tapping, etc.), that sit over various varieties of material stuffs and continue smoothly into one another.[17]

Physical concepts such as *hardness* do not have a fixed content which fixes their domain of application once and for all - as a classical view of concepts would propose. On the contrary, the initial content of a concept adheres to some local patch of the physical world in certain definite limiting conditions (not necessarily explicitly known). Later, as new contexts of use and practical purposes arise, the initial content is extended from this first local patch to others. Such gradual extensions of an initially limited conceptual content are not pre-determined, but rather follow concrete needs that arise in the course of the development of scientific and industrial practices. As noted by Michael Friedman in his review of Wilson's book, "[...] it is typically only after such a process of conceptual evolutionary adaptation that we are in a position to articulate clearly what the content of our concept actually is – or, more precisely, what this content has now, for the moment, become" (Friedman, 2010, p. 534).

Note that an unambiguous categorization of objects is still possible, depending on the practical goals at issue, but this categorization may vary over time; we may classify a given object in a particular area of significance, and then discover its affinity and relations with other areas, in a gradual process of extension of knowledge. Indeed, the ontological position behind the Wilsonian framework seems to be a form of what Christopher Pincock calls "patient realism":

> Wilson's patient realism [...] urges us to wait and see how a given facade can be improved and clarified by later developments. There is no general test that can assure us that we are getting things right. This sort of limited optimism is focused on the specifics of this or that case and is reluctant to extrapolate any further.[18]

Wilson compared the evolution over time of concepts to the extension of real-valued functions on the complex plane, where they behave differently as displaying multi-valuedness[19]; the representation of these functions on Riemann surfaces is taken to offer, in Wilson's word, an "evocative picture" of

[17](Wilson, 2006, p. 338).

[18](Pincock, 2010, p. 120).

[19]A multi-valued function is a function that assumes two (or more) distinct values in its range for at least one point in its domain.

their behavior (Wilson, 2006, pp. 312-19). The mathematical analogy with the extensions of real-valued functions is used by Wilson to show how features of concepts that we observe locally (e.g., the single-valuedness of a function) can loose importance in the passage to a more extended domain.
In order to see what such an extension process could mean for mathematical concepts, consider the concept of square root function, an example that has been used by Wilson (2006, pp. 312-19) and recently revived by Pincock (2012, pp. 268-75). Despite the original definition of $\sqrt{x}$ as a function on non-negative real numbers, the definition of the square root function can be given for complex numbers. The new definition $\sqrt{z}$ introduces some remarkable changes in the function's behavior: in particular, the function is discontinuous in the complex plane when the argument is $-\pi$: the optimal setting to study it turns out to be a Riemann surface.[20]
When representing $\sqrt{z}$ on a two-dimensional plane, we have to draw four different functions using each quadrant of the plane - so that we may come to think of the square root function as multiply realized by four different functions.

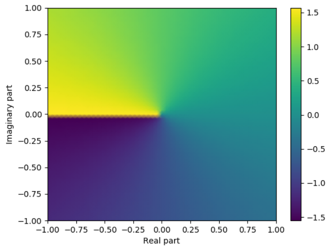


Figure III. Representation on a two-dimensional plane.

Rather, the use of a Riemann surface allows us to represent the square root function not as a multi-valued function, but as a single-valued function whose derivatives exist in a domain as wide as possible.

[20] For the example's details, see Pincock (2012, pp. 269-75)

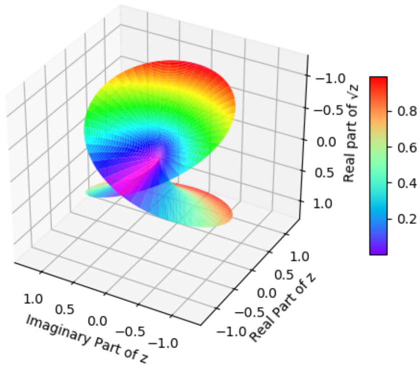


Figure IV. Representation on a Riemann surface.

The generalization introduced by the representation on a Riemann surface thus helps us to enrich our understanding of the features of the square root function. Both Wilson (2006, pp. 318-9) and Pincock suggest that the Riemann surface is to be considered "the proper setting for the study of the square root function" (Pincock, 2012, pp. 274):

> The three other functions are derivative on the function defined on the Riemann surface. The relationship between our original real-valued function and the function defined on the Riemann surface is then not simply that they are two instances of a single concept. It is instead the relationship between a partial account of some domain and a deeper understanding of that domain. [21]

The representation on a Riemann surface provides mathematicians with what is taken to be the best explanation of the peculiar behavior of $\sqrt{z}$ (Pincock, 2012, p. 295-7). Note that the evidence for this explanation does not come from empirical evidence, but from the subsequent developments of pure mathematical reasoning - so that this kind of conceptual expansion is consistent with the *a priori* character of mathematical knowledge.

2.5 Open Texture and Logic

Note that each definition of most concepts will inevitably involve reliance on some logical notions. A way of explaining OT could therefore consist

[21] Pincock (2012, p. 274).

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in arguing that concepts display OT because the logical notions employed in their definitions display OT in their turn.
This option would come with a couple of advantages. First, since each concept virtually involve reliance on some logical notion for its definition, this route would allow one to maintain that most concepts are OT without any further collateral assumptions about empirical concepts themselves and/or the physical world. Second, it would allow one to predict that a great variety of concepts, including, crucially, mathematical ones, will display OT, since all those concepts rely on some logical notions for their definition.
Here are two ways in which this fifth route might be implemented. First, one might claim that the notion of *logical consequence* display OT itself. For example, Beall e Restall (2006, Section 3.1.2) submit that the concept of logical consequence is *unsettled* in the following sense: Beall and Restall claim that the ordinary notion of logical consequence is captured by the Generalized Tarski Thesis (GTT), which states that an argument Σ is valid$_x$ if and only if in every case$_x$ in which the premises are true, so is the conclusion. They claim that admissible “precisifications” of the notion of logical consequence may arise from different specifications of ‘case$_x$’ in GTT. While those precisifications would agree on most arguments, there will be cases which are valid according to one specification, but invalid according to another. Beall e Restall (2006, p. 20) offer the following example:

(1) x is red, if x is red then x is coloured; so x is coloured.

(1) is valid if ‘case$_x$’ in GTT is specified in terms of possible worlds, but invalid if ‘case$_x$’ is specified in terms of Tarskian models. Those cases might be seen as borderline cases of *logical consequence*.
More controversially, one might argue that *specific* logical concepts, e.g. negation, disjunction, etc., display OT. One might argue, for example, that the meaning of logical concepts is exhausted by the rules that govern the introduction and elimination of the corresponding logical constants. At the same, it might be stressed that the introduction of a logical concept is always relative to a given language; so, while our introductory stipulations fixes all the relevant logical validities in the base language, borderline cases are possible with respect to other languages extending the base language. For example, consider McGee (1985)’s celebrated counterexample to *modus ponens*:

If a Republican wins the elections, then if it is not Reagan who wins it will be Anderson.
A Republican will win the elections.

Therefore, if it is not Reagan who wins it will be Anderson.

McGee asks us to imagine that "the Republican Ronald Reagan [is] decisively ahead of the Democrat Jimmy Carter, with the other Republican in the race, John Anderson, a distant third" (1985, p. 462); given these polls, McGee argues that we would have have good reasons to believe the two premises, but not the conclusion. One might argue that the meaning of the natural language conditional leaves it unsettled whether some of the instances of *modus ponens*, and in particular those involving (nested) conditionals in their premises or conclusion, are valid – while at the same time each instance of the same rule whose premises and conclusion do *not* contain the conditional just introduced is definitely valid.

Anyone who wished to take this fifth route should also argue, however, that logical concepts will inevitably display OT. This should be done while avoiding, at the same time, the circularity that seems to be involved in Waismann's first and second options, and the commitments carried by the third option. A possible way of doing that would be to merge the fourth and the fifth options together, and to contend that logical concepts which display OT are better than formal logical concepts. Moreover, we do not assume that the two implementation of the fifth option that we described above will not eventually collapse on each other.

3. Conclusion: An Open Problem for Open Texture

We contended that Waismann's philosophical arguments requires *Strong OT*, namely that (at least empirical) concepts inevitably display open texture. We examined the three arguments that Waismann offers, namely:

(i) the concepts involved in the definition of an (empirical) concepts display OT in their turn;

(ii) the definition of an (empirical) concept takes the form of a *ceteris paribus* conditional, where the notion of *ceteris paribus* is itself open ended; and

(iii) empirical reality fails to settle in each case whether the conditions for the application of our empirical concepts are satisfied.

We showed, however, that *(i)*-*(iii)* seem to support only *Weak OT*, according to which only the (empirical) concepts *that we actually employ* happen to display OT.

We suggested two more routes to (Strong) OT:

(iv) *conceptual engineering* – concepts which display OT are better/ more useful than concepts that do not display OT because the former concepts can be sharpened in order to settle borderline cases/expanded to new domains;

(v) *open texture and logic* – our concepts display OT because the logical concepts involved in their definitions display OT themselves.

As regards *(iv)*, this option has been modelled upon mathematical concepts; further work would be required in order to show that the same is true for concepts in other fields of inquiry. As regards *(v)*, the view that logical concepts display OT requires further articulation and defence. Moreover, as pointed out at the end of the last section, it is questionable whether a full implementation of *(iv)* and *(v)* – or a combination of the two – does avoid the circularity involved in *(i)* and *(ii)* and the commitments carried by *(iii)*.

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